TOPOLOGY OPTIMIZATION: HYBRIDIZATION OF PARTIAL SOLUTIONVERSUS TRADITIONAL MULTI-GOAL METHODS

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ABSTRACT

In a recent project [8] the authors have developed an approach to assist the identification of the optimal topology of a technical system capable to overcome geometrical contradictions that arise from conflicting design requirements. The suggested method is based on the hybridization of partial solutions obtained from mono-objective topology optimization tasks. In order to investigate efficiency, robustness and potentialities of hybridization, a comparison among the proposed approach and the traditional Topology Optimization methods is here presented. The application of the proposed hybridization approach to several case studies of multi-objective optimization problems available in literature has been performed with the aim to evaluate the robustness of the method, through a direct benchmark between the hybridized topology and the traditional methods. The obtained results demonstrate that the proposed method is computationally definitely less expensive than the conventional application of Genetic Algorithms to topological optimization, still keeping the same robustness in terms of searching the global optimum solution. Moreover, the comparison among the hybridized solutions and the solutions obtained through traditional topology optimization methods, shows that the proposed approach often leads to very different topologies having better performance.

KEYWORDS

Computer-aided innovation, computer-aided conceptual design, embodiment design, genetic algorithms, topological optimization, TRIZ

1. INTRODUCTION

The embodiment design of a technical system can be assimilated to a multi-objective problem that the designer tries to solve by finding geometrical solutions able to satisfy different conflicting requirements. Due to the nature of this task improving the performance of a technical system often involves worsening another performance. This kind of conflicts cannot be solved using the traditional approaches based on Topology Optimization, since these methods are able to focus the design task only to one specific performance to be improved. More precisely, they allow to manage multiple goals problems just by defining complex objective functions where a weight must be assigned to each specific goal. Thus, the best compromise solution is usually generated on the base of an initial assumption made by the designer about the relative importance of the requirements, without taking into account the reciprocal interactions among them.

In [8] the authors have presented an approach for the hybridization of optimized density distributions based on TRIZ manipulation, named DAeMON (hybri**D**iz**A**tion of **M**ono **O**bjective optimizatio**N**s); DAeMON has demonstrated the capability to produce solutions that are able to overcome geometrical contradictions [9], a specific type of TRIZ physical contradiction [2]. The effectiveness of the proposed approach has been investigated by performing several case studies related to 2D design problems.

Further developments of the research have revealed that the solutions obtained from mono-objective topology optimizations can be considered as elementary customized modeling features for a specific design task. According to these results in [7] the application of the DAeMON technique to a general 3D design space was presented together with the application of the proposed approach to geometrical contradictions obtained by the comparison of more than two conflicting boundary conditions.

Within the above research activities some issues arose about the efficiency of the proposed approach. More in particular two main interesting evidences still require a deeper investigation:

- the topology resulting from the application of the DAeMON approach is an original solution able to meet the design requirements, that cannot be obtained through traditional optimization methods;
- moreover, the proposed approach seems to be computationally less expensive than Genetic Algorithms (GAs), still keeping the same robustness in terms of searching the global optimum solution.

In this paper, in order to investigate the above mentioned just partially-explained properties, a direct benchmark between hybridized topologies and those obtained through traditional topology optimization systems is performed, with the aim to systematically analyze the potentialities of the hybridization method. In section 2 a review of topology optimization techniques is presented in order to highlight their strength and weakness and the developed hybridization approach is briefly introduced; in section 3 the original method and tools used to perform the above described investigations are presented. The case studies and the benchmark to compare the DAeMON approach with traditional topology optimization techniques are described in section 4. Eventually, section 5 reports a final discussion and the conclusions of the present work.

2. STATE OF THE ART

Continuum Topology Optimization [17] has received extensive attention and experienced considerable progress over the past few years to support design tasks related to structural analyses. It has been recently applied to address design problems also in other fields such as fluid dynamics, heat transfer and non linear structure behavior: examples of the new applications of topological optimization can be found in [12, 4, 5].

Topology Optimization determines the optimal material distribution within a given design space, by modifying the apparent material density assumed as design variable. The design domain is subdivided into finite elements and the optimization algorithm alters the material distribution within the design space at each iteration, according to the Objective and Constraints defined by the designer.

The Objective is constituted by one or more system performances that the optimization should improve. Each system performance is quantitatively assessed by an evaluation parameter that is assumed as metric. According to this statement, a mono-goal optimization task tries to improve a single system performance, while a multi-goal optimization task aims at improving a combination of performances.

The Constraints of the optimization task represents the operating conditions and the requirements the system has to satisfy. Among them, manufacturing constraints may be set in order to take into account the requirements related to the manufacturing process. Also the regions of the design domain defined as “functional” by the designer, are preserved from the optimization process and considered as “frozen” areas by the algorithm. The topology at the end of the optimization process is identified by filtering the resulting material density distribution through a proper threshold having a value included within the interval [0,1].

2.1. Traditional topology optimization techniques

Until now, various families of structural topology optimization methods have been developed [17, 6]. One of the most established families of methods is based on the Homogenization Approach and Optimality Criteria algorithm [3] that has gained a general acceptance in recent years because of its computational efficiency and conceptual simplicity. However it very often brings to local optimal topologies or converges to an infeasible, i.e. not manufacturable solution.

Instead of searching for a local optimum, one may want to find the globally best solution in the design domain. For this purpose GAs have become an increasingly popular optimization tool for many areas of research. More recently, GAs have been gradually recognized as a powerful and robust stochastic global search method for structural topology optimization [18, 19, 15]. Besides, in order to guarantee the robustness of the solution, GAs require more computational resources than the mathematical methods based on Optimality Criteria. As stated in [20], this is due to the high number of design variables that are typically involved in the topology optimization task and this is one of the main reasons for which GAs have not still implemented in commercial CAE tools.

