Chapter 6

The SLTI Method

As previously mentioned, the objective of the SLTI method is to process each radar sweep pertinent to a single transmission pulse in order to detect the presence of a sea/land transition within the radar footprint. This is possible exploiting the HF reflectivity variation related to such transition. The reason of employing a pulse-based approach is to provide a transition detection in time intervals compatible with the ionospheric coherence time that in the Mediterranean area, in normal conditions of the medium, ranges from 23 to 100seconds (see chapter 4 and in particular table 4.2). The analysis for transition detection is based on a basic cross-correlation procedure between the received signal and a reference signal derived from a *"surface mask"* based on geographic information and on the operational radar antenna parameters and pointing direction. The reference signal, in fact, is generated by transforming the surface mask into the time domain through the variations of the equivalent ionospheric reflection height within the -3dB antenna beam. In the following section the SLTI method is outlined and the procedure employed to generate the surface masks is described, together with the calculation required to convert the masks into the time domain. Then the geometry of the problem, that is the model of the simulated OTHR-SW scenario is provided and commented.
Note that most of the principles and equations expressed in the following had been resumed also in one of our papers titled *"Coordinate Registration Method based on Sea/Land Transitions Identification for Over The Horizon Sky-Wave Radar: numerical model and basic performance requirements"* published in 2011 by the IEEE's journal "Transactions on Aerospace and Electronic Systems" [39].

6.1 Surface mask and transformation into time domain

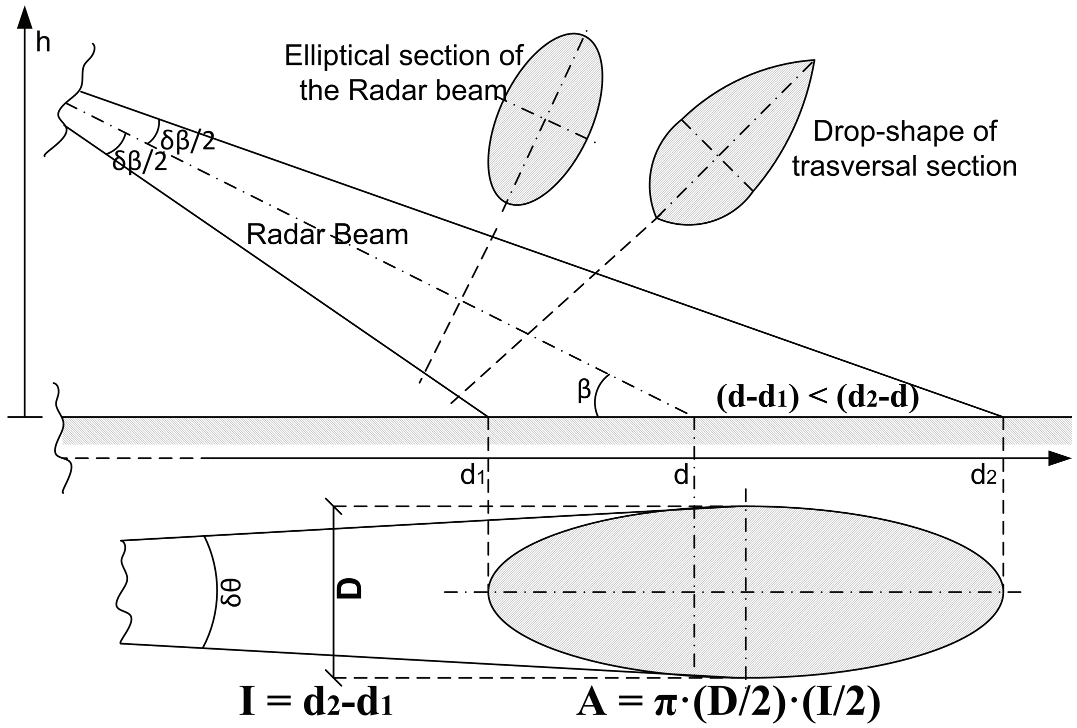

Figure 6.1. Simplified scheme of the geometry assumed for the radar beam (side-view on the top of the figure) and the radar footprint (top-view on the figure's bottom). I indicates the down-range length of the footprint, while A represents its approximated surface.

The *"surface clutter mask"* is a Geo-referenced matrix associated with a given resolution (higher than the one that characterizes the consider OTHR-SW system) to the surveillance area of the OTHR-SW system. In the previous chapters some basic concepts of the clutter model are provided together with simplified sketches of the model's geometry. In figure 6.1 some of these concepts are resumed, while in figure 6.2 the main geometric parameters and some method for the generation of the surface clutter mask are proposed. Assuming a binary classification of the surface (as described in the final part of section 3.3.6), three different methods are considered for the evaluation of the surface clutter mask in the considered rectangular sea/land region:

- **I**: a binary (sea/land) value of the normalized backscattering coefficient is assumed considering the nature of the surface along the radar pointing direction (that is the main axes of the rectangle);

– **II**: a binary value of the normalized backscattering coefficient is assumed depending on the percentage of sea/land surface in the relative rectangular region delimited in range by the pulse projection to the ground δI;

– **III**: a multivalued representation of the backscattering coefficient is assumed following the same principle indicated in **II** and according to the chromatic scale proposed an the bottom of the figure.

