

EXPERIMENTAL INVESTIGATION OF LOW PRESSURE TURBINE NOISE: RADIAL MODE ANALYSIS FOR SWIRLING FLOWS

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ABSTRACT

The well known Radial Mode Analysis (RMA) technique, developed to derive the modal content of an in-duct measured acoustic pressure field, has been applied to the experimental investigation of tone noise generation and transmission mechanisms within low pressure turbine stages.

To improve the accuracy of the modal decomposition when dealing with highly 3D flows, such as intermediate LPT rows and turbine off-design operating conditions, the classical RMA technique for uniform axial flows has been extended to account for flow swirl.

This paper describes the RMA method implemented at University of Florence and presents the first experimental results from the LPT tone noise investigation carried out at Avio acoustic cold flow, for which the RMA has been developed.

INTRODUCTION

In the context of turbomachinery noise studies, the experimental investigation still plays a fundamental role to gain the physical understanding of the noise generation and transmission mechanisms, as well as to support the validation of computational aeroacoustic analysis (CAA).

This is especially true for LP turbines applications, where multiple rows interaction significantly increases the complexity of the acoustic problem.

The work presented in this paper is part of an wider effort, on-going at Avio and University of Florence, to generate a high quality experimental database on LPT tone noise, in which the problem is addressed through a step by step approach, that starts from the characterisation of two-rows interaction noise, then increasing the level of complexity by subsequently increasing the number of rows and noise generating / transmission interactions.

At this aim a series of cold flow turbine experiments are under way at Avio, with rows arrangements ranging from single stage to two stages, with and without TRF; the noise database being represented by in-duct measured acoustic pressure fields, processed through the well known Radial Mode Analysis (RMA) technique to provide the circumferential and radial component modes.

To improve the accuracy of the modal decomposition, the classical RMA technique for uniform axial flows has been extended at University of Florence to properly model the real flow field in which the acoustic pressure field is measured, that is typical for intermediate LPT rows and off-design LPT operating conditions.

In the following the RMA method implemented is described and the experimental results of the first test campaign carried out at Avio acoustic cold flow are given.

In the first section a short review of the theoretical background of in-duct acoustic propagation and RMA technique principles is presented.

In the second section the software implementation is described, with emphasis to the non-uniform flow

formulation based on the numerical solution of the in-duct acoustic modes, and followed by a few results of numerical tests investigating the relevance of flow swirl on the modal decomposition.

In the last section Avio cold flow experimental results are presented, together with a description of the test rig and measurement set-up.

THEORETICAL BACKGROUND

Assuming the flow is inviscid and isentropic, the sound propagation in a constant-area duct with uniform axial flow, can be described as a superposition of single frequency components p_ω that can be expressed in the form (Munjaj, 1997; Rienstra and Hirschberg, 2004):

$$p_\omega(x, r, \varphi) = \sum_{m=-\infty}^{+\infty} \sum_{n=0}^{+\infty} \left(\hat{A}_{mn}^+ e^{ik_{mn}^+ x} + \hat{A}_{mn}^- e^{ik_{mn}^- x} \right) f_{mn}(r) e^{im\varphi} \quad (1)$$

where the subscripts m and n represent the circumferential and radial mode orders, and the superscripts \pm refer to the downstream (+) or upstream (-) traveling waves; the coefficient A_{mn} representing the amplitude of the modes.

For rigid-walled ducts, the axial wavenumber k_{mn} of the fundamental mode (m, n) is defined as:

$$k_{mn}^\pm = \frac{1}{1-M_x^2} \left(-k_0 M_x \pm \sqrt{k_0^2 - (1-M_x^2) \frac{\sigma_{mn}^2}{r_{ip}^2}} \right) \quad (2)$$

and an analytic solution exists for the modes shape functions $f_{mn}(r)$.

In the more general case of swirling and sheared mean flows, the acoustic and vortical disturbances propagating in the duct are coupled, and no analytic solution exists (Golubev and Atassi, 1996; Golubev and Atassi, 1998); equation (1) can still be applied, but the wavenumbers and shape functions have to be solved numerically.

In this case the solution of the system of coupled linearized equation of motion will provide both acoustic and vortical disturbances.

Once the wavenumbers and the corresponding shape functions are available from analytical or numerical method, the modal amplitudes A_{mn} can be recovered from the single frequency complex pressure distributions $p_\omega(x, r, \varphi)$.

RMA METHOD

RMA (Radial Mode Analysis) is an established technique to compute in-duct modal amplitudes of the propagating acoustic modes, based on single frequency complex pressure values measured at discrete locations inside the duct.

Let's assume the acoustic pressure distribution p_ω is available through a discrete set of values, N_x , N_φ and N_r being the number of unsteady pressure values in the axial, azimuthal and radial directions.

By applying a DFT in the azimuthal direction the complex modal amplitudes A_m at each axial and radial

measurement positions can be expressed in the form:

$$\hat{A}_m(x_i, r_j) = \sum_{n=0}^{N_{max}^m} \left(\hat{A}_{mn}^+ f_{mn}(r_j) e^{ik_{mn}^+ x_i} + \hat{A}_{mn}^- f_{mn}(r_j) e^{ik_{mn}^- x_i} \right) \quad (3)$$

where the maximum circumferential order m that can be solved is defined by the Nyquist theorem as

$|m_{max}| = (N_\varphi/2) - 1$, and the maximum radial order n that can be solved is $n_{max} < N_r N_x / 2 - 1$.

