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# 3D Model Retrieval from mechanical drawings analysis

R. Furferi, L. Governi, M. Palai and Y. Volpe

**Abstract** — Computer Aided Engineering (CAE) techniques provide effective solutions for automating the whole product development chain process. Designers, engineers, manufacturing professionals and researchers can now leverage solid modeling data and multi-physics analysis in ways that were inconceivable just few years ago. Among CAE techniques, Computer Aided Design (CAD) has been the most effective in providing methodologies capable of compressing product design and manufacturing cycles, assuring faster turnaround time between design and simulation and improving product quality. Designers and manufacture companies reap the rewards of 3D CAD modelling; as a consequence, research is unceasingly stimulated to look forward. On one hand, research aims to improve capabilities of existing CAD methods and tools; on the other hand novel approaches are extensively investigated with the ambition of carrying out innovative CAD techniques capable of lighting sparking design innovation and creativity. This is particularly true for mechanical design: fast and robust 3D retrieval from 2D drawings that was considered future trend few years ago, is now a key target for commercial software houses like Dassault Systems® and Autodesk® as well as a vigorous focus from an academic outlook. Unfortunately, even if a number of works have been carried out during the last decades, these are mainly described by a conceptual point of view. To derive an orderly procedure covering the necessary steps for retrieving 3D models from mechanical drawings could provide a dramatic boost to researchers and practitioners that introduce this issue on their research. Therefore, the main aim of the present work is to carry out a systematic clear and concise step-by-step procedure for 3D retrieval starting from wireframe models. Since the intent is to afford an as clear as possible, guided, procedure for 3D reconstruction, mathematical description is limited to the simplest case of polyhedral objects. The proposed procedures, inspired by state of the art works, can be effectively contribute to speed-up the possible implementation of methodologies confronting the 3D reconstruction problem.

**Keywords** — Pseudo-wireframe, 3D Retrieval, Mechanical Drawings, Computer Aided Design, Computational Geometry.

## I. INTRODUCTION

**T**ODAY, Computer-aided engineering (CAE) techniques afford effective solutions for automating the whole product development chain process. Designers, engineers, manufacturing professionals and researchers can now leverage solid modeling data and multi-physics analysis in ways that were inconceivable just few years ago [1]. Among CAE techniques, such as computer-aided analysis (CAA), computer-integrated manufacturing (CIM), computer-aided

manufacturing (CAM), material requirements planning (MRP), and computer-aided planning (CAP), Computer Aided Design (CAD) is a central issue in the mechanical design field since it provides designers with a series of tools for streamlining design processes such as drafting, visualization, simulation, documentation, and manufacturing processes. Designers and manufacture companies reap the rewards of 3D CAD modelling; as a consequence, research is unceasingly stimulated to look forward. On one hand, research aims to improve capabilities of existing CAD methods and tools; on the other hand novel approaches are extensively investigated with the ambition of carrying out innovative CAD techniques capable of lighting sparking design innovation and creativity. This is particularly true for mechanical design: fast and robust 3D retrieval from 2D drawings that was considered future trend few years ago, is now a key target for commercial software houses like Dassault Systems® and Autodesk® as well as a vigorous focus from an academic outlook.

As a consequence the conversion from 2D orthographic view engineering drawings to 3D CAD models (known as “reconstruction” problem, see Fig. 1) is still a crucial task in a wide range of applications [2-6].

In order to cope with this issue a number of works have been proposed since first 1970s, providing a series of methodologies for solving the reconstruction problem.

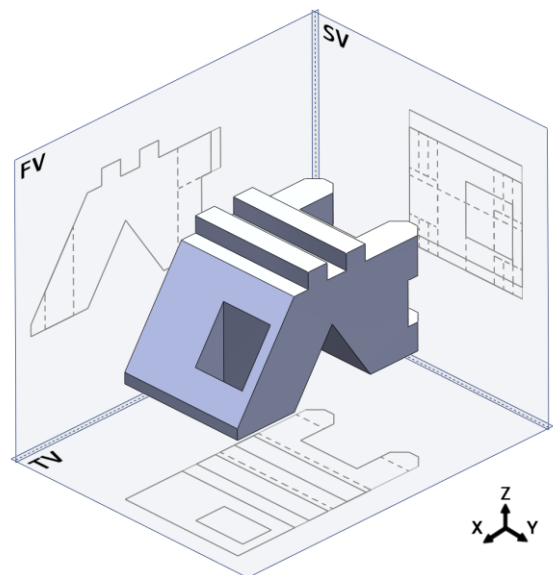


Fig. 1 - typical “reconstruction” problem: from mechanical drawing to 3D solid model.

Generally speaking, the works proposed at the state of the art can be divided in two different families:

1. wireframe-oriented approaches, that are also known as B-rep (Boundary representation) methods;
2. volume-oriented approaches, also called CSG (Constructive Solid Geometry) methods.

A useful review of relevant published works, regarding both B-rep and CSG approaches, is provided by two recent publications [2,7]. Recently the preferred approach for performing 3D reconstruction has been the B-rep based one [8-15]. This is mainly due to the fact that the CSG approach is less suitable to support complex shapes and usually requires heavier user interaction compared to the B-rep one. For such reasons, the present work confronts with the B-rep based reconstruction methodologies.

