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Collusion and the Political Differentiation of Newspapers

Marco Antonielli and Lapo Filistrucchi

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Collusion and the political differentiation of newspapers[°]

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Abstract

We analyse a newspaper market where two editors first choose the political position of their newspaper, then set cover prices and advertising tariffs. We build on the work of Gabszewicz, Laussel and Sonnac (2001, 2002), whose model of competition among newspaper publishers we take as the stage game of an infinitely repeated game, and investigate the incentives to collude and the properties of the collusive agreements in terms of welfare and pluralism. We analyse and compare two forms of collusion: in the first, publishers cooperatively select both prices and political position; in the second, publishers cooperatively select prices only. We show that collusion on prices reinforces the tendency towards a *Pensée Unique* discussed in Gabszewicz, Laussel and Sonnac (2001), while collusion on both prices and the political line would tend to mitigate it. Our findings question the rationale for Joint Operating Agreements among US newspapers, which allow publishers to cooperate in setting cover prices and advertising tariffs but not the editorial line. We also show that, whatever the form of collusion, incentives to collude first increase, then decrease as advertising revenues per reader increase.

JEL classification: L41, L82, D43, K21

Keywords: collusion, newspapers, two-sided markets, indirect network effects, pluralism, spatial competition

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"[T]he newspaper industry [...] serves one of the most vital of all general interests: the dissemination of news from as many different sources, and with as many different facets and colors as is possible. That interest [...] presupposes that right conclusions are more likely to be gathered out of a multitude of tongues, than through any kind of authoritative selection."

Judge Learned Hand, US vs. Associated Press, 1945[1]

1. Introduction

Media industries are well known examples of two-sided markets.[2] Newspaper publishers sell their products to two different categories of buyers, namely readers and advertisers. Readers are interested in news while advertisers aim at reaching potential consumers by buying advertising space in the newspaper. Newspaper publishers know that the more readers their newspaper has the higher the willingness to pay of advertisers for a slot in the newspaper. Vice versa readers may be affected by advertising in the newspaper.[3] Publishers therefore choose cover prices and advertising tariffs taking into account this link between the demands on the two sides of the market. Yet, differently from the case of complement products, this interdependency among the two demands and the resulting link between prices is not recognised by advertisers or readers as they buy only one of the two products sold by the publisher.

This particular characteristic of media markets has always been known to those working in the field and has thus been recognised in the economic literature ever since the first studies of these industries[4], while the literature on two-sided markets itself has developed only in the last ten years, as economists became aware of the fact that other, apparently very different, markets share this basic features with media markets.[5]

The issue of concentration in media markets has been debated both in academic circles as well as among policy makers since at least the 1960's. On the one hand the debate centred on the reasons why a high concentration has often been observed in the industry[6], on the other

1 Learned Hand, J., in United States v. Associated Press, 52 F. Supp. 362, 372 (D.C. S. D. N. Y. 1943), aff'd, 326 U.S. 1 (1945).
2 See Anderson and Gabszewicz (2006) on media markets as two-sided markets.
3 Whether and to what extent readers/viewers/listeners are instead negatively or positively affected by the amount (or concentration) of advertising in a given media is a debated issue. Some theoretical models assume that consumers are advertising-averse, e.g. Gabszewicz, Laussel and Sonnac (2004), Kind, Nillssen and Sorgard (2007) and Peitz and Valletti (2008) for the TV industry. Others specify a (variable) proportion between ad-lovers and ad-haters, e.g. Gabszewicz, Laussel and Sonnac (2005) for the press industry. Yet other models assume that consumers are advertising indifferent, e.g. Gabszewicz, Laussel and Sonnac (2001, 2002) for newspapers, or advertising-lovers, e.g. Gabszewicz, Garella and Sonnac (2007) again for newspapers.
4 See already Corden (1953) and Reddaway (1963). More recently, but still before the theory of two-sided markets was developed, Blair and Romano (1993) and Chaundri (1998).
5 The seminal papers in the field are those by Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006), Evans (2003), Parker and Van Alstyne (2005), and Armstrong (2006). More recently, Weyl (2010) proposed a new way to model firms' pricing in two-sided markets.
6 See Reddaway (1963) and Rosse (1967), Rosse, Owen and Dertouzos (1975), Rosse (1977), Rosse (1978), Rosse (1980) and Bucklin, Caves and Lo (1989) for studies which highlight the importance of economies of scale. See instead Gabszewicz, Garella and Sonnac (2007) and Häckner and Nyberg (2008) for the role played by the indirect network effects.

hand it focused on whether high concentration in the market is detrimental to pluralism, i.e. on whether high concentration leads to duplication of content or, on the contrary, higher product differentiation.[7] With regard to the latter question, results are somewhat ambiguous but surprisingly it has been shown that competition can lead to duplication of content when media outlets are mainly financed through advertising.[8]

Despite the fears of a possible lack of pluralism in the media due to high concentration in the market, not the same attention has been devoted to the possible effects of collusion. Yet in principle joint profit maximization by colluding publishers is able to reproduce the same market outcome as a merger among those same publishers.

On the contrary, the US Newspaper Preservation Act[9], passed in 1970, allows so-called joint operating agreements, which permitted newspapers within the same market area to jointly set cover prices and advertising rates. Hence, under a JOA newspapers are able to collude on both sides of the market.[10] The idea behind the introduction of this block-exemption was to help newspapers to survive, given the trend of market exit initiated by the appearance of radio and continued with the introduction of TV.[11] Interestingly the reason why it was judged important to keep different newspapers alive was to guarantee different editorial lines. Hence, JOAs were supposed to reduce operating costs by combining business aspects of newspapers but maintain editorial independence.

Possibly due also to a similar attitude from antitrust authorities, there have been almost no cases concerning collusion in the newspaper market.

Also in the academia relatively little attention has been paid to the topic of collusion in media markets. More generally, despite the rapid growth of the literature on two-sided markets,

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[7] See Polo (2007) for a discussion.

[8] See Steiner (1952) for the radio industry and both Beebe (1977) and Spence and Owen (1977) for the TV industry. More recently, see Gabszewicz, Laussel and Sonnac (2001, 2002) for the newspaper industry and Gabszewicz, Laussel and Sonnac (2004) for the TV industry. See Anderson and Coate (2005) for why the relationship between competition and the amount of different programming is ambiguous.

[9] See http://law.justia.com/cfr/title28/28-2.0.1.1.6.html for the text of the Newspaper Preservation Act, which in particular states "The term joint newspaper operating arrangement means any contract, agreement, joint venture (whether or not incorporated), or other arrangement entered into between two or more newspaper owners for the publication of two or more newspaper publications, pursuant to which joint or common production facilities are established or operated and joint or unified action is taken or agreed to be taken with respect to any of the following: Printing; time, method, and field of publication; allocation of production facilities; distribution; advertising solicitation; circulation solicitation; business department; *establishment of advertising rates; establishment of circulation rates and revenue distribution*: Provided, that there is no merger, combination, or amalgamation of editorial or reportorial staffs, and that editorial policies be independently determined." Interestingly, the act contains a prohibition to charge a very low price if advertisers bought space in both newspapers since that would be predation.

[10] More precisely, according to the newspaper Preservation Act, "The term joint newspaper operating arrangement means any contract, agreement, joint venture (whether or not incorporated), or other arrangement entered into between two or more newspaper owners for the publication of two or more newspaper publications, pursuant to which joint or common production facilities are established or operated and joint or unified action is taken or agreed to be taken with respect to any of the following: Printing; time, method, and field of publication; allocation of production facilities; distribution; advertising solicitation; circulation solicitation; business department; *establishment of advertising rates; establishment of circulation rates and revenue distribution*: Provided, that there is no merger, combination, or amalgamation of editorial or reportorial staffs, and that editorial policies be independently determined." Interestingly, the act contains a prohibition to charge a very low price if advertisers bought space in both newspapers since that would be predation.

[11] Importantly, the act states that new agreements need to be approved and one of the conditions for approval is that one of the newspapers would not survive without the JOA.

only very recently the issue of collusion has been investigated in empirical as well as in theoretical works and the literature on the topic is still scarce.

Ruhmer (2011) shows how in two-sided markets the presence of indirect network externalities affects the incentives to collude and the welfare implications of collusion. She uses the single-homing model in Armstrong (2006) as a stage game of an infinitely repeated game to model a two-sided market where firms are differentiated on both sides and simultaneously choose both prices. Assuming firms adopt grim trigger strategies she finds that higher network externalities have two opposite effects: on the one hand they tend to raise incentives to collude as they increase the gain from collusion (collusive profits increase and competitive profits decline); whilst on the other hand they tend to lower incentives to collude as they increase the gain from deviation. In her model the latter effect is always found to dominate. As a result, collusion becomes harder to sustain as indirect network effects between the two sides of the market increase. Furthermore, she finds that a higher asymmetry in the indirect network effects reduces the incentives to collude.

Dewenter, Haucap and Wenzel (2011) analyse instead the welfare consequence of collusion on the advertising tariffs only, in a duopoly newspaper market where firms first choose the advertising quantity and then the cover prices, while readers like advertising. Under these assumptions and the additional assumption of a linear demand for differentiated products, they find that collusion on the advertising tariffs may not only lead to an increase in readers' welfare (since it may reduce readers' prices more than it reduces the value of the newspaper to readers by decreasing the quantity of ads) but it can also lead to a higher advertisers' welfare (as it increases advertising tariffs less than it increases the newspaper's value to advertisers due to higher circulation).

Most recently, Boffa and Filistrucchi (2011) discuss an interesting particular case in which firms in a two-sided market raise prices above the monopoly price on one side of the market in order to be able to sustain collusion when perfect joint profit maximization is not sustainable.

In all these theoretical works however product differentiation is exogenous.

Among empirical works, Argentesi and Filistrucchi (2007) provide econometric evidence that daily newspapers in Italy have been colluding on the cover price but not on the advertising tariffs. Flath (2011) analyses how resale price maintenance is used to sustain collusion in the Japanese newspaper market and estimates the welfare loss due to the cartel agreement. None of the two addresses however the issue of whether collusion affected the political position of newspapers.

More recently, Fan (2011) proposes a structural econometric model which endogenise also the choice of newspaper quality. She then evaluates the effects of mergers among competing US newspapers, modelling newspapers engaged in JOAs as cooperatively setting cover prices and advertising tariffs but not newspaper quality. However, her model focuses on vertical rather than horizontal product differentiation. As such it does not endogenise the political position of the newspapers.

Whereas the issue of endogenous product positioning in two-sided markets is per se interesting, concerns about pluralism imply that product differentiation plays a much more crucial role in media markets than in standard markets, at least in as much as pluralism plays the role of a positive externality in the political process.[12]

Indeed, Gabszewicz, Laussel and Sonnac (2001, 2002) develop a model of duopolistic competition among publishers who choose first political position, then cover prices and advertising tariffs. Under the assumption that readers are indifferent to advertising, they show that advertising financing can lead to minimum product differentiation, i.e.to the emergence of the so-called *Pensée Unique.*

A similar result is obtained, under the assumptions that viewers dislike advertising and advertising is informative, by Gal-Or and Dukes (2003) in a model of duopolistic competition among broadcasters that bargain over advertising tariffs with two firms competing in a differentiated product market. Again Gabszewicz, Laussel and Sonnac (2004) obtain the same result in a model of duopolistic competition among broadcasters who choose first their product mix, then advertising tariffs, under the assumption that viewers dislike advertising..[13]

All in all, no paper so far has looked at the impact of collusion on product differentiation in two-sided markets, not even in media markets.

We fill the gap by building on a non-cooperative sequential game developed by Gabszewicz, Laussel and Sonnac (2001, 2002), which we take as the stage game of an infinitely repeated game, modelling a newspaper market where two publishers compete for advertising as well as for readership and decide whether and how to collude. Publishers first choose the political position of their newspaper, then set cover prices and advertising tariffs. Whereas

[12] While Gentzkow (2006) shows that the introduction of TV decreased voter turnout, Gentzkow, Shapiro and Sinkinson (2011) find that entry of newspapers has a robust positive effect on political participation, but newspaper competition is not a key driver of turnout as the effect is driven mainly by the first newspaper to enter the market, and the effect of a second or third paper is significantly smaller. Della Vigna and Kaplan (2007) show instead that the introduction of the Fox News in the US lead to a significant increase in votes for Republicans.

[13] There is also another strand of literature looking at the related issue of media bias rather than pluralism. Whereas the pluralism approach looks at whether media firms provide different opinions of the same story, the media bias approach generally assumes that there exists an objective state of the world and explores reasons why in equilibrium we may not observe media firms report truthfully on that state of the world. Reasons put forward in the literature for the existence of media bias range from (readers) demand side factors, as in Mullathainan and Shleifer (2005) and Gentzkow and Shapiro (2006), to supply side factors, like in Baron (2006). There are also models where media bias is due to the preferences of the advertising side, as in Ellman and Germano (2009). Gentzkow and Shapiro (2008) provides a nice review of the relationship between competition and media bias.

readers single-home, i.e. they buy only one copy of only one newspaper, advertisers may multi-home, i.e. they can buy ad spaces from one, both or none of the two newspapers.

We investigate the incentives to collude using grim trigger strategies and report the properties of the potential collusive agreements in terms of welfare and pluralism. More precisely, we analyse and compare two types of collusion: in the first, publishers cooperatively select both prices and political position; in the second, publishers cooperatively select prices only.

Whereas the first leads to intermediate product differentiation, the second leads, as in Gabszewicz, Laussel and Sonnac (2001, 2002), to minimal product differentiation. However, in the latter case, differently from Gabszewicz, Laussel and Sonnac (2001, 2002), equilibrium prices are positive. Whatever the type of collusion, despite the two-sided nature of the market, our findings confirm the traditional idea that the more competition there is in the market, the better off the consumers will be. In addition, as expected, collusion on both the cover price and the political line yields higher publishers surplus and lower readers' surplus than collusion on cover prices only, but, interestingly, also higher total welfare.

Our findings question the rationale for Joint Operating Agreements among US newspapers. The objective of allowing firms to cooperate in setting cover prices and advertising tariffs but not the editorial line was to keep different newspapers alive and thus guarantee different editorial lines. Our model however predicts that in such a situation editorial lines would tend to converge much more than if political positions were set cooperatively. This in line with the empirical finding of George (2007) and Sweeting (2010). Using data on the assignment of reporters to topical areas at 706 newspapers in the US, George (2007) shows that differentiation increase with ownership concentration. Consistently Sweeting (2010) shows that firms that buy competing stations tend to differentiate them more among themselves and also from their other stations. An additional result of his empirical analysis is that merging firms tend to reposition their stations closer to their competitors, a finding consistent with the competitive model we draw upon when advertising revenues are high enough.

Finally, we show that, whatever the form of collusion, a larger advertising market has a non-monotone effect on the incentives to collude: when the advertising market is small, an increase in its size favours collusion, but when the advertising market is sufficiently large, a further increase will make collusion less likely.

The paper proceeds as follows. The next section (section 2) presents the main features of the duopoly model developed by Gabszewicz, Laussel and Sonnac (2001, 2002). Section 3 proposes a model of collusion: first it introduces the two types of collusion (subsections 3.1 and 3.2), then analyses their welfare consequences (subsection 3.3) and the incentives to collude

(subsection 3.4) and, finally, discusses the role of relocation costs in determining the type of collusion (subsection 3.5). Section 4 concludes.

2. Competition in the newspaper market

We first introduce the model of competition in the newspaper market developed by Gabszewicz, Laussel, and Sonnac (2001, 2002), which we take as the stage game of an infinitely repeated game. We also make explicit the condition on the demand parameters which guarantees that the market is, as in their work, always covered in competition. Such a condition is necessary to compare the competitive outcome with the collusive ones.

2.1. The stage game

The model of Gabszewicz, Laussel and Sonnac (2002) consists of three steps:

- first, publishers choose the political orientation of their newspaper out of a unit interval representing the political spectrum from extreme left to extreme right;
- second, given the political position of their newspaper, publishers compete for readers; readers are assumed to buy one copy of one newspaper, i.e. they single-home;
- third, publishers compete in the advertising market[14]; advertisers are assumed to buy ad spaces from one, two or neither newspaper, i.e. they multi-home.

At any step, choices are made simultaneously and are common knowledge at every subsequent step.

Such a game corresponds to a Hotelling spatial duopoly with a further step for advertising competition.[15]

2.1.1. Readers demand

Readers have political opinions ranging from extreme left to extreme right; they are thus located uniformly on a unit interval [0,1] with every reader ideally corresponding to a point on this line. The market size is 1.

As readers care both about the price of the newspaper and about the political orientation, utility of reader r is defined as follows

$$U_r = \bar{u} - t x_{ir}^2 - p_i \qquad (1)$$

where x_{ir} is the distance between the political orientation of newspaper i with $i = 1,2$, and the political opinion of reader r and p_i denotes the price to be paid for newspaper i. Readers therefore bear a cost when buying a newspaper, which is proportional to the square of the

[14] Note that, in fact, given the particular assumptions on the advertising side, whether firms first set cover prices and then advertising tariffs or they do these simultaneously is irrelevant.
[15] See Hotelling (1929) as corrected by D'Aspremont, Gabszewicz and Thisse (1979).

distance between their political opinion and the political line of the newspaper. Thus the sum $tx_{ir}^2 + p_i$ is the *total cost* sustained by reader r when buying newspaper i. The parameter $\bar{u}$ represents instead the reservation price of readers when buying a copy of a newspaper, i.e. the maximum willingness to pay for a copy of a newspaper. In other words, this parameter is the *intrinsic value* of the newspaper. Such a value is assumed equal across consumers and newspapers.

It is also assumed that readers are indifferent to advertising on daily newspapers.[16]

Without loss of generality, let us assume that publisher 1 chooses a political orientation denoted by a on the unit interval, where a will be the distance between the selected point and 0, while publisher 2 chooses a political opinion denoted by b on the unit interval, where b will be the distance between the selected point and 1.

Gabszewicz, Laussel and Sonnac (2001, 2002) assume that every consumer in the market buys a copy of a newspaper or, in other words, that the reader market is always entirely covered; more formally, the following condition is assumed to be always satisfied at equilibrium:

$$\bar{u} - tx_{ir}^2 - p_i \geq 0 \tag{2}$$

From now on, we will refer to this inequality as *market coverage condition*.

This condition implies the following restrictions on the parameters:

$$\begin{cases} \bar{u} \geq \dfrac{t}{4} \\ \bar{u} \geq \dfrac{5t}{4} + c - k \end{cases} \tag{3}$$

which guarantee that the reservation price is high enough for the market to be entirely covered in both equilibria of the Gabszewicz, Laussel and Sonnac (2002) model. It is important to make this condition explicit as we move to analyse the repeated game and the possibility of collusive behaviour. Indeed, without a finite reservation price, publishers could collude at an indefinitely high price at no cost.

As in a standard Hotelling model with quadratic transportation costs, one can derive the demand from readers by first identifying the indifferent consumer y for each couple of prices p_1 and p_2 and locations a and b.