The approach employed so far is similar to the one described by **II**, but the considered footprint is elliptically shaped (not rectangular), hence the surface of the region delimited in range by the pulse projection δI changes according to its range position within the footprint.

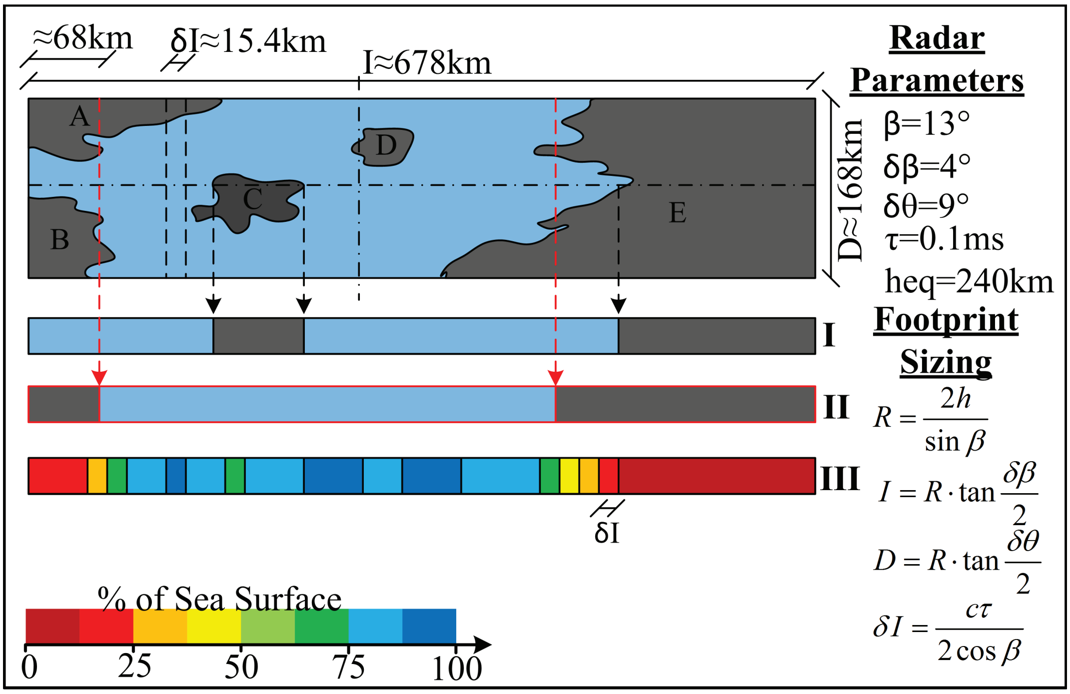

Figure 6.2. Example of the application of three possible methods for the definition of the surface clutter mask.

In order to present and describe the theoretical and mathematical principle of the proposed SLTI method we employ a simplified version of the OTHR-SW's geometry that is represented by the sketch in fig. 6.3.

Let us define the sea/land classification function of the Earth surface with respect to the radar position as:

$$SL\left(R_G, \varphi\right) = \begin{cases} 0 \rightarrow iftheEarthsurfacein\left(R_G, \varphi\right) island \\ 1 \rightarrow iftheEarthsurfacein\left(R_G, \varphi\right) issea \end{cases}$$

