PRAMANA
— journal of
physics

Vol. 84, No. 5
May 2015
pp. 747–755

Freeze-out dynamics in heavy-ion collisions: Recent advances

FRANCESCO BECATTINI
Department of Physics and Astronomy, Università di Firenze, via G. Sansone 1,
I-50019, Sesto F.no (Firenze), Italy
E-mail: becattini@fi.infn.it

DOI: 10.1007/s12043-015-0977-9; ***e*Publication:** 1 May 2015

Abstract. We briefly review recent advances in the subject of hadron production in relativistic heavy-ion collisions. We focus on the issues of chemical freeze-out, chemical equilibration and the role of post-hadronization inelastic collisions. From the observations collected in elementary and heavy-ion collisions, a picture emerges in which hadrons are born in chemical equilibrium at hadronization, thereafter undergoing inelastic and elastic collisions whose impact on the primordial distribution depends on the system size.

Keywords. Relativistic heavy-ion collisions; freeze-out; statistical model.

PACS Nos 25.75.Ag; 12.40.Ee

1. Introduction

The study of hadroproduction in relativistic heavy-ion collisions has been the subject of intense research for more than 20 years by now. Accurate measurements of multiplicities of different species and their spectra over a large span of centre-of-mass energies have been a major test bench for the models which have led to a concrete verification of the QCD phase diagram [1]. Lately, the excellent measurements provided by the LHC experiments at a nucleon–nucleon centre-of-mass energy of $\sqrt{s}_{NN} = 2.75$ TeV have elicited a renewed interest in the dynamics of the bulk hadron production, revived old ideas and stimulated new ones. In this short review, we summarize the status of the field and discuss the recent advances.

2. Freeze-out in an expanding hadron gas

In an expanding sytem of interacting particles freeze-out occurs when the mean scattering time $\tau_{\rm scatt}$ exceeds the mean collision time $\tau_{\rm exp}$:

$$\tau_{\text{scatt}} = \frac{1}{n\sigma \langle v \rangle} > \tau_{\text{exp}} = \frac{1}{\partial \cdot u}, \tag{1}$$

u being the hydrodynamical velocity field and $\langle v \rangle$ is the mean velocity of particles. If the cross-section σ is inelastic, the freeze-out is called chemical, whereas if it includes elastic processes, the freeze-out is called kinetic. Chemical freeze-out of course precedes the kinetic freeze-out as the inelastic cross-section is smaller than the total cross-section.

We can obtain a gross approximation of the expansion time with the ratio $V/\dot{V}$ where $V(t)$ is the volume of the fireball at time t. For a fireball which is spherical in shape with radius R, this is $R/3\dot{R}$ and if the radius increases at approximately the mean particle velocity $\langle v \rangle$, we have the condition:

$$\frac{1}{n\sigma \langle v \rangle} \approx \frac{R}{3\langle v \rangle} \implies \frac{1}{n\sigma} \approx \frac{R}{3}. \tag{2}$$

For a given number of particles N within the volume, this inequality yields the radius at which freeze-out occurs as a function of N and of the average cross-section:

$$R_{\text{fo}} \approx \sqrt{\frac{N\sigma}{4\pi}} \tag{3}$$

and the density at which freeze-out occurs, which decreases with N according to:

$$n_{\text{fo}} \approx \frac{N}{(4\pi/3) R_{\text{fo}}^3} \approx 3\sqrt{\frac{4\pi}{N}} \frac{1}{\sigma^{3/2}}. \tag{4}$$

For instance, for $N = 1000$, typical of heavy-ion collisions, and $\sigma = 30\,\text{mb} = 3\,\text{fm}^2$, one has $R_{\text{fo}} \simeq 15$ fm and $n_{\text{fo}} \simeq 0.06\ \text{fm}^{-3}$, which are in the right ballpark taking into account the drastically made approximations. The estimates (3) and (4) are obviosuly crude, but they tell us that the freeze-out radius, for each particle, approximately scales with the square root of the number of scattering centres a particle can interact with and the related cross-section.

