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A MODIFIED FOREST KINEMATIC MODEL

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1 Introduction

Conservation of forest resources is one of the main subjects in environmental issues. The fundamental problems in the theoretical studies of this subject may be to know the mechanical laws of growing of individual trees, trees in a plot of forest, trees in a forest and even complex systems consist of the forest system and other different systems like soils, water, weather with various interactions between those systems, and to know mathematical structures for these growing processes.

In fact, many researchers have already challenged these problems. The work due to Botkin et al. [4] (cf. also [3]) may be the first and most basic model in mathematical forestry. They were concerned with a plot ($100m^2 \sim 300m^2$) of forest and presented a growth equation for each tree in the plot considering various interactions among trees. Such a model is called the Individual-Based Model (Agent Model). Afterward, Pacala et al. [10, 11] extended this method to describe growing of forest in a full scale. This model is called the Individual- Based Continuous Space Model. In the meantime, macroscopic forest models concerning with the age-dependent tree relationship have been introduced by many authors, e.g., Antonovsky [1] and Antonovsky et al. [2].

Such a model is called the Age-Structured Model. In this paper, we are concerned with the Age-Structured Continuous Space Model. Among others we consider a prototype model describing growth of forest by age-dependent tree relationships and by regeneration processes, which was proposed by Kuznetsov et al. [8].

Recently, the author et al. [15] have tried to introduce a model for describing growth of mangrove geoecosystem. To the forest model presented by [8], they incorporated the soil system with considering interactions operating between the mangrove forest system and the geological system.

In 1994, Kuznetsov, Antonovsky, Biktashev and Aponina [8] introduced a prototype Age-Structured Continuous Model. In a two-dimensional bounded domain Ω, they consider a forest system consisting of mono-species and assume that the tree generations can be simply divided into two age generations, namely, the young age trees and the old age trees. Considering three constituents of system, young age trees, old age trees and seeds in the air, they presented a kinematic model describing the growing process of the system. The initial-boundary value problem for their model is written by

$$\begin{cases} \dfrac{\partial u}{\partial t} = \beta\delta w - \gamma(v)u - fu & \text{in} \quad \Omega \times (0, \infty), \\ \dfrac{\partial v}{\partial t} = fu - hv & \text{in} \quad \Omega \times (0, \infty), \\ \dfrac{\partial w}{\partial t} = d\Delta w - \beta w + \alpha v & \text{in} \quad \Omega \times (0, \infty), \\ u(x, 0) = u_0(x),\ v(x, 0) = v_0(x),\ w(x, 0) = w_0(x) & \text{in} \quad \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^2$ is a two-dimensional bounded domain. Here, Ω denotes the area in which a forest can grow. The unknown functions $u(x, t)$ and $v(x, t)$ denote the tree densities of young and old age classes, respectively, at a position $x \in \Omega$ and at time $t \in [0, \infty)$, and the third unknown function $w(x, t)$ denotes the density of seeds in the air at $x \in \Omega$ and at $t \in [0, \infty)$. The first equation denotes growth of young trees and the second growth of old trees. The third equation describes kinetics of seeds; $d > 0$ is a diffusion constant of seeds, and $\alpha > 0$ and $\beta > 0$ are seed production and seed deposition rates, respectively. While, $0 < \delta \leq 1$ is a seed establishment rate, $\gamma(v) > 0$ is a mortality of young trees which is allowed to depend on the old-tree density v, $f > 0$ is an aging rate, and $h > 0$ is a mortality of old trees. A typical function of $\gamma(v)$ is a square function of the form $\gamma(v) = a(v - b)^2 + c$, where $a > 0$, $b > 0$ and $c > 0$. On w, some boundary conditions should be imposed on the boundary $\partial\Omega$. Nonnegative initial functions $u_0(x) \geq 0$, $v_0(x) \geq 0$ and $w_0(x) \geq 0$ are given in Ω.

We have already studied the problem (1.1). In [5, 6, 7], under the Neumann boundary conditions on w, we constructed global solutions and a dynamical system. It was also shown that its dynamical system possesses a Lyapunov function which furthermore implies non emptiness of ω-limit sets of trajectories and even convergence of trajectories to stationary states as $t \to \infty$. In [12, 13, 14], we considered the case when the Dirichlet boundary conditions are imposed on w. The subsequent paper [16] studied the forest boundary. The model equations in (1.1) certainly reproduce the forest boundaries.