[16] Empirical evidence on the effect of advertising can be found in Sonnac (2000), who reports that the effect of advertising on readers depends on the type of media and on the country, Kaiser and Wright (2006), who find a positive but small effect of advertising on the sales of magazines in Germany and Argentesi and Filistrucchi (2007), who find no effect of advertising on the sales of daily newspapers in Italy. A similar finding is reported by Fan (2011) for US daily newspapers and by van Cayselee and Vanormelingen (2010) for Belgian daily newspapers. Kaiser and Song (2009) find that readers of magazines do not dislike advertising but depending on the type of magazine may also like it. Finally, Wilbur (2008) and Jeziorski (2011) respectively find that TV viewers and radio listeners in the US dislike advertising.
The conclusion we draw from the literature above is that on average consumers like advertising in magazines (when it is relatively targeted and can be avoided), dislike it on TV (when it is not targeted and cannot be avoided) and are indifferent to it on daily newspapers (where it is not targeted but can be avoided). As a consequence, we maintain the assumption in Gabszewicz, Laussel and Sonnac (2001, 2002) that readers are indifferent to advertising on daily newspapers.

Figure 1 is a diagrammatic representation of the problem as in Economides (1984). The horizontal axis represents the unit interval on which the readers' opinions are listed. Publisher 1 and 2 locate respectively at a and $1-b$. Instead, the vertical axis displays the reservation price, newspaper cover prices and the total costs faced by readers. The intersection between the total cost curves gives the location of the consumer y who is indifferent between buying newspaper 1 or newspaper 2. Consumers to the left of y will sustain a lower total cost when buying newspaper 1 while consumers to the right of y will sustain a lower total cost when buying newspaper 2. In other words, y splits the market into the demand for newspaper 1, n_1, and the demand for newspaper 2, n_2.

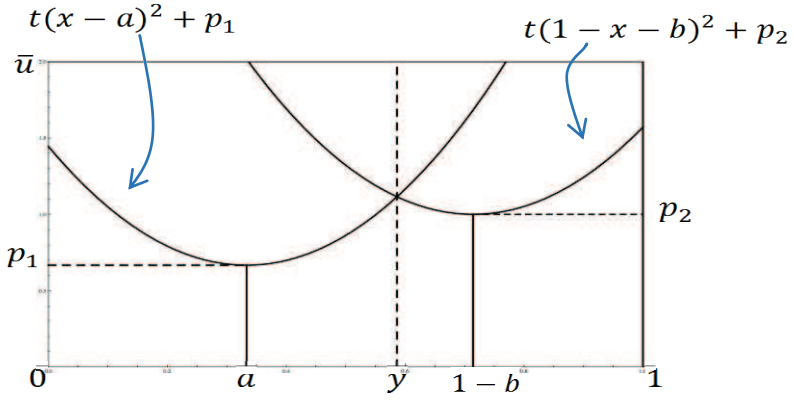

Figure 1

In Figure 1, $y = a + \frac{p_2 - p_1}{2t(1-a-b)} + \frac{1-a-b}{2}$

One can thus obtain the demand of newspapers as functions of prices p_1 and p_2 and location a and b[17]:

$$n_1 = \begin{cases} 0, & if\ a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2} < 0 \\ a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2}, & if\ \ 0 \le a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2} \le 1 \\ 1, & if\ a + \dfrac{p_2 - p_1}{2t(1-a-b)} + \dfrac{1-a-b}{2} > 1 \end{cases} \quad (4)$$

$$n_2 = \begin{cases} 0, & if\ b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2} < 0 \\ b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2}, & \text{if } 0 \le b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2} \le 1 \\ 1, & b + \dfrac{p_1 - p_2}{2t(1-a-b)} + \dfrac{1-a-b}{2} > 1 \end{cases} \quad (5)$$

[17] Note that, as the reservation price is the same for both newspapers, it does not alter the decision of which newspaper to buy.

Having normalized the mass of readers to 1, n_i can also be seen as the market share or the mass of readers of publisher *i*.

2.1.2.Advertising demand

Following Gabszewicz, Laussel and Sonnac (2001,2002), for convenience, the size of the advertising market is assumed to be $4k$, i.e. there are $4k$ advertisers in the market. Each advertiser's preferences depend on the price of ad spaces in a newspaper and on the mass of readers of that newspaper. The intensity of preference for the mass of readers is assumed to depend on a parameter, which characterizes each advertiser. Formally, this intensity is represented by the parameter θ, $\theta \in [0,1]$ and is referred to an ad space in a newspaper; the advertisers' population is uniformly distributed on $[0,1]$.

Thus, the utility of buying an ad in newspaper *i* for an advertiser a of type θ is measured by:

$$U_a = n_i\theta - s_i, \qquad i = 1,2;\ \theta \in [0,1], \tag{6}$$

where n_i represents the amount of readers of newspaper i, and s_i is the advertising tariff applied by publisher i.

Each advertiser is willing to buy an ad space in a newspaper as long as her utility is higher or equal to 0. Therefore, each advertiser has three possible choices: i) not to place an ad in any newspaper; ii) to place an ad in a single newspaper; iii) to place an ad in both. Advertisers can therefore multi-home. Each newspaper can carry as many ad spaces as demanded. Since each advertiser can place only one ad in each newspaper, the quantity of ads in a newspaper will equal the number of advertisers placing ads in that newspaper[18].

If an advertiser a of type θ multi-homes, her utility is measured by:

$$U_a = n_1\theta - s_1 + n_2\theta - s_2 \tag{7}$$

that is each advertiser buys an ad space in a newspaper if her utility to do so is positive. It is important to notice that the newspaper market is split into readers of newspaper 1 and readers of newspaper 2 because of single-homing on that side. Accordingly, advertisers' utility from advertising on one newspaper is independent of whether or not she advertises also on the other.[19]

2.2. The competitive equilibria

[18] In fact an advertiser which chooses two advertising slots in a newspaper is here seen as two advertisers each characterized by a different draw θ from the distribution of types. Note that this assumes no complementarily nor substitutability between the two slots and no quantity discounts. The latter assumption is unverifiable but the former is consistent with the assumption of no complementarity nor substitutability when buying one slot in different newspaper.

[19] Note that this assumption on the advertising side, though surely debatable, has been verified empirically by Rysman (2004) for yellow pages in the US and Fan (2011) for US daily newspapers.

Gabszewicz, Laussel and Sonnac (2002) solve the model by backward induction to find sub-game perfect equilibria. Firstly, they identify the optimal pricing in the advertising market; the results are shown in the previous paragraph. Secondly, by differentiating the profits with respect to cover prices, they obtain the reaction functions for the second step of the game; from them they derive the equilibrium prices as functions of the locations. Thirdly, they demonstrate that both a minimal opinion differentiation equilibrium and a maximal opinion differentiation equilibrium exist for given sets of the parameters.

Starting from equation (6) and (7), Gabszewicz, Laussel and Sonnac (2002) show that publishers select $s_i^* = n_i/2$ as equilibrium tariff, leading to revenues equal to kn_i. Thus, the equilibrium revenues are directly proportional to the mass of readers; in other words, for any newspaper sold, the publisher receives a fixed sum from the advertisers. Hence, k also identifies the *advertising revenues per reader*.

As shown in Gabszewicz, Laussel and Sonnac (2002), the demand for ad spaces in a newspaper is independent from the demand of ad spaces in the other newspaper. Hence, publishers enjoy monopoly power on the advertising side for providing access to their readers.[20]

From the previous analysis and supposing that both publishers produce newspapers at a unit cost per copy $c > 0$, we can easily derive the profit functions, namely:

$$\pi_i = (p_i + k - c)n_i \quad i = 1,2 \tag{8}$$

where n_i is defined in either equation (4) or (5).

The best reply functions are then:

$$p_1 = \max\left\{0, \frac{1}{2}(c - k + p_2 + t - 2tb + tb^2 - ta^2\right\} \tag{8a}$$

$$p_2 = \max\left\{0, \frac{1}{2}(c - k + p_1 + t - 2ta + ta^2 - tb^2\right\} \tag{8b}$$

which, depending on the parameters c, t and k and the political positions chosen in the first step, lead to the optimal prices. Substituting the latter into the profit equation allows to write profits as a function of political locations.

Gabszewicz, Laussel and Sonnac (2002) demonstrate that both a minimal opinion differentiation equilibrium and a maximal opinion differentiation equilibrium exist for given sets of the parameters.

Minimal opinion differentiation equilibrium

For $k \geq c + 25t/72$, both publishers choose to locate in the middle of the political spectrum ($a^* = b^* = 1/2$) and to set a common price equal to

[20] A similar situation is represented in Armstrong's (2006) competitive bottleneck model. However, here, it is assumed that advertising does not affect readers' utility. As such the model is a particular case of the one in Armstrong (2006).

$$p^* = 0 \tag{9}$$

Consequently, they split the market in two equal parts and equilibrium profits are:

$$\pi_{N1} = \frac{(k-c)}{2} \tag{10}$$

From now on, π_{N1} identifies profits arising from a one-shot competition in which publishers decide to locate in the middle of the political spectrum. Figure 2 displays the minimal differentiation equilibrium.

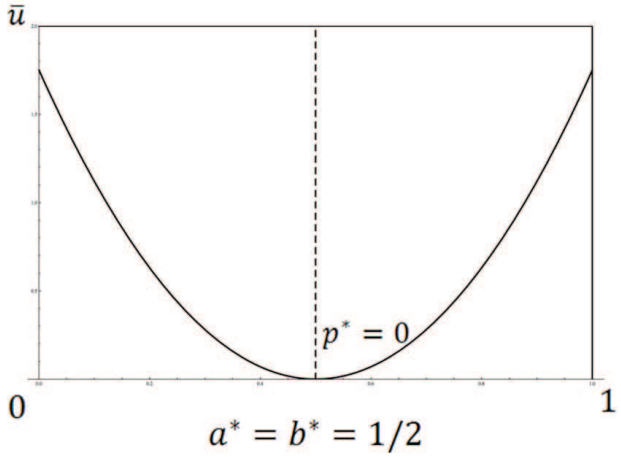

Figure 2

Maximal opinion differentiation equilibrium

For $k \leq c + t/2$, both publishers choose to locate at the endpoints of the political spectrum ($a^* = b^* = 0$) and to set a common price equal to

$$p^* = c - k + t \tag{11}$$

Consequently, they split the market into two equal parts and equilibrium profits are:

$$\pi_{N2} = \frac{t}{2} \tag{12}$$

From now on, π_{N2} identifies profits arising in a one-shot competitive equilibrium in which publishers decide to locate at the endpoints of the political spectrum. Figure 3 displays the maximal differentiation equilibrium.

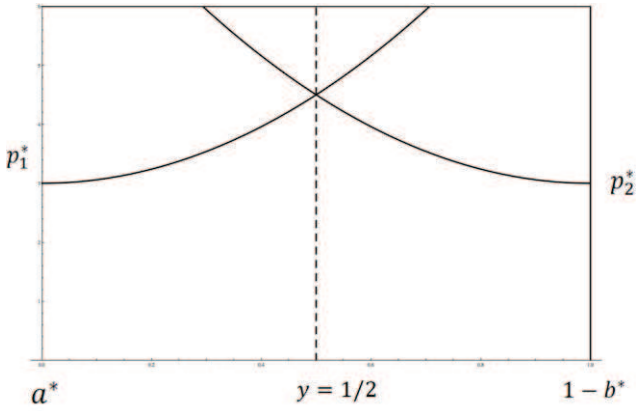

Figure 3

One can check that if $c + \frac{25t}{72} \leq k \leq c + \frac{t}{2}$, both equilibria exist. By comparing (12) and (10), we can see that for this subset of the parameters $\pi_{N2} > \pi_{N1}$: profits made in the maximal differentiation equilibrium are higher than profits made in the minimal differentiation equilibrium. In other words, the maximal differentiation equilibrium Pareto dominates, from the point of view of publishers, the minimal differentiation equilibrium when both equilibria exist. One could thus argue that publishers might be able to coordinate on the maximal differentiation equilibrium. Interestingly, as we will show in Section 3.3, for these parameter values, the minimum differentiation equilibrium is preferable for consumers. In fact, total welfare is exactly the same in the two equilibria.

Note that the maximal differentiation equilibrium is the classical outcome of a one-sided Hotelling model with quadratic transportation costs, as first proposed by D'Aspremont, Gabszewicz and Thisse (1979). It can be explained in the same way: firms relax competition by locating at the endpoints. On the other hand, the existence of the minimal differentiation equilibrium is the main contribution of Gabszewicz, Laussel and Sonnac (2002) and the one on which Gabszewicz, Laussel and Sonnac (2001) focus their discussion: this equilibrium arises because of the presence of the advertising side, which makes stealing customers from the rival more profitable. Indeed, it can be seen that the equilibrium is sustainable if the advertising market is large enough: in this case gaining a reader is more profitable because advertising revenues per reader are higher. Furthermore, if the advertising market is very large, only the minimal differentiation equilibrium remains: due to advertising, competition for readers is very harsh and the publishers do not choose to locate at the endpoints anymore.

Thus, Gabszewicz, Laussel and Sonnac (2002) conclude that convergence in the political orientation of newspapers could result from a rise in the importance of advertisements as source of revenues. In other words, as argued by Gabszewicz, Laussel and Sonnac (2001), the

growth of advertising as a source of revenues for newspapers can help explain the emergence of the so-called *Pensée Unique.*

3. Collusion in the newspaper market

We employ the sequential game described above as a stage game of an infinitely repeated game. In this multi-period framework, publishers may choose to cooperate in order to obtain higher profits.

We assume that publishers take into account as collusive agreements only Pareto optimal agreements, i.e. pairs of strategies that cannot be changed without decreasing at least one of the two publishers' payoffs.

We then assume that the agreement is implemented over time by using grim trigger strategies, i.e. each publisher cooperates as long as the other publisher cooperates, punishes forever any defection from the agreement and believes the other publisher will behave in the same way.

As already discussed, in our model publishers enjoy monopoly power on advertisers for access to their readers. In addition advertising has no impact on newspapers sales. Accordingly, acting cooperatively cannot improve this already optimal behaviour. As a result, any optimal strategy of the repeated game cannot include a different advertising tariff than the competitive one. In practice, publishers can only collude on the cover price and on the political orientation. As we will see, this does not imply that advertising is not relevant anymore, but only that publishers will not need to collude on that side of the market.

We thus take into consideration two kinds of agreements: in the first one, publishers coordinate both the political orientation and the prices of the newspapers; in the second one, publishers coordinate prices only. Whereas the first is a case of full collusion[21], the second one is a case of semi-collusion and fits well with an environment in which publishers find it difficult to coordinate on the political orientation of their newspapers.

When publishers are colluding on both political orientation and prices, we assume that if a publisher defects when choosing political position for its newspaper, punishment starts already at the next price stage.

The main aims of the analysis are: firstly, investigating the factors facilitating collusion; secondly, inspecting the properties of the collusive agreements in terms of welfare and pluralism.

[21] To be precise, to the extent that collusion does not take place on the advertising tariff both are cases of semi-collusion. However, as noted above, in this model collusion on the advertising tariff would not change the profit maximizing advertising tariff.

Given the assumptions above, and following Friedman and Thisse (1993), we first look for possible collusive agreements among the Pareto optimal outcomes of the stage game. When more than one Pareto optimum is identified, publishers select the Pareto optimum that implies setting common prices; if none of the Pareto optima imply common prices, publishers select the one which implies splitting the market equally. Indeed, in our model setting a common price is a plausible outcome since the firms are symmetric. Furthermore, coordination on a common price is easier to achieve and a defection from a common price is easier to detect, thus facilitating collusion.

Publishers adhere to a collusive agreement supported by grim trigger strategies as long as the discounted sum of profits associated with collusion is higher than the discounted sum of profits associated with defection.

Let us define π_C as the collusive profits, π_N as the Nash profits, and π_D are the defection profits. The incentive constraint for collusion can be formalized as follows:

$$\frac{\delta}{1-\delta}(\pi_C - \pi_N) \geq (\pi_D - \pi_C) \tag{13}$$

The left hand side of the inequality represents the sum of discounted future losses due to defection, while the right hand side represents the one-time gain from optimal defection. The critical discount factor is easily derived as:

$$\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)} \tag{14}$$

For all discount factors above $\hat{\delta}$, publishers will find it more profitable to collude than to defect. This critical discount factor will depend on the parameters of the game; as a consequence, any change in one of the parameters implies altering the profitability and sustainability of collusion. For example, if a change in parameters makes the critical discount factor increase, the set of discount factors supporting collusion will become smaller and therefore collusion will become harder to sustain.

3.1. Collusion on prices and political orientation

In this paragraph collusion on prices and political orientation is analysed; firstly, the collusive agreement is investigated; secondly, the optimal defection strategy is found. Collusive profits and defection profits are then obtained.

3.1.1. Collusive agreement

As stated above, we first characterize the Pareto optima of the game. Then we select as possible collusive agreement the Pareto optimum with common price.

We will call *state* a particular pair of strategies $(a, p_1), (b, p_2)$.[22] A state can be improved if moving to a different state allows to increase the payoff of one publisher without decreasing the payoff of the other. When a state can be improved, such a state cannot be a Pareto optimum.

Lemma 1*. Any state for which all readers obtain a utility strictly higher than 0 can be improved.*

Proof. See Appendix.

Figure 4 summarizes what happens in Lemma 1. The publishers can mutually increase their payoff by simply increasing their prices by the same amount $\Delta p_1 = \Delta p_2$ so that the lowest utility consumer still buys a copy: thus, all the readers are buying a copy in this new state $(a, p_1 + \Delta p_1), (b, p_2 + \Delta p_2)$, while y is constant because the change in prices is the same for both newspapers.

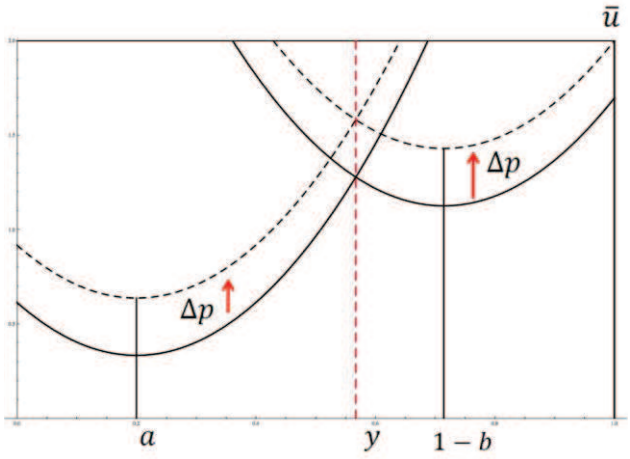

Figure 4

Lemma 1 is useful not only because it helps to characterize the Pareto optima of the game, but also because it suggests how publishers can easily improve a state: they just need to increase prices so that the lowest utility consumer is taken from positive utility to utility 0.

Definition 1. *A state for which at least one consumer obtains utility 0 is called a touch state. This consumer is called reservation price consumer and can be in 0, 1 or y.*

Therefore at least one of the following conditions is to be satisfied by the touch state:

if reservation price consumer is at 0: $\bar{u} - ta^2 - p_1 = 0,$
if reservation price consumer is at y: $\bar{u} - t(y - a)^2 - p_1 = \bar{u} - t(1 - y - b)^2 - p_2 = 0,$
if reservation price consumer is at 1: $\bar{u} - tb^2 - p_2 = 0.$

Pareto optima are to be found among the touch states. Lemma 2 helps to select some touch states.