It is commonly accepted that B-rep reconstruction can be split into two main phases: the first is the reconstruction of the pseudo-wireframe model (set of all possible wireframe models that can be originated by an assigned set of orthographic views); the second is the reconstruction of the 3D solid (or surface) model(s) from the obtained pseudo-wireframe model and coherent with the assigned orthographic views [16, 17].

Unfortunately, even if a number of works have been carried out during the last decades, these are mainly described by a conceptual point of view. To derive an orderly procedure covering the necessary steps for retrieving 3D models from mechanical drawings could provide a dramatic boost to researchers and practitioners that introduce this issue on their research. Therefore, the main aim of the present work is to carry out a systematic clear and concise step-by-step procedure for 3D retrieval starting from wireframe models. Since the intent is to afford an as clear as possible, guided, procedure for 3D reconstruction, mathematical description is limited to the simplest case of polyhedral objects. The proposed procedures, inspired by state of the art works, can be effectively contribute to speed-up the possible implementation of methodologies confronting the 3D reconstruction problem.

A comprehensive, orderly, unambiguous and automatic procedure meant to help researchers and practitioners who want to deal with the first phase of the reconstruction problem has been provided by the authors in a previous work [18]. Starting from the wireframe model obtained with this methodology, the aim of the present work is to retrieval a 3D model starting from wireframe one obtained from mechanical drawings.

The reconstruction procedure involves a number of software routines; by means of them, an initial 3D vectorial wireframe model is processed and a set of 3D solutions, consistent with the initial wireframe model, is extracted. The obtained 3D models are subsequently output according to the most common 3D exchange formats (e.g. IGES, STEP, Parasolid, etc.). The proposed procedure has been implemented using MatLab® programming language to assess its functionality. Extensive testing, carried out on a number of case studies, has demonstrated the effectiveness of the presented approach. Based on the results obtained in the

testing phase, it is possible to state the suitability of the proposed procedure to automate the reconstruction of polyhedral objects by a set of orthographic projections. The procedure can be effectively used as a common basis to speed-up the possible implementation of methodologies confronting the reconstruction problem.

After a brief description of the first main phase (wireframe model reconstruction), provided in section 2, the paper focuses on 3D model reconstruction tasks and validation (section 3). In order to help readers in seeing the proposed approach through, a case study is carried on along the entire reconstruction procedure.

## II. WIREFRAME RECONSTRUCTION

As already stated, the main aim of the present work is to reconstruct a 3D model when a wireframe one is obtained starting from a set of orthographic projections. Wireframe model can be obtained according to the following tasks [18]:

- 1) 2D orthographic data extraction.
- 2) 2D vertexes and edges labeling.
- 3) 2D edges and vertexes manipulation.
- 4) 3D wireframe reconstruction.

### A. 2D orthographic data extraction

The first step of the wireframe reconstruction procedure consists of extracting, for instance from a DXF file, the end-point coordinates of each geometric feature (e.g. lines) and storing them into a database. The result is an ordered matrix (size  $2n \times 3$ ) of  $n$  edges, each one defined by two triplets of coordinates representing the endpoint vertexes. Moreover, each 2D geometric feature is automatically assigned to its orthographic projection thus obtaining three different sets of entities (edges).

### B. 2D vertexes and edges labeling

Once known the set of orthographic projections a conversion of the geometric data into topological ones is performed in order to reduce the information to be processed. Therefore, a topological data structure is accomplished. In detail, each vertex of each projection is labeled with a progressive number.

Each 2D edge can be now represented by the label of its endpoints rather than the actual endpoints coordinates.

Thus, only 3 parameters (a progressive number identifying 2D edge and the label of its two endpoints) are now used to properly identify each edge instead of 7 parameters (a progressive number identifying 2D edge and the coordinates of its two endpoints).

### C. 2D vertexes and edges manipulation

When orthographic projections come out from a digital CAD format, e.g. DXF, even if an object is represented by a univocal set of projections, these can be drawn by using different combination of geometric entities. The segment

highlighted in fig. 1a can be made up of a number of straight vectors (from 1 to 8 as shown in fig. 1c).

Such combination of vectors, anyway, is uncorrelated with the one which would be generated by the projection of the object's 3D edges lying on the plane orthogonal to the view and whose trace contains the original segment (fig. 1c). In other words, the same projection can be represented by different digital CAD files.

As a consequence, in order to obtain a univocally defined vectorial representation, comprising all the possible configurations, a procedure, called "segmentation" is recommended.

First, for each edge, an iterative procedure checks for the possible existence of intermediate vertices. If no intermediate vertex is found, the procedure stops. Otherwise the found intermediate vertex causes the creation of two new edges (unless one of them already exists). This task is performed for each set of edges belonging to a projection, thus adding new edges to the original set.

After the "segmentation" task, a check of collinearity of edges is performed. This phase is fundamental when two non-contiguous vertices are linked by two or more edges (all collinear one with each other). If this check is inaccurate it could happen that two visually identical projections are described by two different digital representations. When a collinearity of edges is detected, a new set of edges is added to the original one. The collinearity can be detected as the logical product of concatenation (two edges sharing the same vertex) and parallelism.