The model equations however seem to be incomplete, for the stationary solutions u, v of (1.1) have a full support Ω even though they are discontinuous functions in Ω. We can see that a clear discontinuity in the density of young and old age trees which corresponds to the forest natural boundary. But, as the profiles [18, Fig. 11.1-2] of stationary solutions shows, even outside the domain surrounded by the forest boundary, the density of trees is positive. See also [6, Fig. 3] and [14, Fig. 3]. In order to have stationary solutions u, v with compact support, we want to modify the model equations. Our modified equations indeed take the form:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \beta\delta(w - w_*)_+ - \gamma(v)u - fu & \text{in} \quad \Omega \times (0, \infty), \\ \dfrac{\partial v}{\partial t} = fu - hv & \text{in} \quad \Omega \times (0, \infty), \\ \dfrac{\partial w}{\partial t} = d\Delta w - \beta w + \alpha\widetilde{v} & \text{in} \quad \mathbb{R}^2 \times (0, \infty), \\ u(x, 0) = u_0(x), \ v(x, 0) = v_0(x), \ w(x, 0) = w_0(x) & \text{in} \quad \Omega \quad \text{and} \quad \mathbb{R}^2. \end{cases} \tag{1.2}$$

Here, $w_* > 0$ is a fixed number and the notation $(w - w_*)_+$ denotes the positive part of $w - w_*$, namely, when $w \geq w_*$, $(w - w_*)_+ = w - w_*$ and when $w < w_*$, $(w - w_*)_+ = 0$. So, w_* is a minimal density of seeds on the ground which is necessary for trees to sprout. Now the unknown function w denoting a density of seeds in the air is defined in the whole plane $\mathbb{R}^2$. And $\widetilde{v}$ denotes a null extension of functions of $L_\infty(\Omega)$ to those of $L_\infty(\mathbb{R}^2)$, namely, $\widetilde{v}(x) = v(x)$ for $x \in \Omega$ and $\widetilde{v}(x) = 0$ for $x \in \mathbb{R}^2 - \Omega$.

As seen, our equations in (1.2) are improved in the twofold senses. First, we extend the domain where w is formulated to the whole plane $\mathbb{R}^2$, since w denotes a density of seeds in the air and the seeds can disperse beyond the boundary $\partial\Omega$ of Ω. Naturally, we need no longer consider boundary conditions on w. Second, we introduce a threshold w_* for sprouting. If $w \leq w_*$, no young age trees are reproduced; of course neither old age trees. This effect may then make the support of stationary solution u, v be compact in Ω yielding a clear forest boundary.

This paper is devoted to announcing that the problem (1.2) equally possesses global solutions and generates a dynamical system in the similar function space as (1.1) equipped with the Neumann or Dirichlet boundary conditions. We can construct a Lyapunov function, too, which implies non emptiness of ω-limit sets and convergence of trajectories to stationary solutions. Some numerical results show that, if the threshold w_* is suitably chosen, then (1.2) admits stationary solutions u, v, having a compact support in Ω.

2 Abstract formulation

2.1 Abstract evolution equation

Let us set the function space in which we handle our problem (1.1). As in [5, 6, 7, 12, 13, 14], we want to take $L_\infty(\Omega)$ for the equations on u and v and $L_2(\mathbb{R}^2)$ for the equation on w. So formulation of using an abstract evolution equation is most convenient.

Set

$$X = \left\{ \begin{pmatrix} u \\ v \\ w \end{pmatrix} ;\ u \in L_\infty(\Omega),\ v \in L_\infty(\Omega) \quad \text{and} \quad w \in L_2(\mathbb{R}^2) \right\} \tag{2.1}$$

as an underlying space. We rewrite (1.2) into the Cauchy problem for an evolution equation

$$\begin{cases} \dfrac{dU}{dt} + AU = F(U), \qquad 0 < t < \infty, \\ U(0) = U_0, \end{cases} \tag{2.2}$$

in X. Here, A is a linear operator acting in X which is given by $A = \operatorname{diag}\{f, h, \Lambda\}$ with

$$\mathcal{D}(A) = \left\{ \begin{pmatrix} u \\ v \\ w \end{pmatrix} ;\ u,\, v \in L_\infty(\Omega) \quad \text{and} \quad w \in H^2(\mathbb{R}^2) \right\},$$

where Λ is the realization of $-d\Delta + \beta$ in $L_2(\mathbb{R}^2)$ with domain $\mathcal{D}(\Lambda) = H^2(\mathbb{R}^2)$. It is clear that A is a sectorial operator of X with angle $\omega_A = 0$. For $0 \le \theta \le 1$, the fractional power A^θ of A is given by $A^\theta = \operatorname{diag}\{f^\theta, h^\theta, \Lambda^\theta\}$ with

$$\mathcal{D}(A^\theta) = \left\{ \begin{pmatrix} u \\ v \\ w \end{pmatrix} ;\ u,\, v \in L_\infty(\Omega) \quad \text{and} \quad w \in H^{2\theta}(\mathbb{R}^2) \right\}.$$