The linear system expressing the A_m coefficients as functions of the modal amplitudes A_{mn} can be written in the form:

$$\mathbf{A}_m = \mathbf{W}_m \mathbf{A}_{mn} \rightarrow [\hat{A}_m(x_i, r_j)] = \left[f_{mn}(r_j) e^{ik_{mn}^+ x_i} \mid f_{mn}(r_j) e^{ik_{mn}^- x_i} \right] \begin{bmatrix} \hat{A}_{mn}^+ \\ \hat{A}_{mn}^- \end{bmatrix} \quad (4)$$

and finally the complex modal amplitudes A_{mn} can be derived by computing the pseudo-inverse matrix \mathbf{W}_m^+ and resolving the inverse system:

$$\mathbf{A}_{mn} = \mathbf{W}_m^+ \mathbf{A}_m \quad (5)$$

A singular value decomposition (SVD) technique can be applied to solve the system (Enghardt, Tapken, Neise, Kennepohl and Heing, 2001; Holste and Neise, 1997; Enghardt and Tapken 2006).

To be noticed here that the knowledge of the complex pressure amplitude A_{mn} of the fundamental modes also enables the computation of the radiated sound power (Tyler and Sofrin, 1962, Goldstein 1976).

SOFTWARE IMPLEMENTATION

The RMA software developed at University of Florence is composed of two modules, the first one resolving the general eigenvalue problem to derive the acoustic modal basis for a given duct and flow conditions, and the second one performing the modal decomposition of the measured in-duct acoustic field, assigned through a given number of axial, circumferential and radial pressure values.

The first module implements the formulation proposed by Kousen (Kousen, 1999), that provides wavenumbers and shape functions of the pressure and vortical waves that propagates in the given duct at given flow conditions, and is based on the assumption that the axial velocity is a generic function of the radial coordinate, while the radial distribution of the swirl velocity can be expressed as a superposition of a rigid and a free vortex model:

$$\mathbf{v}_\varphi(\mathbf{r}) = \Omega \mathbf{r} + \frac{\Gamma}{\mathbf{r}} \quad (6)$$

This assumption allows to use simpler analytical relations to compute radial distribution of mean flow thermodynamic quantities (pressure and speed of sound), as well as to bound the set of nearly-convected eigenvalues as shown by Golubev & Atassi (Golubev and Atassi, 1998).

The second module performs the modal decomposition of the measured acoustic field over the modal basis numerically computed by the first module by standard SVD routines, as reported in literature.

To be noticed that only propagating acoustic modes are used for the RMA, while decaying pressure modes, nearly convected disturbances and spurious numerical solutions are filtered out following the concepts proposed by Golubev & Atassi (Golubev and Atassi, 1998).

Additionally a conditioning analysis module has been implemented to define best sensor positioning based on error propagation, as proposed in (Enghardt and Tapken, 2006). Condition number providing the upper limit for the error on A_{mn} due to perturbations imposed on the vector A_m (Enghardt and Tapken, 2006).

The SW has been validated on test cases available in literature (Golubev and Atassi, 1998; Golubev and Atassi, 1996), to evaluate the accuracy of computed eigenvalues and corresponding eigenfunctions, to verify the completeness of the computed set of acoustic modes and to assess the effectiveness of the implemented filtering in discarding spurious numerical solutions and vortical modes.

In the context of the experimental investigation on turbine tone noise underway at Avio and described in the following paragraphs, dedicated numerical tests have been performed to verify the applicability of simpler flow models, for which an analytic solution of the acoustic modal basis exist. A few results of these tests are proposed below.

To be noticed here that the flow model implemented admits coupled acoustic-vorticity modes which are not orthogonal, and makes sound power analysis more complex and less detailed with respect to the simpler models considered, in which the acoustic modes are orthogonal.

In the numerical tests described below, the modal content of an artificial pressure field corresponding to the swirl model in (6) has been compared to the decomposition obtained on the modal basis of the simplified models; both uniform axial flow and rigid swirl approximations have been considered.

A background flow field representative of real test conditions has been imposed, as reported in Table 1 (mean flow) and Figure 1 (tangential velocity profile).

The relevant differences found between the real and reconstructed modal content for the uniform axial flow case can be easily explained by the consistent errors introduced in the cut-off boundaries and propagation properties of cut-on modes when the flow deviates from the uniform axial flow approximation, see the cut-on modes plots in Figures 2 and 3.

In the rigid swirl flow model the radial distribution of the tangential velocity is given by:

$$v_{\varphi}(r) = \Omega r \tag{7}$$

This model still allows the acoustic modal basis to be derived analytically (J.L. Kerrebrock, 1977; Enghardt and Tapken 2006), while allowing to preserve the cut-off boundaries with better accuracy.

Figures 4 to 6 compare the real and reconstructed modal content at an intermediate frequency; in particular Figure 6 reports the differences between the two solutions expressed in dB.