Lemma 2. *Any touch state for which only one consumer obtains utility 0 can be improved.*

[22] Advertising tariffs are kept out of the definition of state for the sake of simplicity, as the optimal tariff does not change during the analysis.

Proof. See Appendix.

Like Lemma 1, Lemma 2 helps to restrict the set of eligible Pareto optima. Lemma 2 also suggests how the publishers can relocate to improve a touch state. Instead, if in a touch state a newspaper is not located in the middle of its demand, a relocation of this newspaper permits to both publishers to increase their price. Indeed, it can be noticed that a reservation price consumer is located at one of the endpoints of the demand of that newspaper (see Figure 5 for a visual representation). If the publisher of this newspaper shifts her location slightly towards the reservation price consumer, she can increase her price so that the reservation price consumer obtains again utility zero after the relocation. At the same time, the other publisher can increase her price so that the marginal consumer is kept at the same location: the demands are the same and the prices have increased. The procedure can be repeated until the newspaper converges towards the middle of its readership: in that case both the consumers on the endpoints are reservation price consumers. Since the utility of readers depends on prices and transportation costs (the reservation price is constant), decreasing the transportation costs of the readers permits to increase the price. In fact, each publisher relocates in the middle of her demand in order to reduce the transportation costs sustained by her readers; this allows publishers to impose higher prices without losing consumers.

Thanks to Lemma 2 we can introduce the following definition.

Definition 2. *A state for which consumers in 0, 1 and y are reservation price consumers is called a complete touch state.*

Therefore the following conditions are to be satisfied by the complete touch state:

$$\begin{gathered} \bar{u} - ta^2 - p_1 = 0, \\ \bar{u} - t(y-a)^2 - p_1 = \bar{u} - t(1-y-b)^2 - p_2 = 0, \\ \bar{u} - tb^2 - p_2 = 0. \end{gathered}$$

The strategies supporting this kind of state are the following

$$\begin{gathered} \text{publisher 1 applies: } \left(\frac{y}{2}\ ,\ \bar{u} - t\frac{y^2}{4}\right) \\ \text{publisher 2 applies: } \left(\frac{1-y}{2}\ ,\ \bar{u} - t\frac{(1-y)^2}{4}\right) \end{gathered} \tag{16}$$

Publishers locate in the middle of their demand and set prices that keep consumers at the extremes of their demands at utility 0. Every complete touch state is thus characterized only by the marginal consumer y, $0 \leq y \leq 1$.

Lemma 2 and Definition 2 can be better understood in the light of the graphical representation in Figure 5. The dashed curves correspond to the new complete touch state.

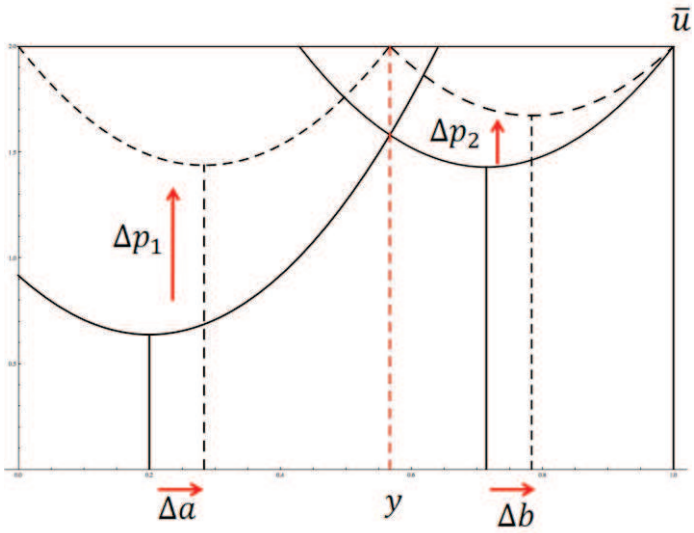

Figure 5

In order to find the Pareto optima of the game, one last question has to be answered: can a complete touch state be improved? Lemma 3 identifies the Pareto optima of the game by answering this question.

Lemma 3*. Every complete touch state is a Pareto optimum.*

Proof. See Appendix.

Once Pareto optima are identified, publishers select the state, which implies a common price: in practice, the following expression has to be satisfied:

$$\bar{u} - t\frac{y^2}{4} = \bar{u} - t\frac{(1-y)^2}{4}$$

which in turn implies:

$$y = \frac{1}{2}.$$

Thus, the complete touch state with $y = 1/2$ forms the agreement between the publishers when they cooperate on both prices and locations. Besides, this agreement implies common prices and profits and splitting the market in the middle. These features suggest that it is actually plausible for the publishers to find an agreement on such a pair of strategies.

Taking (16) with $y = 1/2$, we can verify that publishers agree to locate at ¼ and at ¾ of the unit interval and to set a common price equal to $\bar{u} - \mathrm{t}/16$.

This leads to

Proposition 1. *When collusion takes place on both the political orientation and the cover price of the newspaper, the collusive agreement implies medium political differentiation. Publishers choose $a = b =$ ¼ and set a common price $p_i = \bar{u} - \frac{t}{16}$.*

The corresponding collusive profits for each publisher will be:

$$\pi_{CLP} = \pi_1 = \pi_2 = \frac{\left(\bar{u} - \frac{t}{16} + k - c\right)}{2} \tag{17}$$

From now on, we will refer to expression (17) as collusive profits π_{CLP} when publishers are allowed to collude on prices and locations. The collusion outcome is graphically represented in Figure 6.

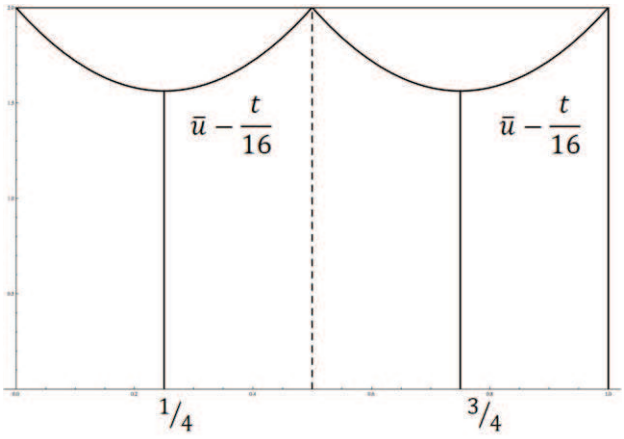

Figure 6

During the analysis, we have assumed the market to be covered. Indeed it is not difficult to check that:

Corollary 1. *If the market is covered in competition, it will be covered also under collusion on both prices and political location.*

Proof. See Appendix.

3.1.2.Defection

Each publisher has two alternatives to defect: first, she could select a political orientation different from the agreed one at the first step of a stage game and then compete in prices in the second step; second, she could stick with the agreed political orientation and defect on the cover price at the second step. Clearly she will select the defection that offers the higher payoff.

In order to find the optimal defection strategy it is necessary to find the optimal defection strategies of the two cases separately and then compare the payoffs. We should keep in mind that the non-deviant publisher applies the strategy

$$\left(\frac{1}{4}, \bar{u} - \frac{t}{16}\right) \tag{19}$$

Proposition 2. *For* $k > \mathrm{c} + \left(\frac{121}{16(13-4\sqrt{3})}\right)\mathrm{t}$ *or* $k < c + \frac{243}{256}t$ *the optimal defection strategy consists of keeping political position unchanged and undercutting the rival by setting a price equal to:*

$$p^* = \begin{cases} \tilde{p} = \dfrac{\bar{u} + c - k + \dfrac{7}{16}t}{2}, & \text{if } \bar{u} \leq c - k + \dfrac{25}{16}t \\ \bar{p} = \bar{u} - \dfrac{9}{16}t, & \text{if } \bar{u} > c - k + \dfrac{25}{16}t \end{cases} \qquad (20a)$$

Defection profits are then

$$\pi(\tilde{p}) = \frac{(\bar{u} + k - c + \frac{7}{16}t)^2}{4t} \qquad (20b)$$

$$\pi(\bar{p}) = \bar{u} - \frac{9}{16}t + k - c \qquad (20c)$$

For $c + \frac{243}{256}t \leq k < \mathrm{c} + \left(\frac{121}{16(13-4\sqrt{3})}\right)\mathrm{t}$ *the optimal defection strategy consists of adopting the same political line of the other newspaper, i.e. choosing* $a = \frac{3}{4}$, *and then setting the competitive price*

$$\hat{p} = 0 \qquad (21a)$$

Defection profits are then

$$\pi(a = 3/4) = \tfrac{3}{4}(k - c) \qquad (21b)$$

Proof. See Appendix.

3.2. Collusion on prices only

3.2.1. Collusive agreement

When publishers are allowed to collude on prices only, at the first step of the stage game, they locate independently while at the second step they set prices cooperatively. Therefore, publishers locate on the political opinion spectrum taking into account that they will apply an agreed rule on prices.

Like the collusive agreement on prices and political orientations, the collusive agreement on prices only has to be a state for which it is not possible to increase the payoff of one publisher without decreasing the payoff of the other publisher, i.e. a Pareto optimum. However, here publishers cannot choose a state within the entire set available, because that would mean they cooperatively select locations as well. Instead they choose a rule on prices that will take the pair of locations as given. Hence, the rule identifies a state for every possible pair of locations.

Thus we first identify the cooperative rule on prices for the second stage and then find the equilibrium behaviour for the first stage taking into account this rule.

The cooperative rule for prices is to be optimal and one could start by characterizing the Pareto optima. It is important to remember, however, that publishers are assumed to set a common price when more than one Pareto optimum is identified. Therefore, we can proceed in a different way: we first take the best common-price-rule into account and then check if it selects Pareto optima.

The best common-price-rule can be identified starting from Lemma 1: indeed, Lemma 1 shows how every state for which all the readers obtain a utility higher than 0 can be improved. This is valid also for the current case in which locations are held constant.[23] Consequently, given the pair of locations (a, b), publishers have to choose among all the touch states compatible with such pair of locations. This result is summed up in Lemma 4.

Lemma 4. *Given a pair of locations* (a, b) *publishers choose a state among the touch states.*

Lemma 4 however does not fully characterize Pareto optima for the case of cooperative selection of prices only. Yet, it selects the only candidate states for Pareto optima: applying the same-price-rule to touch states we therefore obtain the best possible same-price-rule.

The common price has to be set according to the consumer who pays the maximal transportation cost in the market; indeed, this consumer will be the one obtaining utility 0 in the outcome. As stated above, her location can be 0, $(1 + a - b)/2$ or 1. The most distant one among them from the opinion a or b of the newspaper bought by that consumer characterizes the price.[24]

Corollary 2. *Any touch state* $(a, \tilde{p}), (b, \tilde{p})$ *with common pricing is characterized as follows:*

$$\tilde{p} = \bar{u} - t\frac{(1-a-b)^2}{4} \quad if \begin{cases} \frac{1-a-b}{2} > a \\ \frac{1-a-b}{2} > b \end{cases} \tag{22}$$

$$\tilde{p} = \bar{u} - ta^2 \quad if \begin{cases} \frac{1-a-b}{2} \leq a \\ a \geq b \end{cases} \tag{23}$$

$$\tilde{p} = \bar{u} - tb^2 \quad if \begin{cases} \frac{1-a-b}{2} \leq b \\ b > a \end{cases} \tag{24}$$

Figure 7 summarizes equation (24) of the pricing rule.

[23] In the proof of Lemma 1, locations are held constant.

[24] Once the pair (a, b) is constant, publishers can set a common price, select a touch state, and cover the entire market at the same time, only by setting the price according to the largest values among a, b, and $(1 - a - b)/2$, which are the distance from the newspaper bought by respectively consumer 0, 1, and $(1 + a - b)/2 = y$.

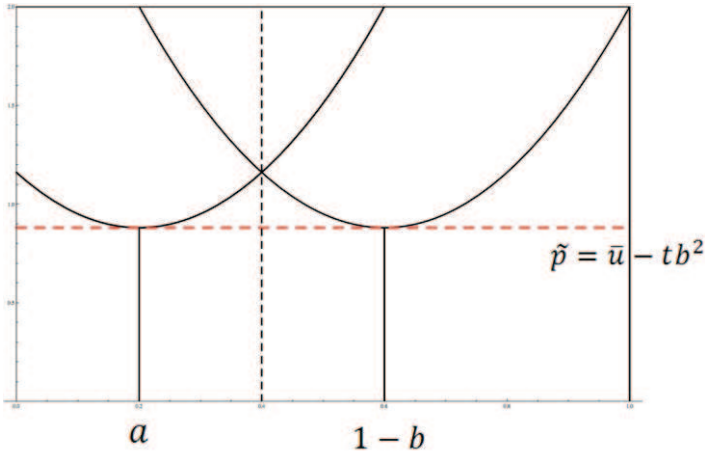

Figure 7

In brief, once (a, b) is given from the first step, publishers cooperatively select the touch state with common price compatible with such couple (a, b); it is easy to note that such a state is unique and therefore the pricing rule is well defined. What remains to be demonstrated is that this pricing rule, i.e. the best common-price-rule, always selects Pareto optima. In other words, it should not be possible to find a state for which a publisher increases her profits without the profits of the other to decrease starting from the touch state selected and changing prices only. This result is derived in Proposition 3.

Lemma 5. *The best common-price-rule is Pareto optimal.*

Proof. See Appendix.

We now have all the instruments to find the Nash equilibrium.

Proposition 3. *When collusion takes place on the cover price of the newspaper only, the collusive agreement implies minimal political differentiation. Publishers choose* $a = b = {}^{1}/_{2}$ *and set a common price* $p_i = \bar{u} - \frac{t}{4}$.

The corresponding collusive profits for each publisher will be:

$$\pi_{CP} = \pi_1 = \pi_2 = \frac{\left(\bar{u} - \frac{t}{4} + k - c\right)}{2} \qquad (25)$$

Proof. See Appendix.

The result is rather straightforward: once a rule on pricing has been agreed, publishers behave like the two firms of a differentiated duopoly with common price (the famous ice-cream sellers on a beach when the price of the ice-cream is set by the local authorities). The only difference is that this common price changes with the resulting pair of locations but the incentive to gain market share is very similar: the only possible Nash equilibrium consists in the pair of locations (½, ½).

From now on, we will refer to the expression in (25) as collusive profits π_{CP} when publishers are allowed to collude on prices only.

The equilibrium is represented in Figure 8. For the sake of simplicity, the reader market is split in the middle, i.e. newspaper 1 caters to readers to the left of ½ and newspaper 2 caters to readers to the right of ½.

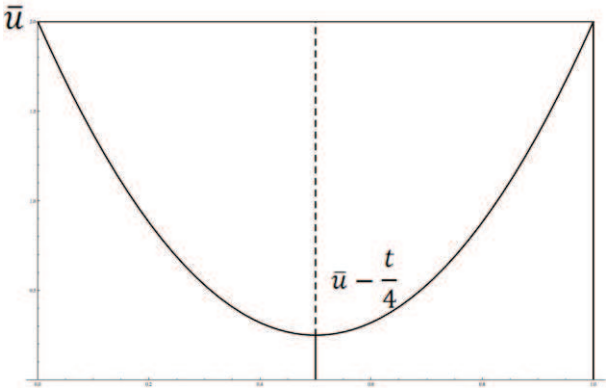

Figure 8

During the analysis, we assumed the market to be covered, exactly like in the case of collusion on both prices and political orientation.

It is not difficult to check:

Corollary 4 *- If the market is covered in competition, it will be covered also under collusion on prices.*

Proof. See Appendix.

3.2.2.Defection

At the first stage, the non-deviant publisher chooses location ½: whatever the location of the defecting publisher, the common price will be set at $\bar{u} - \frac{t}{4}$ since the most distant consumers will be ½ distant from the non defecting publisher. Therefore, the non-deviant publisher plays the equilibrium strategy in any case.

In order to find the optimal defection strategy we can use the procedure employed in the previous paragraph. The same reasoning on punishment and expectations is assumed to hold for this case as well. We should keep in mind that the non-deviant publisher applies the strategy:

$$\left(\frac{1}{2}, \bar{u} - \frac{t}{4}\right) \tag{27}$$

Proposition 4*. When the publishers collude on prices only, a publisher optimally defects by locating at the same point of the collusive strategy and applying a slightly lower price.*

Proof. See Appendix.

The optimal defection consists in not differentiating and undercutting the other publisher. Then all readers simply buy the less expensive newspaper.

Profits related to this strategy are:

$$\pi_{DP} = \bar{u} - \frac{t}{4} + k - c \tag{28}$$

The optimal defection strategy is displayed in Figure 9:

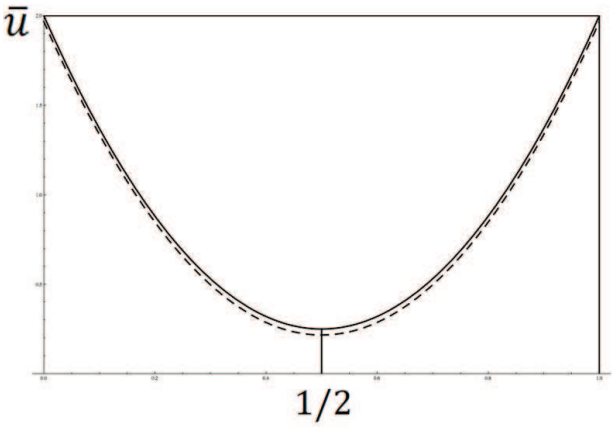

Figure 9

3.3. Welfare analysis

In a two-sided market, we have two distinct groups of consumers (readers and advertisers in our case) and the firms which act as platforms. There exist therefore two consumers welfare and one producer welfare. The relative weight that should be given to the two consumer welfare is a matter of debate in theory and has been chose differently in practice by different competition and regulatory authorities.[25]

Given the assumption that readers are indifferent to advertising, which we have extensively justified above, the particular outcome just described could take place only on the advertiser side of the market.

Yet, in our model, as in Gabszewicz, Laussel and Sonnac (2001, 2002), advertisers always pay the monopoly price, even in competition. Thus, only the total mass of readers affects the utility of advertisers; however, since the market is always covered, the mass of readers is always 1 and the advertisers' surplus does not change throughout the analysis.

As a consequence, we will focus on the component of total welfare, which is equal to the sum of the readers' surplus plus the publishers' profits. For the sake of simplicity we will redefine this to be total welfare.

[25] See Filistrucchi, Geradin and van Damme (2012) for a discussion in the context of mergers involving two-sided platofrms.