The nonlinear operator F is given by

$$F(U) = \begin{pmatrix} \beta\delta(w - w_*)_+ - \gamma(v)u \\ fu \\ \alpha\widetilde{v} \end{pmatrix}, \quad U = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \in \mathcal{D}(A^\eta),$$

where η is an arbitrarily fixed exponent in such a way that $\frac{1}{2} < \eta < 1$. Initial value U_0 will be taken in X.

2.2 Local solutions

As mentioned above, A in (2.2) is a sectorial operator of X with angle $\omega_A = 0$; in particular, A is the generator of an analytic semigroup on X. So it is possible to apply the theory of semilinear abstract parabolic equations to obtain the local existence theorem.

Theorem 2.1. *For any triplet u_0, $v_0 \in L_\infty(\Omega)$ and $w_0 \in L_2(\mathbb{R}^2)$, (2.2) possesses a unique local solution in the function space*

$$\begin{cases} u,\, v \in \mathcal{C}([0, T_0]; L_\infty(\Omega)) \cap \mathcal{C}^1((0, T_0]; L_\infty(\Omega)),\\ w \in \mathcal{C}([0, T_0]; L_2(\mathbb{R}^2)) \cap \mathcal{C}^1((0, T_0]; L_2(\mathbb{R}^2)) \cap \mathcal{C}((0, T_0]; H^2(\mathbb{R}^2)). \end{cases}$$

Here, $T_0 > 0$ is determined by the norm $\|u_0\|_{L_\infty} + \|v_0\|_{L_\infty} + \|w_0\|_{L_2}$ alone.

The procedure of proof of this theorem is quite similar to that of [18, Section 11.2]. See also [5, Theorem 3.1] and [12, Theorem 3.1].

Nonnegativity of local solutions is also verified. It is shown that, if $u_0 \geq 0$, $v_0 \geq 0$ and $w_0 \geq 0$, then the local solution obtained in Theorem 2.1 satisfies $u(t) \geq 0$, $v(t) \geq 0$ and $w(t) \geq 0$ for every $0 < t \leq T_0$. This fact is proved by the method of truncation of using the $\mathcal{C}^{1,1}$ cutoff function given by $H(u) = \frac{u^2}{2}$ for $-\infty < u < 0$ and $H(u) \equiv 0$ for $0 \leq u < \infty$.

3 Dynamical system

3.1 Global solutions

In order to prove the global existence for (2.2), we have to establish a priori estimates for the local solutions.

Proposition 3.1. *Let $0 \leq u_0, v_0 \in L_\infty(\Omega)$ and $0 \leq w_0 \in L_2(\mathbb{R}^2)$. Let (u, v, w) be any local solution of (2.2) on an interval $[0, T_{u,v,w})$ such that*

$$\begin{cases} 0 \leq u, v \in \mathcal{C}([0, T_{u,v,w}); L_\infty(\Omega)) \cap \mathcal{C}^1((0, T_{u,v,w}); L_\infty(\Omega)),\\ 0 \leq w \in \mathcal{C}([0, T_{u,v,w}); L_2(\mathbb{R}^2)) \cap \mathcal{C}^1((0, T_{u,v,w}); L_2(\mathbb{R}^2)) \cap \mathcal{C}((0, T_{u,v,w}); H^2(\mathbb{R}^2)). \end{cases}$$

Then, the estimate

$$\begin{aligned} &\|u(t)\|_{L_\infty} + \|v(t)\|_{L_\infty} + \|w(t)\|_{L_2}\\ &\qquad \leq C[e^{-\rho t}(\|u_0\|_{L_\infty} + \|v_0\|_{L_\infty} + \|w_0\|_{L_2}) + 1], \qquad 0 \leq t < T_{u,v,w}, \end{aligned}$$

holds true with some constant $C > 0$ and some exponent $\rho > 0$ independent of (u, v, w).