Looking at these results, and noting that the sensors arrangement adopted for the RMA was defined in order to optimize the problem conditioning for both swirl models in (6) and (7) (see Figure 7 for the comparison of the condition number), it was concluded that on Avio test rig the simpler swirl model is unable to accurately reproduce the real modal content, even for modes which have low condition number, and the more general swirl model is needed.

Flow parameters	Value
Mx,avg	0.35
Mt,avg	0.19
αavg [°]	27.7.

Table1

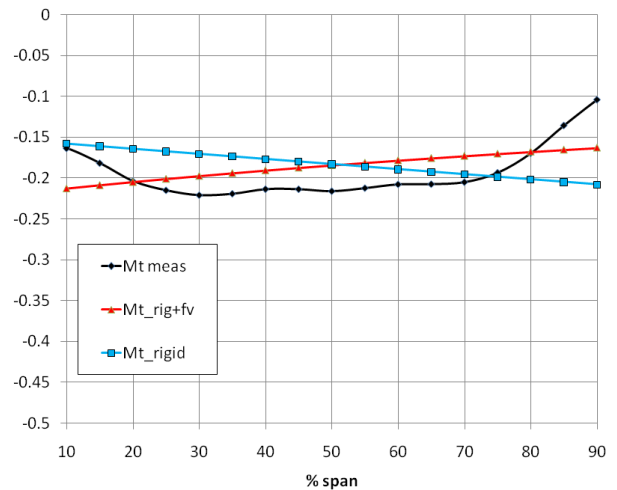


Figure 1: Tangential Mach number profile, rigid swirl plus free vortex model and rigid swirl approximation. The measured profile is reported for comparison.

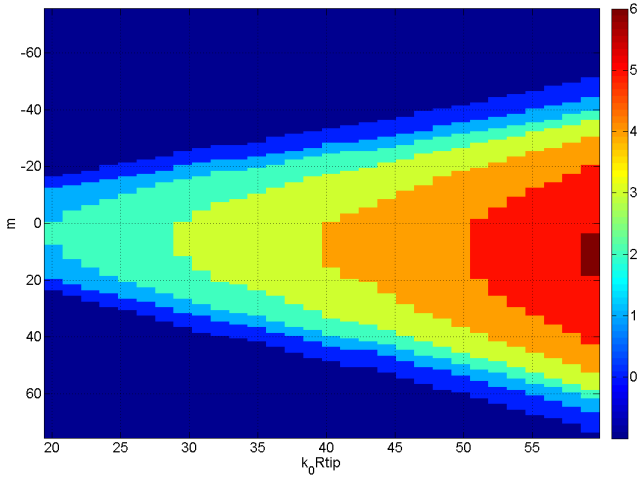


Figure 2: Cut-on modes plot, 3D flow model.

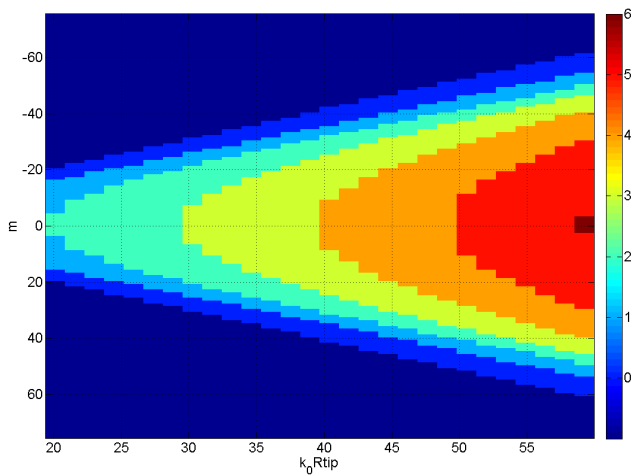


Figure 3: Cut-on modes plot, uniform flow approximation.

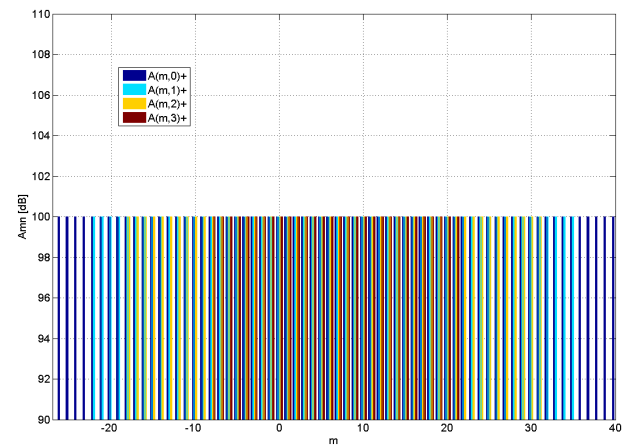


Figure 4: Artificial pressure field, spectrum of modal amplitudes A_{mn} ; $k_0R_{tip}=34$.