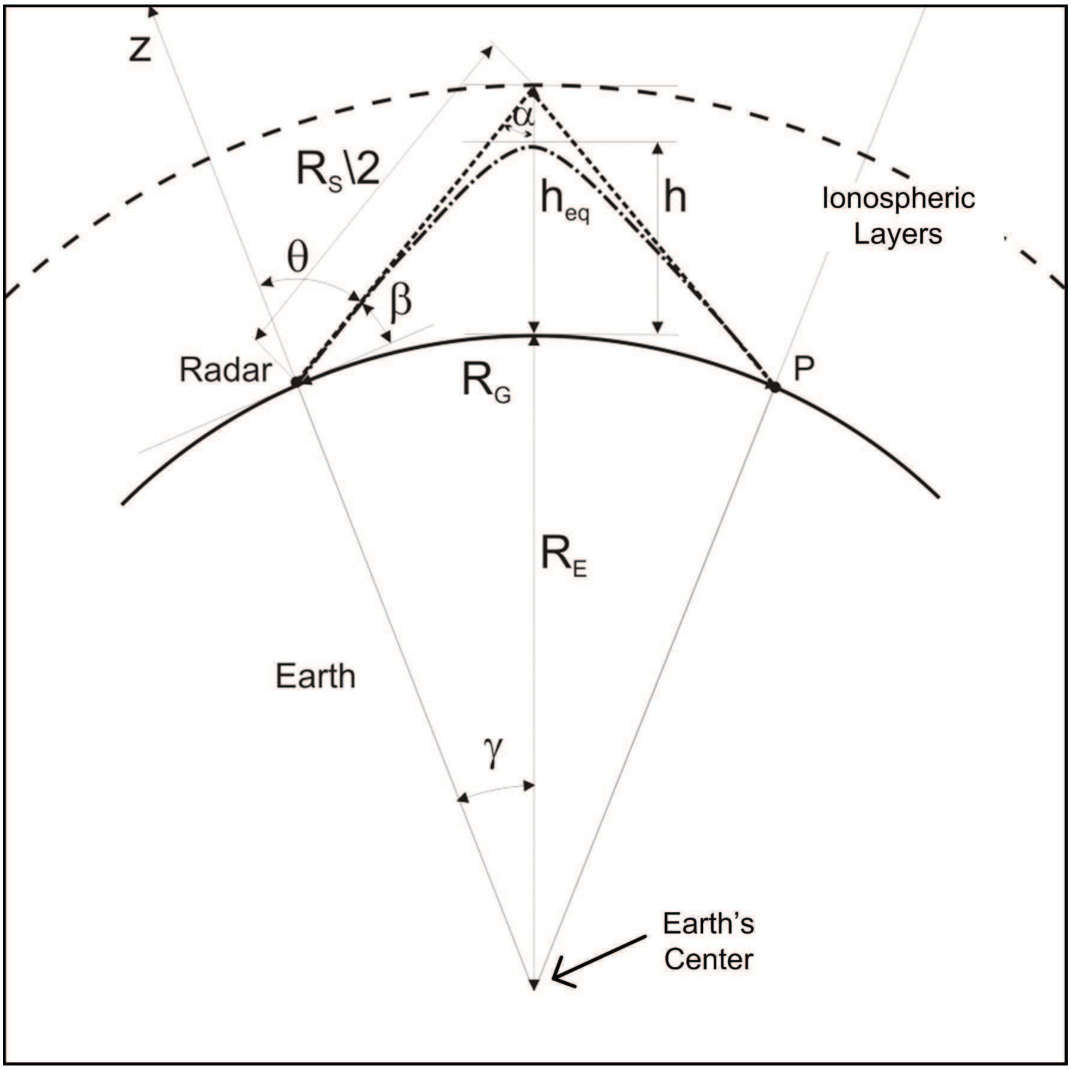

Figure 6.3. *Sketch of the side-view of the simulated OTHR-SW's scenario with description of geometric parameters. Note that the scheme is not on scale in order to simplify the visualization of the geometric parameters. Consider that the average radius of Earth is about 20 times the height of the maximum of the ionospheric e^- density.*

where R_G and φ are respectively the ground-range distance and the azimuthal angle. Then, let us define the *"Sea/Land Fractional Mask"* (SLFM) as:

$$SLFM\left(R_G,\varphi\right)=\frac{1}{\varphi_3}\cdot\int_{\varphi-\frac{\varphi_3}{2}}^{\varphi+\frac{\varphi_3}{2}}SL\left(R_G,\alpha\right)d\alpha$$

where φ_3 is the azimuthal antenna beam-width. In a given position (R_G,φ) on the Earth surface, the SLFM gives the fraction of sea along an ideal arc as wide as the -3 dB azimuthal beam and centered in (R_G, φ). Hence the $SLBM$ (Sea/Land Binary Mask), that is the binary version of the SLFM, is given by:

$$SLBM\,(R_G, \varphi) = \begin{cases} 0 \rightarrow if SLFM\,(R_G, \varphi) < 1/2 \\ 1 \rightarrow if SLFM\,(R_G, \varphi) \geq 1/2 \end{cases}$$

The round-trip delay of a signal impacting on the elementary surface in (R_G, φ) is given by:

$$\tau\,(R_G, h_{eq}) = \frac{2R_S\,(R_G, h_{eq})}{c}$$

where, according to the scheme provided by fig. 6.3:

- R_S is the equivalent ionospheric distance;
- h_{eq} is the equivalent ionospheric reflection height.

It appears evident that, for a given h_{eq}, the signal's round-trip delay $\tau\,(R_G, h_{eq})$ and the ground-range distance $R_G\,(R_G, h_{eq})$ are bi-uniquely related. In respect of the previous equation, the formula that gives the SLBM can be expressed as a function of h_{eq} in the time delay domain:

$$SLBM\,(\tau, h_{eq}, \varphi) = SLBM\,(R_G\,(\tau, h_{eq})\,, \varphi)$$

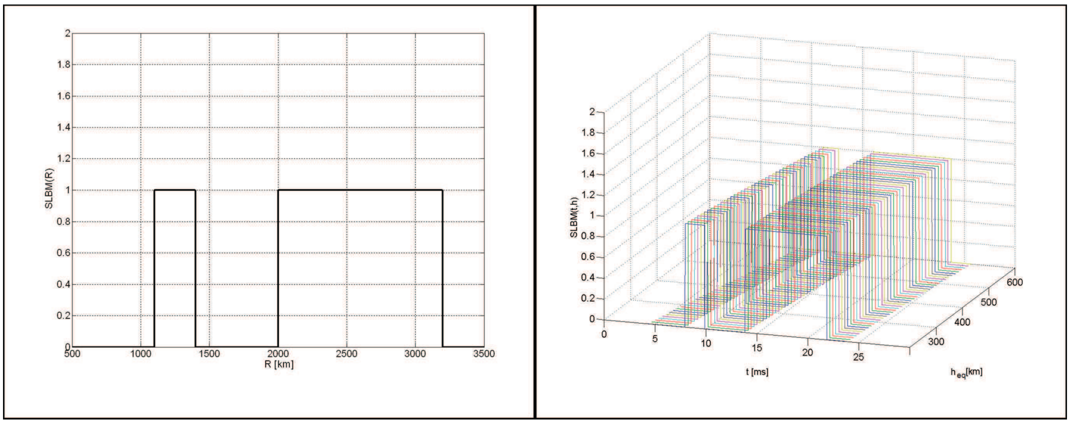

Figure 6.4. *An example of SLBM for an azimuthal direction that gives four sea/land transitions.*

