Perturbation theory for the dynamics of mean-field systems

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Abstract

This work concerns the study of perturbed N-body mean-field interacting systems. Mean-field interactions belong to the general class of long-range interactions and show intriguing equilibrium and out-of-equilibrium features, many of them absent for short-range interactions. One of the most peculiar out-of-equilibrium feature is the fact that these systems get trapped in long-living states, called Quasi-Stationary States (QSSs). Their lifetime diverges algebraically with the size of the system, N. We are here interested in studying the influence of a small external perturbation acting on a stable QSS.

The time evolution of QSSs is well described in the large N limit by the Vlasov equation. In the mean-field limit ($N \to \infty$), Vlasov dynamics fully describe the time evolution of the system, in absence of singular interactions. This equation posses an infinite number of stationary solutions that can be either homogeneous or inhomogeneous in space. We have developed a linear response theory for homogeneous QSSs using Fourier-Laplace techniques, analogous to the ones used in the theory of linear Landau damping. The theory allows us to compute the variation of any observable when a small perturbing field is added to the Hamiltonian of the system. Both the time dependent and the asymptotic response are accessible to the theory. The Hamiltonian Mean-Field (HMF) model is a paradigmatic model for mean-field and long-range interactions. It describe the motion of particle on a unitary circle which interact all to all with an attractive of repulsive potential. Alternatively, the model can be interpreted as representing XY spins with global coupling. Within this interpretation attractive (repulsive) interactions correspond to ferromagnetic (anti-ferromagnetic) couplings. We have shown that when an external field is applied to a stable QSS, magnetization changes as a result of both the

variation of the distribution function and of the mean-field. Comparing our theoretical prediction with numerical simulations we have obtained a good agreement at linear order. Second order corrections are also calculable and can affect the variation of the observables.

The theory for homogeneous state can be obtained without taking into account a peculiar feature of the Vlasov equation, the presence of an infinity of conserved quantities, called Casimirs. This fact cannot be neglected when facing in the problem of inhomogeneous states. Moreover, inhomogeneity introduces a coupling between the mean-field modes and the the modes of the distribution. The treatment of Landau damping for inhomogeneous states is still an open and difficult problem. However, something can be done within our linear response theory approach. Indeed, we have developed an "approximate" linear response theory imposing by hand the constancy of a finite number of Casimirs, besides the usual conserved quantities related with symmetries. Other authors have been able to derive an exact linear response theory for integrable systems, including the HMF model when restricted to stationary states.

In order to check the validity of our approximate linear response theory we compare our result for the response of an observable to the exact result obtained using this second type of linear response theory. Overall, the agreement between the two theories is good. However, close to the phase transition of the second order of the HMF model, the disagreement can become quite sharp, because the presence of additional conserved quantities here plays a crucial role. This is why our approximate theory in unable to produce the correct values of critical exponents, which we have obtained using the exact integrable theory. It is interesting to remark that the exponent in the inhomogeneous phase is not the "classical" mean-field exponent, but depends on the chosen class of distribution, here the one of Jeans' states.

Our perturbative approach can be also used to discuss the mean-field interaction between a small system and a big one, considered as a reservoir in canonical ensemble theory. We studied in particular how a kinetically-defined temperature changes by this contact and found a non-trivial result that a hotter system can become even hotter by the contact.