

# Effects of cutting conditions on forces and force coefficients in plunge milling operations

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## Abstract

The modeling of milling forces is a crucial issue to understand milling processes. In the literature, many force models and experiments to identify force coefficients are found. The objective of this article is to develop a new approach, based on the traditional average force method, able to measure and compute the cutting coefficients for end mills used in plunging operations. This model has been used to evaluate the effect of the radial engagement on the cutting coefficients themselves, proposing a new strategy to update these values for different cutting parameters. This dependency of the cutting coefficient is particularly important for the determination of the stability lobe diagrams, used to predict the chatter conditions. In this article, the method to assess the cutting coefficients, the results of the experimental tests, and the effect of condition-dependent cutting coefficients on process stability are presented.

## Keywords

Force coefficient, plunge milling, average force method, chatter, stability lobe diagram

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## Introduction

The milling processes are continuously increasing their market penetration due to arising request of product with high surface finish and dimensional accuracy, such as the die or mold for the housewares and automotive sectors and for the products for the medial and aerospace industries. The widespread application for this technology is sometimes constrained by challenging cases that could suffer from excessive cutter deflection of chatter, which could appear when the cutting parameters are stressed to increase the process productivity. Among the arising machining processes, particular interest is played by plunge milling because it allows to work deep pocket or other difficult-to-access features maintaining a good productivity and accuracy also with slender tools. The advantage of the plunging operation is that the cutting forces are more oriented along the axis of the tool, where the tool proves its highest stiffness.

The first step to model a machining process is the definition of a cutting force model. An analysis of the literature shows that there exist mainly three cutting force models: lumped mechanism model, dual mechanism model, and ternary mechanism model. In the lumped mechanism model, the effect of both the shearing and friction at the cutting edge are described with a single coefficient. This is a simple model that produces reasonably good results with the advantage of its fast implementation in an industrial context. This approach has been developed for flat-end mill and ball-end mill by many researchers; among these, it could be reported

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by Kline et al.,<sup>1</sup> Altintas and Spence,<sup>2</sup> Lazoglu,<sup>3</sup> who extended this concept to the prediction of cutting forces to ball-end mill. Later, more complex models have been developed, taking into account different coefficients for both the shearing and friction contributions at the cutting edge, these are usually called dual mechanics approaches. In particular, Budak et al.<sup>4</sup> developed a general approach that could feed the model with data extracted from orthogonal cutting tests. Then, Altintas and Lee,<sup>5,6</sup> Yucesan and Altintas,<sup>7</sup> and Engin and Altintas<sup>8</sup> presented general models that could be used for the reliable prediction of cutting forces for a general end mill, often used to obtain stability lobe diagram (SLD) with zero-order approximation. The last is still the basis for more recent models that include other features such as the introduction of the effect of the run-out.<sup>9</sup> A detailed comparison of the most actual approaches to model the cutting force has been carried out by Wan and Zhang,<sup>10</sup> which include also different solutions to carry out one of the most crucial steps in force modeling, the calibration of the model. Some authors, such as Imani et al.,<sup>11</sup> also studied the influence of different process parameters such as the axial depth on the cutting coefficients. At the end, the ternary mechanism model<sup>12</sup> identifies and models the contributions of flank shearing, flank rubbing, and bottom cutting effects; the coefficients could be calculated considering together chip load, chip width, and bottom contact width.

It must be reported that although the analytical methods are still the most used approaches to determine the cutting forces, due to the ever-increasing computing capabilities of modern workstations and the availability on the market of commercial software able to simulate the cutting process using finite element (FE) approach, some researchers<sup>13–15</sup> used simulation to predict the cutting force. Another interesting and arising approach, able to provide a faster result than FE modeling, is provided by the use of slip line field.<sup>16</sup> This approach is based on the strain, strain rate, and temperature-dependent flow stress of the material and friction coefficient.

