

# Lower Bound Limit Analysis of Masonry Arches with CFRP Reinforcements: A Numerical Method

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**Abstract:** This paper presents a numerical method to predict the ultimate load of masonry arches strengthened with carbon-fiber-reinforced polymer (CFRP) strips bonded to the intrados. The voussoirs of the arch and the CFRP strip, ideally divided into the same number of parts as the voussoirs, are modeled as rigid blocks. A finite set of stress resultants represents the stress state acting on interfaces of the rigid blocks. The local failure modes at the block interfaces are defined according to experimental evidence. The model is developed within an associated framework in such a way that the normality rule is satisfied: the upper- and lower-bound theorems of classical limit analysis apply. The ultimate load is predicted by a lower bound approach. The feasible domain is defined by the equilibrium equations and by the linear constraints imposed on the stress resultants. All the relations defining the model are linear, so that a linear programming problem is imposed. The predictions of the numerical model compare well with experimental results. DOI: 10.1061/(ASCE)CC.1943-5614.0000350. © 2013 American Society of Civil Engineers.

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## Introduction

Recent decades have seen a considerable increase in the use of composite materials such as carbon-fiber-reinforced polymer (CFRP) strips to reinforce masonry structures. This is because the use of these light and easily applied strips affords great benefits in terms of increased strength and ductility.

As is well known, proper analysis of masonry structures with CFRP reinforcements requires the processing of complex nonlinear relations and the identification of mechanical parameters, which are often extremely difficult to determine by experimental tests (Cancelliere et al. 2010; Elmalich and Rabinovitch 2010).

At the same time, in normal practice, these composite materials are applied to masonry structures according to the results obtained by simplified models. For instance, technicians involved in the strengthening of an arch using CFRP strips generally need a simple calculation tool, designed to predict solely the failure load and corresponding mechanism. It is precisely for this reason that limit analysis represents a useful approach, as it involves common computational tools and requires the determination of only a limited number of mechanical parameters. Heyman is the leading proponent of the use of limit analysis on unreinforced masonry arches (Heyman 1966, 1982). Livesley (1978) was the first to apply limit analysis theory to unreinforced masonry structures by modeling

them as rigid blocks and using the static approach and linear programming. Gilbert and Melbourne (1994) later followed a kinematic approach to the same problem.

An ample body of literature, by various researchers, exists regarding the application of limit analysis to the study of masonry structures reinforced with CFRP strips (e.g., Caporale et al. 2006; Luciano et al. 2001; Milani et al. 2006, 2009; Orduna and Lourenco 2003; Drosopoulos et al. 2007). The key aspect characterizing masonry structures strengthened with CFRP strips is the capacity that they acquire to bear tensile stresses. Therefore, the structural behavior of an arch reinforced with CFRP strips is completely different from that of an unreinforced arch: the presence of the reinforcement profoundly changes both the value of the collapse load and especially the corresponding failure mechanism. Unreinforced masonry arches generally collapse primarily by the opening of four hinges, or five in symmetrical arches (Lucchesi et al. 1997). Instead, masonry arches with CFRP strips bonded on their intrados may collapse due to different local mechanisms: hinged modes, masonry crushing, debonding or peeling of the CFRP reinforcement, sliding, or CFRP rupture (Foraboschi 2004). In the literature, there are many studies that investigate, by experimental tests, the structural behavior of masonry arches reinforced by CFRP. For instance, in Briccoli Bati et al. 2007, a wide experimental investigation was reported. This research was conducted on masonry arches having mechanical and geometrical characteristics similar to the one analyzed in this paper and showed, among others things, that the collapse of this kind of reinforced masonry arches is not associated with local sliding or CFRP rupture mechanisms.

This paper presents a numerical procedure able to predict the failure load and the corresponding collapse mechanism of CFRP-reinforced masonry arches. The collapse load is evaluated using limit analysis by the lower bound approach applied to a rigid-block model. The feasible domain of the static variables is defined by imposing equilibrium conditions and taking into account only the local collapse mechanisms that occurred in the experimental campaign reported in Briccoli Bati et al. 2007, i.e., hinged modes, masonry crushing, and debonding and peeling

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