An interesting question, which is relevant for a hadronic gas, is whether multibody collisions are significant at some stage in the expansion process. One can write a simple generalization of the two-body collision rate formula:

$$\frac{\mathrm{d}R}{\mathrm{d}^4x} = n_1 n_2 \sigma v_{\text{rel}} = n_1 n_2 \sigma \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{\varepsilon_1 \varepsilon_2}, \tag{5}$$

where p_i are the particle four-momenta and m_i their masses, n_i their densities and v_{rel} the relativistic relative velocity, to a three-body collision assuming that the particle 3 collides with a cluster (12) made of particles 1 and 2:

$$\left.\frac{\mathrm{d}R}{\mathrm{d}^4x}\right|_3 = n_3 n_{(12)} \sigma_{3(12)} v_{\text{rel}} P_{(12)}, \tag{6}$$

where $n_{(12)}$ is the density of clusters, v_{rel} is a suitable extension of the relativistic relative velocity function for the 2-body problem and $P_{(12)}$ is the cluster formation probability. The latter can be assumed to be proportional to $n_1 n_2 \sigma_{12}^{3/2}$, i.e. proportional to the probability that 1 and 2 find themselves within a range of $\approx \sqrt{\sigma}$. Replacing $\sigma_{3(12)}$ and σ_{12} with a typical hadronic cross-section σ, one has

$$\left.\frac{\mathrm{d}R}{\mathrm{d}^4x}\right|_3 \approx \frac{n_1 n_2 n_3}{\varepsilon_1 \varepsilon_2 \varepsilon_3} \sigma^{5/2} f_{\text{rel}}(m_1, m_2, m_3, \sqrt{s}, ...), \tag{7}$$

where f is a function of relativistic invariants pertaining to a three-particle system. This approximated formula can be extended to N-body collisions:

$$\left.\frac{\mathrm{d}R}{\mathrm{d}^4x}\right|_N \approx \left(\prod_{i=1}^{N}\frac{n_i}{\varepsilon_i}\right)\sigma^{(3N-4)/2} f_{\mathrm{rel}}(m_1,\ldots,m_N,\sqrt{s},\ldots). \tag{8}$$

Binary collisions prevail when the left-hand side of (5) largely exceeds the one in (7). Assuming that the ratio of the relativistic functions f_{rel} average to $\mathcal{O}(1)$, this happens when

$$n\sigma^{3/2} \ll 1 \implies \lambda = \frac{1}{\sigma n} \gg \sqrt{\sigma}. \tag{9}$$

This is physically a very clear condition: the particle mean free path λ should be much larger than its effective interaction distance as determined by the square root of the cross-section. This is by no means a trivial requirement; for an expanding hadron gas to have a stage where binary collisions prevail before freeze-out, this implies, according to eq. (2):

$$R_{\mathrm{fo}} \approx \sqrt{\frac{N\sigma}{4\pi}} \approx 3\lambda \gg 3\sqrt{\sigma} \tag{10}$$

that is the system should contain a large number of particles, $N \gg \mathcal{O}(100)$. If this condition is not fulfilled, the system decouples when ternary and, in general, N-nary collisions are still relevant. This is an important conclusion, to be emphasized: in relativistic ion collisions, for the decoupling to occur in the binary collision regime, the number of particles should largely exceed $\mathcal{O}(100)$. Thus, this may happen only in heavy-ion collisions.

There is one more important condition to be addressed: for the freeze-out to be in the kinetic regime, meaning a system of quasifree colliding particles, it is necessary that the mean free path of particles largely exceeds their mean (thermal) wavelength, that is,

$$\lambda = \frac{1}{\sigma n} \gg \frac{1}{\langle p\rangle} = \frac{1}{m\langle v\rangle} = \frac{1}{\sqrt{3Tm}}, \tag{11}$$

where the last expression assumes the non-relativistic regime of most hadrons, which is a good approximation for a hadron gas in the actual observed range of temperatures. The motivation of the condition (11) is that the collisional approximation of a strongly interacting system makes sense only if particles exist, i.e. their quantum wavelength is much smaller than its mean free path. Otherwise, the so-called particles would interact after a collision before being 'formed' and the whole kinetic picture would break down. Hence, writing $n = N/(4\pi R^3/3)$ and using (3):

$$\frac{4\pi}{3}R^3 \gg N\sigma\frac{1}{\sqrt{3Tm}} \implies R \gg R_{\mathrm{fo}}^{2/3}\left(\frac{3}{Tm}\right)^{1/6} \tag{12}$$