The procedure of proof of this theorem is quite similar to that of [18, Section 11.3]. See also [5, Proposition 5.1] and [12, Proposition 5.1].

As an immediate consequence of Proposition 3.1, we can prove the existence and uniqueness for (2.2).

Theorem 3.1. *Let* u_0, $v_0 \in L_\infty(\Omega)$ *and* $w_0 \in L_2(\mathbb{R}^2)$ *with* $u_0 \geq 0$, $v_0 \geq 0$ *and* $w_0 \geq 0$. *Then,* (2.2) *possesses a unique global solution such that*

$$\begin{cases} 0 \leq u, v \in \mathcal{C}([0,\infty); L_\infty(\Omega)) \cap \mathcal{C}^1((0,\infty); L_\infty(\Omega)), \\ 0 \leq w \in \mathcal{C}([0,\infty); L_2(\mathbb{R}^2)) \cap \mathcal{C}^1((0,\infty); L_2(\mathbb{R}^2)) \cap \mathcal{C}((0,\infty); H^2(\mathbb{R}^2)). \end{cases}$$

The global solution also satisfies the estimate

$$\begin{aligned} &\|u(t)\|_{L_\infty} + \|v(t)\|_{L_\infty} + \|w(t)\|_{L_2} \\ &\qquad \leq C[e^{-\rho t}(\|u_0\|_{L_\infty} + \|v_0\|_{L_\infty} + \|w_0\|_{L_2}) + 1], \qquad 0 \leq t < \infty. \end{aligned} \tag{3.1}$$

Furthermore, it is immediate to construct a dynamical system generated by the Cauchy problem (2.2). Set

$$K = \left\{ \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix} \in X; \quad u_0 \geq 0,\ v_0 \geq 0 \quad \text{and} \quad w_0 \geq 0 \right\}$$

as the phase space. As shown by Theorem 3.1, for each $U_0 \in K$, there exists a unique global solution $U(t; U_0) = {}^t(u(t), v(t), w(t))$ to (2.2). Therefore, we can define a nonlinear semigroup $\{S(t)\}_{t\geq 0}$ acting on K by $S(t)U_0 = U(t; U_0)$. We see that $S(t)$ is continuous with respect to the X-topology, see [5, Proposition 5.3] and [12, Proposition 5.2]. Hence, $(S(t), K, X)$ defines a dynamical system in the universal space X, K being phase space, which is called the dynamical system generated by (2.2).

3.2 Lyapunov function

As for the case of the old model (1.1), we can construct a Lyapunov function for the dynamical system $(S(t), K, X)$.

In fact, put $\varphi = fu - hv$. Then,

$$\begin{aligned} \frac{\partial \varphi}{\partial t} &= f\frac{\partial u}{\partial t} - h\frac{\partial v}{\partial t} = f[\beta\delta(w - w_*)_+ - \gamma(v)u - fu] - h\varphi \\ &= f\beta\delta(w - w_*)_+ - [\gamma(v) + f + h]\varphi - h[\gamma(v) + f]v. \end{aligned}$$

Multiply this by φ and integrate the product in Ω. Then, since $\varphi = \frac{\partial v}{\partial t}$, it follows that

$$\frac{1}{2}\frac{d}{dt}\int_\Omega \varphi^2 dx + h\frac{d}{dt}\int_\Omega \Gamma(v)dx - f\beta\delta\int_\Omega \frac{\partial v}{\partial t}(w-w_*)_+dx = -\int_\Omega [\gamma(v)+f+h]\left(\frac{\partial v}{\partial t}\right)^2 dx,$$

where $\Gamma(v) = \int_0^v [\gamma(v)+f]dv$ is a forth order function in v. Meanwhile, multiple the third equation of (1.2) by $\frac{\partial}{\partial t}(w-w_*)_+$ and integrate the product in $\mathbb{R}^2$. Then,

$$\begin{aligned}\int_{\mathbb{R}^2}\frac{\partial}{\partial t}(w-w_*)\cdot\frac{\partial}{\partial t}(w-w_*)_+dx &= d\int_{\mathbb{R}^2}\Delta(w-w_*)\cdot\frac{\partial}{\partial t}(w-w_*)_+dx\\ &-\beta\int_{\mathbb{R}^2}(w-w_*)\frac{\partial}{\partial t}(w-w_*)_+dx - \beta w_*\int_{\mathbb{R}^2}\frac{\partial}{\partial t}(w-w_*)_+dx\\ &+\alpha\int_{\mathbb{R}^2}\widetilde{v}\frac{\partial}{\partial t}(w-w_*)_+dx.\end{aligned}$$