#### D. 3D wireframe reconstruction

Once a database of edges and vertices for each projection view is built, it is possible to reconstruct a pseudo vertex skeleton represented by a matrix  $\Lambda$ :

$$\Lambda = \begin{bmatrix} w_1 & x_1 & y_1 & z_1 & v_1^1 & v_1^2 & v_1^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ w_q & x_q & y_q & z_q & v_q^1 & v_q^2 & v_q^3 \end{bmatrix} \quad (1)$$

Each row vector  $\lambda_i$  of  $\Lambda$  describes a 3D vertex where  $q$  is the total number of 3D vertexes. The first element ( $\lambda$ ) of each row represents the label of the 3D vertex; the following three elements ( $x, y, z$ ) are the coordinates of the 3D vertexes while the remaining elements ( $v^1, v^2, v^3$ ) are the labels of the 3D vertexes projection on, respectively, the three coordinate planes TV, FV and SV<sup>1</sup>. The definition of the pseudo-skeleton matrix allows to build the pseudo-wireframe. In other words, once 3D vertexes have been identified it is possible to reconstruct the set of 3D edges coherent with the starting orthogonal views.

First the set of 3D edges that are not orthogonal to any projection is assessed; then a similar procedure is carried out for the 3D edges that are orthogonal respectively to TV, FV

and SV.

The result is a matrix  $E$  containing the entire set of 3D edges:

$$E = \begin{bmatrix} r_1 & \lambda_1^a & \lambda_1^b \\ \vdots & \vdots & \vdots \\ r_s & \lambda_s^a & \lambda_s^b \end{bmatrix} \quad (2)$$

Each row vector  $e_i$  identifies a 3D edge where  $s$  is the total number of 3D edges. First element of each row is the label of the 3D edge; the other two elements represents the labels of the two 3D edge endpoints.

Matrixes  $\Lambda$  and  $E$  define the pseudo-wireframe model.

In fig. 2 a graphic representation of the above defined pseudo-wireframe model referred to its set of orthographic projections is depicted.

### III. 3D MODEL RECONSTRUCTION

A number of methods for 3D reconstruction are in literature; unfortunately, to the best of authors' knowledge, these are mainly oriented towards a theoretical approach and a comprehensive, orderly, unambiguous and automatic procedure is still required.

The main contribute of the present work is to provide a practical approach for 3D model retrieval based on a straightforward mathematical description.

In detail 3D reconstruction is carried out (starting from the mathematical description of the pseudo-wireframe provided in

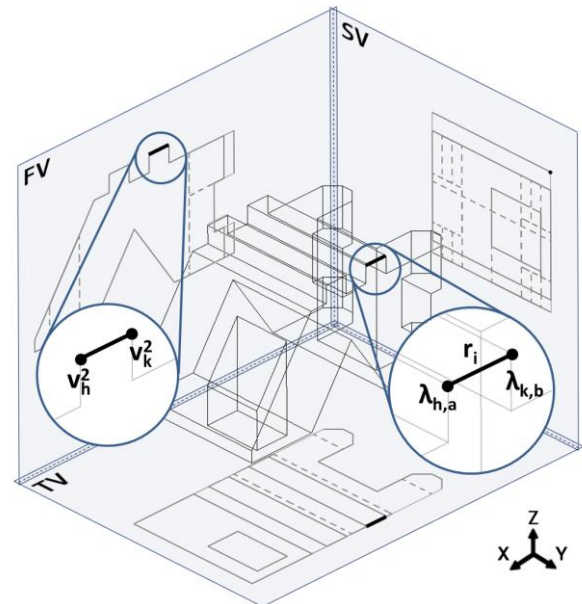


Fig. 2 – pseudo-wireframe model obtained from a set of orthographic projections. Labels for 3D vertexes and edges and label for 2D vertexes (referred to FV) are depicted.

section 2) according to the following tasks:

<sup>1</sup> Top View, Front View and Side View.

- Detection of planar edge cycles.
- Detection of face cycles.
- Creation of virtual block cycles.
- Solution(s) validation.

#### A. Detection of planar edge cycles

The first step for reconstruction consists of the identification of all the possible 3D faces defining the boundary of the final solution. In order to achieve such a goal, the following three phases have been devised:

- Detection of the planes defined by all the couples of pseudo-wireframe edges sharing an endpoint.
- Identification of all the edge cycles lying on each plane.
- Definition of the faces by means of the analysis of each edge cycle.

#### Planes detection

Each 3D edge must lie on at least two planes and each plane must include at least three edges. So, in order to detect planes, these two conditions have to be respected. The first step is to list all the possible planes obtainable by means of normalized cross products between couples of edges sharing an endpoint (which results in the vector normal to the plane defined by the two edges).