with the assumption that the number of particles N does not change till freeze-out. The last inequality dictates that we have a kinetic stage in hadron gas expansion, that is, $R < R_{\mathrm{fo}}$, provided that the radius at freeze-out is very much larger than the thermal wavelength of the hadrons. As a numerical example, for pions at $T = 160$ MeV, the last factor equals 1.3 fm$^{1/3}$ and for $R_{\mathrm{fo}} = 15$ fm, the above inequality demands $R \gg 8$ fm. We stress again that these numbers are crude approximations and only have an illustrative purpose. They should not be compared as such with actual measurements of the system size with e.g., pion interpherometry.

Another relevant question is whether multibody collisions can occur in what we can properly call a kinetic regime. According to (9), multibody collisions are relevant if the mean free path is of the order of the square root of the cross-section. Therefore, we are led to demand that

$$\lambda \approx \sqrt{\sigma} \gg \sqrt{\frac{1}{3mT}}. \tag{13}$$

Is this condition fulfilled? For pions at the typical temperature $T = 160$ MeV, the right-hand side is about 1 fm, which is not much smaller than the square root of the typical hadronic cross-section. Under this circumstance, the hadron gas cannot be considered as made of individual colliding particles, at least as far as pions are concerned. This is quite a peculiar situation compared to other, more familiar, kinetic systems. For instance, for liquid water at $T = 0$°C, the mean free path can be estimated to be 0.3 nm, which is approximately equal to $\sqrt{\sigma}$. Yet the mean thermal wavelength of a H_2O molecule is of the order of 5 pm, mostly due to the large molecular mass. Hence, in liquid water we have multibody collisions and yet molecules retain their particle identity which is necessary to have a kinetic description. This special feature of the hadron gas, close to the critical QCD temperature, is related to the very low viscosity/entropy density ratio which reflects the absence of quasiparticles, hence of kinetic description, along with the other so-called strongly interacting fluids [2].

3. The statistical model

Among the models which have been proposed to account for the quantitative features of hadron production in relativistic heavy-ion collisions, the statistical hadronization model can be considered as a reference. Besides its remarkable success in reproducing particle multiplicities with a few parameters, this model is used in nearly all hydrodynamical calculations to turn the fluid cells into hadrons through the Cooper–Frye prescription, enforcing local thermodynamical equilibrium. Furthermore, it has been shown that this model can reproduce hadronic multiplicities in elementary collisions, from e^+e^- to pp and from low to high energy [3]. Therefore, it turns out that the statistical model grasps a peculiar universal feature of the hadronization process, hence of QCD in the non-perturbative regime, regardless of the kind of collision.

Indeed, the statistical model has proved to be successful in relativistic heavy-ion collisions throughout the explored centre-of-mass energy range [3]. The interpretation of its success is, however, still a subject of debate. Some proposed that thermalization is achieved at the level of hadrons through multiple collisions [4]. Others [5], including the author, advocate that hadronization itself gives rise to an equilibrated system of hadrons (modulo the extra strangeness suppression in elementary collisions, not fully understood). In Hagedorn's words *hadrons are born in equilibrium* because it would be otherwise impossible to explain the statistical distributions observed in elementary collisions where multiplicities are such that freeze-out coincides with hadron formation, and, chiefly, the apparent full chemical equilibrium of strange particle abundances.

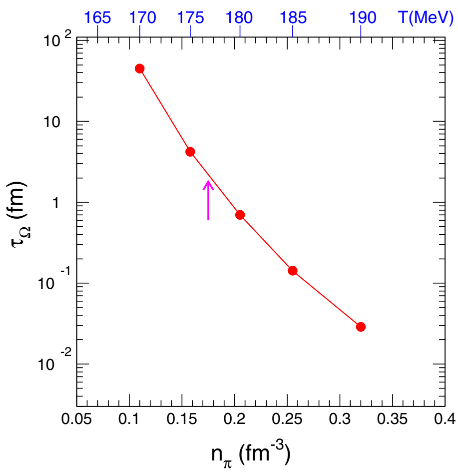

Figure 1. Estimated equilibration time for Ω hyperons as a function of pion density and temperature in a hadron gas through multibody collisions (from ref. [4]).