The readers' surplus can be defined as follows:

$$W_r = \int_0^y [\bar{u} - t(x-a)^2 - p_1]dx + \int_y^1 [\bar{u} - (1-x-b)^2 - p_2]dx \quad (29)$$

Joint profits are:

$$W_p = \pi_1 + \pi_2 = (p_1 + k - c)y + (p_2 + k - c)(1-y) \quad (30)$$

Clearly total welfare is:

$$W_r + W_p = W_t \quad (31)$$

It is easy to derive readers', publishers' and total welfare for the several cases analysed:

Competition - Minimal political differentiation equilibrium

Readers' W_r	$\bar{u} - \frac{t}{12}$
Publishers' W_p	$k - c$
Total W_t	$\bar{u} - \frac{t}{12} + k - c$

Table 1

Competition - Maximal political differentiation equilibrium

Readers' W_r	$\bar{u} - \frac{13t}{12} + k - c$
Publishers' W_p	t
Total W_t	$\bar{u} - \frac{t}{12} + k - c$

Table 2

Collusion on prices and locations (medium political differentiation equilibrium)

Readers' W_r	$\frac{t}{24}$
Publishers' W_p	$\bar{u} - \frac{t}{16} + k - c$
Total W_t	$\bar{u} - \frac{t}{48} + k - c$

Table 3

Collusion on prices only (minimal political differentiation equilibrium)

Readers' W_r	$\frac{t}{6}$
Publishers' W_p	$\bar{u} - \frac{t}{4} + k - c$

Total W_t	$\bar{u} - \frac{t}{12} + k - c$

Table 4

Using the tables above, several comparisons can be made, both from a readers' welfare perspective and from a total welfare perspective.

First, however, it is worth noting that in this framework prices are never so high that they prevent any consumer from buying a newspaper as the market is always covered. Consequently, no possible increase in prices can reduce the total demand of newspapers, meaning that any shift in prices only implies a redistribution of surplus between readers and publishers. Nevertheless, shifts in prices are often associated with shifts in the political orientations of newspapers, with positive or negative effects on each reader's utility and in turn on readers' surplus.

From a reader's perspective, as already mentioned, for the subset of the parameters for which both competitive equilibria are sustained, the equilibrium with minimal differentiation is better than the equilibrium with maximal differentiation. However, the competitive outcomes always outperform both collusive outcomes. Moreover, collusion on prices only is better than collusion on both prices and locations. Thus no gain in the readers' welfare due to relocation of newspapers in the political spectrum is large enough to offset the price increase. These results confirm the idea that the more competition there is in the market, the better off readers are.

Therefore, in our model, the two-sided nature of the market, albeit present, is such that it makes no exception to the general rule that competition is good for consumers.

From a total welfare perspective, it is easy to check that collusion on prices and political orientations outperforms collusion on prices only. Compared to competitive equilibria, whether the minimal differentiation or the maximal differentiation one, collusion-on-prices-only provides the same total welfare while the collusion-on-everything outcome is superior. So that the gains provided to the publishers by collusive agreements respectively just offset or exceed the losses in the readers' surplus. The former effect is due to the assumption that the market is covered in competition and to the fact that once the market is covered in competition, it is covered also in collusion. Hence, prices go up while locations remain constant, so only a redistribution of surplus takes place. Instead in the latter case, locations improve from the point of view of readers[26] but firms extract even more surplus from readers through higher prices.

In conclusion, the following corollary holds

[26] Indeed, it is easy to note that the new location pair minimizes the sum of the transportation costs sustained by all readers

Corollary 5 *- A collusive agreement on prices and locations decreases readers' welfare but increases total welfare, while a collusive agreement on prices leaves total welfare unchanged but harms readers. Furthermore, while total welfare is higher with a collusive agreement on prices and locations than with a collusive agreement on prices only, the opposite holds for readers' welfare.*

3.4. Incentives to collude

We now recover the critical discount factor for each type of collusion and each of the stage game equilibria, which would be used as a punishment in case of defection.

It is then possible to analyse the change in the incentives to collude by differentiating each discount factor with respect to each parameter. Any change in a parameter which decreases a critical discount factor, enlarges the set of discount factors supporting collusion and therefore makes collusion more likely.

3.4.1. Collusion on prices only

The critical discount factors when publishers collude on prices only are

Punishment triggered	Critical discount factor
Minimal differentiation equilibrium $k \geq c + 25t/72$	$\hat{\delta}_{\min P}$
Maximal differentiation equilibrium $k \leq \mathrm{c} + \mathrm{t}/2$	$\hat{\delta}_{\max P}$

Table 5

where, recalling that $\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)}$ and substituting the relevant expressions for profits as a function of the parameters,

$$\hat{\delta}_{\max P} = \frac{4c - 4k + t - 4\bar{u}}{8c - 8k + 6t - 8\bar{u}}$$

$$\hat{\delta}_{\min P} = \frac{4c - 4k + t - 4\bar{u}}{4c - 4k + 2t - 8\bar{u}}$$

Table 6 below, we report the effect of an increase in the exogenous parameters on the critical discount factor. Clearly, a higher discount factor makes collusion less likely while a lower one makes collusion more likely.

Parameter	Punishment triggered	
	Minimal differentiation	**Maximal differentiation**
Political sensitiveness (t)	positive	positive
Advertising market dimension (k)	positive	negative
Marginal cost (c)	negative	positive
Reservation price ($\bar{u}$)	negative	negative

Table 6

The results shown in Table 6 can be explained as follows.

Political sensitiveness. As readers become more sensitive to political messages, i.e. the transportation cost increases, collusion among publishers is less likely. The reason for this is not straightforward: defection as well as collusive profits decrease, but the former effect outweighs the latter so that the one-time gain from defection decreases. However, punishment profits are either increasing or constant in the political sensitiveness: future losses due to current defection are thus enlarged. This effect cancels out the previous one.

The intuition is that the more readers are politically conscious, the easier it is for newspapers to find their own readership: in fact readers will be more attached to their "closer" newspaper. Indeed the equilibrium of maximal differentiation is sustained by high transportation parameters, everything else being equal: readers are easier to be targeted and competition can be relaxed by locating at the endpoints of the unit interval. Therefore, the newspapers do not need to coordinate decisions to cater to different segments of the market.

Reservation price. Collusion is in general easier to sustain if the reservation price increases. Two countervailing effects take place: on one hand, defection profits increase more than the collusive profits, exactly like in the case of the transportation cost; on the other hand the punishment profits are steady in respect to the reservation price while the collusive profits increase. The second effect offsets the first one. In fact, this confirms the idea that when two firms collude they can exploit the market more than they could do competing. An increase in the

reservation price shifts the demand outward and allows publishers to gain more from collusion.[27]

Advertising market dimension. Interestingly the effect of a larger advertising market (and thus higher revenues per reader) on collusion strongly depends on the punishment triggered and, in turn, on the parameters. This means that a shift in the parameter k can encourage as well as discourage collusive behaviour depending on the starting value of the parameter itself.

In order to obtain a complete picture of how the incentives to collude depend on the advertising parameter k, a graphical representation of the results of Table 6 is provided below in Figure 10. The graph displays the critical discount factor (on the vertical axis) as a function of the dimension of the advertising market (on the horizontal axis).[28] Note that, given the reservation price, the marginal cost of the newspaper, and the political sensitiveness, the advertising market dimension k can take values above k_0 only, as also the conditions in (3) need to be satisfied.[29] For low values of k, $k_0 \leq k < k_1$, only the equilibrium of maximal differentiation is sustainable in one-shot competition. For intermediate values of k, $k_1 \leq k < k_2$, both equilibria are sustainable. For high values of $k, k \geq k_2$, only the equilibrium of minimal differentiation is sustainable.[30] The green curve shows the critical discount factor when the punishment triggered is the maximal differentiation equilibrium ($\hat{\delta}_{2P}$,); instead, the red curve shows the critical discount factor when the punishment triggered is the minimal differentiation equilibrium ($\hat{\delta}_{1P}$). For intermediate values of k, $k_1 \leq k \leq k_2$, i.e. when both the equilibria are sustainable, the effects depend on the punishment chosen. However, since in this parameter interval the maximal differentiation equilibrium Pareto dominates the minimal differentiation one, then it is optimal, in order to sustain collusion, to punish the deviant by reverting to the Pareto inferior Nash equilibrium; hence one can assume that the relevant discount factor is $\hat{\delta}_{1P}$, which increases as K grows.

[27] Note that here the effect is reinforced by the assumption that the market is always covered: publishers can cooperatively set prices as functions of the reservation price itself (see the prices under collusion at (17) and (25)) while, when they are competing, they simply guarantee that the prices are not too high so that everyone is buying a newspaper.
[28] The chosen values for the other parameters are: $\bar{u} = 2,\ c = 2,\ t = 1$, which satisfy the assumptions of the model.
[29] As explained above, these conditions allow the market to be covered in the competitive outcomes of the stage game.
[30] $k_0 = \frac{5}{4}, k_1 = \frac{169}{72}, k_2 = \frac{5}{2}$.

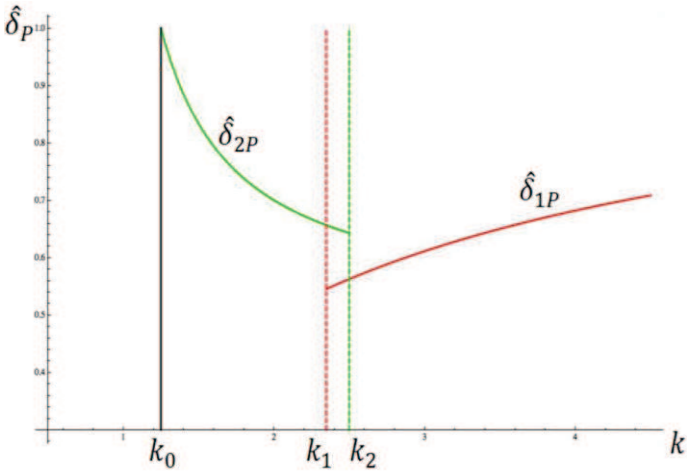

Figure 10

The critical discount factor decreases as long as the starting value is low, i.e. when the punishment triggered can only be the maximal differentiation equilibrium; it increases from intermediate values on, i.e. when the punishment triggered is the minimal differentiation equilibrium.

When k is high enough, so that the punishment is the minimal differentiation equilibrium, any further increase in the advertising market increases the critical discount factor and makes collusion less likely. First, it should be noticed that collusion, defection and punishment profits are all increasing functions of k; in fact, defection profits increase more than collusive profits so that the one-time gain increases; collusive profits and punishment profits increase in the same way so that future losses will be steady. Thus, collusion is less feasible when k grows. An explanation of this can be offered by noting that the advertising parameter represents the sum paid to each publisher for any copy sold: the market is split in the middle when publishers collude as well as when they compete but not when one of them defects; thus, if k rises, defecting becomes more profitable than colluding and than competing because the deviant has more readers than in the other cases. Thus, the trend exhibited for high levels of k seems to be explained by the fact that each publisher gains more and more for any additional reader she can cater to by defecting; this makes defection more likely.

Once could argue that the same reasoning applies when the punishment triggered is a maximal differentiation equilibrium; indeed, defection and collusive profits are identical to the previous case; hence, gaining market share in the defection turn is more profitable when the advertising market dimension is higher. Nevertheless, as shown in the graph, when the one-stage equilibrium is the maximal differentiation one, punishment profits do not depend on the dimension of the advertising market. This may sound surprising. However, recall from (11) that competitive prices are in that case:

$$p^* = t + c - k$$

So that the revenue per reader k is entirely passed on to readers in form of a discount on the cover price: the publishers internalize the indirect network effects running from the advertisers to the readers; they bridge the two sides perfectly and make no money on it. Indeed, the respective profits as in (12) are:

$$\pi_{N2} = \frac{t}{2}$$

Conversely, collusive prices do not depend on the advertising market dimension, while the profits do (see (17) and (25)). Thus, when the publishers collude, they do not subsidise the readers with the advertising receipts they earn. When passing from competition to collusion, the publishers pass from a situation in which the advertising receipts are totally passed on to readers to a situation in which the advertising receipts are totally retained. Therefore, if the advertising revenues per reader increase, collusion becomes more and more desirable and therefore more likely.

To conclude, the following corollary holds

Corollary 6 *– When firms collude on prices only, a larger advertising market has a non-monotone effect on the incentives to collude: when the advertising market is small, an increase in the size of the advertising market favours collusion, but when the advertising market is sufficiently large, a further increase will make collusion less likely.*

Proof. See Appendix.

Marginal cost of a copy. The analysis of the change in the collusive incentives due to a change in the marginal cost is in fact symmetric to the previous one. The reason is that they enter the profit function in (8) in additive way. The only difference is the sign.

3.4.2. Collusion on both prices and political position

Given defection profits, competitive profits and collusive profits, it is easy to calculate the critical discount factors when publishers collude on both prices and the political position. They are are reported in Table 7.

Punishment triggered	Parameter areas	Critical discount factor
Minimal differentiation equilibrium $k \geq c + 25t/72$	$\bar{u} \geq c - k + 25t/16$ (defection without relocation, price $\bar{p}$	$\hat{\delta}_{\min 1LP}$
	$\bar{u} < c - k + 25t/16$ and $k \leq c + \frac{243}{256}t$ or $k \geq \left(\frac{121}{16(13-4\sqrt{3})}\right)$t (defection without relocation, price $\hat{p}$)	$\hat{\delta}_{\min 2LP}$
	$\bar{u} < c - k + 25t/16$ and $c + \frac{243}{256}t \leq k <$ c $+ \left(\frac{121}{16(13-4\sqrt{3})}\right)$t (defection with relocation)	$\hat{\delta}'_{min2LP}$
Maximal differentiation equilibrium $k < c + 25t/72$	$\bar{u} \geq c - k + 25t/16$ (defection without relocation, price $\bar{p}$)	$\hat{\delta}_{\max 1LP}$
	$\bar{u} < c - k + 25t/16$ (defection without relocation, price $\hat{p}$	$\hat{\delta}_{\max 2LP}$

Table 7

where, recalling that $\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)}$ and substituting the relevant expressions for profits as a function of the parameters,

$$\hat{\delta}_{\min 1LP} = \frac{c - k + \frac{17}{16}t - \bar{u}}{c - k + \frac{9}{8}t - 2\bar{u}}$$

$$\hat{\delta}_{\min 2LP} = \frac{c - k + \frac{\text{t}}{16} - \bar{u} + \frac{1}{2t}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}{c - k + \frac{1}{2t}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}$$

$$\hat{\delta}_{\max 1LP} = \frac{1}{2}$$

$$\hat{\delta}_{\max 2LP} = \frac{c - k + \frac{\mathrm{t}}{16} - \bar{u} + \frac{1}{2t}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}{-t + \frac{1}{2t}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}$$

$$\hat{\delta}'_{min2LP} = 1 - \frac{2\bar{u} - \frac{t}{8}}{k - c}$$

In this case the study of the effect of an increase in the exogenous parameters on the critical discount factors are more complex. As an example, we thus report only the effect of an increase in the dimension of the advertising market, which is the parameter most directly related to the two-sided nature of the market. As already noted, the effect of an increase in c on the critical discount factor are symmetric.

Punishment triggered	**Parameter areas**	**Effect on the critical discount factor**
Minimal differentiation equilibrium $k \geq c + 25t/72$	$\bar{u} \geq c - k + 25t/16$ (defection without relocation, price $\bar{p}$)	Positive
	$\bar{\mathrm{u}} < c - k + 25t/16$ and $k \leq c + \frac{243}{256}t$ or $k \geq \left(\frac{121}{16(13-4\sqrt{3})}\right)\mathrm{t}$ (defection without relocation, price $\tilde{p}$	positive
	$\bar{\mathrm{u}} < c - k + 25t/16$ and $c + \frac{243}{256}t \leq k < \mathrm{c} + \left(\frac{121}{16(13-4\sqrt{3})}\right)\mathrm{t}$ (defection with relocation)	positive
Maximal differentiation equilibrium $k < c + 25t/72$	$\bar{u} \geq c - k + 25t/16$ (defection without relocation, price $\bar{p}$)	none
	$\bar{u} < c - k + 25t/16$ (defection without relocation, price $\tilde{p}$	**negative**

Table 8

Observing Table 8, we can see that, in case the minimal differentiation equilibrium prevails, an increase in the advertising market dimension will increase the critical discount factor and make collusion less likely. When instead the maximal differentiation equilibrium prevails higher advertising revenues per reader do not affect the likelihood of collusion when the deviant publisher's optimal price allows him to gain all the market (price $\bar{p}$) but would instead increase the critical discount factor and therefore make collusion more difficult if the deviant publisher 's optimal price allows him to appropriate all customers (price $\tilde{p}$). Interestingly, this respectively depends on whether $\bar{u} > \frac{75}{72}t$ or $\bar{u} < \frac{75}{72}$t. Hence there is no effect of a larger advertising market on the incentives to collude if the reservation price is high enough.

Overall, when analysing the incentives to collude on both the cover price and the political position, the effects of an increase in k are very similar to those found analysing the incentives on cover prices only.

To conclude, the following corollary holds

Corollary 7 *– When firms collude on both cover prices and political locations, a larger advertising market has a non-monotone effect on the incentives to collude: when the advertising market is small, an increase in the size of the advertising market favours collusion, but when the advertising market is sufficiently large, a further increase will make collusion less likely. The latter effect however disappears if readers' reservation price is sufficiently high.*

Proof. See Appendix.

3.5. Relocation costs and the form of collusion

A possible weakness of the discussion above is represented by the assumption that publishers can change the political orientation of their newspapers at any repetition of the stage game without any time constraint and without incurring any cost. However, in reality, changing political orientations may in fact be more complicated than changing prices. A publisher wishing to move the newspaper from left to right might for instance need to substitute part of its left-wing journalists who are not ready to change their articles' line.

Let us assume that every time a publisher changes her political orientation, she has to bear a non-negative sunk cost F, no matter how much it is changed or the direction of the change. This cost can well represent the expenses linked to the recruitment of new signatures and to marketing campaigns. The higher the sunk cost, the more difficult will be changing the political orientation with respect to prices. Since it is a cost associated with relocation of the newspaper on the political line, we hereafter call it *relocation cost*.

First of all, let us consider two extreme cases: in the first one the relocation cost is zero, in the second it is infinite. In fact the first case is represented by the model studied until now; instead, the second corresponds to a repeated Gabzsewicz, Laussel and Sonnac (2002) where locations are selected at the beginning, once forever. The game is in fact very similar to the one analysed by Friedman and Thisse (1979). In such a case, collusion on prices is the only possibility: no agreement on political orientations could arise since a defection at the very first step could not be punished. Furthermore, it can be easily derived that the final outcome and the defection strategy found for collusion on prices only and $F = 0$ can apply with F infinite too.

One would need to consider also the intermediate and more general case in which F is finite and positive. Both forms of collusion can be sustained in theory for given sets of parameters and do not change their characteristics in terms of outcomes. However, the sustainability of collusion is now altered by the relocation cost: in some cases, the punishment strategy implies relocation and therefore leads to different payoffs. Accordingly, the sustainability of collusion should be analyzed looking at different critical discount factors.

As before, this critical discount factor depends on the type of collusion and on the prevailing punishment equilibrium. If locations change between collusion, defection and/or punishment, publishers pay the relocation cost F. It is easy to notice that in one case locations do not change: it is when publishers collude on prices only and the prevailing equilibrium is the minimal differentiation one. In all other cases, locations change in the first punishment turn. The critical discount factor can be derived starting from the streams of payoffs associated with defection and cooperation and will be a function of F.