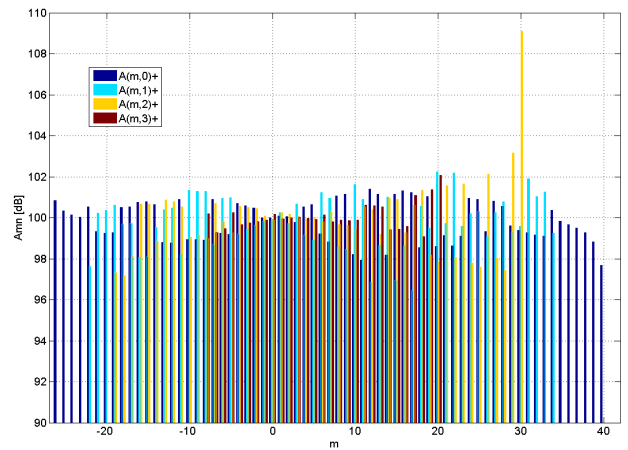


Figure 5: Modal distribution, rigid swirl approximation $k_0R_{tip}=34$.

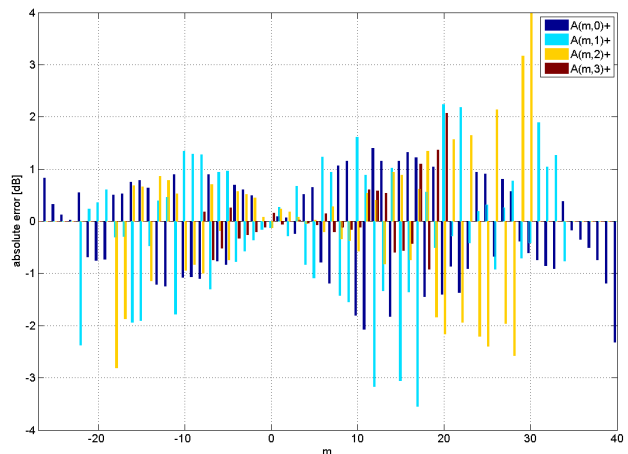


Figure 6: Difference in modal amplitude derived from rigid swirl and rigid swirl plus free vortex approximation, expressed in dB.

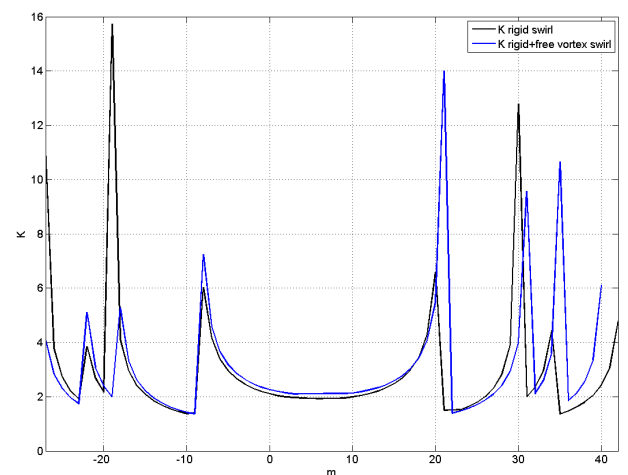


Figure 7: Condition number; comparison between rigid swirl and rigid plus free vortex swirl models; $k_0R_{tip}=34$, $N_x=2$, $N_r=9$.

COLD FLOW INVESTIGATION

In the following the application of RMA software developed at University of Florence on the LPT tone noise investigation currently on-going at Avio Experimental Laboratory is described.

In subsonic LP turbines, representative of commercial aeroengine applications, tone noise generated by rows interaction, propagates along the turbine to the exhaust duct.

The model turbine currently installed on Avio cold flow rig is aimed to investigate tone noise generation and transmission, through a modular design that allows easy replacement of rows, modification of rows axial spacing, add-up of rows (single stage to two stages set-ups, TRF).

In the design of the model turbine, engine-to-rig similitude criteria have been followed. Endwall dimensions have been chosen such that the turbine may be operated at conditions representative of noise certification points, still allowing operation at aerodynamic design point, and duct geometry representative of the reference engine turbine is guaranteed. Rows dimensions and number off are selected to obtain high aspect ratio / low solidity blading, typical of state of the art commercial LP turbine design, while providing aeroacoustic similitude in terms of non-dimensional frequency.

A flat end-wall design is chosen, representative of last turbine stages.

To be noticed that Avio cold flow rig operates at ambient exit pressure; with overpressure at inlet provided by a compressor station. As a consequence, engine-to-rig flow field similitude is guaranteed in terms of Mach number, while engine Reynolds number is not fully reproduced.

ROTOR-STATOR INTERACTION

Rotor-stator interaction is a well-known noise generation mechanism for axial turbomachines, which generates typical tonal components at frequencies given by the blade passing frequency (BPF) and its multiple harmonics.

For two rows interaction tones, the classical theory of Tyler and Sofrin applies (Tyler and Sofrin, 1962), according to which the generated acoustic pressure field at a given harmonic h of the BPF is a superposition of spinning modes with azimuthal order:

$$m = hB - kV, \quad k = \dots, -1, 0, 1, \dots \quad (8)$$

where B and V indicate the airfoil number of the rotor and stator rows.

In (Holste and Neise, 1997) an extension of the classical theory is reported to account for interaction between multiple rotors and stators.

In the RMA processing of Avio test turbine data the rotor-stator interaction theory has been applied to classify the generated tones frequencies and circumferential mode orders.