It is important to note that the cutting force models are dependent on the tool geometry and the machining process approach, so the choice or definition of optimal cutting model could not be a trivial task. To solve this issue, approaches that are oriented to have a unified model for cutting<sup>17</sup> based on a complete description of tool geometry or to create specific cutting models for each operation are arising. Some examples of this trend are the development of a model specific for the threading operation<sup>18</sup> and the development of a mathematical method to geometrically define a general tool as a flat tool with geometry variables or cutting inserts<sup>19</sup> or alternatively a multi-purpose tool.<sup>20</sup>

As stated before, plunge milling is an arising operation because it shows a relatively higher stiffness of the tool in the axial direction, and a larger component of the cutting forces is oriented along the tool axis. As a result, a tool used in plunge milling operation is more stable than the same used in flank milling operation, using the same engagement and cutting parameter values; for this reason, plunge milling has become an interesting strategy because it can often allow higher removal rate than other cutting strategies. Most of the cutting models are specific for the flanking and slotting operations: only few authors investigated the mechanics of plunge milling.<sup>21</sup> Plunge milling is similar to boring (with interrupted cuts),<sup>22</sup> so a different method to measure the cutting force coefficients must be developed. This article proposes a new simple method to obtain experimentally the cutting coefficients for traditional end mills; the idea is to obtain cutting coefficients in a faster way that could be used reliably to feed a plunging chatter model in order to improve the industrial usability of such approach. The chatter model that will be used is the one proposed by Altintas and Ko<sup>21</sup> for plunging.

The proposed method is based on the average force method that is normally used for slotting and flank milling; starting from this background, new relations have been developed to determine the average chip thickness and average coefficients in end mills plunging tasks. Those relationships are used to predict the cutting forces, thanks to a model for the interaction between the material and the main rake angle (in plunge, it is the frontal one, not the peripheral one). Since the developed approach proposes a simple strategy to obtain the coefficients, it is possible to replicate the tests in order to evaluate the effect of the cutting parameters, such as the radial engagement, on their values. Some example of the experimental tests will be provided in this article.

## Average force method

### *Average force model description*

The model developed is based on average cutting force model developed by Altintas and Ko,<sup>21</sup> where the cutting forces are proportional to feed per tooth and to chip thickness. The average cutting force model is one of the most adopted models, thanks to its simplicity and the good accuracy of prediction. However, it must be reported that there are also more complex models that consider nonlinear relationship between force and feed per tooth,<sup>23</sup> obtaining more precise results. The choice of the average cutting force model has been motivated by the larger diffusion; however, the developed geometrical relations could be used also for more

complex models. The average cutting force model uses the following description for the cutting forces

$$F = k_c f_{tooth} h + k_e h \quad (1)$$

where  $k_c$  is the coefficient related to tool cutting actions,  $k_e$  is the coefficient related to tool friction effect,  $f_{tooth}$  is the feed per tooth, and  $h$  is the instantaneous cross section of the chip. Equation (1) is valid for the three components of the forces: tangential (the main one), radial, and axial. To characterize all the components of the cutting forces, six coefficients are needed. These coefficients are affected by lubricant and wear conditions: increase in the tool wear causes a change in the cutting radius and mean angle of the tool,<sup>24,25</sup> while the presence of lubricant alters the friction of the tool on the chip and machined surface. For this reason, they are measured in the most repeatable conditions: dry mill with brand new tooltip. It has been proven by Grossi et al.<sup>26,27</sup> that these coefficients can change with cutting speed in case of slotting. As a result, they provide a reliable estimation of the cutting forces only in a narrow range of speed around the one used to acquire them; it is necessary, if your tool is used at very different spindle speeds, to perform more tests and calculate the effect of cutting speed on these coefficients. In general, the effect of other cutting parameters on the cutting coefficients is not considered; however, the tests carried out in this research highlighted that this could not be assumed true for plunging operations.