Figure 6.4 shows an example of $SLBM\,(R_G, \varphi)\,|_{\varphi=\varphi_0}$ with four sea/land transitions along the azimuth pointing direction φ_0. The left side of the picture, shows the corresponding $SLBM\,(\tau, h_{eq})$, evidencing how the time delay is affected by variations of the equivalent reflection height. This simple example is indicative of how the sea/land transition is shifted in time by a variation of h_{eq}. This is the basic concept of the CR method discussed in the following.

6.2 Correlation for the identification of Sea/Land Transition

If with $P_{rx}(t, f, \beta_0, \varphi_0)$ we indicate the total signal power received after the transmission of a monochromatic pulse with frequency f along the pointing direction (β_0, φ_0) , where β_0 represents the antenna elevation angle, then we can express the time correlation between $SLBM$ and P_{rx} as:

$$C(h_{eq}, f, \beta_0, \varphi_0) = \int_{-\infty}^{\infty} P_{rx}(t, f, \beta_0, \varphi_0) \cdot SLBM(t, h_{eq}, \varphi_0) \cdot rect\left[\frac{t - \tau(\beta_0, h_{eq})}{\delta T}\right] \cdot dt$$

where:

- $\tau(\beta_0, h_{eq})$ is the time delay of the power contribution corresponding to β_0 and h_{eq};
- $rect(t)$ is the unit rectangle function.

Note that the rect function in the last equation limits the correlation to the -3 dB antenna beam-width, by defining the integration interval as:

$$\delta T = \left[\tau(\beta_0 - \frac{\theta_3}{2}, h_{eq}) - \tau(\beta_0 + \frac{\theta_3}{2}, h_{eq})\right]$$

Such interval corresponds to the projection of the -3dB beam-width to the ground.
The value of h_{eq} that maximizes the expression of $C(h_{eq}, f, \beta_0, \varphi_0)$ along the direction (β_0, φ_0) for a given frequency is the *estimated equivalent reflection height*:

$$h_{est}(f, \beta_0, \varphi_0) = max_{h_{eq}}\left[C(h_{eq}, f, \beta_0, \varphi_0)\right]$$

The equation that intrinsically provides the range coordinate registration of the received signal power is finally obtained by inserting this last equation into the transformation formula that expresses $\tau(R_G, h_{eq})$. The result can be written as:

$$P_{rx}(R_G, f, \beta_0, \varphi_0) = P_{rx}(t(R_G, h_{est}), f, \beta_0, \varphi_0)$$

6.3 Numerical Model of the OTHR-SW Received Echo

If we consider a monostatic OTHR-SW system operating at frequency f, with transmit and receive antenna gains $G_T(\theta, \varphi)$ and $G_R(\theta, \varphi)$ respectively, then the power radiated along the direction (θ, φ), at distance r and from the transmitter is generally expressed as:

$$\frac{P_T}{4\pi r^2} \cdot G_T(\theta, \varphi, f)$$

Hence the incident power after the Ionospheric reflection on the P point on the Earths surface (fig. 6.3) can be expressed as:

$$\frac{P_T}{4\pi R^2(h_{eq})} \cdot G_T(\theta, \varphi, f) \cdot L(\theta, \varphi, f)$$

where $L(f)$ accounts for the total atmospheric and ionospheric losses and $R(h_{eq})$ is the true propagation path length (aka "Slant-Range Distance"). Note that, even if not explicitly indicated, h_{eq} depends on time t, on frequency f and on the propagation direction (θ, φ).
Let $\sigma_c(R_G, \varphi, t, f, \theta)$ be the $NRCS$ at time t, for the frequency f and the incidence direction θ of the infinitesimal surface element dS relative to the generic point $P = (R_G, \varphi)$. The infinitesimal expected power at the receiver contributed by this surface element is:

$$d\overline{P_{rx}} = \frac{P_T}{(4\pi)^3 R^4(h_{eq})} \cdot \frac{c^2}{f^2} \cdot G_T(\theta, \varphi, f) G_R(\theta, \varphi, f) \sigma_c(R_G, \varphi, t, f, \theta) L^2(\theta, \varphi, t, f) dS$$