EXPERIMENTAL RESULTS

Avio test rig is equipped with a rotating duct section at turbine exit, to enable the detailed mapping of the generated acoustic field and its propagation pattern. Provision for radial rakes of dynamic pressure sensors or microphones and flush-mounted microphone arrays is available.

For the RMA, the acoustic field mapping is complemented by mean flow field measurements, used in computing the acoustic modal basis.

For the test campaign whose results are given in the following the acoustic pressures have been measured through radial rakes of dynamic pressure sensors, following an approach similar to that reported in (Enghardt, Tapken, Neise, Kennepohl and Heing, 2001).

In particular a two rake arrangement has been used ($N_x = 2$), with uniform sensors spacing in the radial direction. The number of radial sensors for each rake has been fixed in order to allow some margin on the solution of the highest propagating radial mode orders ($N_r = 9$) at the most restrictive test conditions. A picture of the rakes and calibration set-up is shown in Figure 8.

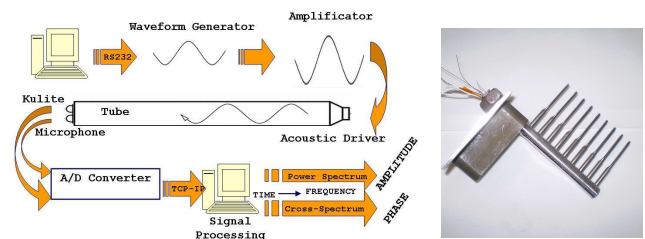


Figure 8: Dynamic Calibration set-up (left) and Measurement Rake (right)

To be noticed that the axial spacing of the rakes has been defined in order to minimize the error of the modal decomposition for the given radial sensors arrangement, according to the optimum conditioning of the system matrix W_m . The results of conditioning analysis for the selected test sensors arrangement at a representative test condition is shown in Figure 9.

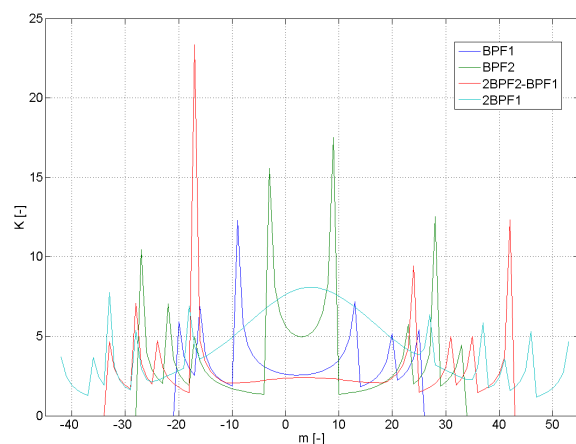


Figure 9: Condition number at optimized axial spacing (OP2).

The number of azimuthal traversing positions of the rakes at the different test conditions were assigned as to enable the solution of the propagating tones, in terms of frequency and circumferential mode orders.

High speed acquisition hardware has been used, allowing the simultaneous acquisition of all unsteady pressure signals. Additionally, a one pulse per revolution trigger and a high resolution encoder have also been acquired for synchronous resampling purposes.

In the following, examples of the results obtained for the two stage turbine configuration shown in Figure 10 are presented.

The airfoil count of the test turbine is listed in Table 2; as can be easily deduced from the table, V1-B1 interaction tone, corresponding to a very low circumferential order, is designed for cut-on over the whole operating range, while B1-V2 and V2-B2 tones are either cut-off or cut-on depending on the test regime.

The results reported below refer to three operating conditions representative of the real engine operation; OP1 corresponding to the turbine design point, OP2 and OP3 corresponding to a high speed and a low speed off-design condition respectively, see Table 3 for more details on the turbine operation.

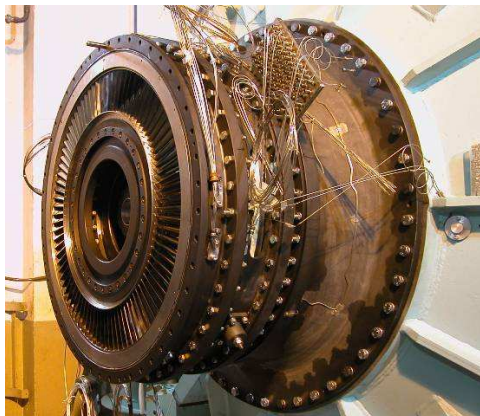


Figure 10: Avio Cold Flow rig- two stage configuration.

stage	Turbine Row	A/F Count
1°	Stator 1, V1	64
	Rotor 1, B1	68
2°	Stator 2, V2	112
	Rotor 2, B2	88

Table 2

	OP1	OP2	OP3
Corrected Mass Flow [kg/sK ^{0.5} /kPa]	2.20	2.05	1.65
PR [-]	2.0	1.8	1.3
Corrected Speed [rpm/K ^{0.5}]	190	180	139
M_{exit,avg} [-]	0.40	0.33	0.20
α_{exit,avg} [°]	-27	-16	2

Table 3

In Figure 11 to 13 the propagating modes plot computed by the RMA SW for the design, high speed and low speed off-design condition are shown. It is easily seen that OP3 mean flow at turbine exit is not far from a uniform flow approximation, characterized by a symmetric $\pm m$ orders propagation; while at OP1 and OP2 the exit swirl increases the number of propagating azimuthal orders with negative sign, i.e. the number of cut-on modes spinning in the direction opposite to the mean flow rotation.