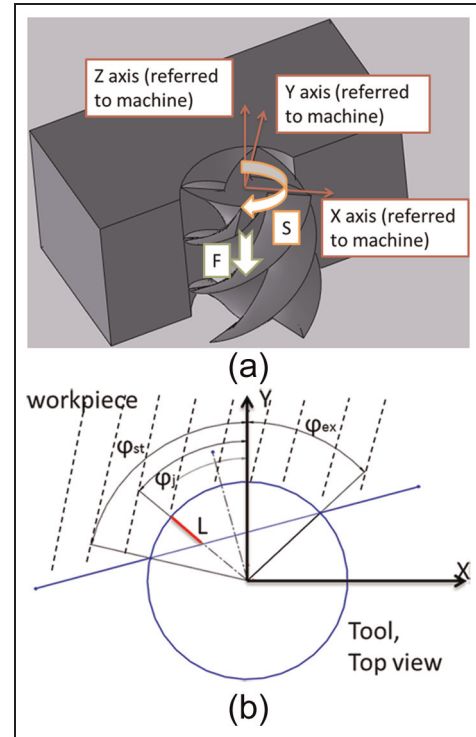
### Plunge average force model

The developed model starts taking into account the geometric position of a tool engaging a workpiece and from the cutting force model. Figure 1(a) and (b) describes a generic configuration of the tool and workpiece with respect to a Cartesian reference system placed with Z-axis coincident with tool axis. The red segment  $h$  indicates the instantaneous chip thickness of the  $j$ th cutter in a generic position that is identified by the angle  $\varphi_j$  with respect to Y-axis. In plunge operation, the main cutters are the frontal ones not the peripheral ones as in slotting or flanking: so they are substantially a straight cutter that operates with a different radial engagement for each angular positioning.

The angles  $\varphi_{st}$  and  $\varphi_{ex}$  are referred to the beginning of the cut and the end of the cut for each tooth in each revolution. It is possible to indicate the angle  $\alpha$  as follows

$$\alpha = \frac{\varphi_{ex} - \varphi_{st}}{2} \quad (2)$$

Consequently, it is possible to indicate the length of segment  $h$  (the instantaneous chip cross section) as a summation of the  $M$ th elements of  $\Delta a$  that are



**Figure 1.** (a) Generic relative tool–workpiece position cutting a vertical wall. The center of the reference system is coincident with the tool axis, while axis direction is coincident to an external fixed coordinate system, for example, machine system. (b) Top view of a tool–workpiece position cutting a vertical wall.

engaging the material.  $h$  can be expressed also analytically, as shown in equation (3) using equation (2)

$$h = \sum_k^M \Delta a = R - \frac{R \cos \alpha}{\cos(\varphi_j - \alpha - \varphi_{st})} \quad (3)$$

where  $R$  is the radius of the tool.  $h$  reaches its maximal value when  $\varphi_j$  is equal to  $\alpha$ , that is, the mean engage angle;  $h$  is defined as zero outside the interval  $\varphi_{st}$  and  $\varphi_{ex}$ . It is also possible to introduce a rotational matrix to switch from tangential, radial, and axial reference systems to the tool Cartesian one. Axial axis is coincident to Z-axis for definition

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \begin{bmatrix} -\cos(\varphi_j) & -\sin(\varphi_j) \\ \sin(\varphi_j) & -\cos(\varphi_j) \end{bmatrix} \begin{Bmatrix} F_t \\ F_r \end{Bmatrix} \quad (4)$$

With equation (4), it is possible to write in discrete form in the Cartesian reference system the forces at the tooltip

$$F_x = \sum_{j=1}^N \sum_{k=1}^M \Delta a \{ -[K_{t_e} f_{t_i} + K_{t_e}] \cos \varphi_j - [K_{a_e} f_{t_i} + K_{a_e}] \sin \varphi_j \} \quad (5)$$

$$F_y = \sum_{j=1}^N \sum_{k=1}^M \Delta a \{ [K_{t_c} f_t + K_{t_e}] \sin \varphi_j - [K_{a_c} f_t + K_{a_e}] \cos \varphi_j \} \quad (6)$$

$$F_z = \sum_{j=1}^N \sum_{k=1}^M \Delta a \{ [K_{f_c} f_t + K_{f_e}] \} \quad (7)$$

Applying the definition of average forces

$$F_{average} = \frac{1}{\varphi_{pitch}} \int_0^{2\pi} F(\varphi) d\varphi \quad (8)$$