Indeed, in the collisional approach to thermalization, the abundance of strange particles, particulary the Ω baryon, is a long-standing problem. In order to reproduce the observed enhancement in relativistic heavy-ion collisions with respect to *pp*, one has to invoke multibody collisions [4,6] as well as the existence of hypothetical broad resonances called Hagedorn states [6]. However, we have seen in §2 that in a hadron gas multibody reactions become relevant in a regime where the system cannot be considered as kinetic, i.e. in a regime where it is still a strongly interacting fluid and its particle or quasiparticle description is questionable. Besides, the estimates of the time needed to achieve equilibrium abundances of Ω hyperon are of the order of 1000 fm/c [4,6] for the chemical freeze-out temperatures of around 160 MeV (see also figure 1) determined at LHC [7,8]. These difficulties, and chiefly the observations in elementary collisions, lead to the conclusion that hadronization produces a chemically equilibrated system.

4. Is there chemical life after hadronization?

If hadronization itself gives rise to an equilibrated population of hadrons, can the later processes, if any, affect it? This is an important question for understanding the hadron production mechanisms in both elementary and relativistic heavy-ion collisions. We can sketch – in a very ideal fashion – the major steps in the process leading to the freeze-out in relativistic heavy-ion collisions as points along a time-oriented axis as in figure 2. In fact, the freeze-out processes are continuous, species-dependent, geometry-dependent and overlapping. Nevertheless, figure 2 is a useful tool for the purpose of illustration.

After hadronization, the system begins to expand, but it can stay a little in the strong coupling regime where it cannot be described as a kinetic system, according to the discussion in §2. At some point, this regime, where chemical equilibrium is presumably kept, ceases and the era of binary collisions sets in, provided the number of particles is $> \mathcal{O}(100)$. For some time, binary collisions can be inelastic (chemical freeze-out), thereafter only elastic till kinetic freeze-out occurs and the system finally decouples.

Is this picture borne out by the data? The existence of a stage where binary collisions prevail is confirmed by at least three major evidences:

(1) The suppression of short-lived resonances compared to long-lived particles in the statistical model fits [9–11], an effect which is not seen in elementary collisions [12]. It can be explained by the rescattering of the resonance decay products in the expanding medium, which is not possible for a small system.
(2) The difference between the temperatures determined in multiplicity fits and those in the fits of transverse spectra [11]. This means that elastic interactions continue after chemical freeze-out. Also this effect is not seen in elementary collisions [12].
(3) The dependence of kinetic freeze-out temperatures as a function of the impact parameter [11,13]. Collisional decoupling depends on particle multiplicity, as we have seen, as well as geometry, and it occurs at a higher density if multiplicity is lower.

The existence of a kinetic stage where hadrons interact elastically is by now generally accepted. Do we have a similar evidence for the abundances? In other words, do hadrons chemically decouple after some stage of binary collisions? It should be stressed that, if such a stage exists, hadronic abundances should show deviations from an initial chemical equilibrium situation [14].

Until recently, there was no major clue of such a phenomenon as statistical model fits were in agreement with the data to a satisfactory degree of accuracy. However, new measurements of antiproton yield at SPS energy [15] and at LHC [16] resulted in significantly lower values compared to the statistical model predictions [17,18]. This result has been soon interpreted as an effect of post-hadronization inelastic rescattering, where antibaryons are annihilated before chemical freeze-out, increasing pion multiplicity [19–21]. We point out that such annihilations should occur in a stage where 2-body collisions prevail (see figure 2) and Monte-Carlo simulations such as UrQMD [22] only include binary collisions. However, as has been mentioned, the separation of N-ary to binary collision stages in fact cannot be sharp and indeed it was pointed out [23] that the back-reaction of N pions colliding to regenerate antibaryons could continue for a longer time.

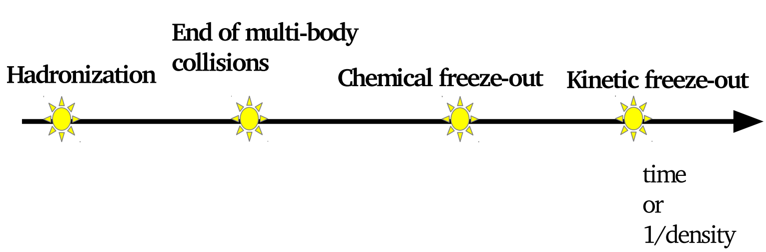

Figure 2. Sketch of the time sequence of processes in an expanding, strongly interacting hadronic system: after hadrons are formed, a strongly interacting stage follows where particles may undergo multibody collisions. Thereafter, mostly binary collisions survive till chemical freeze-out and, later, kinetic freeze-out.