Unsurprisingly, it can be shown that the critical discount factor decreases in F so that the bigger F, the more likely collusion is, no matter which is the type of collusion. Accordingly, as long as F is less than $\pi_D - \pi_N$, both kinds of collusion make sense, with collusion being more likely as F grows. This condition guarantees that the relocation cost is not so high that it offsets the benefits of defection. If it actually does, the case is similar to the one with an infinite relocation cost. When F approaches $\pi_D - \pi_N$, only collusion on prices remains a possibility.

4. Conclusions

We analysed a newspaper market where two editors first choose the political position of their newspaper, then set cover prices and advertising tariffs.

We built on the work of Gabszewicz, Laussel and Sonnac (2001, 2002), who show that in competition advertising financing tends to reduce political differentiation among newspapers and can explain the ascent of the so-called *Pensée Unique.* This is more likely the larger the advertising market and therefore the larger the per reader revenues from advertising.

We took their model as the stage game of an infinitely repeated game and investigated the incentives to collude using grimm trigger strategies and the properties of the collusive agreements in terms of welfare and pluralism.

As in Gabszewicz, Laussel and Sonnac (2001, 2002), we assumed newspapers enjoy monopoly power over advertisers, that may multi-home, for access to their readers, who are instead assumed to single-home. We further assumed that readers are not affected by the quantity of advertising on the newspaper. We justified this assumption referring to the empirical literature on newspaper markets.

We thus analysed and compared two types of collusion: in the first, publishers cooperatively select both cover prices and political position; in the second, publishers cooperatively select cover prices only. In our setting there is in fact no gain for publishers from collusion on the advertising market.

Whereas full collusion leads to intermediate product differentiation, collusion on prices only leads, as in Gabszewicz, Laussel and Sonnac (2001, 2002), to minimal product differentiation. However, in the latter case, differently from Gabszewicz, Laussel and Sonnac (2001, 2002), cover prices are positive and the minimal differentiation outcome does not depend on the size of the advertising market. Hence, collusion on prices reinforces the tendency towards a Pensée Unique discussed in Gabszewicz, Laussel and Sonnac (2001). Indeed, if advertising revenues are so low that maximal differentiation would be the competitive outcome, the higher they are the more likely is collusion. Yet we also showed that, whenever advertising revenues are so high that minimum differentiation arises as a competitive equilibrium, then the higher advertising revenues, the less likely is collusion.

Our findings question the rationale for Joint Operating Agreements among US newspapers, regulated by the Newspaper Preservation Act of 1970, according to which publishers are allowed to cooperate in setting cover prices and advertising tariffs but not the editorial line. The rationale of the exemption from cartel law was to keep different newspapers alive and thus guarantee different editorial lines. Our model however predicts that in such a situation editorial lines would tend to converge much more than if political positions were set cooperatively. More generally, allowing a Joint Operating Agreement as an alternative to a merger among the same newspapers would not seem justified by the above objective. The prediction in our model is in line with the empirical finding of George (2007), who shows that differentiation increases with ownership concentration. Sweeting (2010) that firms that buy competing stations tend to differentiate them more among themselves and also from their other stations. An additional result of his empirical analysis is that merging firms tend to reposition their stations closer to their competitors, a finding consistent with the

competitive model we draw upon when advertising revenues are high enough. In fact, if the objective of the exemption from collusion on cover prices and advertising tariffs under a JOA is to allow newspapers to cover their fixed costs, then collusion on everything (or a merger cleared subject to the guarantee not to close down one newspaper) would be superior. Indeed, such an alternative would guarantee higher variable profits and also a higher total welfare.

Our analysis however shows that despite the two-sided nature of the market the more competition there is the better it is for consumers. In particular, competition yields higher welfare than collusion on prices only, which in turn outperforms collusion on both prices and the political position of newspapers. Note that this is true even if we assumed the market to be covered. Despite this assumption, no gain in the readers' welfare due to relocation of newspapers in the political spectrum is large enough to offset the price increase.

From a total welfare point of view instead a collusive agreement on prices and locations decreases readers' welfare while increasing total welfare, as publishers choose locations that minimize the sum of the political costs sustained by all readers but then extract all the extra surplus of readers through higher prices. A collusive agreement on prices only brings about no consequences for total welfare but harms readers, because prices increase while locations remain constant, so that only a redistribution of surplus takes place. The latter effect is due to the assumption that the market is covered.

We believe our analysis fits well the case of collusion in duopoly newspaper markets. An interesting extension would be to study how collusive outcomes and incentives to collude change as the number of competing newspaper increases. In a model with more than two competing newspapers, it would also be possible to analyse the impact of a collusive agreement among a subset of publishers on overall political differentiation in the market. A starting point for such an analysis would then be the paper Behringer and Filistrucchi (2011) which extends the competitive model of Gabszewicz, Laussel and Sonnac (2001, 2002) to more than two firms. A further extension to collusion might however raise some technical difficulties.

Finally, all our results have been obtained under a set of assumptions which we argued fit quite well the market for daily newspapers. Different sets of assumptions may be necessary to address the same issue in different media markets. For instance, as discussed above, one might want to allow for a negative effect of advertising on viewership of TV and possibly for a positive effect of advertising on readership of magazines. We leave all this to future research.

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Appendix

Proof of Lemma 1.

Take a state $(a, p_1), (b, p_2)$ and let y denote the marginal consumer who splits the market in the demand for 1, y, and the demand for 2, $1 - y$, in such state. Take now the lowest utility consumer as the reader with the lowest utility associated with such state: this consumer can only be in 0, 1 or y because they are the most distant from the location of the newspaper she buys. She is paying a total price lower than her reservation price. The publishers can mutually increase their payoff by simply increasing their prices by the same amount $\Delta p_1 = \Delta p_2$ so that the lowest utility consumer still buys a copy: thus, all the readers are buying a copy in this new state $(a, p_1 + \Delta p_1), (b, p_2 + \Delta p_2)$, while y is constant because the change in prices is the same for both newspapers. To conclude, the demands are unchanged but prices are higher and, as a consequence, payoffs increase. QED.

Proof of Lemma 2.

The proof consists in finding a state, which makes both the publishers better off by taking consumers in 0, y and 1 to utility 0.

To begin with, let us consider how each publisher tends to relocate when she takes into account that her demand is kept fixed by the other publisher. For example, when publisher 1's demand y is fixed, we can derive her profits from (8):

$$\pi_1 = (p_1 + k - c)y.$$

It is easy to see how the publisher will set the highest price compatible with the coverage of her demand; this optimum price will be:

$$p_1^* = \begin{cases} \bar{u} - t(y - a)^2, & \text{if } 0 \leq a < y/2 \\ \bar{u} - ta^2, & \text{if } y/2 \leq a \leq y \end{cases}$$

Her profits can be written as:

$$\pi_1 = \begin{cases} (\bar{u} - t(y - a)^2 + k - c)y, & \text{if } 0 \leq a < \dfrac{y}{2} \\ (\bar{u} - ta^2 + k - c)y, & \dfrac{\text{if } y}{2} \leq a \leq y \end{cases}$$

Taking the first derivative with respect to a, we find that

$$\begin{cases} \dfrac{\partial \pi_1}{\partial a} > 0, & \text{if } 0 \leq a < y/2 \\ \dfrac{\partial \pi_1}{\partial a} < 0, & \text{if } y/2 \leq a \leq y \end{cases} \tag{15}$$

The optimal location is therefore $a^* = y/2$. The same result can be similarly derived for publisher 2, her optimal location is $b^* = (1-y)/2$.

Expression (15) also states that each publisher tends to relocate to the middle of her demand when the other publisher simply accommodates this relocation in order to maintain the indifference at y. In fact both publishers can relocate following this tendency: the final state will imply that the demand is still at y while prices are higher and therefore payoffs are bigger. In this final state, consumers in $0, y$ and 1 are kept at utility 0. To conclude, a state in which consumers in $0, y$ and 1 are kept at utility 0 improves a state in which only one of these consumers is kept at utility 0 and the marginal consumer is at y.[31] QED.

Proof* of *Lemma 3.

The proof consists in checking that starting from any point on the unit interval, any change in y does not increase the payoff of one publisher without decreasing the payoff of the other publisher.

First, profits can be written as:

$$\pi_1 = \left(\bar{u} - t\frac{y^2}{4} + k - c\right)y$$
$$\pi_2 = \left(\bar{u} - t\frac{(1-y)^2}{4} + k - c\right)(1-y)$$

Differentiating the profit functions with respect to y we find that, given the parameters compatible with the assumptions of this model:

$$\frac{\partial \pi_1}{\partial y} > 0 \ \ \forall y \in [0,1]$$
$$\frac{\partial \pi_2}{\partial y} < 0 \ \ \forall y \in [0,1]$$

Therefore in all cases the profit of one publisher cannot be increased without the profit of the other to decrease. QED.

Proof of Corollary 1

Let us consider the profits made by a publisher located at ¼ who increases its price above $\bar{u} - t/16$. The readership associated with this new price cannot be identified through the demand functions in (4). Nevertheless, it is easy to note that the readership shrinks and is distributed for one half to the left of the location and for the other half to the right. The marginal

[31] In the proof, the respective demands are kept constant to the touch state levels. This is only a fiction that helps to characterize Pareto optima: once they are fully characterized, different Pareto optima will imply different market shares and the publishers will be able to choose among them.

consumer on the right endpoint obtains utility zero[32]; recalling (1), we find the location of this reader is

$$x_{mc} = \frac{1}{4} + \sqrt{\frac{\bar{u} - p}{t}}$$

Thus, it is easy to observe that the readership is now

$$n = 2\sqrt{\frac{\bar{u} - p}{t}}$$

Recalling the profit function in (8), we can observe that the new profit function is

$$\pi = 2\sqrt{\frac{\bar{u} - p}{t}}(p + k - c) \tag{18}$$

We can check that the profits in (18) are never higher than the collusive profits in (17) when the parameters set is restricted as in (3) and $p > \bar{u} - t/16$.[33] The argument is parallel for the publisher located at ¾. Accordingly, if the market is covered in competition, it will be covered also under collusion. QED

Proof of Proposition 2. The proof consists of three different steps: a) we first find the best defection strategy in case the location is shifted from ¼; b) we then find the best defection strategy keeping location fixed at ¼; c) finally, we compare the two defection strategies to find the overall optimal one.

a) When a publisher defects at the location step, the other publisher punishes her from the subsequent step onwards; as a result, in the defection turn the deviant selects a location different from ¼ while the other one actually plays ¼ and they play a Nash equilibrium in prices given locations at the second step. If, for each pair of locations in the first step, there is a unique Nash equilibrium in the price game, when choosing the location the deviant knows which Nash equilibrium will follow in the second step; in other words, she will select the preferred Nash equilibrium as she choose the location.

First of all, it is necessary to find the Nash equilibria given the pair of political orientations $(a, ¼)$. Following Gabsewicz, Laussel and Sonnac (2002) and starting from the profit function in (8), we can find the best prices of publisher 1 and 2 respectively:

$$p_1 = \max\left\{0, \frac{1}{2}(c - k + p_2 + \frac{9}{16}t - a^2 t)\right\} \tag{32}$$

[32] In this case, she is indifferent between buying a copy and not buying.
[33] It should be remembered that the restrictions in (3) ensures the competitive equilibria exist once the market coverage condition is assumed.

$$p_2 = \max\left\{0, \frac{1}{2}(c - k + p_1 + \frac{15}{16}t - 2at + ta^2)\right\} \tag{33}$$

By substitution we can find Nash equilibria in prices. The non-negativity constraint on price levels leads us to four regions, as in Gabzewicz, Laussel and Sonnac (2002).

Region 1.

$$p_1 = c - k + \frac{11}{16}t - \frac{2}{3}at - \frac{1}{3}a^2t \tag{34}$$

$$p_2 = c - k + \frac{13}{16}t - \frac{4}{3}at + \frac{1}{3}a^2t \tag{35}$$

Region 2.

$$p_1 = \frac{1}{2}\left(c - k + \frac{9}{16}t - a^2t\right) \tag{36}$$

$$p_2 = 0 \tag{37}$$

Region 3.

$$p_1 = 0 \tag{38}$$

$$p_2 = \frac{1}{2}\left(c - k + \frac{15}{16}t - 2at + a^2t\right) \tag{39}$$

Region 4.

$$p_1 = 0 \tag{40}$$

$$p_2 = 0 \tag{41}$$

Note that the newspaper demands in (4) and (5) and so the profit function in (8) assume that publisher 1 is located on the left hand side of publisher 2; accordingly, the four regions are conditional on $0 \leq a \leq \frac{3}{4}$. On the other hand, publisher 1 would never locate in the interval $\frac{3}{4} < a \leq 1$ since she would be located in the smaller segment of the political spectrum.

We now discuss the restrictions that need to be satisfied in order for the different regions (and thus the different equilibria) to be admissible.

If $c > k$, we have $\frac{1}{2}\left(c - k + p_2 + \frac{9}{16}t - a^2t\right) > 0$ and $\frac{1}{2}\left(c - k + p_1 + \frac{15}{16}t - 2at + ta^2\right) > 0$ in $0 \leq a \leq 3/4$. Thus, only Region 1 is admissible.

If $k > c$. Suppose region 4 is admissible in $0 \leq a \leq 3/4$. Then we need $\frac{1}{2}\left(c - k + \frac{9}{16}t\right) < 0$ and $\frac{1}{2}\left(c - k + \frac{15}{16}t\right) < 0$, therefore, $t \leq \frac{16}{15}(k - c)$.

If $t > \frac{16}{15}(k-c)$, at $a = 0, \frac{1}{2}\left(c - k + \frac{15}{16}t\right) > 0$ and $p_2 > 0$. If t is large(small) enough, region 1(3) will be admissible in the small interval around $a = 0$. We substitute $p_2 = \frac{1}{2}\left(c - k + \frac{15}{16}t - 2at + a^2t\right)$ into $\frac{1}{2}(c - k + p_2 + \frac{9}{16}t - a^2t)$ and let $a = 0$, we have the condition $\frac{3}{2}(c-k) + \frac{33}{32}t < 0 \rightarrow t < \frac{16}{11}(k-c)$, under which region 3 is admissible around $a = 0$. Then we solve a for:

$$p_2 = \frac{1}{2}\left(c - k + \frac{15}{16}t - 2at + a^2t\right) = 0$$

And we obtain:

$$a_1 = 1 - \frac{1}{2}\sqrt{4\frac{(k-c)}{t} + \frac{1}{4}}$$

That is, for $\frac{16}{15}(k-c) < t \leq \frac{16}{11}(k-c)$

Region 3 is admissible for $0 \leq a < a_1$

Region 4 is admissible for $a_1 \leq a \leq 3/4$

If $\frac{16}{11}(k-c) < t$, region 1 is admissible around $a = 0$. We use the fact:

$$p_1 = c - k + \frac{11}{16}t - \frac{2}{3}at - \frac{1}{3}a^2t$$

$$p_2 = c - k + \frac{13}{16}t - \frac{4}{3}at + \frac{1}{3}a^2t$$

$$p_2 - p_1 = \frac{1}{8}t - \frac{2}{3}at + \frac{2}{3}a^2t = 0 \quad \text{with solution } a = \frac{1}{4} \text{ and } a = \frac{3}{4}$$

This means, for $a < \frac{1}{4}$ we have $p_1 < p_2$, while for $\frac{1}{4} < a \leq \frac{3}{4}$ we have $p_2 < p_1$

$$p_1 = c - k + \frac{11}{16}t - \frac{1}{6}t - \frac{1}{48}t = c - k + \frac{1}{2}t, \text{ at } a = \frac{1}{4}$$

So, if $t < 2(k-c)$, region 1 is admissible around $a = 0$ and region 3 is admissible around $a = \frac{1}{4}$. Solving $p_1 = c - k + \frac{11}{16}t - \frac{2}{3}at - \frac{1}{3}a^2t = 0$, we obtain:

$$a_2 = -1 + \frac{3}{2}\sqrt{\frac{4(c-k)}{3t} + \frac{49}{36}}$$

To conclude, when $\frac{16}{11}(k-c) < t \leq 2(k-c)$

Region 1 is admissible for $0 \leq a < a_2$

Region 3 is admissible for $a_2 \leq a < a_1$

Region 4 is admissible for $a_1 \leq a < 3/4$

If $2(k-c) < t$, solving $p_2 = c - k + \frac{13}{16}t - \frac{4}{3}at + \frac{1}{3}a^2t = 0$, we obtain:

$$a_3 = 2 - \frac{3}{2}\sqrt{\frac{4(k-c)}{3t} + \frac{25}{36}}$$

For $a_3 \leq a, p_2 = 0$ and solving $\frac{1}{2}\left(c - k + p_2 + \frac{9}{16}t - a^2t\right) = 0$, we obtain:

$$a_4 = \sqrt{\frac{c-k}{t} + \frac{9}{16}}$$

To conclude , when $t > 2(k-c)$

Region 1 is admissible for $0 \leq a < a_3$

Region 2 is admissible for $a_3 \leq a < a_4$

Region 4 is admissible for $a_4 \leq a \leq 3/4$

Summing up, the four regions are admissible for the following sets of parameters:

- If $c > k$, only Region 1 is admissible.
- If $k > c$ and
  - $0 < t \leq \frac{16}{15}(k-c)$
    - ✓ only Region 4 is admissible in $0 \leq a \leq 3/4$
  - $\frac{16}{15}(k-c) < t \leq \frac{16}{11}(k-c)$
    - ✓ Region 3 is admissible for $0 \leq a < a_1$
    - ✓ Region 4 is admissible for $a_1 \leq a \leq 3/4$
  - $\frac{16}{11}(k-c) < t \leq 2(k-c)$
    - ✓ Region 1 is admissible for $0 \leq a < a_2$
    - ✓ Region 3 is admissible for $a_2 \leq a < a_1$
    - ✓ Region 4 is admissible for $a_1 \leq a < 3/4$
  - $t > 2(k-c)$
    - ✓ Region 1 is admissible for $0 \leq a < a_3$
    - ✓ Region 2 is admissible for $a_3 \leq a < a_4$
    - ✓ Region 4 is admissible for $a_4 \leq a \leq 3/4$

where

$$a_1 = 1 - \frac{1}{2}\sqrt{4\frac{(k-c)}{t} + \frac{1}{4}}$$

$$a_2 = -1 + \frac{3}{2}\sqrt{\frac{4(k-c)}{3t} + \frac{49}{36}}$$

$$a_3 = 2 - \frac{3}{2}\sqrt{\frac{4(k-c)}{3t} + \frac{25}{36}}$$

$$a_4 = \sqrt{\frac{c-k}{t} + \frac{9}{16}}$$

Once the Nash equilibria are fully characterized, we proceed by backward induction and find the optimal location for the deviant newspaper in the first step. Therefore, we need to find the maximum of the profit function for every possible set of parameters. We start by inspecting the sign of the derivative of the profits with respect to location a in every possible region.