The delay with which the contribution from the infinitesimal surface element in P is received is $\tau = 2R_S/c$, where $R_S(h_{eq})$ is the slant range path length. In other words, the last equation gives the infinitesimal power contribution received τ seconds after the pulse transmission along the (θ, φ) direction, backscattered from the infinitesimal surface dS in $P = (R_G, \varphi)$. In order to derive the infinitesimal power contribution from the solid angle $sin\theta d\theta d\varphi$, let us suppose that a radar pulse is transmitted with a duration ΔT. The infinitesimal impact surface is defined by the intersection of an infinitesimal tube-shaped volume with depth $c\Delta T/2$ with the Earth surface. Such intersection may result in the entire projection of the solid angle at ground if ΔT is sufficiently high (*beam-limited surface*) or in a strip within the projection of the solid angle if ΔT is smaller (*range- or pulse-limited surface*) [60]. The condition for the infinitesimal impact surface to be range-limited is:

$$\frac{Rd\theta}{cos\theta} > \frac{c\Delta T}{sin\theta}$$

The infinitesimal surface dS can be written as:

$$dS = R(h_{eq}) \cdot c\Delta T \cdot d\varphi$$

or otherwise:

$$dS = R^2(h_{eq}) \cdot tg(\theta) \cdot d\theta \cdot d\varphi$$

The total received signal in $t = \tau$ is the sum of all surface contributions with the same path length R_S , irrespectively of the direction (θ, φ). This is true also if a pulse of finite length $T = N_p \Delta T$ is transmitted. The equation of the infinitesimal expected power at the receiver contributed by the surface element dS can be converted into a discrete form to be employed in numerical simulations:

$$\overline{P_{i,j,k}} = \frac{P_T}{(4\pi)^3 R^4(h_{i,j,k}(f))} \cdot \frac{c^2}{f^2} \cdot G^2_{i,j,k}(f)\sigma_{i,j,k}(f) \cdot \Delta S_{i,j,k} L^2_{i,j,k}(f)$$

where:

- $i = 1 \dots N_p$ is the *time index* corresponding to the i-th part of the pulse (subdivided in N_p elements of duration $\Delta T = T/N_p$);
- $j = 1 \dots N_e$ is the *elevation index*, corresponding to the j-th part in elevation of the -3 dB radar beam (subdivided in N_e elements centered in $\theta_j = \theta_0 - \theta_3/2 + (j-1) \cdot \theta_3/N_e$, being θ_3 and θ_0 respectively the -3 dB beam-width and pointing direction in elevation);
- $k = 1 \dots N_a$ is the *azimuth index*, corresponding to the k-th azimuthal part of the -3 dB radar beam (subdivided in N_a elements centered in $\varphi_k = \varphi_0 - \varphi_3/2 + (k-1) \cdot \varphi_3/N_a$, being φ_3 and φ_0 respectively the -3 dB beam-width and pointing direction in azimuth);
- $\sigma_{i,j,k}(f) = \sigma_c(R_{i,j,k}, \varphi_k, t_i, f, \theta_j)$;
- $h_{i,j,k}(f) = h_{eq}(f, t_i, \theta_j, \varphi_k)$;
- $L_{i,j,k}(f) = L(f, t_i, \theta_j, \varphi_k)$;
- $\Delta S_{i,j,k}$ is the numerical expression of the infinitesimal surface dS, that is it is the numerical version of one of the two given formula of dS, depending on the range-limited condition.

Summarizing, the last equation gives the discrete expected power contribution related to $t_i = i\Delta T$, $\theta = \theta_j$ and $\varphi = \varphi_k$ reaching the antenna with a delay

$$\tau_{i,j,k} = \frac{2R_{i,j,k}}{c}$$

with $R_{i,j,k} = R_s(h_{eq}(f, t_i, \theta_j, \varphi_k))$.

The complex signal $V_{i,j,k}(t)$ at the receive antenna associated to this power contribution can be expressed as:

$$V_{i,j,k}(t) = \sqrt{2\overline{P_{i,j,k}}} \cdot b\left(t - \tau_{i,j,k}, T_{P_{i,j,k}}, T_{L_{i,j,k}}\right) \cdot e^{j\psi_{i,j,k}}$$

where the expected value of $P_{i,j,k}$ is given by the previous expression of $\overline{P_{i,j,k}}$, while $\psi_{i,j,k}$ is the phase accounting for propagation and surface interaction effects, and $b(t, T_P, T_L)$ is the normalized trapezoidal function, defined according the geometry of fig. 6.5 as:

$$b(t, T_P, T_L) = \begin{cases} \frac{t}{T_P} \leftarrow 0 < t < T_P \\ 1 \leftarrow T_P < t < T_L \\ \frac{-1+T_L}{T_P} + 1 \leftarrow T_L < t < (T_L + T_P) \\ 0 \leftarrow otherwise \end{cases}$$

where T_P is the rise time:

$$T_P = \begin{cases} 2\Delta T \leftarrow R_{i,j,k}\theta_j tan\theta_j > c\Delta T \\ \frac{2R_{i,j,k}\theta_j tan\theta_j}{c} \leftarrow otherwise \end{cases}$$

and T_L is the central duration:

$$T_L = \begin{cases} \frac{2R_{i,j,k}\theta_j tan\theta_j}{c} \leftarrow R_{i,j,k}\theta_j tan\theta_j > c\Delta T \\ 2\Delta T \leftarrow otherwise \end{cases}$$