A new method for Topology Optimization that combines the features of Bi-directional Evolutionary Structural Optimization (BESO) and GAs has been recently investigated in [20]. The proposed approach leads to the same results of classical GAs but it proves to have less computational costs. However, so far it has been tested only for mono-objective optimization task without taking in consideration multi-objective problems. Moreover, even the robustness of the evolutionary algorithms have been improved in the last years, the algorithm proposed in [20] still may suffer of numerical problems due to mesh-dependency and possible non-convergence of the solutions.

In conclusion, the above described literature review shows that the traditional topology optimization techniques based on mathematical methods are more efficient than GAs from a computational point of view, but GAs have a higher robustness in finding the global optimal solution which is a not negligible limitation of traditional mathematical methods.

2.2. The DAeMON approach

The authors have developed a new topology optimization method that tries to merge together the robustness of GAs in finding the global optimal solution with the computational efficiency of the mathematical methods.

The proposed approach is based on the hybridization of the density distributions generated by topological optimizations of mono-objective problems, according to the elementary requirements that should be satisfied. These partial solutions, in the form of density distributions, can be used as elementary customized features to be combined according to the following formula (hybridization of partial solutions) which is a particular case of that extensively described in [8]:

$$\rho\left(x,y,z\right)=\frac{\sum\limits_{i=1}^{N}K_i\rho_i(x,y,z)}{\sum\limits_{i=1}^{N}K_i} \tag{1}$$

where:

- ρ(x,y,z) is the hybridized topology;
- N is the overall number of conflicting mono-objective optimizations (two if a classical TRIZ contradiction model is adopted);
- K_i is the weight assigned to the i-th distribution of density;

The hybridization method has demonstrated its effectiveness in solving a particular case of geometrical contradictions: those arising in systems that experience different static load conditions [9, 7]. In facts, according to the TRIZ System of Inventive Standards [2], and more specifically to the Standard 3.1.4 (convolution of several systems), hybridization is a well known transition to improve the efficiency of a bi-poly system. Further details about hybridization techniques are provided in [16].

3. APPROACH AND METHODS USED IN PROBLEM SOLVING

The output obtained through the application of the hybridization formula (1) to the N density distributions resulting from mono-objective optimizations, is a hybrid density distribution built upon several topologies. Each hybrid topology is determined by soiling the N density distributions through proper density thresholds before performing the combination. The algorithm here proposed allows to systematically “browse” the whole set of hybridized solutions by automatically varying the density thresholds until the best globally hybrid topology is identified according to the objectives of the optimization task.

As stated in section 2, GAs are very robust optimization techniques in identifying the global optimum solution. Thanks to this characteristic, they are often used as alternatives to the mathematical methods in solving complex optimization problems such as related to the shape generation. An example is presented in [14] where a paradigm for the integration of GAs in 3D-CAD environments is suggested in order to perform automatic shape and topology variations. With the aim to demonstrate that the proposed