In Region 1

$$\frac{\partial \pi}{\partial a} < 0 \qquad \text{when applicable}$$

In Region 2

$$\text{If } 2(k-c) \leq t < \frac{50}{9}(k-c),\ \frac{\partial \pi}{\partial a} > 0 \ \ \forall a \in [a_3, a_4)$$

$$\text{If } \frac{50}{9}(k-c) \leq t, \qquad \begin{cases} \frac{\partial \pi}{\partial a} < 0 \ \ \forall a \in [a_3, a_5) \\ \frac{\partial \pi}{\partial a} > 0 \ \ \forall a \in (a_3, a_4) \end{cases}$$

$$\text{where } \ a_5 = \frac{1}{2} + \frac{1}{6}\sqrt{\frac{12(k-c)}{t} + \frac{9}{4}}$$

In Region 3

$$\frac{\partial \pi}{\partial a} < 0 \qquad \text{when applicable}$$

In Region 4

$$\frac{\partial \pi}{\partial a} > 0 \qquad \text{when applicable}$$

We then look for the maximal profits in each of the parameter regions.

If $\mathrm{k} > \mathrm{c}, 0 < \mathrm{t} \leq \frac{16}{15}(\mathrm{k}-\mathrm{c})$, only region 4 is admissible and $\frac{\partial \pi}{\partial \mathrm{a}} > 0$. Maximal profit obtained at $\mathrm{a} = \frac{3}{4}$ and $\pi^4 = \frac{3}{4}(\mathrm{k}-\mathrm{c})$

If $\frac{16}{15}(k-c) < t \leq \frac{16}{11}(k-c)$, we have two potential maximal profit at $\mathrm{a} = \frac{3}{4}$ or 0, with profit $\pi^3|\mathrm{a}=0 = \frac{1}{3\mathrm{t}}(\mathrm{k}-\mathrm{c})(\frac{33}{16}\mathrm{t}+\mathrm{c}-\mathrm{k})$ and $\pi^4|(\mathrm{a}=\frac{3}{4}) = \frac{3}{4}(\mathrm{k}-\mathrm{c})$.

If $\frac{16}{11}(k-c) < t \leq 2(k-c)$, we also have two potential maximal profit at $\mathrm{a} = \frac{3}{4}$ or 0, with profts $\pi^1|(\mathrm{a}=0) = \frac{\mathrm{t}}{18}\left(\frac{11}{4}\right)^2\frac{3}{4}$ and $\pi^4|(\mathrm{a}=\frac{3}{4}) = \frac{3}{4}(\mathrm{k}-\mathrm{c})$

If $t > 2(k-c)$, for $\mathrm{a} \in [0, \mathrm{a}_3)$, Region 1 is admissible and maximal profit is $\pi^1|(\mathrm{a}=0) = \frac{\mathrm{t}}{18}\left(\frac{11}{4}\right)^2\frac{3}{4}$; for $\mathrm{a} \in (\mathrm{a}_3, \mathrm{a}_4]$, Region 2 is admissible and there is no critical value(max) in this interval; for $\mathrm{a} \in (\mathrm{a}_4, \frac{3}{4}]$, Region 4 is admissible and maximal profit in this inverval is $\pi^4|(\mathrm{a}=\frac{3}{4}) = \frac{3}{4}(\mathrm{k}-\mathrm{c})$.

Now, to obtain conditions which make a=0 be the optimal strategy, we have:

a=0, if:

$$\begin{aligned}\left\{\frac{16}{15}(\mathrm{k}-\mathrm{c}) < t \leq \frac{16}{11}(\mathrm{k}-\mathrm{c}) \wedge \frac{1}{3\mathrm{t}}(\mathrm{k}-\mathrm{c})\left(\frac{33}{16}\mathrm{t}+\mathrm{c}-\mathrm{k}\right) > \frac{3}{4}(\mathrm{k}-\mathrm{c})\right\} \\ \vee \left\{\frac{16}{15}(\mathrm{k}-\mathrm{c}) < t \leq 2(\mathrm{k}-\mathrm{c}) \wedge \frac{\mathrm{t}}{18}\left(\frac{11}{4}\right)^2\frac{3}{4} > \frac{3}{4}(\mathrm{k}-\mathrm{c})\right\} \\ \vee \left\{2(\mathrm{k}-\mathrm{c}) < t \wedge \frac{\mathrm{t}}{18}\left(\frac{11}{4}\right)^2\frac{3}{4} > \frac{3}{4}(\mathrm{k}-\mathrm{c})\right\} = \left\{\mathrm{t} > \frac{288}{121}(\mathrm{k}-\mathrm{c})\right\}\end{aligned}$$

a=3/4 if:

$$\left\{\mathrm{k} > c \wedge 0 > t \leq \frac{288}{121}(k-c)\right\}$$

It is therefore easy to see that the critical points are $a = 0$, $a = 3/4$ and $a = a_3$. After calculating the profits made by the deviant publisher in the critical points and checking which is higher in any possible subset of parameters, we can conclude that the optimal defection strategy consists of selecting

$$a = 0 \qquad \text{if } c > k \bigvee (k > c \bigwedge t > \frac{288}{121}(k-c))$$

$$a = \frac{3}{4} \qquad \text{if } k > c \bigwedge 0 < t \leq \frac{288}{121}(k-c)$$

at the first step and then applying the Nash equilibrium strategy in the prices step.

Let us denote $\pi(a=0)$ the profits made by the deviant in the defection turn when she selects $a=0$ in the first step, and $\pi(a=3/4)$ the profits made by the deviant in the defection turn when she selects $a=3/4$ in the first step.

These are respectively

$$\pi(a=0)=\frac{55}{192}t$$

$$\pi(a=3/4)=\frac{3}{4}(k-c)$$

b) Optimal defection with location fixed at ¼ is easier to obtain since one simply needs to find a defection price for the second step of the stage game, given that the pair of locations is fixed at (1/4,1/4) and the non-deviant publisher applies a price equal to $\bar{u}-\frac{t}{16}$. The deviant publisher maximizes the profits in the price.

By maximising the profits in (8) with respect to p_1 with $a=\frac{1}{4}$, $b=\frac{1}{4}$ and $p_2=\bar{u}-\frac{t}{16}$, we obtain that

$$1)\quad \frac{\partial\pi}{\partial p}>0 \text{ for } p<\frac{1}{2}\left(\bar{u}+c-k+\frac{7}{16}t\right)$$

$$2)\quad \frac{\partial\pi}{\partial p}<0 \text{ for } p>\frac{1}{2}\left(\bar{u}+c-k+\frac{7}{16}t\right)$$

$$3)\quad \frac{\partial\pi}{\partial p}=0 \text{ for } p=\frac{1}{2}\left(\bar{u}+c-k+\frac{7}{16}t\right)$$

Therefore, the optimal price is

$$p_1^*=\frac{1}{2}\left(\bar{u}+c-k+\frac{7}{16}t\right) \tag{42}$$

Nevertheless, it should be noticed that this optimal price is meaningful as long as it implies a total demand lower than 1. When defecting, the deviant publisher gets part of the demand of the non-deviant publisher; if she gets all the demand of the other, her demand will be 1 and any smaller price would not imply more demand. It is easy to check that the threshold price for which the deviant's demand is exactly 1 is

$$p=\bar{u}-\frac{9}{16}t \tag{43}$$

There is no reason why the publisher should charge a lower price; hence, if the price in (42) is smaller than the threshold price in (43), the best price is the threshold price itself. Thus following the optimal defection strategy for this case is:

$$p_1^*=\frac{1}{2}\left(\bar{u}+c-k+\frac{7}{16}t\right) \quad \text{if} \quad \bar{u}\leq c-k+\frac{25}{16}t \tag{44}$$

$$p_1^* = \bar{u} - \frac{9}{16} \quad \text{if} \quad \bar{u} > c - k + \frac{25}{16}t \tag{45}$$

Let us denote profits made applying the optimal price in (44) $\pi(\tilde{p})$ and profits made with the optimal price in (45) $\pi(\bar{p})$.

$$\pi(\tilde{p}) = \frac{(\bar{u} + k - c + \frac{7}{16}t)^2}{4t}$$

$$\pi(\bar{p}) = \bar{u} - \frac{9}{16}t + k - c$$

c) Finally we need only compare profits made in the two alternative defections. In other words, we have to compare $\pi(\tilde{p})$ and $\pi(\bar{p})$ with $\pi(a = 0)$ and $\pi(a = 3/4)$ under the conditions

$$\begin{cases} \bar{u} \geq \dfrac{t}{4} \\ \bar{u} \geq \dfrac{5t}{4} + c - k \end{cases} \tag{3}$$

First, we compare $\pi(a = 0)$ with $\pi(\tilde{p})$ and $\pi(\bar{p})$. We need to show that $\pi(\tilde{p}) > \pi(a = 0)$ and $\pi(\bar{p}) > \pi(a = 0)$.

1. If $\bar{u} \leq c - k + \frac{25}{16}t$, to show that $\pi(a = 0) < \pi(\tilde{p})$, and therefore to show that, $\frac{\pi(\tilde{p})}{\pi(a=0)} = \frac{(\bar{u}+k-c+\frac{7}{16}t)^2}{4t^2}\frac{192}{55} > 1$. Since $t > \frac{288}{121}(k - c)$, we have $\frac{5t}{4} + c - k > \frac{t}{4}$, so we have $\frac{5t}{4} + c - k \leq \bar{u} \leq c - k + \frac{25}{16}t$. The minimal value of $\frac{(\bar{u}+k-c+\frac{7}{16}t)^2}{4t^2}\frac{192}{55}$ is obtained at $\frac{5t}{4} + c - k = \bar{u}$ and is > 1.

2. if $\bar{u} > c - k + \frac{25}{16}t > \frac{5}{4}t + c - k$, we then compare $\pi(a = 0)$ with $\pi(\bar{p})$. Since $t > \frac{288}{121}(k - c)$, we also have $c - k + \frac{25}{16}t > \frac{1}{4}t$. So, $\pi(\bar{p}) - \pi(a = 0) = \bar{u} + k - c - \frac{9}{16}t - \frac{55}{192}t > \frac{25}{16}t - \frac{9}{16}t - \frac{55}{192}t > 0$.

In sum, $\pi(\tilde{p})$ and $\pi(\bar{p})$ dominate $\pi(a = 0)$ in any cases. Discussion when $c > k$ is trivial.

Second, we compare $\pi\left(a = \frac{3}{4}\right)$ with $\pi(\tilde{p})$ and $\pi(\bar{p})$. We need to show that $\pi(\tilde{p}) > \pi\left(a = \frac{3}{4}\right)$ and $\pi(\bar{p}) > \pi\left(a = \frac{3}{4}\right)$.

We need to compare $\pi\left(a = \frac{3}{4}\right)$ and $\pi(\bar{p})$ under the condition:

$$\begin{cases} t \leq \dfrac{288}{121}(k - c) \\ \bar{u} \geq \dfrac{t}{4} \\ \bar{u} \geq \dfrac{5}{4}t + c - k \\ \bar{u} > c - k + \dfrac{25}{16}t \end{cases}$$

1. If $\frac{\mathrm{t}}{4} > c - k + \frac{25}{16}\mathrm{t} \leftrightarrow \mathrm{t} < \frac{16}{21}(\mathrm{k} - \mathrm{c})$, the condition reduces to be:

$$\begin{cases} \mathrm{t} \leq \frac{16}{21}(\mathrm{k} - \mathrm{c}) \\ \bar{\mathrm{u}} \geq \frac{\mathrm{t}}{4} \end{cases}$$

$$\pi(\bar{p}) - \pi\left(a = \frac{3}{4}\right) = \bar{u} - \frac{9}{16}t + \frac{1}{4}(k - c) > \frac{t}{4} - \frac{9}{16}t + \frac{1}{4}(k - c) = \frac{1}{4}\left(k - c - \frac{5}{4}t\right)$$
$$> \frac{1}{4}\left(k - c - \frac{5}{4} * \frac{4}{21}(k - c)\right) > 0$$

2. if $\frac{16}{21}(\mathrm{k} - \mathrm{c}) \leq \mathrm{t}$, the condition reduces to be:

$$\begin{cases} \frac{16}{21}(\mathrm{k} - \mathrm{c}) < t \leq \frac{288}{121}(\mathrm{k} - \mathrm{c}) \\ \bar{\mathrm{u}} > c - k + \frac{25}{16}\mathrm{t} \end{cases}$$

$$\pi(\bar{p}) - \pi\left(a = \frac{3}{4}\right) = \bar{u} - \frac{9}{16}t + k - c - \frac{3}{4}(k - c) > \frac{25}{16}t - \frac{9}{16}t - \frac{3}{4}(k - c) = t - \frac{3}{4}(k - c)$$
$$> \left(\frac{16}{21} - \frac{3}{4}\right)(k - c) > 0$$

We then need to compare $\pi\left(a = \frac{3}{4}\right)$ and $\pi(\tilde{p})$ under the conditions:

$$\begin{cases} \mathrm{t} \leq \frac{288}{121}(\mathrm{k} - \mathrm{c}) \\ \bar{\mathrm{u}} \geq \frac{\mathrm{t}}{4} \\ \bar{\mathrm{u}} \geq \frac{5}{4}\mathrm{t} + \mathrm{c} - \mathrm{k} \\ \bar{\mathrm{u}} \leq \mathrm{c} - \mathrm{k} + \frac{25}{16}\mathrm{t} \end{cases}$$

Since, it makes sense only when $\mathrm{c} - \mathrm{k} + \frac{25}{16}\mathrm{t} > \frac{\mathrm{t}}{4}$, the condition above reduces to be:

$$\begin{cases} \frac{16}{21}(\mathrm{k} - \mathrm{c}) < t \leq \frac{288}{121}(\mathrm{k} - \mathrm{c}) \\ \bar{\mathrm{u}} \geq \frac{\mathrm{t}}{4} \\ \bar{\mathrm{u}} \geq \frac{5}{4}\mathrm{t} + \mathrm{c} - \mathrm{k} \\ \bar{\mathrm{u}} \leq \mathrm{c} - \mathrm{k} + \frac{25}{16}\mathrm{t} \end{cases}$$

If $\mathrm{t} > k - \mathrm{c}, \frac{5}{4}\mathrm{t} + \mathrm{c} - \mathrm{k} > \frac{\mathrm{t}}{4}$, we have

$$\begin{cases} \mathrm{k}-\mathrm{c}<t\leq\dfrac{288}{121}(\mathrm{k}-\mathrm{c}) \\ \dfrac{5}{4}\mathrm{t}+\mathrm{c}-\mathrm{k}\leq\bar{\mathrm{u}}\leq\mathrm{c}-\mathrm{k}+\dfrac{25}{16}\mathrm{t} \end{cases}$$

$$\frac{\pi(\tilde{p})}{\pi\left(a=\frac{3}{4}\right)}=\frac{(\bar{\mathrm{u}}+\mathrm{k}-\mathrm{c}+\frac{7}{16}\mathrm{t})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}\geq\frac{(\frac{5}{4}\mathrm{t}+\frac{7}{16}\mathrm{t})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}=\frac{27^2}{16^2}*\frac{t}{3(k-c)}>\frac{27^2}{16^2}*\frac{1}{3}=0.95$$

Since $0.95<1$ than 1. So, we cannot guarantee that $\pi\left(a=\frac{3}{4}\right)<\pi(\tilde{p})$.

We then find the parameter interval such that no incentive to defect exists:

If $\mathrm{t}>k-c$, $\frac{5}{4}\mathrm{t}+\mathrm{c}-\mathrm{k}>\frac{\mathrm{t}}{4}$, we have:

$$\begin{cases} \mathrm{k}-\mathrm{c}<t\leq\dfrac{288}{121}(\mathrm{k}-\mathrm{c}) \\ \dfrac{5}{4}\mathrm{t}+\mathrm{c}-\mathrm{k}\leq\bar{\mathrm{u}}\leq\mathrm{c}-\mathrm{k}+\dfrac{25}{16}\mathrm{t} \end{cases}$$

$$\frac{\pi(\tilde{p})}{\pi\left(a=\frac{3}{4}\right)}=\frac{(\bar{\mathrm{u}}+\mathrm{k}-\mathrm{c}+\frac{7}{16}\mathrm{t})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}\geq\frac{(\frac{5}{4}\mathrm{t}+\frac{7}{16}\mathrm{t})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}=\frac{27^2}{16^2}*\frac{t}{3(k-c)}$$

To make $\frac{27^2}{16^2}*\frac{t}{3(k-c)}>1$, we have $\mathrm{t}>\frac{256}{243}(\mathrm{k}-\mathrm{c})$.

If $\mathrm{t}<k-c$, thus, $\frac{5}{4}\mathrm{t}+\mathrm{c}-\mathrm{k}<\frac{\mathrm{t}}{4}$, we have:

$$\begin{cases} \dfrac{16}{21}(\mathrm{k}-\mathrm{c})<t<\mathrm{k}-\mathrm{c} \\ \dfrac{\mathrm{t}}{4}\leq\bar{\mathrm{u}}\leq\mathrm{c}-\mathrm{k}+\dfrac{25}{16}\mathrm{t} \end{cases}$$

$$\frac{\pi(\tilde{p})}{\pi\left(a=\frac{3}{4}\right)}=\frac{(\bar{\mathrm{u}}+\mathrm{k}-\mathrm{c}+\frac{7}{16}\mathrm{t})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}\geq\frac{(\frac{1}{4}\mathrm{t}+\frac{7}{16}\mathrm{t}+\mathrm{k}-\mathrm{c})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}=\frac{(\frac{11}{16}\mathrm{t}+\mathrm{k}-\mathrm{c})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}$$

To make $\frac{(\frac{11}{16}\mathrm{t}+\mathrm{k}-\mathrm{c})^2}{3\mathrm{t}(\mathrm{k}-\mathrm{c})}>1$, we have

$$(\frac{11}{16}\mathrm{t}+\mathrm{k}-\mathrm{c})^2>3t(\mathrm{k}-\mathrm{c})\leftrightarrow\frac{121}{256}\mathrm{t}^2-\frac{13}{8}\mathrm{t}(\mathrm{k}-\mathrm{c})+(\mathrm{k}-\mathrm{c})^2>0$$

Define $f(t)=\frac{121}{256}\mathrm{t}^2-\frac{13}{8}\mathrm{t}(\mathrm{k}-\mathrm{c})+(\mathrm{k}-\mathrm{c})^2$

$\frac{\partial f(t)}{\partial t}<0$ for $\frac{16}{21}(\mathrm{k}-\mathrm{c})<t<\mathrm{k}-\mathrm{c}$. Solve $f(t)=0$, we have:

$$\bar{\mathrm{t}}=\left(\frac{208}{121}-\frac{64}{121}\sqrt{3}\right)(\mathrm{k}-\mathrm{c})$$

Thus, to make defection invalid, we need $\frac{16}{21}(\mathrm{k}-\mathrm{c})<t\leq\left(\frac{208}{121}-\frac{64}{121}\sqrt{3}\right)(\mathrm{k}-\mathrm{c})$.