Figures 14 to 20 show the radial mode decomposition of the acoustic pressure field measured at turbine exit; in these figures the modal amplitudes are expressed through the modal coefficients A_{mn} in dB scale.

To be noticed that all cut-on azimuthal orders are reported on the x-axis, that is bounded at cut-off.

As can be seen from Figures 14-15-16, reporting RMA results at B1 first BPF for the three operating conditions, the only tone detected at B1 first BPF corresponds to V1-B1 fundamental interaction $m=B1-V1=+4$; at this frequency B1-V2 fundamental interaction is cut-off at all regimes and multirow interactions are not measured.

Figures 17 and 18 show the modal decomposition of the acoustic field at B2 first BPF for OP1 and OP2; at both regimes the dominating mode is V2-B2 fundamental interaction $m=B2-V2=-24$; but another mode is present, even if at a significantly smaller amplitude, corresponding to V1-B2 interaction $m=B2-V1=+24$.

RMA results at a higher frequency corresponding to B1 second BPF at OP2 are reported in Figure 19. At this frequency the interaction of B1 with both V1 and V2 is cut-on, and the amplitude of the two corresponding orders $m=2B1-2V1=+8$ and $m=2B1-V2=+24$ are comparable.

Finally Figure 20 shows that multirow interaction modes are also present, even if at an amplitude far lower than that of fundamental interactions; this graph corresponds to a combined B1 – B2 frequency, dominated by the 3 rows interaction $m=2B2-B1-V2=-4$. The same picture shows that even the mode $m=2B2-B1-V1=+44$, originating by the interaction with V1, can be identified in the modal spectrum.

The results presented above suggest that the interaction of two adjacent rows is by far the dominating noise generation mechanism, when compared to multirow interaction and interaction of non adjacent rows. Nevertheless these results are far from comprehensive, and more extensive testing is needed to draw a general conclusion about noise generation in multirow environment.

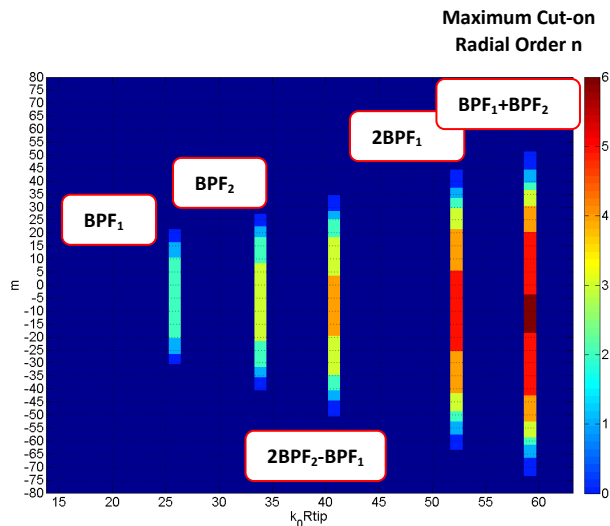


Figure 11: Map of cut-on modes at OP1 flow condition.

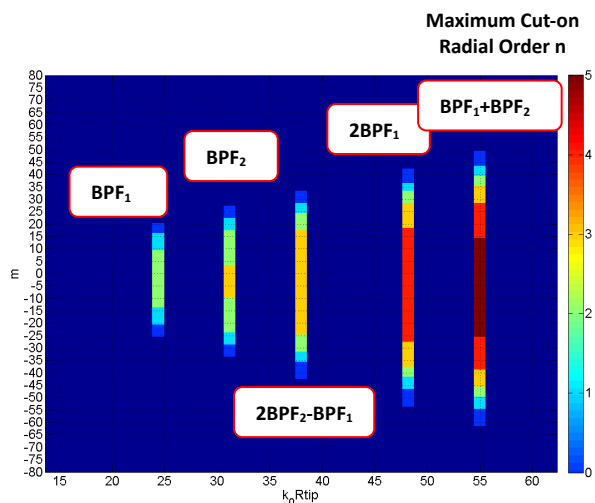


Figure 12: Map of cut-on modes at OP2 flow condition.

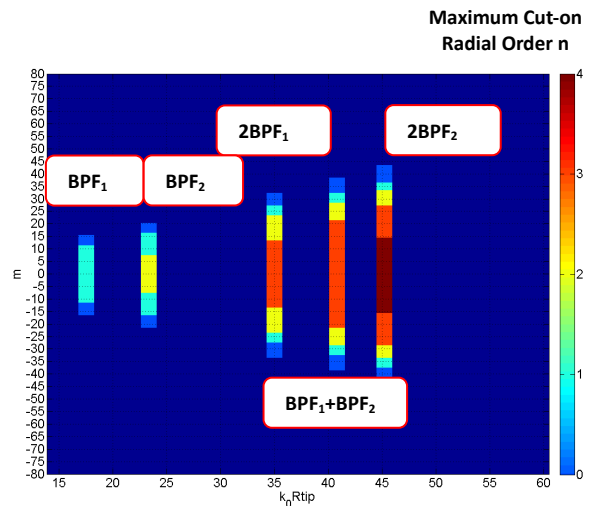


Figure 13: Map of cut-on modes at OP3 flow condition.