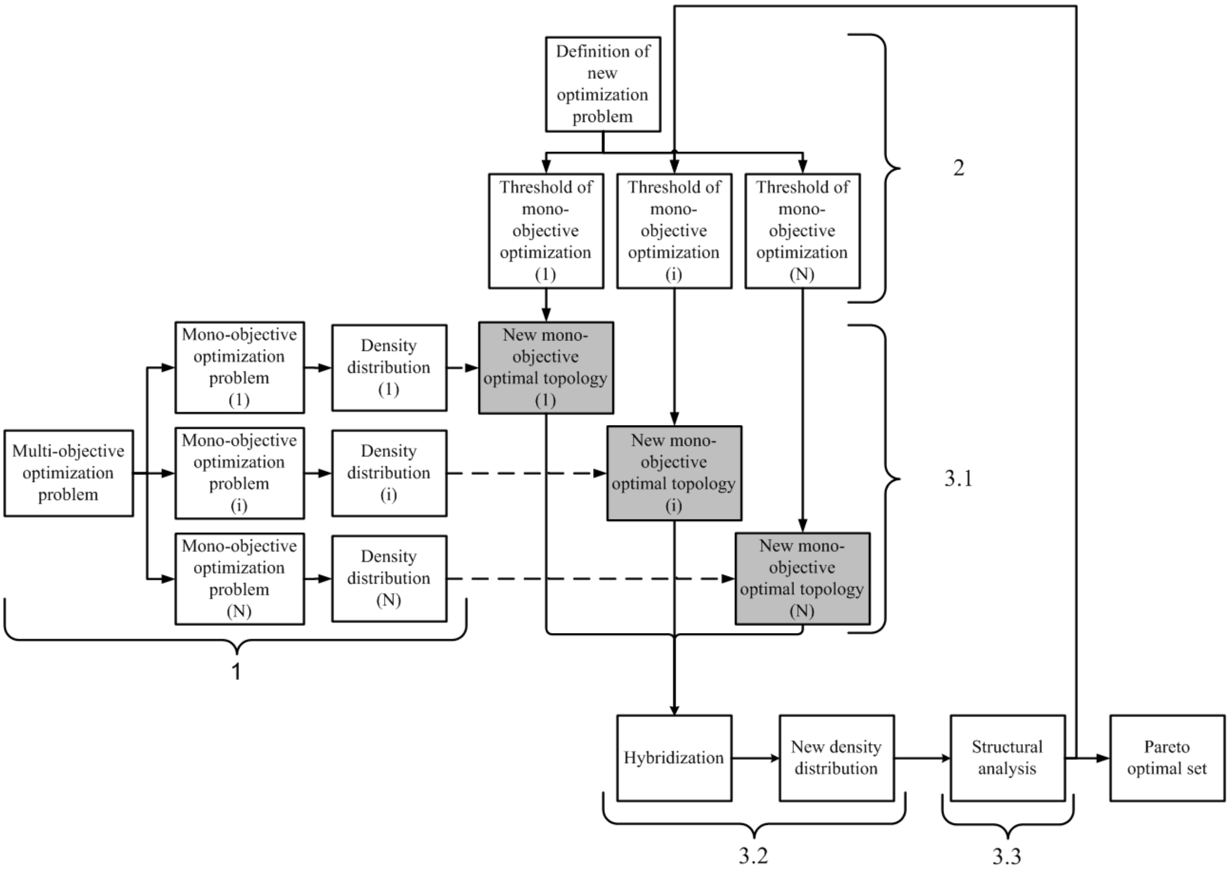


Figure 1 Scheme of the proposed algorithm: (1) identification of N density distributions from a multi-objective optimization problem; (2) definition of new optimization problem and N density thresholds; (3.1) identification of N mono-objective optimal topology by soiling the density distribution by the density threshold; (3.2-3.3) analysis of the hybridized topology. The steps from 3.1 to 3.2 are iterated until global optimum topology is identified.

approach merges the positive features of GAs and mathematical methods, an experimental campaign has been performed: the robustness of the method has been evaluated by a direct benchmark between the hybridized topology and the one obtained through GAs, obviously under the same multi-objective optimization task; moreover, the time-to-solution has been considered as a reference metric to evaluate the efficiency of the proposed algorithm from the computational point of view, with respect to the traditional methods.

The original approach proposed in this paper is schematically represented in Figure 1.

In the following the detailed description of each step of the algorithm is presented:

- The original multi-objective optimization problem is decomposed into N mono-objective optimization tasks that are solved using traditional mathematical methods; the results of this step are N density distributions: one for each mono-objective optimization. It is worth to remember that the density distribution can assume a fuzzy value between 0 and 1.
- A new optimization problem is defined: the objectives and constraints are the same of the multi-objective optimization task, the density thresholds of the N mono-objective density distributions coming from step 1 are defined as design variables instead of the material density of each finite element of the design domain.
- The new optimization problem is solved:
  - The N density distributions coming from step 1 are soiled by varying the density thresholds that have been defined as design variables in step 2. For each of the N density distributions, the algorithm assigns a density value 1 to the finite elements having a density greater than

the selected threshold and a density value “0” to the others. Therefore, after soiling, the N density distributions are characterized by only two discrete values of density that identify void spaces with zero density and filled spaces with density ”1”. In order to clarify the optimal hybridization mechanism let’s consider a trivial exemplary plate, subdivided into 4 finite elements: Figure 2 (a) and (b) shown the density distributions of a plate coming subjected to two mono-objectives optimization task. After soiling, the density distributions of the two resulting plates are modified according to the above described criteria as shown respectively in (c) & (d) and in (e) & (f).

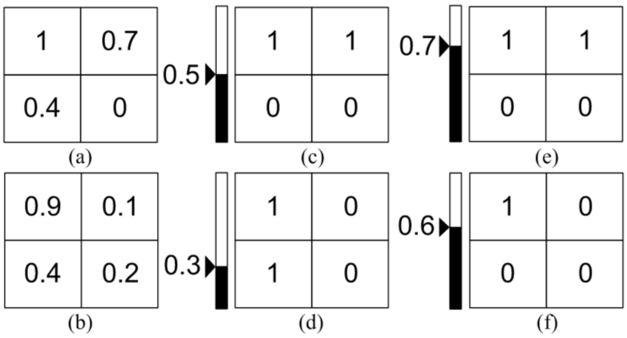


Figure 2 (a) and (b): density distributions of a plate optimized under two different boundary conditions.(c): density distribution of plate (a) with density threshold = 0.5. (d): density distribution of plate (b) with density threshold = 0.3; (e): density distribution of plate (a) with density threshold = 0.7. (f): density distribution of plate (b) with density threshold = 0.6

- The resulting N topologies are combined by the algorithm through the application of the formula (1), with equal weights. Each combination determines a hybridized topology. Thus, the value of density distribution related to the resulting hybridized topology is evaluated by the following formula:

$$\rho\left(x,y,z\right) = \frac{\sum_{i=1}^{N} \rho(x,y,z)_i}{N} \quad (2)$$

where:

- $\rho(x,y,z)_i$ is the soiled value of density of an element in the i-th mono-objective optimization. The numerator coincides with the number of mono-objective optimization for which the finite element FE(x,y,z) has density = 1;
- N is the overall number of conflicting mono-objective optimizations. Since the hybridized topology is the sum of N layers of voids and filled elements, the density distribution of each hybridized topology is composed by 2 + (N-1) different discrete values, i.e. density can assume the values 0, 1 and (N-1) intermediate levels. For example, if N=2, the density distribution of hybridized topology is characterized by values 0, 1 and only one intermediary value, equal to “0.5”. This new density distribution contains more details than necessary to the accomplishment of the following step 3.3, but they will be used to identify the operational zone of the emerging contradiction.

For example, the density distributions showed in Figure 2 (c) and (d) are combined by the algorithm through the application of the formula (2), and the resulting density distribution is shown in Figure 3 (a). In Figure 3 (b) the black areas are the elements for which in any mono-objective task the value of density is “1” (elements “must be”); the white areas identify void voxels having zero density that result in any mono objective task; besides, intermediate values of density, i.e. gray areas, represent voxels where contradictory requirements apply: the voxel should be filled for one or more functional requirements, but the voxel should be void for another set of optimization objectives. In other terms the gray voxels constitute the operational zone of the geometrical contradiction, as defined in [8].