Let  $t_u$  and  $t_v$  be two generic 3D edges:

$$t_u = \begin{bmatrix} \Lambda(E(u,2),2) & \Lambda(E(u,2),3) & \Lambda(E(u,2),4) \\ \Lambda(E(u,3),2) & \Lambda(E(u,3),3) & \Lambda(E(u,3),4) \end{bmatrix} \quad (3)$$

$$t_v = \begin{bmatrix} \Lambda(E(v,2),2) & \Lambda(E(v,2),3) & \Lambda(E(v,2),4) \\ \Lambda(E(v,3),2) & \Lambda(E(v,3),3) & \Lambda(E(v,3),4) \end{bmatrix}$$

The normalized cross product between  $t_u$  and  $t_v$  defines a series of generic plane versor  $\omega$  and a series of plane labels  $n$ , under the following conditions:

$$\begin{aligned} \forall \{u, v\}: (t_u \in E) \wedge (t_v \in E) \wedge (u \neq v) \wedge \dots \\ \dots (\lambda_u^a \equiv \lambda_v^a \vee \lambda_u^a \equiv \lambda_v^b \vee \lambda_u^b \equiv \lambda_v^b \vee \lambda_u^b \equiv \lambda_v^b); \\ \omega = \begin{bmatrix} i & j & k \end{bmatrix} = \frac{t_u \times t_v}{|t_u \cdot t_v|} \end{aligned} \quad (4)$$

$$\begin{aligned} \text{if } \lambda_u^a \equiv \lambda_v^a \vee \lambda_u^a \equiv \lambda_v^b \rightarrow n = \lambda_u^a; \\ \text{if } \lambda_u^b \equiv \lambda_v^a \vee \lambda_u^b \equiv \lambda_v^b \rightarrow n = \lambda_u^b; \end{aligned}$$

$$p = [\delta, i, j, k, n]$$

Where vector  $p$ , contains a progressive label  $\delta$  identifying the plane, the cross product result (in the form of its components  $i, j, k$ ) and the label  $n$  of the generic 3D vertex

shared by  $t_u$  and  $t_v$  represents the generic plane (see Fig. 3).

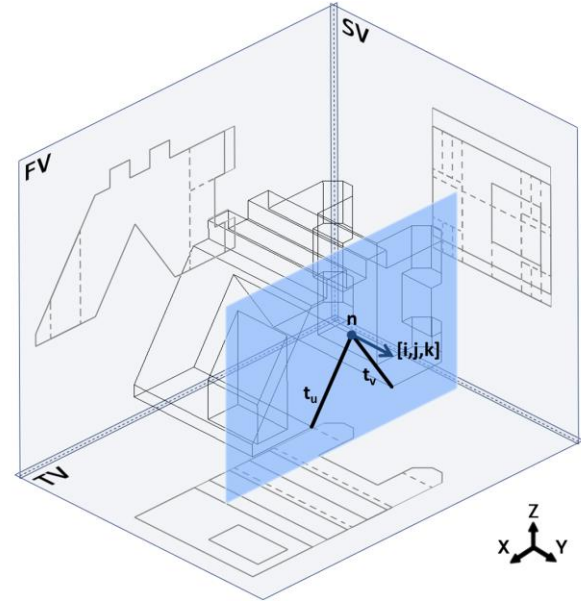


Fig. 3 – a generic plane  $p$  defined by the cross product between 3D edges  $t_u$  and  $t_v$

Once all the generic planes are evaluated it is straightforward to define a matrix  $P$  (called “plane matrix”):

$$P = \begin{bmatrix} \delta_1 & i_1 & j_1 & k_1 & n_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \delta_m & i_m & j_m & k_m & n_m \end{bmatrix} \quad (5)$$

Where  $m$  is the number of detected planes.

#### Identification of edge cycles

Once the “matrix of planes” is compiled, it is possible to compute the edge cycles by inspecting each identified plane. This is performed by means of the following tasks<sup>2</sup>.

In order to make as clear as possible the next tasks, the following definitions will be used hereinafter:

- $\lambda_i = i^{\text{TH}}$  row of  $\Lambda$  (i.e. the  $i^{\text{TH}}$  3D vertex);
- $\lambda_{ij} = j^{\text{TH}}$  element of  $\lambda_i$ ;
- $e_i = i^{\text{TH}}$  row of  $E$  (i.e. the  $i^{\text{TH}}$  3D edge);
- $e_{ij} = j^{\text{TH}}$  element of  $e_i$ ;
- $p_i = i^{\text{TH}}$  row of  $P$  (i.e. the  $i^{\text{TH}}$  3D plane);
- $p_{ij} = j^{\text{TH}}$  element of  $p_i$ .

At the end of the operations (see pseudo-code below), the result is represented by a set of edge cycles  $C$  (see Fig. 4), each one linked to its belonging plane  $p$ :

$$C = [\Theta^1, \dots, \Theta^s]^T \quad (6)$$

<sup>2</sup> The tasks are described in the form of pseudo-codes to make as clear as possible the followed approach.



**PSEUDO – CODE FOR PLANAR CYCLE RETRIEVAL**

**FOR each plane**  $p_i$

**STEP 1** Define a subset  $SE$  of  $E$  composed by the  $c$  vectors  $e_i$  such as:

$$[p_{i2}, p_{i3}, p_{i4}] \cdot e_i = 0 \wedge [p_{i2}, p_{i3}, p_{i4}] \cdot \hat{\omega} = 0, \text{ where:}$$

$$\hat{\omega} = \begin{bmatrix} \lambda_{i,2} & \lambda_{i,3} & \lambda_{i,4} \\ \lambda_{n,2} & \lambda_{n,3} & \lambda_{n,4} \end{bmatrix}, i = e_{i2}$$

The subset contains all the edges lying on the plane  $p_i$ .