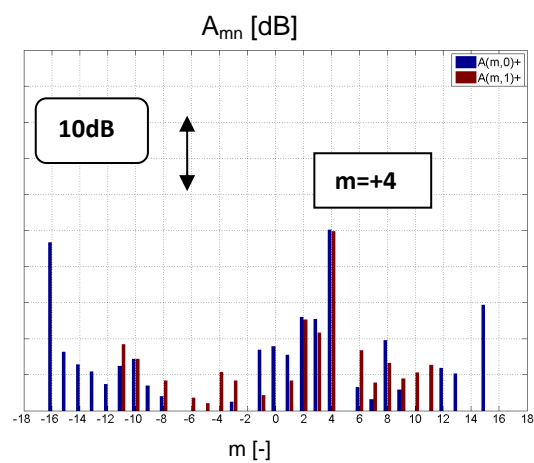


Figure 14: Downstream Propagating Modal Amplitudes at first BPF of B1, operating condition: OP3.

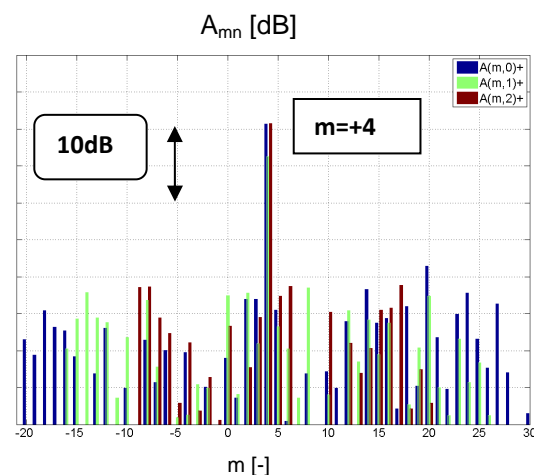


Figure 15: Downstream Propagating Modal Amplitudes at first BPF of B1, operating condition: OP1.

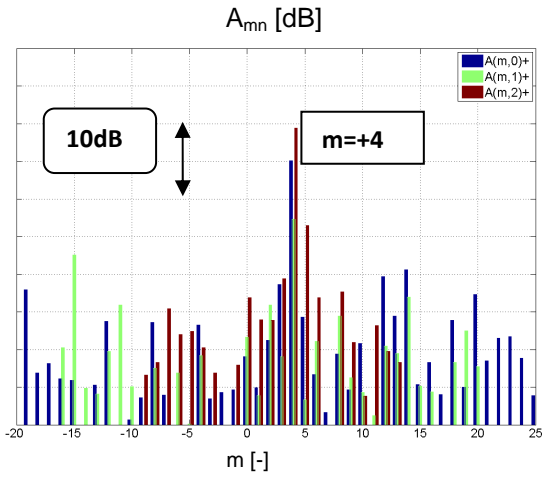


Figure 16: Downstream Propagating Modal Amplitudes at first BPF of B1, operating condition: OP2.

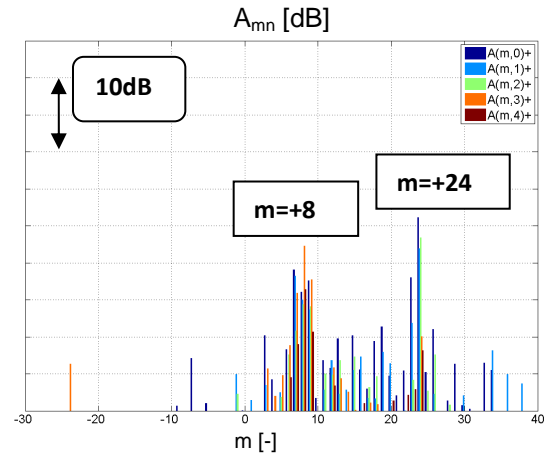


Figure 19: Downstream Propagating Modal Amplitudes at second BPF of B1, operating condition: OP2.

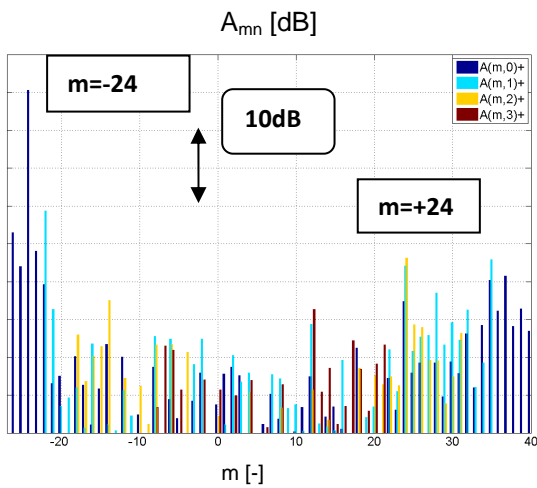


Figure 17: Downstream Propagating Modal Amplitudes at first BPF of B2, operating condition: OP1.