And by substituting the analytical form of equation (3) in equations (5)–(7), knowing that this integral is nonzero only between the angles  $\varphi_{st}$  and  $\varphi_{ex}$ , and that the flutes are evenly spaced by the pitch angle, we can write the final formula of the method

$$F_{x-ave} = -\frac{RN}{2\pi} \varepsilon_1 [K_{t_c} f_t + K_{t_e}] - \frac{RN}{2\pi} \varepsilon_2 [K_{a_c} f_t + K_{a_e}] \quad (9)$$

$$F_{y-ave} = \frac{RN}{2\pi} \varepsilon_2 [K_{t_c} f_t + K_{t_e}] - \frac{RN}{2\pi} \varepsilon_1 [K_{a_c} f_t + K_{a_e}] \quad (10)$$

$$F_{z-ave} = \frac{RN}{2\pi} \varepsilon_3 [K_{f_c} f_t + K_{f_e}] \quad (11)$$

where  $N$  is the number of the flutes and  $\varepsilon$  variables are expressed by the following formulas

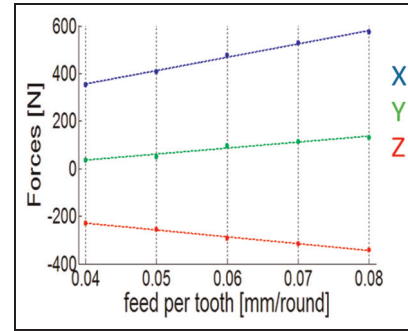
$$\varepsilon_1 = \int_{\varphi_{st}}^{\varphi_{ex}} \cos \varphi - \frac{\cos \alpha \cos \varphi}{\cos(\varphi - \alpha - \varphi_{st})} d\varphi \quad (12)$$

$$\varepsilon_2 = \int_{\varphi_{st}}^{\varphi_{ex}} \sin \varphi - \frac{\cos \alpha \sin \varphi}{\cos(\varphi - \alpha - \varphi_{st})} d\varphi \quad (13)$$

$$\varepsilon_3 = \int_{\varphi_{st}}^{\varphi_{ex}} 1 - \frac{\cos \alpha}{\cos(\varphi - \alpha - \varphi_{st})} d\varphi \quad (14)$$

### Computation of the force coefficient

To compute the force coefficients, a series of plunge tests must be executed with different cutting parameters: these must be chosen in order to achieve a stable cutting. For each test, the three components of the forces must be measured, normally with a dynamometric table. The tests must be carried out with different values of the feed per tooth. According to the model, the forces are proportional to feed per tooth, so if a linear increment of feed per tooth is chosen, the forces (and their average values) must also follow this trend. For these measures, there is no need of compensating the dynamics of the dynamometric table<sup>28</sup> because it



**Figure 2.** Example of measurement and data interpolation to obtain force coefficient.

does not affect the average values of the measured forces.

As shown in Figure 2, the average values of the forces are plotted using the feed per tooth as variable. A regressive model could then be used to compute the six cutting force coefficients, using equations (9)–(11).

### Experimental setup

In order to verify the effectiveness of the proposed model, a set of experimental tests has been carried out. The test piece consists of a rectangular piece of aluminum clamped with a couple of screws to a dynamometric table (Figure 3). Additionally, a microphone was used to verify the stability of the process.<sup>29–31</sup>