The trapezoidal shape of the envelope is consistent with the interaction of a pulse with a discrete (theoretically infinitesimal) surface element. The total instantaneous signal at time t is thus

$$s(t) = \sum_{i,j,k} V_{i,j,k}(t)$$

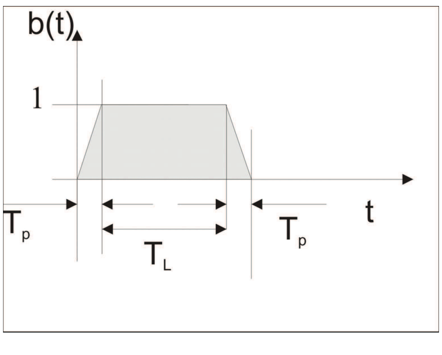

Figure 6.5. *Trapezoidal function: T_P is the rise time and T_L the central duration.*

6.4 Simulation of the OTHR-SW Scenario

In this section we introduce a simplified simulation scenario providing the expected power in reception in the presence of a single sea/land transition. The objective is to provide rough reference requirements (in terms of both SNR and differential sea/land Normalized Radar Cross section (NRCS)) for a given range coordinate registration precision when applying the correlation method described in section 2.2. This analysis is intended both to give a first insight into the potential of the method and to provide the foundation for future quantitative comparisons applying the numerical model described in the previous section in more realistic scenarios involving complex models of the ionosphere (including multipath) and of the surface responses.
The first hypothesis for the simplified reference scenario is that the equivalent reflection height is constant with respect to both time and propagation direction: $h_{eq}(f, t, \theta, \varphi)$. As a consequence, the time delay is independent of the azimuth angle φ and of the transmission instant t_i:

$$\tau_{i,j,k} = \tau_j = \frac{2R_S(\beta_j, H)}{c}$$

In other terms, this first hypothesis implies the total absence of ionospheric multipath and a bi-unique correspondence between the ground distance and time delay. This is certainly an ideal condition for detecting NRCS variations that are orthogonal to the pointing azimuth φ_0. Let τ_{min} and τ_{max} be the minimum and maximum time delay, respectively. Considering only the signal contributions coming from the -3 dB antenna beam in elevation, namely from the elevation interval $[(\theta_0 - \theta_3/2) \leftrightarrow ((\theta_0 + \theta_3/2))]$, we have:

$$\tau_{min} = \frac{2R_S\left(\beta_0 + \frac{\beta_3}{2}, H\right)}{c}$$

and

$$\tau_{min} = \frac{2R_S\left(\beta_0 - \frac{\beta_3}{2}, H\right)}{c}$$

The two previous equations can at a first sight appear wrong, but we need to remember that in OTHR-SW applications the time delay undergoes greater variations when elevation angles are smaller.

The signal backscattered from the surface reaches the antenna with a delay ranging between τ_{min} and $\tau_{max} + T$. Fig. 6 is a schematic sketch of the expected signal power in reception due to the transmission of one pulse in two cases:

- at the top when $\sigma_c(R, \varphi, t, f, \theta)$ presents a constant value;
- at the bottom when $\sigma_c(R, \varphi, t, f, \theta)$ changes sharply from two different constant values in $\theta = \theta_0$ (corresponding to the delay $\tau_T = \frac{2R_S(\beta_0,H)}{c}$)

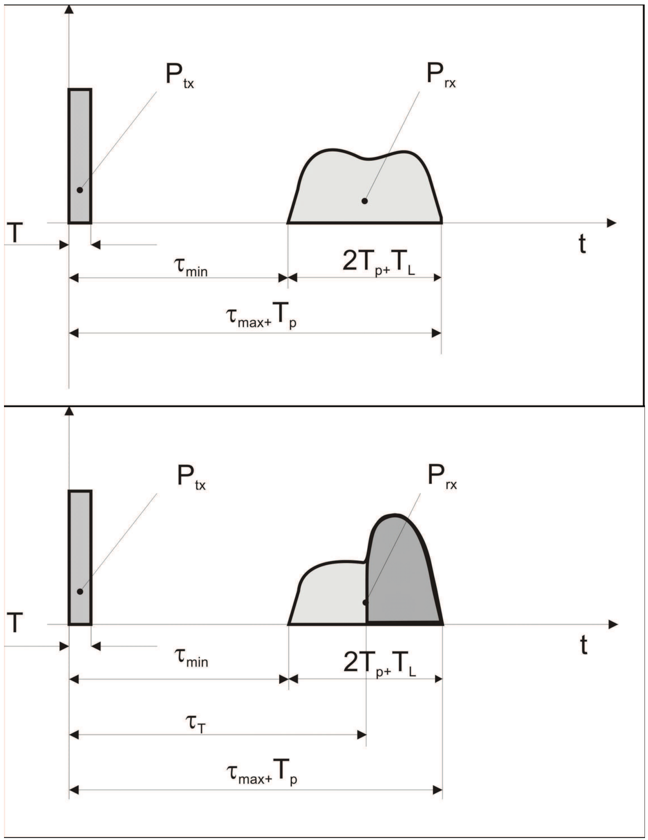

Figure 6.6. Sketch of the expected signal power of the echo after the transmission of a single pulse in two different hypothesis for σ_c.