Similarly, the hybridization of density distributions showed in Figure 2 (e) and (f) brings to the results reported in Figure 3 (c). In (d) it is shown the operational zone of the emerging contradiction.

- The hybridized topology is evaluated with respect to the objectives of the optimization task, through classical FE analysis. It is worth to notice that such hybridized topology is composed by all finite elements having value of density greater than zero (grey and black elements in Figure 3 b and d).

The steps from 3.1 to 3.2 are iterated until global optimum topology is identified. The steps 1 and 3 could be performed through the use of GAs and/or tradi-

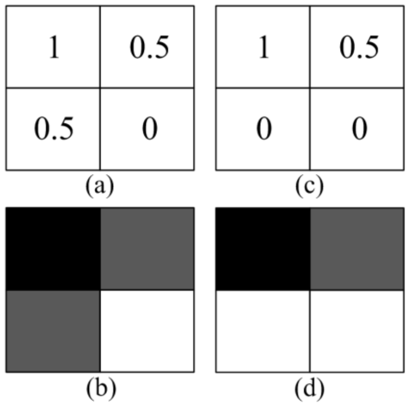


Figure 3 (a): hybridized density distribution for the plates of Figure 2 (c) and (d). (b): black elements = element must be; grey element = contradiction elements. (c): hybridized density distribution for the plates of Figure 2 (e) and (f). (d): black elements = element must be; grey element = contradiction elements.

tional mathematical methods. In the first case, the GA is applied over many generations, and each generation is a population of many individual designs, to attain the optimum chromosome strings and hence the optimum topologies. The chromosome string is a sequence of N genes, where N is the overall number of conflicting mono-objective optimizations and each gene is a threshold density related to a mono-objective optimization.

For the tasks of this paper the mono-objective optimizations have been performed using the mathematical method available in the code Optistruct v. 8.0 [10], while the above described algorithm have been implemented through ModeFrontier v. 4.0, the optimization environment developed by [11].

4. CASE STUDIES

Several case studies have been performed in order to test robustness and efficiency of the hybridization algorithm. In this section three examples are briefly described to clarify the working principle and the potentialities of the proposed approach. The first two problems have been taken from [1] where the multi-objective topology optimization has been performed by means of GAs. They concern the design of a plate having an overall dimension of 400 x 300 mm^2 that is discretized with 1200 (40x30) isoparametric plane stress finite elements. The plate is made of a steel alloy, so the unit cell material is assumed isotropic with Young's modulus equal to 210 GPa and Poisson coefficient equal to 0.3.

The plate undergoes two different combinations of load cases and the optimizations task consists in finding the optimal thresholds density in order to minimize the mass and the displacements of the nodes where the loads are applied.

In [1] the computational times required to solve both problems are absent: more details are postponed to the section 4.1 and Figure 4.

The third case study concern the optimization of a bicycle frame under three different load conditions. It is described in detail in the sub-section 4.3.

4.1. Example 1

The domain geometry and boundary conditions, as well as the loading conditions of the first example are shown in Figure 4. Two load cases are taken into account:

1. A point load in the y direction with a magnitude of 200 N.
2. A point load in the y direction with a magnitude of -200 N.

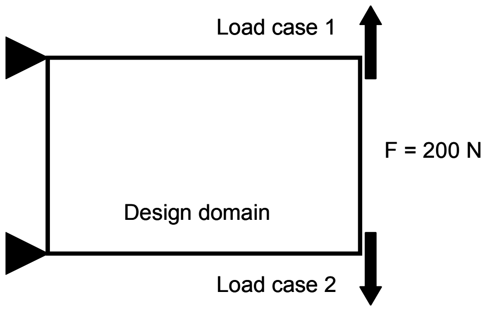


Figure 4 Plate under two different load conditions. The plate is fully constrained at the corners on the left edge and the forces are alternatively applied on the upper and lower corner of the right edge.

As mentioned above, this exemplary case study has been taken from [1], where it was used by the authors to assess the effectiveness of a multi-objective topology optimization method based on Genetic Algorithms (GAs). Initially in such case, the proposed single-objective problems were solved and the resulting solutions were introduced in the initial population. The initial population was composed of 200 individuals (in each generation only 100 individuals were selected for matting). The solutions obtained after 600 generations are presented in Figure 5.

In Figure 5, the values of the deformation energy for the two load cases, here named as external works

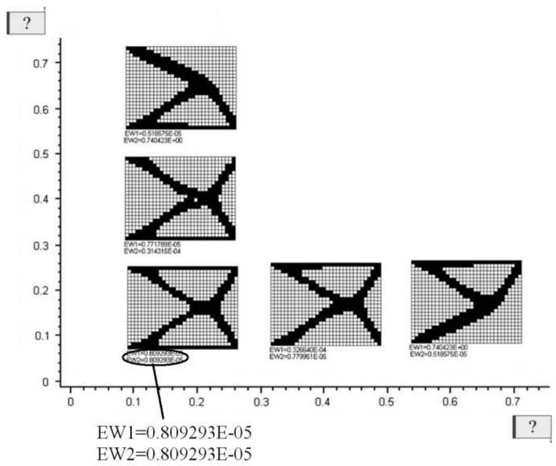


Figure 5 Set of solutions obtained in [1] through GAs. The x and y axes of the diagram extracted from the paper represent the deformation energy EW1 and EW2, related to the two load cases. Despite the specific values don't match the axes scale without any explanation by the authors, the diagram has been used as a reference for the Pareto front of the optimization task.