**STEP 2** Define  $se_i$  as the  $i^{\text{TH}}$  row of  $SE$

**STEP 3** Define a subset  $\Lambda$  of  $\Lambda$  composed by the vectors  $\lambda_i$  such as:  $\lambda_i \in se_{i2} \vee \lambda_i \in se_{i3}, i = 1, \dots, c$

**STEP 4** Define  $s\lambda_i$  as the  $i^{\text{TH}}$  row of  $\Lambda$

**STEP 5** **FOR each**  $s\lambda_i$

**STEP 5a** Define a subset  $SSE^{s\lambda_i}$  of  $SE$  composed by the  $d$  vectors  $se_i$  such as:

$$se_{i2} = s\lambda_i \vee se_{i3} = s\lambda_i$$

**STEP 5b** Define  $ssse_i^{s\lambda_i}$  as the  $i^{\text{TH}}$  row of  $SSE^{s\lambda_i}$

**STEP 5c** Sort  $SSE$  rows such as 3D edges  $ssse_i^{s\lambda_i}$  are counter clockwise (CCW) ordered with respect to  $s\lambda_i$

**END**

**STEP 6** Redefine  $SE$  by horizontally concatenate it with a zero vector:  $SE = [SE | 0]$

**STEP 7** Define a subset  $\overline{SE}$  of  $E$  obtained by reversing second and third columns of  $SE$

**STEP 8** Define  $\overline{se_i}$  as the  $i^{\text{TH}}$  row of  $\overline{SE}$

Now the plane  $p_i$  is ready to be inspected

**STEP 9** **WHILE**  $\prod se_{i4} = 0 \vee \prod \overline{se}_{i4} = 0$

**STEP 9a** Set  $SE_{old} = SE$  and  $\overline{SE}_{old} = \overline{SE}$

**STEP 9b** Find the  $i^{\text{TH}}$  row in  $SE$  (or  $\overline{SE}$ ) such as  $se_{i4} = 0$  or  $\overline{se}_{i4} = 0$

**STEP 9c** Set:  $start = s\lambda_i$  (starting vertex)

**STEP 9d** Set:  $se_{i4} = 0.5$  (starting edge)

**STEP 9e** Define  $tmp = 0$

**WHILE**  $tmp = 0$

**STEP 9e.1** Find in  $SSE^{start}$  the first non “walked” edge  $se_{f_{mw}}$ , i.e. the edge  $se_i$  such as  $se_{i4} = 0$  or the edge  $\overline{se_i}$  such as  $\overline{se}_{i4} = 0$  (depending on the “walking” direction)  
Note that  $se_{f_{mw}} = [r_{f_{mw}}, \lambda_{f_{mw}}^a, \lambda_{f_{mw}}^b, 0]$

**STEP 9e.2** **IF**  $se_{f_{mw}4} = 0$   
**THEN**  $se_{f_{mw}4} = 1$

**STEP 9e.3** **IF**  $se_{f_{mw}4} = 0.5$   
**THEN**  $tmp = 1, se_{f_{mw}4} = 1$

**STEP 9f** **STEP** Reset  $start = se_{f_{mw}3}$  (starting vertex)  
**9e.3**

**STEP 9g** Find the elements of the fourth columns of  $SE$  and  $\overline{SE}$  that are different to the correspondent ones in  $SE_{old} = SE$  and  $\overline{SE}_{old} = \overline{SE}$  and store their row into a matrix  $C$

**END**

**END**

If, for instance, the cycle  $\Theta^1$  belongs to plane  $p_7$  and is bounded by edges  $e_{15}, e_{23}, e_5$ :

$$\Theta^1 = \begin{bmatrix} 0 & p_7 & e_{15,3} & e_{23,3} & e_{5,3} \\ 0 & 0 & e_{15,2} & e_{23,2} & e_{5,2} \end{bmatrix}^T \quad (7)$$

This result has to be further refined by assigning to each cycle the right path direction: clockwise (CW) or counter clockwise (CCW).

In order to achieve this goal, the sum value ( $S$ ) of all the inner angles of each cycle is computed so that:

- if  $S = -360^\circ$  the cycle is CW directed (Fig.5a);
- if  $S = 360^\circ$  the cycle is CCW directed (Fig.5b).

