On the morphodynamic instability of water-ice interface in a brackish lake

L. Solari[1] and G. Parker[2]

[1] Department of Civil & Environmental Engineering, University of Florence, via S. Marta 3, 50139 Firenze, Italy
[2] Department of Civil & Environmental Engineering and Department of Geology, Hydrosystems Laboratory, University of Illinois, 301 N. Mathews Ave., Urbana, IL 61801, U.S.A.

1. Introduction

Periodic morphologies, such as ripples, can be found under ice cover in rivers and lakes, and in glacial meltwater streams (e.g. Ashton and Kennedy, 1972; Parker, 1975; Camporeale and Ridolfi, 2012). Stability analysis suggests that a plane ice surface in contact with water at a temperature slightly above freezing can become unstable, with the appearance of ripples (e.g. Thorsness and Hanratty, 1979).

Investigations of freshwater lakes (e.g. Stefan et al., 1998) suggest that in lakes that have an ice cover in the winter, temperature, and therefore density is stably stratified. In this condition, without any further driving mechanism, convective mixing due to unfavourable density stratification and, also, wind forcing are absent.

In the case of brackish lakes, however, gradual freezing at the interface causes the formation of black ice, from which salt is excluded. As the ice cover thickens, a net downward flux of dissolved salt is generated in the below water at the interface. Both salt exclusion from the ice, as the ice cover thickens, and subsequent freshwater ice melt can modify the seasonal structure of a brackish lake. Pieters and Lawrence (2009) documented, through temperature and conductivity measurements along the water column, the effect of salt exclusion on seasonal circulation in Tailings Lake, a brackish lake located in the Northwest Territories, Canada. In particular, they showed that for Tailings Lake, during winter, the formation of significant black ice, and the resulting flux of salt beneath the ice, was sufficient to break down the stable temperature density stratification to the degree necessary to fully redistribute this salt throughout the water column.

In the present work, we study the morphodynamic instability of the water-ice interface. Results provide a mechanistic explanation for the field observations collected by Pieters and Lawrence (2009).

2. Formulation of the problem

We consider the case of an ice-water interface that moves slowly downward due to freezing in a brackish lake. The freezing excludes salt from the ice, thus forcing a downward flux of dissolved salt in the water at the interface. The flux of salt from the top can reverse the stable stratification due to the temperature until the fluid is rendered top-heavy, and thus unstably stratified.

We tackle this problem by considering the stability of a laminar flow subject to stratification effects due to both temperature and salinity. We consider small amplitude waveforms of the ice-water interface. The governing equations of the system for the hydrodynamics, the convection and diffusion of heat and salinity, and the evolution of the ice-water interface due to the exchange of heat are solved using a linear perturbation approach.

The base state is characterized by a water-ice interface that moves downward at a constant speed, salt is introduced in the lake at a constant rate, temperature profile exhibits a stable stratification and water beneath the ice is in a static condition. The stability of this base state is studied for various combinations of the relevant dimensionless parameter here called, in analogy with Rayleigh-Benard problem, Rayleigh numbers Ra_1 and Ra_2. Ra_1 scales the ratio of the tendency of the buoyancy force due to temperature difference to drive convection, to diffusive effects that damp this convection; Ra_2 scales the corresponding ratio associated with the buoyancy force due to salinity difference.

3. Results and conclusions

Results allow estimating the threshold of the relevant dimensionless parameters, above which the system becomes unstable. In these conditions, the system exhibits the growth of two cells of circulation rotating in opposite directions: a stronger cell confined to a region close to the ice cover, and a much weaker cell distributed over the remaining lower portion of the water column. These cells are expected to drive mixing of salinity and temperature in the lake. These convective cells are inherently linked to the deformations in the ice-water interface.

The under-ice undulations here predicted remain to be observed. The common existence of undulating bedforms at water-ice interfaces are, however, observed in other contexts (e.g. Asthon and Kennedy, 1972; Camporeale and Ridolfi, 2012), suggesting their likely presence in the case considered herein.

References

Ashton, G.D., and Kennedy, J.F. (1972). Ripples on underside of river ice covers. *J. of Hydraulic Division* HY9, 1603-1624.

Camporeale, C., and Ridoldi, L. (2012). Ice ripple formation at large Reynolds numbers. *J. Fluid Mech.*, 694, 225-251

Parker, G. (1975). Meandering of supraglacial melt streams. *Water Resources Research*, 11(4), 551 552.

Pieters, R., and Lawrence, G.A. (2009). Effect of salt exclusion from lake ice on seasonal circulation. *Limnology and Oceanography*, 54(2), 401–412

Stefan, H.G., Fang, X., and Hondzo, M. (1998). Simulated climate change effects on year-roundwater temperatures in temperate zone lakes. *Climatic Change* 40, 547–576.

Thorsness, C, B. & Hanratty, T. 1979. Stability of dissolving or depositing surfaces. AIChE Journal, 25 :697-701.