(EW1 and EW2), are indicated under each solution. According to the proposed algorithm, the first step consists in decomposing the original multi-objective optimization problem into N mono-objective optimization tasks; the overall goal is determining the optimal material distribution that minimizes the deformation energy of the plate according to a target mass reduction.

According to step 2, a new optimization problem is defined: the objectives and constraints of this optimization task are the same of the original one, but the N density thresholds are now defined as design variables.

Within the step 3.1, the N density distributions coming from the step 1, are soiled by varying the density thresholds; two exemplary individuals are showed in Figure 6.

The application of the hybridization algorithm leads to a set of individuals obtained by combining the soiled density distributions: the individual obtained by the hybridization of the topologies shown in Figure 6 is shown in Figure 7 (a).

In Figure 7 (b) it is also shown the same topology with the operational zone of the emerging contradiction: the black areas represent the "must be" elements, while the gray areas are the contradiction elements.

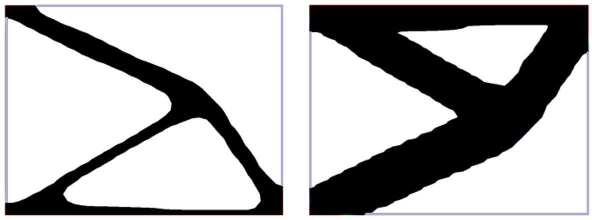


Figure 6 (i):Topology obtained under load case 1 and density threshold = 0.82; (ii): Topology obtained under load case 2 and density threshold = 0.05.

Figure 7 (a): Topology obtained by hybridization of mono-objective topology shown in Fig 3. (b): Operational zone of the geometrical contradiction.

The steps from 3.1 to 3.2 are iterated until global optimum topology is identified; such steps are performed through the use of GAs. In this case the initial population used by the GA was composed by 20 individuals and the solutions are obtained after 50 generation. Hence, the optimization task consists in finding the optimal threshold density in order to minimize the mass and the displacements of the nodes where the loads are applied; each iteration implies performing N structural analyses.

Unlike single-objective optimization, where objective and fitness functions are often identical, both fitness assignment and selection have to address several objectives of a multi-objective optimization problem. Hence, instead of a single optimum, multi-objective optimization problems solutions consist of a Pareto optimal set. In the total absence of information regarding the priority of the objectives, a ranking scheme based upon the Pareto optimality is regarded as an appropriate approach to represent the strength of each individual in an evolutionary algorithm for multi-objective optimization [20]. By applying the proposed algorithm, the solutions obtained after 50 generation are presented in Figure 8.

In this case, the hybridization approach brings approximately to the same topology obtained by the

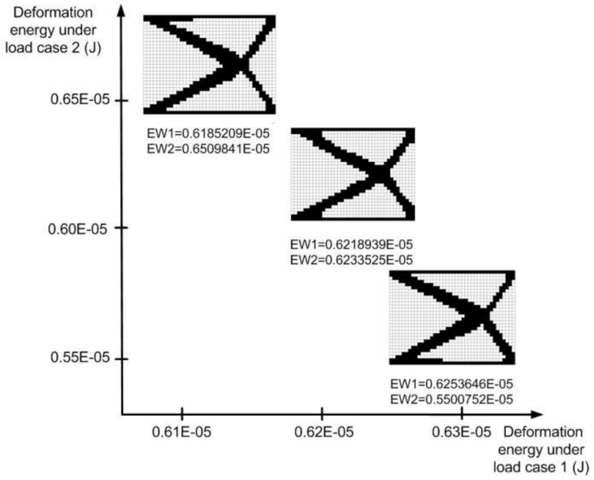


Figure 8 Set of solutions obtained through the proposed method.

GAs. A benchmark among the solutions presented in Table 1 has been performed by evaluating the deformation energy under the two load cases experienced by the plate. The obtained results are shown in Table 1.

Table 1 Comparison of deformation energy among GA solution and hybrid solution for both load cases.

	GA	Hybrid	Δ
EW1 (J)	0.810E-5	0.622E-5	-23%
EW2 (J)	0.810E-5	0.623E-5	-23%

In both cases the material volumes of the optimal solutions are approximately 30% of the initial volume. The topology obtained by the proposed algorithm has a greater stiffness than the GA topology. Time to solution is proportional to the number of generations: the GAs optimal topology is obtained after 600 generations, while the topology obtained by the proposed algorithm is obtained after 50 generation and the time to solution is approximately 100 minutes, so such solution seems to be more efficient than GAs from the computational point of view.

4.2. Example 2

The second example still refers to a plate optimization problem extracted from [1]. The boundary conditions are shown in Figure 9. Two load cases are considered:

1. A point load in the x direction with a magnitude of 200 N.
2. A point load in the y direction with a magnitude of 200 N.

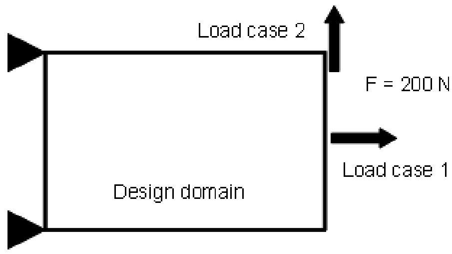


Figure 9 Plate under two different load conditions. The plate is fully constrained at the corners on the left edge and the forces are alternatively applied on the middle and the upper point of the right edge.

In this case, as well as in the previous example, the initial population considered in [20] for GA optimization was composed of 200 individuals and the solutions obtained after 600 generations are presented in Figure 10.