Assuming that, by ideally walking on the cycle boundary, the faces always lie on the left side, it is clear that only the CCW cycles can originate faces, since CW one do not delimit a finite area region. According to this last step all the  $\Theta$  matrixes can be updated by adding the right path direction; for instance, supposing that  $\Theta^1$  is CW cycle, the updated matrix is:

$$\Theta^1 = \begin{bmatrix} 0 & p_7 & e_{15,3} & e_{23,3} & e_{5,3} \\ 0 & 1 & e_{15,2} & e_{23,2} & e_{5,2} \end{bmatrix}^T \quad (8)$$

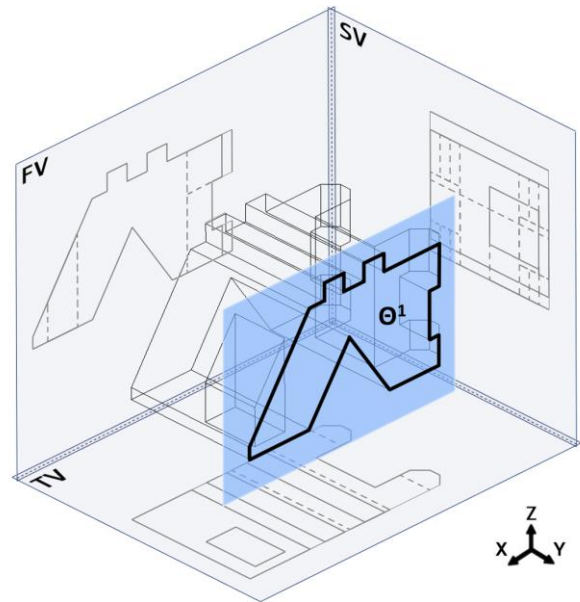


Fig. 4 – a generic cycle  $\Theta$ .

In practice, a conventional value equal to 1 (-1) is assigned for CW cycle (CCW cycle). It has to be noted that first row of each cycle  $\Theta$ , is a zero vector (size 1x2) used as a delimiter between two sequential cycles.

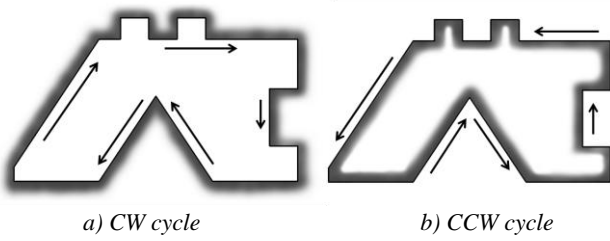


Fig. 5 – CW and CCW cycles.

Once cycles are defined, each face  $f$ , commonly known as “virtual face”, is described by a row vector storing the label of the linked plane and the list of all the edges representing the face boundary. Continuing the example described above and supposing that  $\Theta^1$  is CCW:

$$f = [p_7, e_{15,1}, e_{23,1}, e_{5,1}] \quad (9)$$

All the detected faces defines a matrix  $F$  of size  $n_f \times (n_{\max} + 1)$  where:  $n_f$  is the number of faces, while  $n_{\max}$  is the number of edges surrounding the face delimited by the maximum number of edges<sup>3</sup>. Accordingly, the matrix  $F$  is obtained by appending all the vectors  $f$ :

$$F = \begin{bmatrix} f_1 \\ \vdots \\ f_{n_f} \end{bmatrix}_{n_f \times n_{\max} + 1} \quad (10)$$

### B. Detection of faces cycles

It is known that the set of faces resulting from the previous task generally represents a superset of the face set describing the actual 3D solid [16, 17]. In order to complete the reconstruction process, it is, thereby, necessary to identify the set of elementary 3D blocks (commonly known as “virtual blocks”) whose combinations provide the final 3D geometries.

The block detection task is performed by detecting faces cycles. This is carried out analogously to the face detection task previously described; the main difference between the two is represented by the space dimensionality: blocks are managed in 3D space while faces in 2D one. For completeness, a concise pseudo-code for faces cycles detection is described below.

From the algorithm point of view, this task requires a “warm up” phase for the compilation of two auxiliary matrixes:

$$- F_1 \text{ (size } s \times \max\{\# fe_i\})$$

<sup>3</sup> The number of elements in a generic  $f$  depends on the number of surrounding edges and this last is not fixed *a priori*. As a consequence, in order to append all the face vectors into the faces matrix  $F$  all the vectors  $f$  are required to have the same length. This is obtained by adding a series of 0 elements until the proper length is reached.

$$- F_2 \text{ (size } n_f \times \max\{\# ef_i\})$$

Each row of matrix  $F_1$  is devoted to an edge  $e$  and contains the labels of all the faces  $fe_i$  having  $e$  among their boundary edges.

The second matrix allows a fast detection of the edges  $ef_i$  composing the boundary of each face; each row is devoted to a face  $f$  and contains the labels of all the edges  $ef_i$  bounding it. Once these two auxiliary matrixes have been compiled, the procedure carries on detecting the loops of faces (blocks). The face cycle identification is a close analog of the one devised for the planar edge cycles, described in the previous section. The procedure stops when the two sides of each face has been used in cycle detection process. The implemented procedure related to the cycles of faces is inspired by a work of Yan, Chen and Thang [19]. Referring to Fig. 6, given two generic faces,  $f_1$  and  $f_2$ , so that they share an edge  $e$  and assuming as positive the normal vector pointing out from the sheet four cases are possible:

- in cases A and D the vectors normal to both provenience and destination faces have the same sign; positive in case A and negative in case D.
- in cases B and C the vectors normal to provenience and destination faces have opposite sign; in B the positive normal vector belongs to the destination face while, in C to the provenience one.