In sum, $\pi(\tilde{p}) > \pi\left(a = \frac{3}{4}\right)$ doesn't hold when:

$$\left(\frac{208}{121} - \frac{64}{121}\sqrt{3}\right)(\mathrm{k} - \mathrm{c}) < t \leq \frac{256}{243}(\mathrm{k} - \mathrm{c})$$

It is easy to check that in every possible subset of parameters, $\pi(\tilde{p})$ and $\pi(\bar{p})$ are higher than $\pi(a = 0)$ and $\pi(a = 3/4)$ respectively, except for the parameter range $\left(\frac{208}{121} - \frac{64}{121}\sqrt{3}\right)(\mathrm{k} - \mathrm{c}) < t \leq \frac{256}{243}(\mathrm{k} - \mathrm{c})$. When $\left(\frac{208}{121} - \frac{64}{121}\sqrt{3}\right)(\mathrm{k} - \mathrm{c}) < t \leq \frac{256}{243}(\mathrm{k} - \mathrm{c})$, we have $\pi(a = 3/4) > \pi(\tilde{p})$. QED

Proof of Lemma 5.

We can start analysing the second and third case of Definition 3. The demonstration is parallel and will be given for case 2 only. When this is the case, the consumer located at 0 is the reservation price consumer: for the market to be entirely covered in any different state with (a, b) given, $\mathrm{p}_1 \leq \bar{\mathrm{u}} - \mathrm{ta}^2$. For any $\mathrm{p}_1 < \bar{\mathrm{u}} - \mathrm{ta}^2$, the state can be improved for Proposition 1 (it is not a touch state anymore). Therefore, only p_2 can be modified: it cannot be decreased because with p_1 fixed at $\bar{\mathrm{u}} - \mathrm{ta}^2$ for the just mentioned reasons, this would lead to a decrease in publisher 1's profits; it can be increased on condition that publisher 2's profits increase. In fact, in this case publisher 1's profits would increase. With (a, b) given and $\mathrm{p}_1 = \bar{\mathrm{u}} - \mathrm{ta}^2$, publisher 2's profits can be written as

$$\pi_2 = (\mathrm{p}_2 + \mathrm{k} - \mathrm{c})\left(\frac{1 + \mathrm{b} - \mathrm{a}}{2} + \frac{\bar{\mathrm{u}} - \mathrm{ta}^2 - \mathrm{p}_2}{2\mathrm{t}(1 - \mathrm{a} - \mathrm{b})}\right).$$

Differentiating such function on p_2, we find

$$\frac{\partial \pi_2}{\partial p_2} = \frac{c - k - 2p_2 + t - 2at - b^2 t + \bar{u}}{2t(1 - a - b)}$$

We can check that this derivative is negative for $\mathrm{p}_2 = \bar{\mathrm{u}} - \mathrm{ta}^2$ and therefore increasing p_2 would lead to decrease publisher 2's profits. Consequently, no change in prices can make any publisher better off without making the other publisher worse off for case 2.

In case 1, the reservation price consumer is located in the middle between a and $1 - \mathrm{b}$; this means that any different price pair has to set the consumer on y to be the reservation price consumer; otherwise, either the market is not covered, or the state is not a touch state and can be improved. The new price pair thus needs to satisfy the following conditions

$$\begin{cases} \mathrm{p}_1 = \bar{\mathrm{u}} - \mathrm{t}(\mathrm{y} - \mathrm{a})^2 \\ \mathrm{p}_2 = \bar{\mathrm{u}} - \mathrm{t}(1 - \mathrm{y} - \mathrm{b})^2 \end{cases}$$

For a given y, the new price pair is the one above in (34). Accordingly, profits can be rewritten as

$$\begin{cases} \pi_1 = (\bar{u} - t(y-a)^2 + k - c)y \\ \pi_2 = (\bar{u} - t(1-y-b)^2 + k - c)(1-y) \end{cases}$$

Differentiating such profits functions on y, we obtain:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial y} = -c + k - a^2 t + u + 4aty - 3ty^2 \\ \dfrac{\partial \pi_2}{\partial y} = c - k + 3t - 4bt + b^2 t - u - 6ty + 4bty + 3ty^2 \end{cases}$$

We can check that with $y = \frac{1+a-b}{2}$

$$\begin{cases} \dfrac{\partial \pi_1}{\partial y} > 0 \\ \dfrac{\partial \pi_2}{\partial y} < 0 \end{cases}$$

Therefore, it is not possible to increase one's profits without decreasing the other's profits. As a consequence, the best same-price-rule is optimal, i.e. selects Pareto optima only.

To conclude, the publishers agree to follow this pricing rule at any second stage every turn when they are allowed to collude on prices only. Going backward, at the first stage they take into account this rule in order to select the optimal location. As a result, we can investigate a Nash equilibrium outcome for the stage game in order to fully characterize the collusion outcome.

Without loss of generality, it can be assumed that $a \leq 1 - b$. This does not mean that, for example, publisher 1 cannot select a location on the right hand side of publisher 2's one, but only that when she does it, she can be thought to have changed her profits to that of publisher 2.

Before moving to equilibrium behaviour, it is important to note that each case identifies a different region on $[0,1] \times [0,1]$ when $a \leq 1 - b$, $(a, b) \in [0,1] \times [0,1]$.

To do this, we firstly derive the profit functions for every of the three regions identified by the three cases of Definition 3:

$$\text{region 1)} \quad \begin{aligned} \pi_1 &= \left(\bar{u} - \frac{t}{4}(1-a-b)^2 + k - c\right)\left(\frac{1+a-b}{2}\right) \\ \pi_2 &= \left(\bar{u} - \frac{t}{4}(1-a-b)^2 + k - c\right)\left(\frac{1-a+b}{2}\right) \end{aligned} \tag{44}$$

$$\text{region 2)} \quad \begin{aligned} \pi_1 &= (\bar{u} - ta^2 + k - c)\left(\frac{1+a-b}{2}\right) \\ \pi_2 &= (\bar{u} - ta^2 + k - c)\left(\frac{1-a+b}{2}\right) \end{aligned} \tag{45}$$

[34] In fact, the starting price pair verifies the condition and represents the case of $y = \frac{1+a-b}{2}$.

$$\text{region 3)} \qquad \begin{matrix} \pi_1 = (\bar{u} - tb^2 + k - c)(\frac{1 + a - b}{2}) \\ \pi_2 = (\bar{u} - tb^2 + k - c)(\frac{1 - a + b}{2}) \end{matrix} \tag{46}$$

Given this set of profit functions depending on locations only, equilibrium behaviour is easily derived for publisher 1. Given symmetry, the same result holds for publisher 2. It can be shown that given b, differentiating the profits functions in (44), (45) and (46) on a and comparing the results with the set of parameters compatible with the model and with the respective set of variables represented by each region as in (22), (23) and (24) results in:

$$\frac{\partial \pi_1}{\partial a} > 0 \quad \text{and} \quad \frac{\partial \pi_2}{\partial b} > 0, \ \ \forall (a, b) \in [0,1] \times [0,1], \text{ with } a \leq 1 - b \tag{47}$$

Therefore the rule only selects Pareto optima. QED.

Proof of Proposition 3.

The expression in (27) implies that publisher 1, given an expectation of publisher 2's location b, whatever the region is, tends to locate at $1 - b$ because it is the maximum value of a compatible with the limit $a \leq 1 - b$; the same can be said for publisher 2: she tends to locate at $1 - a$. This behaviour derives from the fact that setting a common price at the second stage of the game implies the split of the demand between a and $1 - b$ in two equal parts: moving towards the other's location, each publisher gains demand; this is a common outcome when the prices are fixed (beach). This happens in every region and in particular, between the regions so that publishers face the same incentives moving their location towards the one of the other publisher.

Every pair of locations $(a, 1 - a)$ (or $(1 - b, b)$, that is the same), is a Nash equilibrium candidate. Only $(1/2,1/2)$ is left when we consider that when the publisher 1 locate in $1 - b$, she faces the incentive to jump on the other side of the $1 - b$ if $1 - b < 1/2$; indeed, given the demand function when the publishers locate at the same point of the unit interval, publisher 1 takes the demand on the left hand side of the point while publisher 2 takes the demand on the right hand side. The symmetric incentive is faced by publisher 2 when $1 - a < 1/2$. Therefore, the publishers can expect the exact location of the other only locating in the middle of the unit interval. QED.

Proof of Corollary 4

The proof is similar to the one given for Corollary 1.

Let us consider the joint profits made by publishers located at ½ and setting a common price above $\bar{u} - t/4$. The respective readerships associated with this new price cannot be

identified through the functions in (4) and (5). Nevertheless, it is easy to note that publisher 1 sells to readers located to the left of ½ and publisher 2 sells to readers located to the right of ½. Naturally, the readerships are equal. The marginal consumer on the right endpoint obtains utility zero[35]; recalling (1), we find the location of this reader is

$$x_{mc} = \frac{1}{2} + \sqrt{\frac{\bar{u} - p}{t}}$$

Thus, it is easy to observe that the readership of each newspaper is now

$$n = \sqrt{\frac{\bar{u} - p}{t}}$$

Recalling the profit function in (8), we can observe that the new profit function is

$$\pi = \sqrt{\frac{\bar{u} - p}{t}}(p + k - c) \tag{26}$$

We can check that the profits in (26) are never higher than the collusive profits in (25) when the parameters set is restricted as in (3) and $p > \bar{u} - t/4$.[36]

Accordingly, if the market is covered in competition, it will be covered also under collusion. QED

Proof of Proposition 4. The best reply to $(1/2, \bar{u} - t/4)$ can be found calculating the profit function of publisher 1 when publisher 2 plays such strategy, differentiating firstly with respect to the price charged by publisher 1 and then with respect the location taking into account the optimal price.

First, the profit function from (8) can be found for $(b, p_2) = \left(\frac{1}{2}, \bar{u} - \frac{t}{4}\right)$; if we take

$$\tilde{n} = \frac{a}{2} + \frac{1}{4} + \frac{\bar{u} - \frac{t}{4} - p_1}{2t\left(\frac{1}{2} - a\right)} \tag{48}$$

we obtain:

$$\pi_1 = \begin{cases} 0, & \tilde{n} < 0 \\ (p_1 + k - c)\left(\frac{a}{2} + \frac{1}{4} + \frac{\bar{u} - \frac{t}{4} - p_1}{2t\left(\frac{1}{2} - a\right)}\right), & 0 \le \tilde{n} \le 1 \\ (p_1 + k - c), & \tilde{n} > 1 \end{cases} \tag{49}$$

[35] In this case, she is indifferent between buying a copy and not buying.
[36] It should be remembered that the restrictions in (3) allow the competitive equilibria to exist once the market coverage condition is assumed.

Differentiating (42) with respect to p_1 we find that:

$$\frac{\partial \pi_1}{\partial p_1} = \begin{cases} 0, & \tilde{n} < 0 \\ \dfrac{c - k - 2p_1 - a^2 t + \bar{u}}{t - 2at}, & 0 \le \tilde{n} \le 1 \\ 1, & \tilde{n} > 1 \end{cases} \tag{50}$$

We can now analyse the optimal behaviour in the sub-interval $\left[0, \frac{1}{2}\right]$; indeed, the optimal behaviour in the subinterval $\left[\frac{1}{2}, 1\right]$ is exactly specular to the former.

As long as the demand for newspapers is between 0 and 1 (second row in (49) and (50)), publisher 1 chooses an optimal price equal to:

$$\tilde{p}_1^* = \frac{c - k - a^2 t + \bar{u}}{2} \tag{51}$$

It is important to observe that $\frac{\partial \tilde{p}_1^*}{\partial a} < 0$ in the entire unit interval, except 1/2.

This pricing rule is valid as long as the demand for newspapers is between 0 and 1. Indeed, recalling (49) with $p_1 = \tilde{p}_1^*$, we can easily obtain that:

$$0 \le \tilde{n} \le 1 \text{ if and only if } a \in [0, \bar{a}], \text{ where } \bar{a} = \frac{2t - \sqrt{-ct + kt + 2t^2 + t\bar{u}}}{t} \tag{52}$$

One can verify that:

$$0 \le \bar{a} < 1/2 \tag{53}$$

Using $\tilde{p}_1^*$ as pricing rule, publisher 1 chooses the best location $\tilde{a}^*$. Differentiating the profit function with respect to a, we obtain that:

$$\frac{\partial \pi_1}{\partial a} > 0, \qquad \forall a \in [0, \bar{a}] \tag{54}$$

As explained in Proposition 5 for a similar case, (54) makes sense as long as the pricing in (51) leads to a demand between 0 and 1.

Therefore, for $a \in \left[\bar{a}, \frac{1}{2}\right]$, the pricing $\tilde{p}_1^*$ leads to a demand equal to 1 for which this pricing is not optimal anymore: as can be seen from the third row of (43), in this case the publisher optimally selects the highest price possible. We can start identifying the price for which the demand is exactly 1 for $a \in \left[\bar{a}, \frac{1}{2}\right)$:

$$\bar{p}_1^* = \bar{u} - t(1 - a)^2 \tag{55}$$

It is easy to observe that any lower price will bring lower profits and hence is not optimal. Any higher price does not guarantee demand equal to 1: in this case we can check from (28) and (29) that profits increase when p decreases. As a consequence, $\bar{p}_1^*$ is the optimal pricing for $a \in [\bar{a}, 1/2]$. Using $\bar{p}_1^*$ as pricing rule we can check that

$$\frac{\partial \pi_1}{\partial a} > 0, \qquad \forall a \in \left[\bar{a}, \frac{1}{2}\right) \tag{56}$$

Summarizing results in (44) and (56),

$$\frac{\partial \pi_1}{\partial a} \begin{cases} > 0, & \forall a \in [0, \bar{a}) \\ > 0, & \forall a \in \left[\bar{a}, \frac{1}{2}\right) \end{cases} \tag{57}$$

It remains to show what happens when $a = 1/2$; here the pricing $\bar{p}_1^*$ leads exactly to the collusion strategy where demand for publisher 1 is ½ while the pricing $\tilde{p}_1^*$ leads to a demand equal to 1 for which $\tilde{p}_1^*$ is not optimal.

In fact, with $a = 1/2$ publisher 1 locates at the same point as publisher 2 or, from a different perspective, she does not differentiate her product: in this framework publisher 1 takes the whole market simply applying a price slightly lower than the price applied by publisher 2, in this case equal to $p_2 = \bar{u} - \frac{t}{4}$.

We can now compare this result with the one in (57): this shows that publisher 1 is incentivized to locate as close as possible to ½ applying a price that sets the consumer in 1 to utility 0; such pricing would lead to a higher price in $a = ½$, but the relative demand there would be ½ only, because publisher 2 is located at the same point and applies the same price. As just observed, here publisher 1 optimally applies a slightly lower price. Therefore, publisher 1's optimal defection strategy consists of $a = 1/2$ and $p = \bar{u} - \frac{t}{4} - \varepsilon$, with ε infinitely small. QED.

Proof of Corollary 6

From (28) and tables 1-4, we obtain

$$\pi_D = \bar{u} - \frac{t}{4} + k - c$$

$$\pi_C = \frac{\bar{u} - \frac{t}{4} + k - c}{2}$$

$$\pi_{Nmin} = \frac{k - c}{2}$$

$$\pi_{Nmax} = \frac{t}{2}$$

Substitute into $\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)}$:

$$\hat{\delta}_{max\,P} = \frac{\bar{u} - \frac{t}{4} + k - c}{2} * \frac{1}{\bar{u} - \frac{t}{4} + k - c - \frac{t}{2}} = \frac{4c - 4k + t - 4\bar{u}}{8c - 8k + 6t - 8\bar{u}}$$

$$\hat{\delta}_{minP} = \frac{\bar{u} - \frac{t}{4} + k - c}{2} * \frac{1}{\bar{u} - \frac{t}{4} + k - c - \frac{k-c}{2}} =$$

$$\frac{\bar{u} - \frac{t}{4} + k - c}{2} * \frac{1}{\bar{u} - \frac{t}{4} + \frac{k}{2} - \frac{c}{2}} = \frac{4c - 4k + t - 4\bar{u}}{4c - 4k + 2t - 8\bar{u}}$$

Taking derivatives with respect to k one obtains:

$$\frac{\partial \hat{\delta}_{\max P}}{\partial k} = (-4)(8c - 8k + 6t - 8\bar{u})^{-1} + (4c - 4k + t - 4\bar{u})(-1)(8c - 8k + 6t - 8\bar{u})^{-2}(-8)$$

$$= \frac{-4(8c - 8k + 6t - 8\bar{u})}{(8c - 8k + 6t - 8\bar{u})^2} + \frac{8(4c - 4k + t - 4\bar{u})}{(8c - 8k + 6t - 8\bar{u})^2}$$

$$= \frac{(32c - 32k + 8t - 32\bar{u}) - (32c - 32k + 24t - 32\bar{u})}{(8c - 8k + 6t - 8\bar{u})^2}$$

$$= \frac{8t - 24t}{(8c - 8k + 6t - 8\bar{u})^2} = \frac{-16t}{(8c - 8k + 6t - 8\bar{u})^2} < 0$$

$$\frac{\partial \hat{\delta}_{minP}}{\partial k} = (-4)(4c - 4k + 2t - 8\bar{u})^{-1} + (4c - 4k + t - 4\bar{u})(-1)(4c - 4k + 2t - 8\bar{u})^{-2}(-4)$$

$$= \frac{-4(4c - 4k + 2t - 8\bar{u}) + 4(4c - 4k + t - 4\bar{u})}{(4c - 4k + 2t - 8\bar{u})^{-2}}$$

$$= \frac{-8t + 32\bar{u} + 4t - 16\bar{u}}{(4c - 4k + 2t - 8\bar{u})^{-2}}$$

$$= \frac{16\bar{u} - 4t}{(4c - 4k + 2t - 8\bar{u})^{-2}} \geq 0$$

since, from (3), $\bar{u} \geq \frac{t}{4} = \frac{4t}{16}$

The derivatives with respect to c have the opposite signs, while the derivatives with respect to ū and t are also easily taken to obtain the other results in Table 6. QED.