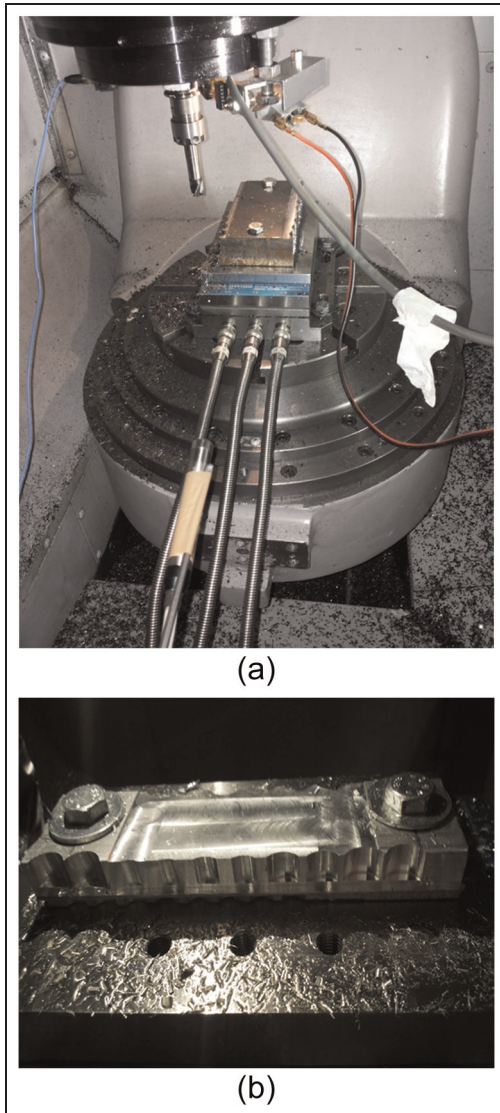
The chosen tool was a Mitsubishi Carbide Mill I MX1n 2S3A 12010 ET2020 12 that has 12 mm of diameter and three flutes, the shank was a Mitsubishi EF i MX12—U12N017L080C, the tool holder was a Big Mega New Baby Chuck MGN13 Daishowa Seiki Co. Ltd, and the milling machine was a Mori Seiki NMV 1500. It has been chosen to use a spindle speed of 8000 r/min and to maintain it constant in every test to avoid the problem of coefficient changes with cutting speed.<sup>32</sup> It has also been chosen to perform the coefficient measurements for four different radial depths of cut: 0.5, 1, 2, and 3 mm. For each radial depth of cut, a set of five different feeds per tooth have been tested: from 0.03 to 0.07 mm/tooth spaced by 0.01 mm/tooth.

### Results and discussions

The results in terms of average forces and force coefficients are presented in Table 1. Every test was checked to ensure the stability of all of them, with no chatter evidence not on the surface not in signal analysis. The first result is that the average force method provides a good approximation of the mean cutting force, as proven by the very good accuracy of the linear regression model used to interpolate the experimental data. In this case,

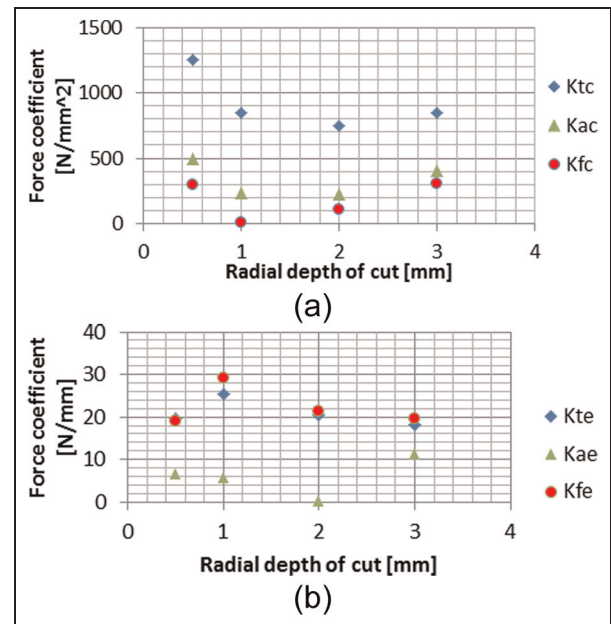
**Table 1.** Force coefficient values.

AE (mm)	$K_{tc}$ (N/mm <sup>2</sup> )	$K_{te}$ (N/mm)	$K_{ac}$ (N/mm <sup>2</sup> )	$K_{ae}$ (N/mm)	$K_{fc}$ (N/mm <sup>2</sup> )	$K_{fe}$ (N/mm)
0.5	1249.04	19.49199	496.0457	6.663548	298.9652	18.96992
1	843.1247	25.52482	230.6405	5.775643	9.895345	29.11762
2	750.193	20.60815	222.4607	0.148353	112.8966	21.32309
3	847.9039	18.23892	404.52	11.11235	303.0117	19.52395

**Figure 3.** (a) Example of setup with a steel workpiece on dynamometer and (b) detailed view of plunge tests on aluminium block.