Therefore,

$$\frac{d}{2}\frac{d}{dt}\int_{\mathbb{R}^2}|\nabla(w-w_*)_+|^2dx + \frac{\beta}{2}\frac{d}{dt}\int_{\mathbb{R}^2}[(w-w_*)_+]^2dx + \beta w_*\frac{d}{dt}\int_{\mathbb{R}^2}(w-w_*)_+dx - \alpha\int_{\mathbb{R}^2}\widetilde{v}\frac{\partial}{\partial t}(w-w_*)_+dx = -\int_{\mathbb{R}^2}\left[\frac{\partial}{\partial t}(w-w_*)_+\right]^2dx.$$

As a consequence we obtain that

$$\begin{aligned}&\frac{d}{dt}\Big\{\int_\Omega\left[\frac{\alpha}{2}\varphi^2 + h\alpha\Gamma(v) - (f\alpha\beta\delta)v(w-w_*)_+\right]dx\\ &+\int_{\mathbb{R}^2}\left[\frac{df\beta\delta}{2}|\nabla(w-w_*)_+|^2 + \frac{f\beta^2\delta}{2}[(w-w_*)_+]^2 + (f\beta^2\delta w_*)(w-w_*)_+\right]dx\Big\}\\ &= -\alpha\int_\Omega[\gamma(v)+f+h]\left(\frac{\partial v}{\partial t}\right)^2dx - f\beta\delta\int_{\mathbb{R}^2}\left[\frac{\partial}{\partial t}(w-w_*)_+\right]^2dx \le 0.\end{aligned}$$

This means that the function

$$\begin{aligned}\Psi(U) &= \int_\Omega\left[\frac{\alpha}{2}(fu-hv)^2 + h\alpha\Gamma(v) - (f\alpha\beta\delta)v(w-w_*)_+\right]dx\\ &+\int_{\mathbb{R}^2}\left[\frac{df\beta\delta}{2}|\nabla(w-w_*)_+|^2 + \frac{f\beta^2\delta}{2}[(w-w_*)_+]^2 + (f\beta^2\delta w_*)(w-w_*)_+\right]dx \qquad (3.2)\end{aligned}$$

defined for $U \in \mathcal{D}(A^{\frac{1}{2}})$ becomes a Lyapunov function for $(S(t), K, X)$.

3.3 ω-limit sets

For each trajectory $S(t)U_0$, $U_0 \in K$, the ω-limit set is defined by

$$\omega(U_0) = \bigcap_{t\geq 0} \overline{\{S(\tau)U_0;\ t \leq \tau < \infty\}} \qquad \text{(closure in the topology of } X).$$

We cannot expect, however, that $\omega(U_0)$ is a nonempty set for every initial value $U_0 \in K$ (cf. [7, Section 4] and [14, Section 5]). So, we will introduce some ω-limit set in a weak topology.

We define the L_2 topology of X as follows. A sequence $\{^t(u_n, v_n, w_n)\}$ in X is said to be L_2 convergent to $^t(u_0, v_0, w_0) \in X$ as $n \to \infty$, if $u_n \to u_0$ strongly in $L_2(\Omega)$, $v_n \to v_0$ strongly in $L_2(\Omega)$, and $w_n \to w_0$ strongly in $L_2(\mathbb{R}^2)$. Then, using this topology we define the L_2-ω-limit set of $S(t)U_0$, $U_0 \in K$, by

$$L_2\text{-}\omega(U_0) = \bigcap_{t\geq 0} \overline{\{S(\tau)U_0;\ t \leq \tau < \infty\}} \qquad \text{(closure in the } L_2 \text{ topology of } X).$$

In addition, we may equip X with the weak* topology. A sequence $\{^t(u_n, v_n, w_n)\}$ in X is said to be weak* convergent to $^t(u_0, v_0, w_0) \in X$ as $n \to \infty$, if $u_n \to u_0$ weak* in $L_\infty(\Omega)$, $v_n \to v_0$ weak* in $L_\infty(\Omega)$, and $w_n \to w_0$ strongly in $L_2(\mathbb{R}^2)$. Using this topology, we define also the w*-ω-limit set of $S(t)U_0$, $U_0 \in K$, by

$$\text{w*-}\omega(U_0) = \bigcap_{t\geq 0} \overline{\{S(\tau)U_0;\ t \leq \tau < \infty\}} \quad \text{(closure in the weak* topology of } X).$$

By definition it is clear that

$$\omega(U_0) \subset L^2\text{-}\omega(U_0) \subset \text{w*-}\omega(U_0), \qquad U_0 \in K.$$

It is possible to prove the following results. In view of (3.1) of Theorem 3.1, Banach-Alaoglu's theorem provides non emptiness of w*-$\omega(U_0)$.