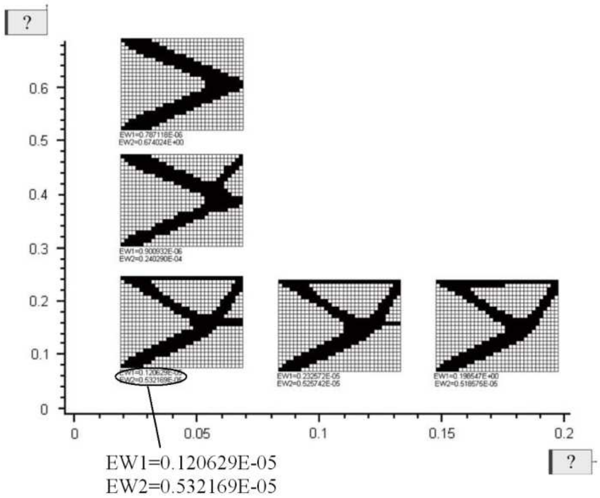


Figure 10 Set of solutions obtained in [20] through GA.

The optimization task is to minimize the deformation energy of the structure under the two load cases shown in Figure 9. The two individuals obtained through mono-objective optimization tasks are shown in Figure 11; the individual obtained by the hybridization of such topologies is shown in Figure 12 (a).

The steps from 3.1 to 3.2 are iterated until global optimum topology is identified; such steps are performed through the use of GAs and also in this case the initial population used by GA was composed by

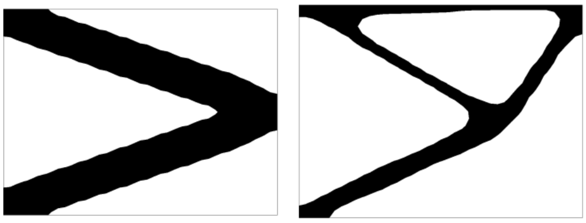
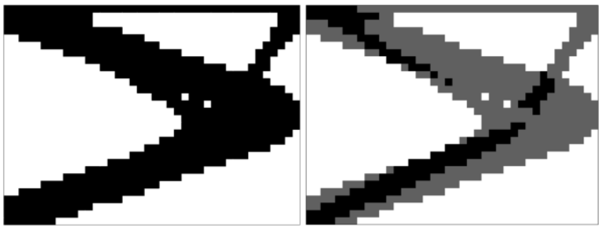


Figure 11 (i): Topologies obtained under load case 1 and density threshold = 0.27; (ii): Topology obtained under load case 2 and density threshold = 0.83.



(a) (b)

Figure 12 (a): Topology obtained by hybridization of mono objective topology shown in Figure 8; (b): Operational zone of the geometrical contradiction.

20 individuals and the solutions, obtained after 50 generations, are presented in Figure 13.

In both cases the material volumes of the optimal solutions are approximately 30% of the initial volume and unlike the previous example, in this case the hybridization approach brings to a different topology than the one obtained by the GAs. Also in this case, a benchmark among the solutions presented in Figure 13 has been performed by evaluating the deformation energy under the two load cases experienced by the plate. The obtained results are shown in Table 2.

Table 2 Comparison of deformation energy among GA solution and hybrid solution for both load cases.

	GA	Hybrid	Δ
EW1 (J)	0.121E-5	0.080E-5	-33%
EW2 (J)	0.532E-5	0.565E-5	+6%

The topology obtained by the proposed algorithm has a greater stiffness for the load case 1 and a slightly smaller stiffness for load case 2 than the GA topology. Besides, it must be observed that load case 2 implies higher deformation energy. Moreover, also

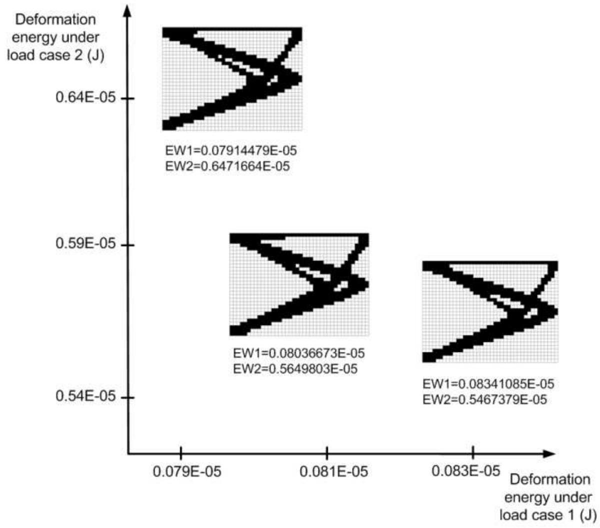


Figure 13 Set of solutions obtained through proposed method.

this second example demonstrates that the proposed approach seems to be more efficient than traditional GAs from the computational point of view: also in this case the GAs optimal topology is obtained after 600 generations, while 50 generations are sufficient to achieve the results shown in Table 2 with the DAeMON technique.

4.3. Example 3

The third case study concerns the topology optimization of a bicycle frame, where the forces applied to that structure are very different depending on the position of the rider, which in turn heavily depends on the slope of the road. As already mentioned, this example is taken from [13], where it was used by the authors to assess the effectiveness of a multi-objective topology optimization method based on Genetic Algorithms (GAs). For the sake of simplicity only the following three different cases have been considered by the authors:

1. On flat landscape: the greatest force is applied on the saddle.
2. On up-hill ground: the rider pushes hard on the pedals and pulls on the handlebars.
3. On steep roads: the rider doesn't sit on the saddle any more, thus transferring his entire load to the pedals.