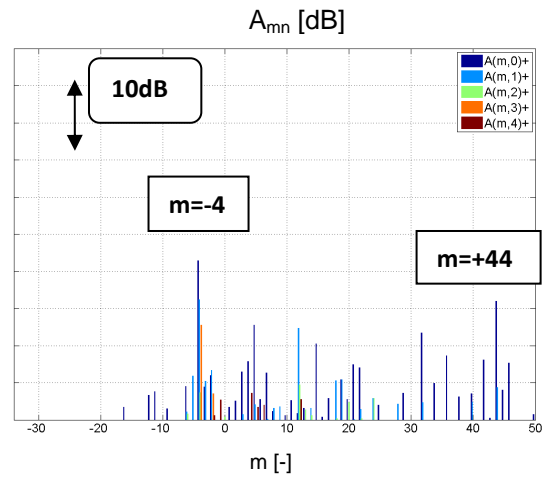


Figure 20: Downstream Propagating Modal Amplitudes at 2B2-B1, operating condition: OP1.

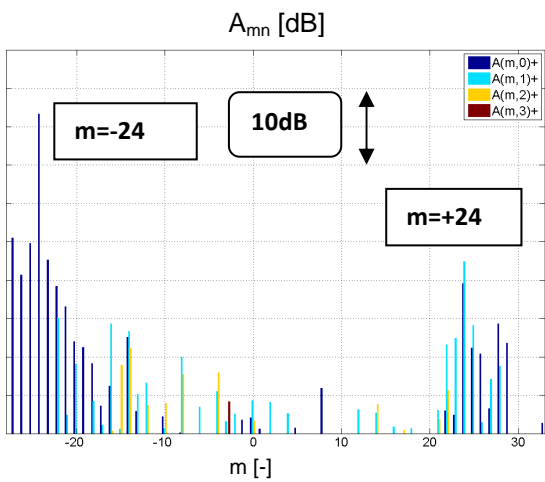


Figure 18: Downstream Propagating Modal Amplitudes at first BPF of B2, operating condition: OP2.

CONCLUSIONS

In order to investigate tone noise generation and propagation mechanisms within low pressure turbine stages, an extensive test campaign is on-going at Avio Group cold flow test facility in cooperation with University of Florence.

The first experimental results of LPT tone noise investigation, carried out on a two-stage turbine at three operating points, were presented in this paper.

Unsteady pressure data were collected using in-duct sensor rakes and were analyzed by means of RMA technique to gain modal amplitudes of propagating modes at frequencies which were typical of tone noise.

A software was implemented to perform modal decomposition of the measured sound pressure fields. To take into account flow conditions representative of rig testing, the set of in-duct propagating modes were computed numerically, basing on a swirling mean flow model; the tangential velocity radial distribution was given

by a superposition of a free vortex rotation and solid body rotation. The software was validated on literature test cases.

A set of numerical tests was performed to investigate the capability of simpler models, for which an analytic solution of the acoustic modal basis exists, to provide an accurate radial mode analysis for swirling flows, in the context of the experimental investigation underway at Avio.

Results showed that the modal decomposition is strongly affected by the swirl model assumed. Such result suggests that extending the investigation to a more general flow model, including a generic swirl distribution and accounting for shear, may be worthwhile.

Radial mode analyses were carried out for the dominant tones at operating points representative of the design, high speed and low speed off-design conditions. Modal distributions showed that the fundamental interaction between adjacent blade and vane is the main noise source. Non adjacent rows interactions (B2-V1) and multirow interactions (B1-B2-V2) were also detected, but with significant lower modal amplitudes.

Results reported in the present work, will be appended to a wider database of measured acoustic data that are being collected at Avio cold flow test facility. The modular design of the model turbine will allow to investigate the effects on noise generation of multistage turbine design parameters such airfoil count and design and rows axial spacing. For each configuration, the fundamental tonal noise sources will be detected by the modal decomposition of the measured sound fields.

NOMENCLATURE

x	Coordinate in direction of duct axis
r	Radial coordinate, with subscripts tip or hub, indicates tip or hub duct radius, respectively
φ	Azimuthal coordinate
p	Pressure
ω	Circular frequency ($\omega=2\pi f$)
p_ω	Acoustic pressure at ω
f	Frequency
m	Azimuthal modal order, number of lobes or cycles of circumferential pressure variation
n	Radial modal order, number of zeroes of pressure functions along radial direction
$f_{mn}(r)$	Radial pressure function of order m and n
k	Index for Tyler & Sofrin relation
k_0	Free field wavenumber
k_{mn}^\pm	Axial wavenumber for m - n mode, + and - indicate, respectively, downstream and upstream propagation
A_{mn}^\pm	Complex modal pressure amplitude
A_m	Azimuthal mode amplitude

M	Mach number
σ_{mn}/r_{tip}	Radial wavenumber
v_φ	Tangential velocity component
W_m	Linear system matrix
W_m^+	Pseudo inverse of matrix W_m
Ω	Solid body rotation angular frequency
Γ	Free-vortex circulation
h	Blade passage frequency harmonic index
B	Number of rotor blades
V	Number of stator vanes
R	Radius
N_x	number of axial measurement positions
N_r	number of radial measurement positions
N_φ	number of azimuthal measurement positions

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