Observe that if h_{eq} were not constant, the transition may be less evident and that the signals may not exhibit the same duration $T_L + 2T_P$ in both cases. In order to evaluate the detectability of a sharp NRCS transition in $R_G = R_{GT}$, the signal received after one pulse transmission was simulated as:

$$r(t) = s(t) + n(t)$$

where $s(t)$ is the surface signal of interest, while $n(t)$ is white Gaussian noise. The elementary contributions $V_{i,j,k}(t)$ are statistically independent complex processes, being $P_{i,j,k}$ and $\psi_{i,j,k}$ modelled as independent random variables. $P_{i,j,k}$ and $\psi_{i,j,k}$ are respectively assumed as exponentially distributed and uniformly distributed in the interval $[0, 2\pi]$. The expected value of $P_{i,j,k}$ may take two values in correspondence of the two different NRCS values assumed for land (σ_{L0}) or sea (σ_{S0}):

$$\sigma_C(R_G, \varphi, t, f, \theta) = \begin{cases} \sigma_{L0} \leftarrow for R_G < R_{GT} \\ \sigma_{S0} \leftarrow for R_G > R_{GT} \end{cases}$$

Parameter	Value]
f	15 MHz
B_n	$1/T$
G	10 dB
H	260 km
L	-10 dB
θ_0	16
θ_3	10
φ_3	5
σ_{S0}	-27 dB

Table 6.1. Values of the main Radar and Geometric parameters used in the simulations.

The values adopted for all the involved radar, clutter and noise parameters are listed in Table 6.1.
In order to quantify the error affecting the SLTI retrieval in this reference scenario, a Monte Carlo simulation was carried out. For given values of T and σ_{S0} , 1000 runs were performed for different values of ($\Delta\sigma = \sigma_{S0}/sigma_{L0}$). 1000 samples of the received signals $r(t)$ were generated accordingly and the mean and standard deviation of the errors made in the estimation of the equivalent Ionospheric reflection height were recorded.

Fig. 6.7 shows the mean error on the estimate of the sea/land transition position (left) and its standard deviation (right), for:

- pulse duration $T = 0.1$ ms (left side) and $T = 0.05$ ms (right side);
- sea backscattering coefficient $\sigma_{S0} = -27$ dB;
- transmitted power P_T varying in the range $[5 - 100]$ kW with a step of 5 kW;
- difference between sea and land backscattering coefficients $\Delta\sigma$ varying in the range $[2 - 13]$ dB with a step of 1 dB.

A mean error comprised between -5 and 5 km and a standard deviation smaller than 10 km for $\Delta\sigma > 7$ dB are observed in correspondence of the highest peak power levels. Moreover, notice that the standard deviation keeps fairly constant when $\Delta\sigma > 7$ dB, which means that sharper NRCS transitions are not needed to further improve the transition position estimate.
Fig. 6.8 shows the average Clutter-to-Noise Ratio (CNR) versus the mean and standard deviation of the error on the estimate of the sea/land transition position, for $\Delta\sigma > 7$ dB and for $T = 0.1$ ms (left) and $T = 0.05$ ms (right). The figure shows that for $CNR > 3$ dB the standard deviation keeps steadily at its minimum level (10 km), while the mean error floats steadily between -5 and 5 km. Note that the vertical clustering visible at the lowest CNR levels is due to the constant step of progression applied to P_T.