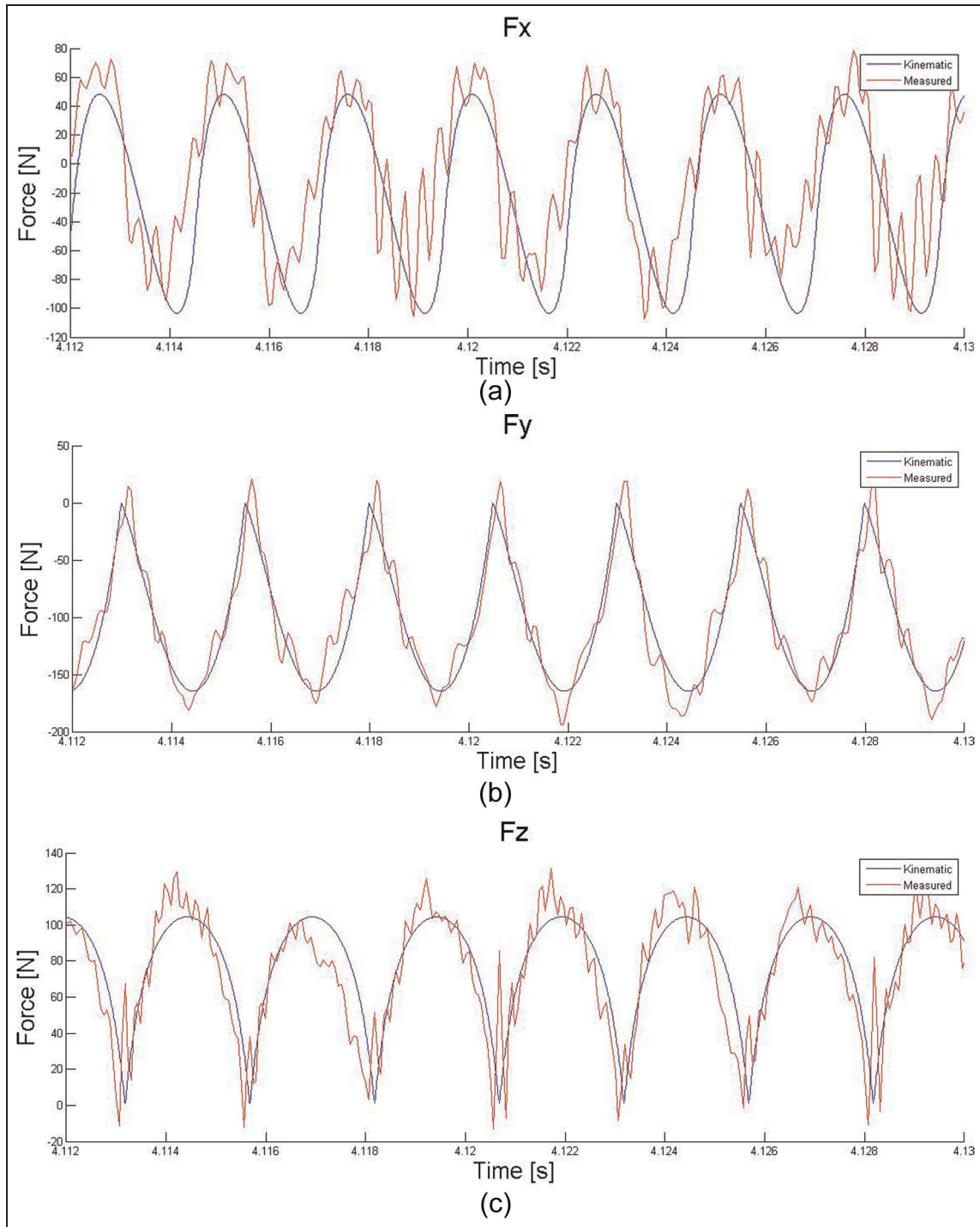
the behavior of the cutting force could be considered linear, with respect to the chip area.