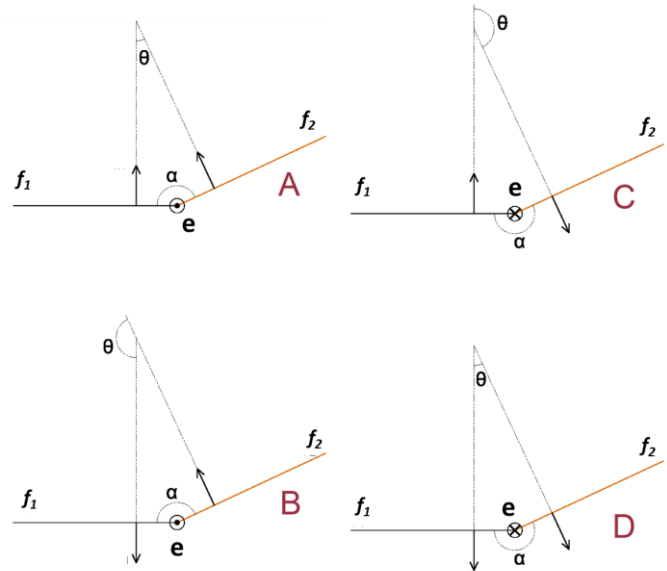


Fig. 6 – possible cases in face cycles detection.

By examining Fig. 6 no particular difficulty seems to arise in order to perform the face cycles retrieval; actually, the correct identification of the normal vectors referring to the edge  $e$  it is not trivial from a mathematical point of view.

Accordingly, the qualitative approach presented in [19] has been formalized as follows, in order to overcome possible ambiguities and to obtain a strict, unequivocal management of

the loop detection. By means of a series of cross product comparisons involving the two face normals and the edge  $e$ , the authors provide a method that allows to correctly identify the right configuration among the four ones depicted in Fig. 6.

The final result of this task is a matrix  $B$  of blocks  $b$  defined as follows:

$$B = \begin{bmatrix} b_1 \\ \vdots \\ b_{n_b} \end{bmatrix} \quad (11)$$

Where each block is represented by a vector  $b$  given by the label of the block ( $\xi$ ) and the label list of the faces composing the block itself ( $f_{\xi i}$ ):

$$b = [\xi \quad f_{\xi 1} \quad \dots \quad f_{\xi \text{end}}] \quad (12)$$

### C. Creation of virtual block cycles

The last task of the reconstruction procedure deals with block management in order to provide complete set of block combinations leading to 3D objects whose projections are coherent with the starting ones.

This task starts by computing the volume  $V$  of each single block so that the one with the maximum volume is discarded. In fact, it is clear that the biggest block is the external shell (see for instance Fig.7 referred to the example provided in Fig. 1). The other  $n_b$  blocks are combined in order to obtain all the possible  $c_b$  block combinations regardless to their order:

$$c_b = \sum_{k=1}^n C_{n,k} \quad (13)$$

where  $C_{n,k}$  is the binomial coefficient.

Discussing Fig. 8 (which is also referred to the example of Fig. 1), the possible block configurations are: block A, block B, blocks A and B.

### D. Solution(s) validation

In order to evaluate the solution correctness all the  $c_b$  obtained configurations are verified according to a two-steps procedure.

#### First Validation

First, each configuration is compared with the original set of 2D views. All the 3D geometric entities (composing the combination) are re-projected onto the coordinate planes; the resulting 2D projections are then compared with the original ones. If the block configuration originates a set of projections matching the original ones, it is candidate to be a correct solution. This first validation, however, is not sufficient to establish the solution correctness since the 3D blocks are still

disjoint; in fact, geometric entities belonging to the shared faces between two adjacent blocks have to be discarded for constructing the 3D model. These, are internal to the 3D model boundary and so they cannot belong to the 3D final model i.e. they are “false” entities). If this operation is not carried out, such false entities, generate edges in 2D projection that may, in some cases, overlap real ones. As a consequence first validation allows, on one hand, to find a subset of block configurations that is coherent with the original projections. On the other hand it needs a further validation process.

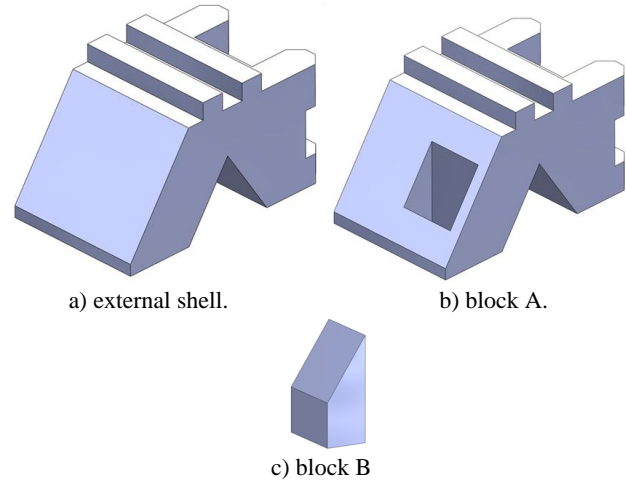


Fig. 8 – virtual block configurations.