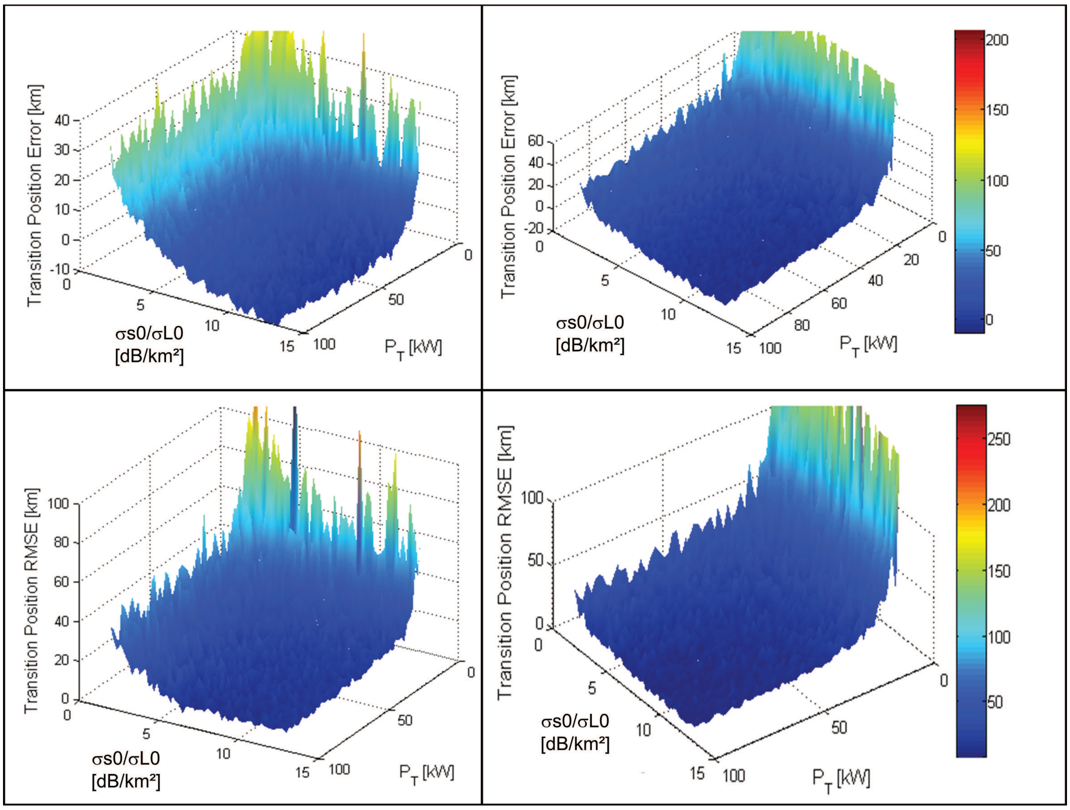

Figure 6.7. *Mean error* and *Error's Standard Deviation* in the estimate of the Sea/land transition position for $T = 0.1$ ms (left side) and $T = 0.05$ ms (right side).

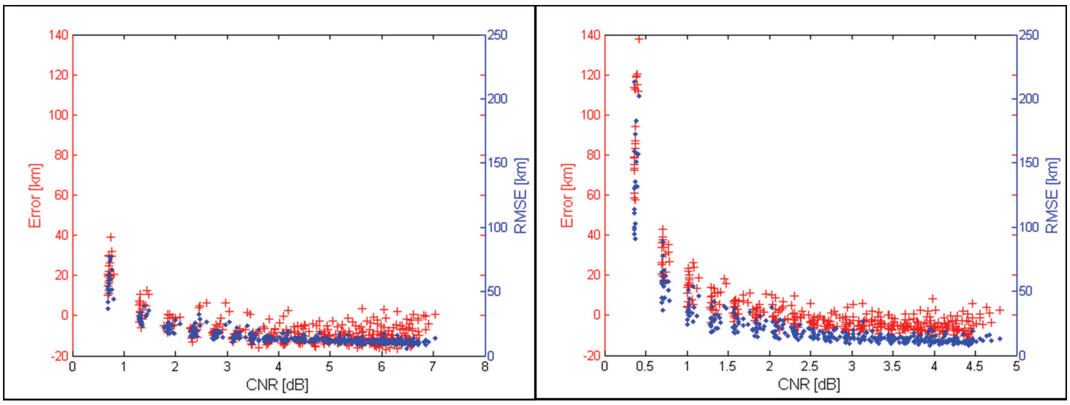

Figure 6.8. *Mean Positioning Error* vs *Clutter-to-Noise Ratio* (CNR) for $T = 0.1$ ms (left side) and $T = 0.05$ ms (right side).

6.5 References for the SLTI method

For any eventual further investigation upon the Sea-Land Transition Identification (SLTI) method, besides the already mentioned paper published on the IEEE "AESS" magazine:

- F. Cuccoli, L. Facheris, F. Sermi - *Coordinate Registration Method based on Sea/Land Transitions Identification for Over The Horizon Sky-Wave Radar: numerical model and basic performance requirements* - IEEE Transaction on AESS, Vol 47, Iss. 4, Oct 2011 - pp. 2974-2985.

we also provide the following list that includes the papers related to the SLTI presented to international congresses during the period of the present research. The entire work has been conducted in a team together with professors Dino Giuli and Luca Facheris (University of Florence) and doctor Fabrizio Cuccoli.

- F. Cuccoli, L. Facheris, D. Giuli, F. Sermi - *Over The Horizon Sky-Wave Radar: Coordinate Registration by Sea-Land Transitions Identification* - PIERS 2009 in Moscow;

- F. Cuccoli, L. Facheris, D. Giuli, F. Sermi - *Over The Horizon Sky-Wave Radar: Simulation Tool for Coordinate Registration Method based on Sea-Land Transitions Identification* - European Microwave Conference EuMW 2009 in Rome;

- F. Cuccoli, F. Sermi, L. Facheris, D. Giuli - *Sea-Land Transitions Identification for Coordinate Registration of Over The Horizon Sky-Wave Radar: numerical model for performance analysis* IEEE IRS 2010 in Vilnius;

- F. Cuccoli, F. Sermi, L. Facheris, D. Giuli - *OTHR-SW Coordinate Registration method based on Sea-Land Transitions Identification: Clutter Model Definition* European Microwave Conference EuMW 2010 Paris;

- L. Facheris, F. Cuccoli, F. Sermi *Real-Time Correction of Distributed Ionospheric Model by OTHR Coordinate Registration based on Sea/Land transition Identification: Method Outline* PIERS 2011 in Marrakech;

- F. Cuccoli, L. Facheris, F. Sermi - *Over The Horizon Sky Wave Radar Simulator for Ionosphere and Earth Surface Sounding* IEEE International Geosciences And Remote Sensing Symposium IGARSS-11 Vancouver, 24-29/07/2011 - Page(s): 277 280;

- F. Cuccoli, L. Facheris, F. Sermi - *Sea/Land transition Identification for Coordinate Registration of OTH Sky Wave Radar: end to end software simulator and performance analysis* - Progress In Electromagnetic Research Symposium PIERS 2012 Kuala Lumpur 27-30/03/2012.