The tests carried out highlighted also another trend in the cutting force data: the cutting force coefficients change for different radial engagements. This trend is

**Figure 4.** (a) Trend of cutting coefficients and (b) trend of edge coefficients.

reported in Figure 4(a) and (b), where the nonlinear behavior of the coefficients is evident.

It is possible to note that for reduced radial engagement, the cutting coefficient becomes higher when the dimension of radial depth of cut is small; this is probably due to the influence of the corner radius of the cutter.<sup>33</sup> It is possible to confirm that forces are proportional to feed per tooth, but the value of the constant changes with the cutting parameter. In this case, the cutting coefficients show a minimum value for radial depth of cut within the range of 1–2 mm. Edge coefficients are usually relevant on the computation of the cutting forces only in case of low feed per tooth; when using greater feed per tooth, their contribution to the total cutting force is reduced. However, these coefficients do not have a trend as relevant as the cutting coefficients. The values of force coefficient have been used in a kinematic simulation of the cut to test their capacity of describing and foresee the cutting force. Here is reported as an example one of the results obtained for spindle speed of 8000 r/min, feed of 960 mm/min, and radial immersion of 3 mm (Figure 5).

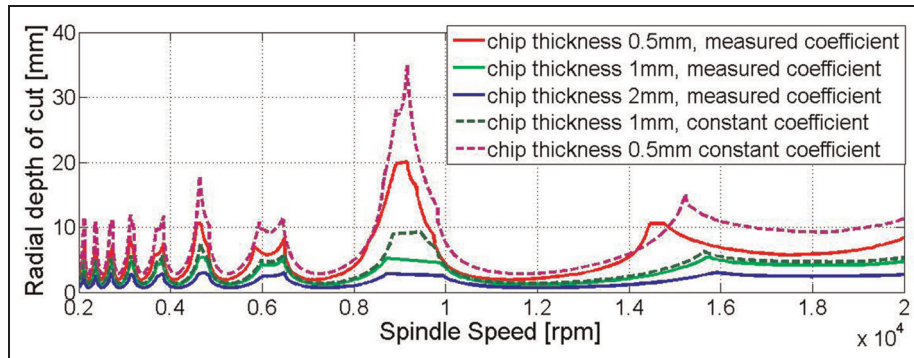


**Figure 5.** (a) Comparison between kinematic X forces (blue) and measured X forces (red), (b) comparison between kinematic Y forces (blue) and measured Y forces (red), and (c) comparison between kinematic Z forces (blue) and measured Z forces (red).

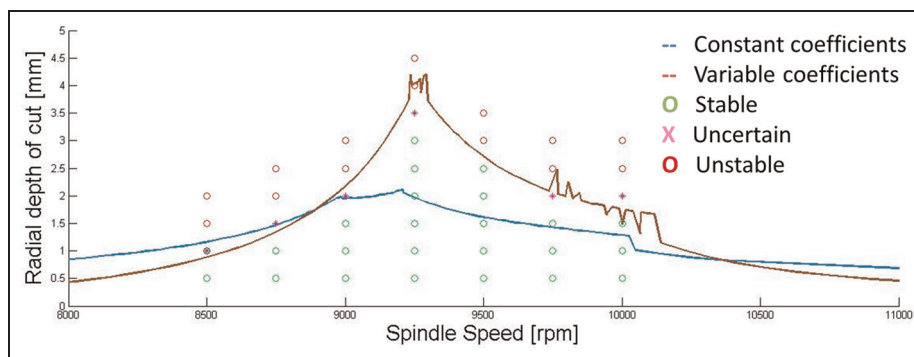
This comparison have been carried out just using a kinematic model, that it is not able to consider the dynamics of the system, responsible for the high frequency variation of the measured forces. However, the general trend of the cutting forces is correctly represented by the proposed model.

