Preface

The study of geometric properties of partial differential equations has always attracted the interest of researchers and is now a broad and well-established research area, with contributions that often come from experts from disparate areas of mathematics, such as differential and convex geometry, functional analysis, calculus of variations, mathematical physics, to name a few.

The interplay between partial differential equations and geometry has two main aspects: on the one hand, the former is classically a powerful tool for the investigation of important problems coming from differential geometry and, on the other hand, the latter gives useful and often decisive insights in the study of PDE's. Now that basic questions about PDE's, such as existence, uniqueness, stability and regularity of solutions for initial/boundary value problems, have been fairly understood, research on topological and/or geometric properties of their solutions have become more vigorous.

Research on geometric aspects for parabolic and elliptic PDE's provides a vast variety of possibilities. Issues currently and actively studied comprehend among others: positivity of solutions; critical points: their structure, possible occurrence and evolution; spike-shaped solutions; symmetry and non-symmetry for ground states and overdetermined boundary value problems; stability of symmetric configurations; convexity, quasi-convexity or starshape of level sets; estimates on geometrically or physically relevant quantities such as surface area and curvature of level surfaces or torsional creep, eigenvalues and eigenfunctions; impact of curvature of the domain on the relevant solutions and their possible behavior for large or short times, and so on. Similarly wide is the assortment of mathematical tools and techniques, analytic and geometric, employed to analyze such issues: functional inequalities such as isoperimetric, Hardy or Brunn-Minkowski inequalities; Pohozaev-type identities; maximum principles; Harnack inequalities; asymptotics for solutions; moving-planes or sliding methods; Bernstein and Liouville-type theorems, viscosity-solutions techniques, et cetera.

This volume aims to promote scientific collaboration in this very active area of research, by presenting recent results and informative surveys and by exploring new trends and techniques. It contains original papers and a few survey articles.

As it appears from the table of contents, apart for one or two exceptions, all authors are either Italian or Japanese. The Italian and Japanese mathematical schools have a long tradition of research in PDE's and count various research groups active and steadily collaborating in the study of geometric properties of their solutions. For this reason, the successful idea of E. Yanagida and K. Ishige, at the time (2008) in Sendai at Tohoku University, to have a (regularly meeting) joint conference on these topics was enthusiastically welcome by the first Editor of this book. Consequently, a first workshop met in Sendai in 2009. Contributors to this volume are some of the participants to the *Second Italian-Japanese Workshop on Geometric Properties for Parabolic and Elliptic PDE's* that met in Cortona (Italy) on June 20–24, 2011, that was kindly sponsored by the Istituto Nazionale di Alta Matematica "F. Severi" (INdAM) and, besides the Editors of this book, was organized by A. Cianchi (Università di Firenze), F. Gazzola (Politecnico di Milano), K. Ishige (Tohoku University) and E. Yanagida (Tokyo Institute of Technology). This meeting was a great occasion to blend common and different experiences in the field both at senior and junior level.

The Editors wish to thank INdAM and its President Vincenzo Ancona who made possible both the Cortona workshop and the publication of this book of articles.

The meeting took place only a few months after the catastrophic earthquake and tsunami that hit Japan at the beginning of 2011, particularly in the Sendai and Fukushima area. The option of a cancellation was seriously taken into account by the organizers. Thanks to the serene courage of the Japanese part, the conference finally took place and was hailed as a promising sign in the way to normality.

This book is certainly the best confirmation of that sign.

Firenze, Italy   Rolando Magnanini
Sendai, Japan   Shigeru Sakaguchi
Napoli, Italy   Angelo Alvino