As a matter of fact, the reanalysis of the data assuming equilibrium at hadronization and subsequent post-hadronization inelastic rescattering stage using UrQMD predictions as correction factors (see figure 3) gives a better fit to the data and a hadronization (or latest chemical equilibrium point) in agreement with the extrapolated QCD lattice line (see figure 4) [7]. Even though UrQMD does not include N-nary collisions with N>2, this is a strong indication that inelastic processes following hadronization do play a role and that chemical freeze-out does not coincide with hadronization. Still, taking these processes properly into account, it is possible to reconstruct the original hadronization conditions.

5. Outlook

We are at just the beginning of a new generation of analyses aimed at accurately determining the hadronization conditions of the quark gluon plasma. For this purpose, as has been mentioned, we shall need reliable calculations of inelastic hadronic collisions processes in an expanding system, both binary and multibody. One of the crucial tests in this respect will be the analysis at different centralities. As can be seen from the

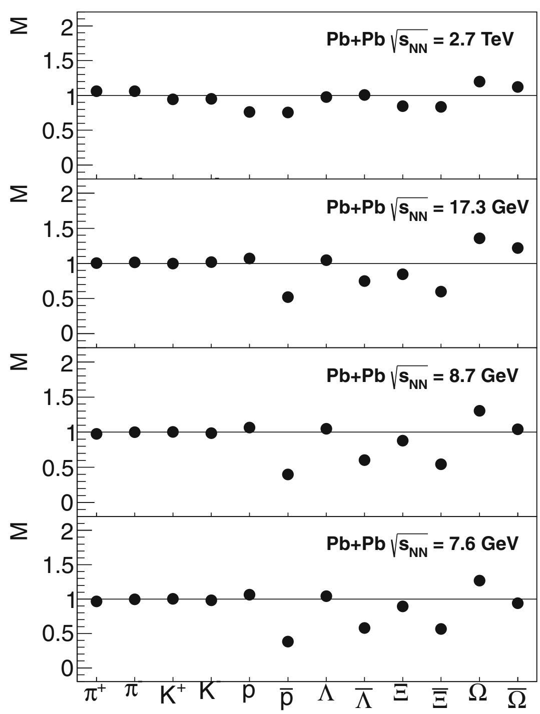

Figure 3. Ratios between hadronic multiplicities after chemical freeze-out and those at hadronization calculated with UrQMD [22] at several centre-of-mass energies in Pb–Pb collisions (from ref. [7]).

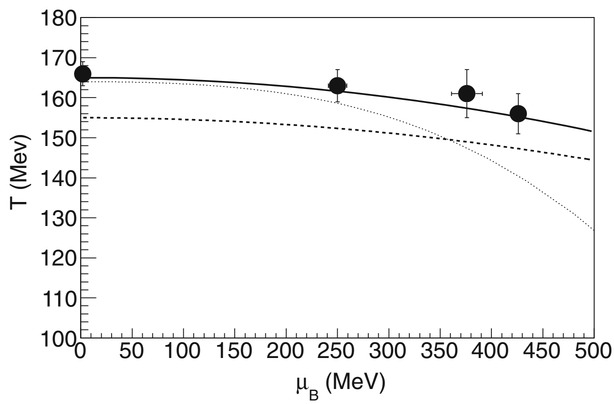

Figure 4. Reconstructed hadronization points in the (T, μ_B) plane compared with lattice QCD extrapolations (solid and dashed lines) and the chemical freeze-out line (dotted curve) (from ref. [7]).

simple formula (3), freeze-out occurs earlier for lower multiplicities. Hence some dependence of sensitive quantities like e.g. antibaryon annihilation on centrality is expected. At the same time, one would expect to see some slight dependence of the chemical freeze-out temperature obtained with plain statistical model fits on centrality, particularly a slight rise from central to peripheral collisions. Up to RHIC energies, results are not conclusive: at top energy, no significant dependence was seen [13,24], whereas at lower energies (in the beam scan programme) the centrality dependence changes slope according to the statistical ensemble used [25]. More effects may play a role in determining chemical freeze-out conditions as a function of centrality (a change of QGP hadronization conditions if baryon density changes etc.) and those should be taken into account. A major clue will come from the analysis of LHC data, where hadronization is expected to occur at vanishing baryon density throughout the centralities. Therein, owing to the increased multiplicities, freeze-out radii have a larger spread, possibly giving rise to an observable dependence of chemical freeze-out temperature on the impact parameter.