### Second Validation

Only the block configurations that verify the first validation procedure are merged by means of a Boolean union operation with the aim of obtaining a single 3D block. Let  $\psi$  be a generic block configuration among the  $\Psi$  block configurations satisfying the first validation and  $n_\psi$  be the number of blocks composing the  $\psi^{\text{th}}$  block configuration. The Boolean union operation allows the definition of a new set of  $BU_\psi$  blocks:

$$BU_\psi = \bigcup_{i=1}^{n_\psi} b_i \quad (14)$$

By re-projecting these  $BU_\psi$  blocks onto the three coordinate planes a further comparison with the original views is thereby performed.

The accomplishment of this merging phase only on the candidate configurations detected by using the first validation, leads to a considerable reduction in computation time since the merging of a large number of block configurations is avoided. The result of this second validation is the set of the correct 3D solutions (Fig.8).

## IV. SOFTWARE IMPLEMENTATION

The entire set of algorithms, performing all the procedures



described in section III, is implemented by means of two software packages, Matlab® and Rhinoceros®. The first one, which is widely spread in the scientific community, is mainly used as a mathematical kernel, while Rhinoceros® (by means of custom scripts) allows to easily translate Matlab® output results into 3D models and operate them according to the described procedure.

### CONCLUSIONS

In this work an orderly, unambiguous and automatic procedure covering the necessary steps for retrieving 3D models from mechanical drawings is provided. Since the intent was to afford an as clear as possible, guided, procedure for 3D reconstruction, mathematical description has been fully developed referring to the simplest case of polyhedral objects.

The proposed procedures, inspired by state of the art works, can be effectively contribute to speed-up the possible implementation of methodologies confronting the 3D reconstruction problem. In fact, the procedure has been designed like an open source tool for researchers who want to deal with the “reconstruction problem”; in other word strictly following the proposed procedure steps, researchers will be able to quickly introduce themselves in the reconstruction problem field.

In order to assess the effectiveness of the devised procedures, these have been implemented and tested on a number of case studies.

If, for instance, the example provided by Figs. 9 and 10 is examined, starting from the orthographic projections of Fig. 9, three blocks can be obtained. It is straightforward that both the upper blocks in Fig. 10 successfully pass the first validation while only the upper right is able to satisfy the second validation. For this reason it is the only solution that is coherent with the original set of projections.

Though the present work is focused on mechanical drawings, future work will be addressed to develop a more general methodology dealing with the reconstruction of 3D CAD models starting from 2D views of objects defined by edges with arbitrary geometry. This objective is aimed by the authors’ desire of dealing with free-form sketches typically used in the Artistic or Fashion Design field.

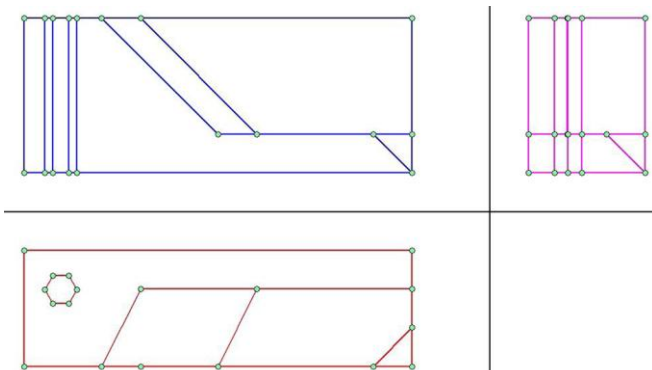


Fig. 9 – orthographic projections.

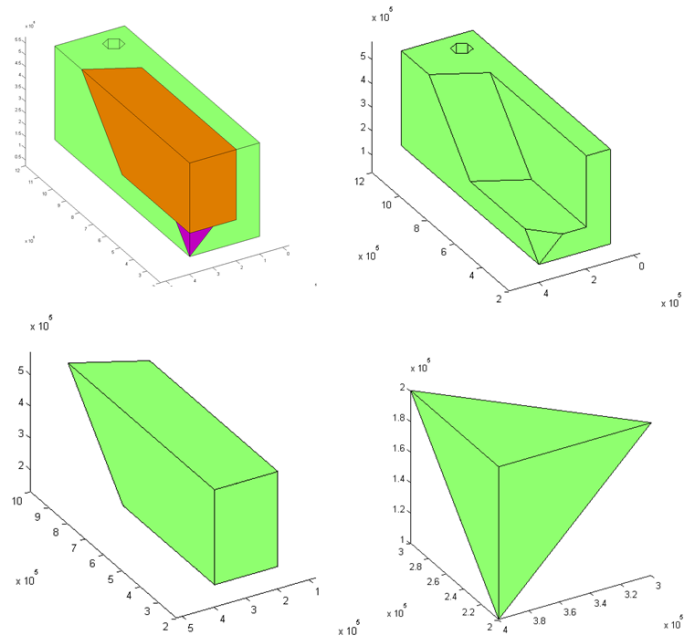


Fig. 10 – virtual blocks obtained by processing projections in Fig. 9; only the upper right configuration satisfies the original projections.

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