Theorem 3.2. *For each $U_0 \in K$, w*-$\omega(U_0)$ is a nonempty set.*

We do not know whether L_2-$\omega(U_0) =$ w*-$\omega(U_0)$ or not in general. We can however utilize the Lyapunov function (3.2) to show that every L_2-ω-limit set consists of equilibrium points.

Theorem 3.3. *The set L_2-$\omega(U_0)$ consists of equilibrium points for every $U_0 \in K$.*

The procedure of proof of this theorem is quite analogous to that of [18, Theorems 11.2-4]. Note that, for any $\varepsilon > 0$, there exist a radius $R > 0$ and time $T > 0$ such that $\|w(t)\|_{L_2(|x|>R)} < \varepsilon$ for every $t > T$. See also [6, Theorem 5.2] and [13, Theorem 5.1].

It is however unknown whether the L^2 ω-limit set is nonempty for any initial value $U_0 \in K$ or not. It is similarly unknown whether the weak* ω-limit set consists of equilibrium points for any $U_0 \in K$ or not. For the details, see [17].

4 Some numerical results

In this section, we will present some numerical example of solution to the problem (1.2). Set $\Omega = [0, 1] \times [0, 1]$. As for the unknown function w, we set $\Omega_0 = [-2, 3] \times [-2, 3]$ and assume that w satisfies the diffusion equation in this wider domain Ω_0 and satisfies the Dirichlet boundary conditions on $\partial\Omega_0$. (It is impossible to take $\Omega_0 = \mathbb{R}^2$ in actual computations.)

More precisely, we consider the following problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = (w - 1.7)_+ - [(v-3)^2 + 0.2]u - u & \text{in} \quad \Omega \times (0, \infty), \\ \dfrac{\partial v}{\partial t} = u - 0.048v & \text{in} \quad \Omega \times (0, \infty), \\ \dfrac{\partial w}{\partial t} = 0.05\Delta w - w + \widetilde{v} & \text{in} \quad \Omega_0 \times (0, \infty), \\ w = 0 & \text{on} \quad \partial\Omega_0 \times (0, \infty), \\ u(x, 0) = 0.9, \ v(x, 0) = 6.0, \ w(x, 0) = 0 & \text{in} \quad \Omega \quad \text{and} \quad \Omega_0. \end{cases} \tag{4.1}$$

As t increases, the numerical solution to (4.1) converges to a stationary solution. At $t = 50.000$, the solution $(u(t), v(t), w(t))$ is completely stabilized in numerical sense. Profiles of u and v at that moment are illustrated in Figure 1. We see that u and v vanish identically in the corners of Ω. At the vertexes of Ω the value of w is, according to profile of w in Figure 2, almost equal to 0.8. But the threshold w_* is now 1.7. Therefore, the forest has no trees in the corners.

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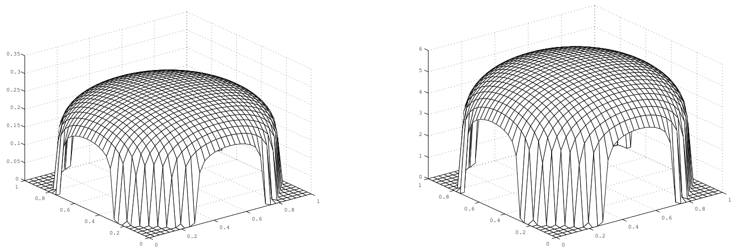
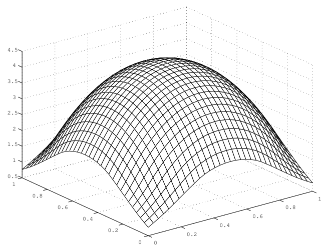


(a) Graph of u (b) Graph of v

Figure 1: Graph of u and v at $t = 50.000$



(a) Graph of w

Figure 2: Graph of w at $t = 50.000$

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