Acknowledgements

The author is grateful to M Bleicher, J Steinheimer and R Stock for useful discussions.

References

[1] J Cleymans, H Oeschler, K Redlich and S Wheaton, *Phys. Rev. C* **73**, 034905 (2006)
[2] J Casalderrey-Solana, H Liu, D Mateos, K Rajagopal and U A Wiedemann, arXiv:1101.0618 [hep-th]
A Adams, L D Carr, T Schäfer, P Steinberg and J E Thomas, *New J. Phys.* **14**, 115009 (2012)
[3] F Becattini, *An introduction to the statistical hadronization model*, arXiv:0901.3643 [hep-ph]
F Becattini and R Fries, in: *Relativistic heavy ion physics*, Landolt-Börnstein 1–23, arXiv:0907.1031 [nucl-th] and references therein

[4] P Braun-Munzinger, J Stachel and C Wetterich, *Phys. Lett. B* **596**, 61 (2004)
[5] F Becattini and U W Heinz, *Z. Phys. C* **76**, 269 (1997)
U Heinz, *Nucl. Phys. A* **661**, 140 (1999)
R Stock, *Phys. Lett. B* **456**, 277 (1999)
P Castorina, D Kharzeev and H Satz, *Eur. Phys. J. C* **52**, 187 (2007)
[6] J Noronha-Hostler, M Beitel, C Greiner and I Shovkovy, *Phys. Rev. C* **81**, 054909 (2010)
[7] F Becattini, M Bleicher, T Kollegger, T Schuster, J Steinheimer and R Stock, *Phys. Rev. Lett.* **111**, 082302 (2013)
[8] A Andronic, P Braun-Munzinger, K Redlich and J Stachel, *Nucl. Phys. A* **904–905**, 535c (2013)
[9] F Becattini, M Gazdzicki, A Keranen, J Manninen and R Stock, *Phys. Rev. C* **69**, 024905 (2004)
[10] C Markert, *J. Phys. G* **31**, S169 (2005)
[11] C Blume, *Acta Phys. Polon. B* **43**, 577 (2012)
[12] F Becattini and G Passaleva, *Eur. Phys. J. C* **23**, 551 (2002)
[13] U Heinz and G Kestin, *PoS CPOD* **2006**, 038 (2006)
[14] S A Bass and A Dumitru, *Phys. Rev. C* **61**, 064909 (2000)
[15] NA49 Collaboration: T Anticic *et al*, *Phys. Rev. C* **83**, 014901 (2011)
[16] ALICE Collaboration: B Abelev *et al*, *Phys. Rev. C* **88**, 044910 (2013)
[17] F Becattini, J Manninen and M Gazdzicki, *Phys. Rev. C* **73**, 044905 (2006)
[18] A Andronic, P Braun-Munzinger and J Stachel, arXiv:0707.4076 [nucl-th]
[19] Y Pan and S Pratt, arXiv:1210.1577 [nucl-th]
[20] J Steinheimer, J Aichelin and M Bleicher, *Phys. Rev. Lett.* **110**, 042501 (2013)
[21] F Becattini, M Bleicher, T Kollegger, M Mitrovski, T Schuster and R Stock, *Phys. Rev. C* **85**, 044921 (2012)
[22] H Petersen *et al*, *Phys. Rev. C* **78**, 044901 (2008) and references therein
[23] R Rapp and E V Shuryak, *Phys. Rev. Lett.* **86**, 2980 (2001)
[24] J Manninen and F Becattini, *Phys. Rev. C* **78**, 054901 (2008)
[25] STAR Collaboration: S Das, *Nucl. Phys. A* **904–905**, 891c (2013)