Proof of Corollary 7

From proposition 2, we know that:

$$\pi_D^1 = \bar{u} - \frac{9}{16}t + k - c$$

$$\pi_D^2 = \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{4t}$$

$$\pi_C = \frac{\bar{u} - \frac{t}{16} + k - c}{2}$$

$$\pi_{Nmin} = \frac{k - c}{2}$$

$$\pi_{Nmax} = \frac{t}{2}$$

Substitute these into $\hat{\delta} = \frac{(\pi_D - \pi_C)}{(\pi_D - \pi_N)}$, one obtains:

$$\hat{\delta}_{\min 1LP} = \frac{(\pi_D^1 - \pi_C)}{(\pi_D^1 - \pi_{Nmin})} =$$

$$= \left[\bar{u} - \frac{9}{16}t + k - c - \frac{\bar{u} - \frac{t}{16} + k - c}{2}\right] / [\bar{u} - \frac{9}{16}t + k - c - \frac{k - c}{2}]$$

$$= \frac{c - k + \frac{17}{16}t - \bar{u}}{c - k + \frac{9}{8}t - 2\bar{u}}$$

$$\hat{\delta}_{min2LP} = \frac{(\pi_D^2 - \pi_C)}{(\pi_D^2 - \pi_{Nmin})}$$

$$= \left[\frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{4t} - \frac{\bar{u} - \frac{t}{16} + k - c}{2}\right] / [\frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{4t} - \frac{k - c}{2}]$$

$$= \frac{c - k + \frac{t}{16} - \bar{u} + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}}{c - k + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}}$$

$$\hat{\delta}_{\max 1LP} = \frac{1}{2}$$

$$\hat{\delta}_{max2LP} = \frac{(\pi_D^2 - \pi_C)}{(\pi_D^2 - \pi_{Nmax})}$$

$$= \left[\frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{4t} - \frac{\bar{u} - \frac{t}{16} + k - c}{2}\right] / [\frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{4t} - \frac{t}{2}]$$

$$= \frac{c - k + \frac{t}{16} - \bar{u} + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}}{-t + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}}$$

The maximal differentiation equilibrium requires that $\mathrm{k} \leq \mathrm{c} + \frac{\mathrm{t}}{2}$, which is equivalent to $2(\mathrm{k} - \mathrm{c}) \leq \mathrm{t}$. Since $\left(\frac{208}{121} - \frac{64}{121}\sqrt{3}\right)(\mathrm{k} - \mathrm{c}) < t \leq \frac{256}{243}(\mathrm{k} - \mathrm{c})$ is not included in $2(\mathrm{k} - \mathrm{c}) \leq \mathrm{t}$, deviation by relocation cannot take place when the punishment is the maximal differentiation equilibrium, but only when the punishment is the minimal differentiation equilibrium.

Now, for $\left(\frac{208}{121} - \frac{64}{121}\sqrt{3}\right)(\mathrm{k} - \mathrm{c}) < t \leq \frac{256}{243}(\mathrm{k} - \mathrm{c})$, we have

$$\pi_{\mathrm{D}}^{2\prime} = \frac{3}{4}(k - c)$$

$$\pi_{\mathrm{C}} = \frac{\bar{\mathrm{u}} - \frac{\mathrm{t}}{16} + \mathrm{k} - \mathrm{c}}{2}$$

$$\pi_{\mathrm{Nmin}} = \frac{\mathrm{k} - \mathrm{c}}{2}$$

$$\pi_{\mathrm{Nmax}} = \frac{\mathrm{t}}{2}$$

so that

$$\hat{\delta}'_{min2LP} = \frac{\frac{3}{4}(k - c) - \frac{\bar{u} - \frac{t}{16} + k - c}{2}}{\frac{3}{4}(k - c) - \frac{k - c}{2}} = 1 - \frac{2\bar{u} - \frac{t}{8}}{k - c}$$

We now discuss the signs of the derivative of the different with respect to k.

Trivially

$$\frac{\partial \hat{\delta}_{max1LP}}{\partial \mathrm{k}} = 0$$

It is possible to rewrite $\hat{\delta}_{\min 1LP}$ as

$$\hat{\delta}_{\min 1LP} = \frac{\mathrm{c} - \mathrm{k} + \frac{17}{16}\mathrm{t} - \bar{\mathrm{u}}}{\mathrm{c} - \mathrm{k} + \frac{9}{8}\mathrm{t} - 2\bar{\mathrm{u}}} = \frac{\mathrm{c} - \mathrm{k} + \frac{18}{16}\mathrm{t} - 2\bar{\mathrm{u}} + \bar{\mathrm{u}} - \frac{1}{16}\mathrm{t}}{\mathrm{c} - \mathrm{k} + \frac{9}{8}\mathrm{t} - 2\bar{\mathrm{u}}} = 1 + \frac{\bar{\mathrm{u}} - \frac{1}{16}\mathrm{t}}{\mathrm{c} - \mathrm{k} + \frac{9}{8}\mathrm{t} - 2\bar{\mathrm{u}}}$$

Then

$$\frac{\partial \hat{\delta}_{min1LP}}{\partial \mathrm{k}} = \left(\bar{\mathrm{u}} - \frac{1}{16}\mathrm{t}\right)(-1)\frac{1}{\left(\mathrm{c} - \mathrm{k} + \frac{9}{8}\mathrm{t} - 2\bar{\mathrm{u}}\right)^2}(1) > 0$$

since from (3) $\bar{\mathrm{u}} \geq \frac{1}{4}\mathrm{t} = \frac{4}{16}\mathrm{t}$.

One can rewrite $\hat{\delta}_{min2LP}$ as

$$f(k) = \hat{\delta}_{min2LP} = \frac{c - k + \frac{t}{16} - \bar{u} + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}}{c - k + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}}$$

$$= 1 + \frac{\frac{t}{16} - \bar{u}}{c - k + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}}$$

Then differentiating with respect to k yields

$$\frac{\partial f(k)}{\partial k} = \left(\frac{t}{16} - \bar{u}\right)(-1)\frac{-1 + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)}{t}}{\left[c - k + \frac{\left(\bar{u} + k - c + \frac{7}{16}t\right)^2}{2t}\right]^2}$$

The first term $\frac{t}{16} - \bar{u} < 0$, because $\bar{u} \geq \frac{t}{4}$ from (3). So, we only need to discuss the sign of $-1 + \frac{\left(\bar{u}+k-c+\frac{7}{16}t\right)}{t}$.

If $0 > c - \bar{u} + \frac{9}{16}t$, we have $k > 0 > c - \bar{u} + \frac{9}{16}t$ and $1 - \frac{\left(\bar{u}+k-c+\frac{7}{16}t\right)}{t} < 0$. So, the first order derivative is positive, that is, $\frac{\partial f(k)}{\partial k} > 0$.

If $c - \bar{u} + \frac{9}{16}t > 0$, we have $\frac{\partial f(k)}{\partial k} > 0$ when $c - \bar{u} + \frac{9}{16}t \leq k$ and $\frac{\partial f(k)}{\partial k} < 0$ when $k < c - \bar{u} + \frac{9}{16}t$. Since in minimal opinion differentiation equilibrium $c + \frac{25}{72}t \leq k$, then $c + \frac{25}{72}t \leq k < c - \bar{u} + \frac{9}{16}t$.

However, the parameter restriction $c + \frac{25}{72}t \leq k < c - \bar{u} + \frac{9}{16}t \rightarrow c + \frac{25}{72}t < c - \bar{u} + \frac{9}{16}t \rightarrow \bar{u} \leq \frac{936-25}{72}t = \frac{11}{72}t$ is not compatible with the restriction $\bar{u} \geq \frac{1}{4}t = \frac{18}{72}t$ from (3).

So that

$$\frac{\partial \hat{\delta}_{min2LP}}{\partial k} > 0$$

Differentiating $\hat{\delta}'_{min2LP}$ one gets

$$\frac{\partial \hat{\delta}'_{min2LP}}{\partial k} = \frac{2\bar{u} - \frac{t}{8}}{(k - c)^2} > 0$$

because from (3) we have $\bar{u} \geq \frac{t}{4} > \frac{t}{16} \rightarrow 2\bar{u} - \frac{t}{8} > 0$.

One can rewrite $\hat{\delta}_{max2LP}$ as

$$f(k) = \hat{\delta}_{max2LP} = \frac{c - k + \frac{t}{16} - \bar{u} + \frac{1}{2t}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}{-t + \frac{1}{2t}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2}$$

$$= \frac{c - k + \frac{17}{16}t - \bar{u}}{-t + \frac{1}{2t}\left(k - c + \frac{7}{16}t + \bar{u}\right)^2} + 1$$

Setting $k - c + \frac{7}{16}t + \bar{u}$=X, we have:

$$f(X) = \frac{\frac{3}{2}t - X}{\frac{1}{2t}X^2 - t} + 1$$

For $X \neq 0$[37]

$$f(X) = \left(\frac{3}{2}t - X\right)\left(\frac{1}{2t}X^2 - t\right)^{-1} + 1$$

Then,

$$\frac{\partial f(X)}{\partial X} = \left(\frac{3}{2}t - X\right)\left[-\left(\frac{1}{2t}X^2 - t\right)^{-2}\right]\left(\frac{2}{2t}X^{2-1}\right) - \left(\frac{1}{2t}X^2 - t\right)^{-1}$$

$$= \left(\frac{1}{2t}X^2 - t\right)^{-1}\left[-\left(\frac{1}{2t}X^2 - t\right)^{-1}\left(\frac{X}{t}\right)\left(\frac{3}{2}t - X\right) - 1\right]$$

The first term is positive (FT>0) if $\frac{1}{2t}X^2 - t > 0$. This is the case if $\frac{1}{2}X^2 > t^2$, i.e. if $X^2 > 2t^2$. So if either $X > \sqrt{2}t$ or $X < -\sqrt{2}t$. This happens if $k - c + \frac{7}{16}t + \bar{u} > \sqrt{2}t$ or $k - c + \frac{7}{16}t + \bar{u} < -\sqrt{2}t$ or equivalently if either $k > c - \bar{u} - \frac{7}{16}t + \sqrt{2}t$ or $k < c - \bar{u} - \frac{7}{16}t - \sqrt{2}t$.

The first term is instead negative (FT<0) if $c - \bar{u} - \frac{7}{16}t - \sqrt{2}t < k < c - \bar{u} - \frac{7}{16}t + \sqrt{2}t$.

Finally it is infinite (FT→∞) if $k = c - \bar{u} - \frac{7}{16}t - \sqrt{2}t$ or $k = c - \bar{u} - \frac{7}{16}t + \sqrt{2}t$. Here the function $f(X)$ has two asymptotes.

The second term ST is instead positive (ST>0) if

$$-\left(\frac{1}{2t}X^2 - t\right)^{-1}\left(\frac{X}{t}\right)\left(\frac{3}{2}t - X\right) > 1$$

[37] If $X = 0$, $f(X) = -\frac{1}{2}$ and $\frac{\partial f(X)}{\partial X} = 0$

This is the case if

$$-\frac{X\left(\frac{3}{2}\mathrm{t}-X\right)}{\left(\frac{1}{2}X^2-t^2\right)}>1$$

If $t\left(\frac{1}{2}X^2-1\right)>0$ (FT>0), the inequality above holds if

$$-X\left(\frac{3}{2}\mathrm{t}-X\right)>\left(\frac{1}{2}X^2-t^2\right)$$

$$-\frac{3}{2}\mathrm{t}X+X^2>\frac{1}{2}X^2-t^2$$

$$\frac{1}{2}X^2-\frac{3}{2}\mathrm{t}X+t^2>0$$

$$X^2-3tX+2t^2>0$$

$$(\mathrm{X}-2\mathrm{t})(\mathrm{X}-\mathrm{t})>0$$

Therefore,

$$X>2t$$

or

$$X<t$$

Since $k-c+\frac{7}{16}\mathrm{t}+\bar{u}$=X, then it needs to be either

$$k>\mathrm{c}-\bar{u}+\frac{25}{16}t$$

or

$$k<c-\bar{u}+\frac{9}{16}t$$

If instead $\left(\frac{1}{2t}X^2-t\right)<0$ (FT<0) the second term ST is positive if

$$-\frac{X\left(\frac{3}{2}\mathrm{t}-X\right)}{\left(\frac{1}{2}X^2-t^2\right)}>1$$

which now is satisfied if

$$-X\left(\frac{3}{2}\mathrm{t}-X\right)<\left(\frac{1}{2}X^2-t^2\right)$$

$$-\frac{3}{2}\mathrm{t}X+X^2<\frac{1}{2}X^2-t^2$$

$$\frac{1}{2}X^2 - \frac{3}{2}\mathrm{t}X + t^2 < 0$$

$$X^2 - 3tX + 2t^2 < 0$$

$$t < X < 2t$$

Since $k - c + \frac{7}{16}\mathrm{t} + \bar{u}$=X, then it needs to be

$$t < k - c + \frac{7}{16}t + \bar{u} < 2t$$

or equivalently

$$c - \bar{u} + \frac{9}{16}t < k < \mathrm{c} - \bar{u} + \frac{25}{16}t$$

The second term ST is instead negative (ST<0) if

$$-\left(\frac{1}{2t}X^2 - t\right)^{-1}\left(\frac{\mathrm{X}}{\mathrm{t}}\right)\left(\frac{3}{2}\mathrm{t} - X\right) < 1$$

This is the case if

$$-\frac{X\left(\frac{3}{2}\mathrm{t} - X\right)}{\left(\frac{1}{2}X^2 - t^2\right)} < 1$$

If $t\left(\frac{1}{2}X^2 - 1\right) > 0$ (FT>0), it happens if

$$-X\left(\frac{3}{2}\mathrm{t} - X\right) < \left(\frac{1}{2}X^2 - t^2\right)$$

$$-\frac{3}{2}\mathrm{t}X + X^2 < \frac{1}{2}X^2 - t^2$$

$$\frac{1}{2}X^2 - \frac{3}{2}\mathrm{t}X + t^2 < 0$$

$$X^2 - 3tX + 2t^2 < 0$$

$$c - \bar{u} + \frac{9}{16}\mathrm{t} < k < \mathrm{c} - \bar{u} + \frac{25}{16}t$$

If instead $\left(\frac{1}{2t}X^2 - t\right) < 0$ (FT<0), the second term (ST<0) is negative if

$$-\frac{X\left(\frac{3}{2}\mathrm{t} - X\right)}{\left(\frac{1}{2}X^2 - t^2\right)} < 1$$

which is equivalent to

$$-X\left(\frac{3}{2}\mathrm{t} - X\right) > \left(\frac{1}{2}X^2 - t^2\right)$$

$$-\frac{3}{2}\mathrm{t}X + X^2 > \frac{1}{2}X^2 - t^2$$

$$\frac{1}{2}X^2 - \frac{3}{2}\mathrm{t}X + t^2 > 0$$

$$X^2 - 3tX + 2t^2 > 0$$

$$k > \mathrm{c} - \bar{u} + \frac{25}{16}t \text{ or } k < c - \bar{u} + \frac{9}{16}\mathrm{t}$$

Finally, the second term ST is zero (ST=0) if

$$-\left(\frac{1}{2t}X^2 - t\right)^{-1}\left(\frac{\mathrm{X}}{\mathrm{t}}\right)\left(\frac{3}{2}\mathrm{t} - X\right) = 1$$

This is the case if

$$-\frac{X\left(\frac{3}{2}\mathrm{t} - X\right)}{\left(\frac{1}{2}X^2 - t^2\right)} = 1$$

that is if

$$X^2 - 3tX + 2t^2 = 0$$

which is true if

$$k = \mathrm{c} - \bar{u} + \frac{9t}{16}$$

or

$$k = c - \bar{u} + \frac{25t}{16}$$

Now, using above results, we can conclude the conditions for the sign of first order derivative.

$\frac{\partial f(X)}{\partial X} > 0$, if either FT>0 and ST>0, or if FT<0 and ST<0.

To have FT>0 and ST>0, we need

$$\begin{cases} \mathrm{k} \neq \mathrm{c} - \bar{\mathrm{u}} - \frac{7}{16}\mathrm{t} \\ \mathrm{k} > c - \bar{\mathrm{u}} - \frac{7}{16}\mathrm{t} + \sqrt{2}\mathrm{t} \text{ or } \mathrm{k} < c - \bar{\mathrm{u}} - \frac{7}{16}\mathrm{t} - \sqrt{2}\mathrm{t} \\ \mathrm{k} > c - \bar{\mathrm{u}} + \frac{25}{16}\mathrm{t} \text{ or } \mathrm{k} < c - \bar{\mathrm{u}} + \frac{9}{16}\mathrm{t} \end{cases}$$

$$\rightarrow \ \mathrm{k} > c - \bar{\mathrm{u}} + \frac{25}{16}\mathrm{t} \text{ or } \mathrm{k} < c - \bar{\mathrm{u}} - \frac{7}{16}\mathrm{t} - \sqrt{2}\mathrm{t}$$

To have FT<0 and ST<0, we need

$$\begin{cases} k \neq c - \bar{u} - \frac{7}{16}t \\ c - \bar{u} - \frac{7}{16}t - \sqrt{2}t < k < c - \bar{u} - \frac{7}{16}t + \sqrt{2}t \\ k > c - \bar{u} + \frac{25}{16}t \text{ or } k < c - \bar{u} + \frac{9}{16}t \end{cases}$$

$$\rightarrow\ c - \bar{u} - \frac{7}{16}t - \sqrt{2}t < k < c - \bar{u} + \frac{9}{16}t$$

$\frac{\partial f(X)}{\partial X} < 0$, if either FT>0 and ST<0, or if FT<0 and ST>0.

To have FT>0 and ST<0, we need

$$\begin{cases} k \neq c - \bar{u} - \frac{7}{16}t \\ k > c - \bar{u} - \frac{7}{16}t + \sqrt{2}t \text{ or } k < c - \bar{u} - \frac{7}{16}t - \sqrt{2}t \\ c - \bar{u} + \frac{9}{16}t < k < c - \bar{u} + \frac{25}{16}t \end{cases}$$

$$\rightarrow\ c - \bar{u} - \frac{7}{16}t + \sqrt{2}t\ < k < c - \bar{u} + \frac{25}{16}t$$

To have FT<0 and ST>0, we need

$$\begin{cases} k \neq c - \bar{u} - \frac{7}{16}t \\ c - \bar{u} - \frac{7}{16}t - \sqrt{2}t < k < c - \bar{u} - \frac{7}{16}t + \sqrt{2}t \\ c - \bar{u} + \frac{9}{16}t < k < c - \bar{u} + \frac{25}{16}t \end{cases}$$

$$\rightarrow c - \bar{u} + \frac{9}{16}t < k < c - \bar{u} - \frac{7}{16}t + \sqrt{2}t$$

To conclude, we have

- $\frac{\partial f(k)}{\partial k} < 0$ if $c - \bar{u} + \frac{9}{16}t < k < c - \bar{u} + \frac{25}{16}t$

  and $k \neq c - \bar{u} - \frac{7}{16}t + \sqrt{2}t$,
- $\frac{\partial f(k)}{\partial k} > 0$ if $k > c - \bar{u} + \frac{25}{16}t$

  or $k \neq c - \bar{u} - \frac{7}{16}t - \sqrt{2}t$ and $k<c - \bar{u} + \frac{9}{16}t$,

However, the parameter restriction $k > c - \bar{u} + \frac{25}{16}t$ is not compatible with the parameter area for $\hat{\delta}_{max2LP}$, which requires $\bar{u} < c - k + \frac{25}{16}t$. Also $k<c - \bar{u} + \frac{9}{16}t \rightarrow \bar{u} < c - k + \frac{9}{16}t$ is not compatible with the restriction $\bar{u} >c - k + \frac{5}{4}t = c - k + \frac{20}{16}t$ from (3).

So that

$$\frac{\partial \hat{\delta}_{max2LP}}{\partial k} < 0$$

To sum up

$$\frac{\partial \hat{\delta}_{min1LP}}{\partial k} > 0$$

$$\frac{\partial \hat{\delta}_{min2LP}}{\partial k} > 0$$

$$\frac{\partial \hat{\delta}'_{min2LP}}{\partial k} > 0$$

$$\frac{\partial \hat{\delta}_{max1LP}}{\partial k} = 0$$

$$\frac{\partial \hat{\delta}_{max2LP}}{\partial k} < 0$$

QED