Figure 14 shows the results of the three mono-objective optimizations, while Figure 15 shows the solution of the multi-loading optimization problem.

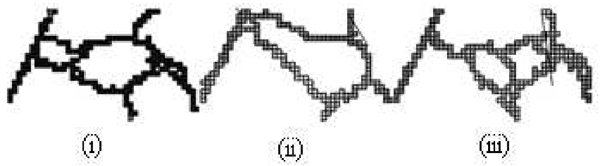


Figure 14 Solutions of the mono-objective optimization problems. (i): Steady ground. (ii): Heavy slop. (iii): Sitting up-hill position [13].

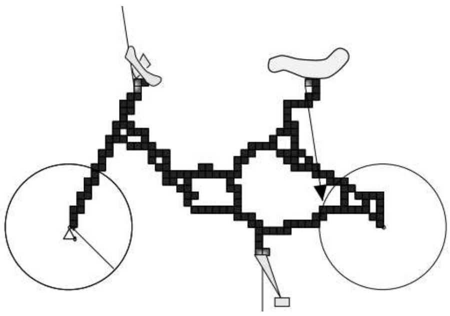


Figure 15 Solution of the multi-loading bicycle [13].

The optimization task is to minimize the deformation energy of the frame under the previously mentioned load cases. The design domain was not available in [13], so it has been determined by measuring a racing bicycle frame.

Figure 16 shows the chosen domain geometry: in gray the "non design" areas are shown, respectively the saddle zone and the handlebars zones. It is necessary to mention that due to the aim of the present research, the design task has been simplified by taking into account a two dimensional design domain, despite a real bicycle frame is subjected to forces outside of the plane of the wheels. Besides, the case study still constitutes a relevant test for the proposed algorithm, also due to the possibility of performing comparisons with [13].

The 2D design domain has been subdivided into 6563 isoparametric plane stress finite elements; the structure is made of a steel alloy, so the unit cell material is assumed isotropic with Young's modulus equal to 210 GPa and Poisson coefficient equal to 0.3. The boundary conditions have been extrapolated by the figures reported in [13].

The loading conditions for the three load cases are:

1. On flat landscape, Figure 17:
   a. The saddle loaded in the x direction with a magnitude of -28N and in the y direction with

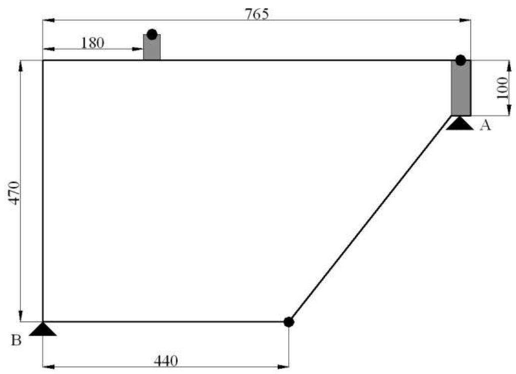


Figure 16 Domain geometry of the bicycle frame and its functional surfaces: the black circles are the application points of the forces. A and B are the constraints.

   magnitude of -363N.
   b. The handlebar loaded in the y direction with a magnitude of 44 N.
   c. The pedals loaded in the x direction with a magnitude of -489N and in the y direction with a magnitude of 481N.

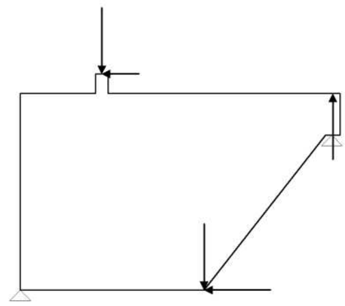


Figure 17 Load case "flat landscape".

2. On up-hill ground, Figure 18:
   a. The handlebar loaded in the x direction with a magnitude of -28N and in the y direction with a magnitude of 488N.
   b. The pedals loaded in the x direction with a magnitude of -1600N and in the y direction with a magnitude of -1200N.
3. On steep roads, Figure 19:
   a. The pedals loaded in the x direction with a magnitude of -2000N and in the y direction with a magnitude of -2000N.

By applying the proposed algorithm, a Pareto optimal set is obtained and it is shown in Figure 20. In this case, in order to find the optimal threshold den-

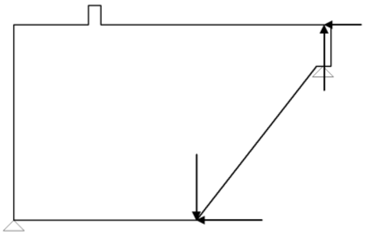


Figure 18 Load case “up-hill”.

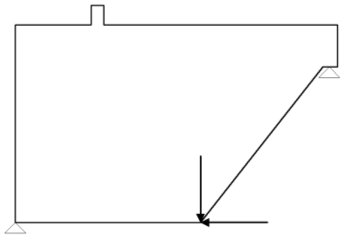


Figure 19 Load case “steep road”.

sity that minimizes the mass and the compliance for the tree load cases, the initial population used by GA was composed by 20 individuals and the solutions are obtained after 50 generations.

Unlike previous examples, in [13] the quantitative results of the optimized topology have not been reported, so in this paper the comparison among GAs results and the results of the proposed algorithm has not been made. In order to make a quantitative comparison, a traditional multi-objective optimization performed by a mathematical method has been accomplished by assigning the same relevance to the three objectives, i.e. considering as objective function the sum of the deformation energy of the load cases.

Moreover, in order to analyze the potentialities of the hybridization method, a traditional bicycle frame has been analyzed under the three load cases. The topologies of the bicycle structures can heavily change varying the value of the frame mass, so for the sake of simplicity, the comparison is made only for one value of the mass. Figure 21 (a) and Figure 22 show the three compared topologies; Figure 21 (b) shows the DAeMON Hybrid topology (Figure 21 (a)) with the operational zone of the emerging contradiction. Also in this case, benchmark among such solution has been performed by evaluating the deformation energy under the three load cases. The obtained results are shown in Table 3.

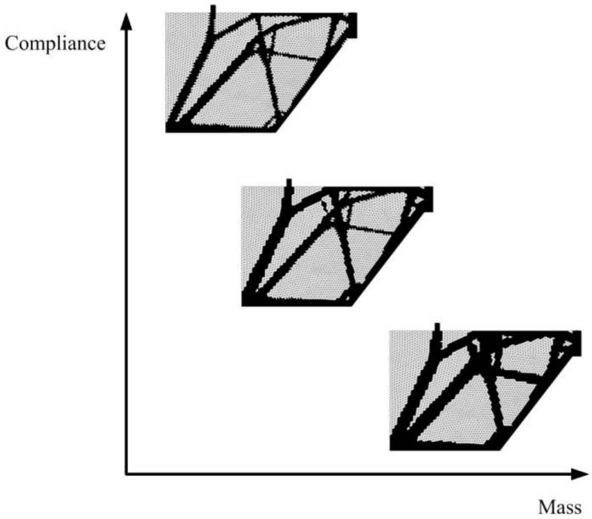


Figure 20 Set of solutions obtained through proposed method.

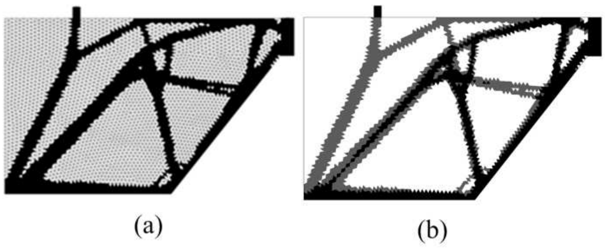


Figure 21 (a): DAeMON hybrid solutions; (b): operational zone of the geometrical contradiction.

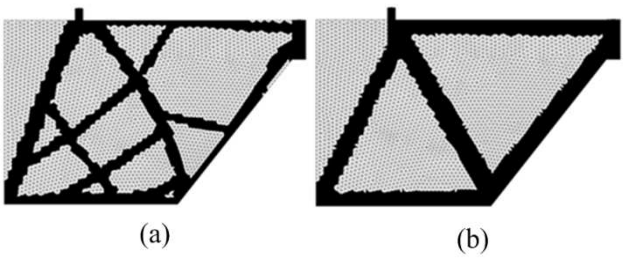


Figure 22 (a): Multi-objective topology; (b): Traditional bicycle frame.

Table 3 Comparison of deformation energy among hybrid solution, traditional solution and multi-objective solution for the three load cases.

	Hybrid	Traditional	Multi
EW 1 (J)	3,02E-05	3,21E-05	3,28E-05
EW 2 (J)	1,67E-04	1,74E-04	1,92E-04
EW 3 (J)	3,25E-04	3,34E-04	3,85E-04

The topology obtained by the hybridization algorithm proposed in this paper has a greater stiffness for all the three load cases.

5. DISCUSSION AND CONCLUSION

The proposed algorithm based on hybridization of partial solutions allows to systematically “browse” the whole set of hybridized topologies by automatically varying the density thresholds until the best globally hybrid topology is identified. The results so far described, demonstrate that the proposed method is computationally less expensive than the conventional application of Genetic Algorithms (GAs) to topological optimization, still keeping the same robustness in terms of searching the global optimum solution. Moreover, the comparison among the DAeMON hybrid solutions and the solutions obtained through traditional GAs or with numerical methods, shows that the proposed approach often leads to very different topologies having better performance.

Furthermore, the conventional GAs methods may lead to topologies with “checkerboard” patterns (alternating elements of material and void) and “floating” elements (elements “floating” in space and not connected to the main structural body). Such geometries may be invalid or impractical, consequently the overall procedure may not be robust. In order to overcome these limits, specific algorithms have been developed aimed at making the optimization procedure more robust, but these algorithms make it also more expensive in terms of computing resources. DAeMON is able to preserve the connections among adjacent elements avoiding the “floating” elements problem. Moreover it avoids the “checkerboard” effect, since the mono-objective optimizations used for hybridization are performed with traditional numerical methods that are immune with respect to this problem.

The hybridization based on the formula (1) has demonstrated its effectiveness to identify original solutions only with a specific class of geometrical contradictions for structural design tasks: those arising from different static load conditions applied to the system.

Beside, according to what has been stated in [7], a more general form of the hybridization formula takes into account also other kinds of combinations, which are based on rotations and translations of partial solutions. Further developments of the research will go towards the investigation of these hybridization rules in order to systematically identify what kinds of geometrical contradictions they are able to solve. In such a way, standard rules of combination of partial solutions aimed at solving each class of geometrical contradictions will be defined.

The identification of the parameters governing rotations and translations of partial solutions will be performed through the algorithm presented in section 3, which will be extended still preserving a logic based on a small number of variables to manage. At the current state of development, the proposed approach is able to deal with constraints related to the mass or the volume of the optimized geometry, but it is not able to manage other kinds of optimization constraints such as those related to manufacturing requirements. This is another important issue that should be addressed in the future developments of the method. Eventually, the application of DAeMON also in contexts different from the structural fields (such as thermal, fluidynamics, etc.) will be studied in order to investigate the possibility to generalize the overall technique.

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