### Effect of variable cutting coefficients on SLD

The importance to have a reliable cutting force coefficient lays in the possibility to predict correctly the cutting forces, possibility that could be used to design the



**Figure 6.** Continuous lines: SLD with measured coefficients; dashed lines: SLD computed with 2 mm coefficients taken as a constant standard.



**Figure 7.** Comparison between SLD with constant coefficient and variable coefficients.

fixture and the tooling, and to have a reliable estimation of the stability of the process. Using an analytical solution to forecast the arising of chatter,<sup>21</sup> it is possible to create a SLD to represent graphically the stable and unstable zones. Using the radial engagement-dependent cutting force coefficients, it is possible to evaluate how this parameter affects the stability of the process. In order to solve this system, it is necessary to use an iterative approach: for each cutting speed, a first guess of threshold radial depth of cut is provided and then the value of the cutting force coefficients is updated till the convergence of the result. In Figure 6, the SLD of a plunging operation is reported, as obtained with model in Altintas and Ko<sup>21</sup> with three different radial engagements in which it is possible to see the difference in taking constant cutting coefficients referred to a standard group of coefficient measured at 2 mm of radial depth of cut (blue line in Figure 6).

Observing Figure 6, it's easy to understand how to predict stability limits obtained with the standard approach (without taking into account the variation of cutting coefficients with radial engagement) leads to errors that can be relevant.

To assess the better accuracy provided by the model-proposed coefficients, some experimental tests of

chatter detection have been carried. The stability of the operation has been verified, thanks to the use of different sensors: dynamometer, accelerometers, and a microphone.<sup>29–31</sup> The results are reported in Figure 7.

From the data in Figure 7, it is possible to evaluate how the use of the model-proposed coefficients improves the predictive capability of the SLD because it takes into account how the forces are affecting the model at different radial engagements.

## Conclusion

In this article, a novel method to compute cutting coefficients has been shown. The average force model and method was applied to plunge milling operations. The obtained coefficients have been used in a kinematic model to have a comparison between predicted and measured forces: this analysis has demonstrated that this method can really be used for force prediction, and the data show a good accord with the measured ones even if the kinematic model does not consider the dynamics of the system. The influence of the radial depth of cut in the determination of the value of the cutting coefficients has also been shown. This trend of the force coefficients shows the possibility to choose a

set of parameters to minimize tooltip forces. The variation of cutting coefficient must also be taken into account for the development and for the correct use of analytical plunge chatter models to obtain an accurate SLD.

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### Declaration of conflicting interests

The authors declare that there is no conflict of interest.

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### References

1. Kline WA, DeVor RE and Lindberg JR. The prediction of cutting forces in end milling with application to cornering cuts. *Int J Mach Tool Des Res* 1982; 22: 7–22.
2. Altintas Y and Spence A. End milling force algorithms for CAD systems. *CIRP Ann Manuf Technol* 1991; 40: 31–34.
3. Lazoglu I. Sculpture surface machining: a generalized model of ball-end milling force system. *Int J Mach Tool Manuf* 2003; 43: 453–462.
4. Budak E, Altintas Y and Armarego EJA. Prediction of milling force coefficients from orthogonal cutting data. *J Manuf Sci E Trans ASME* 1996; 118: 216–224.
5. Altintas Y and Lee P. A general mechanics and dynamics model for helical end mills. *J Manuf Sci E Trans ASME* 1998; 120: 684–692.
6. Lee P and Altintas Y. Prediction of ball-end milling forces from orthogonal cutting data. *Int J Mach Tool Manuf* 1996; 36: 1059–1072.
7. Yucesan G and Altintas Y. Prediction of ball end milling force. *J Eng Ind Trans ASME* 1996; 118: 95–104.
8. Engin S and Altintas Y. Mechanics and dynamics of general milling cutters. Part 1: helical end mills. *Int J Mach Tool Manuf* 2001; 41: 2195–2212.
9. Wan M, Lu MS, Zhang WH, et al. A new method for identifying the cutter runout parameters in flat end milling process. *Mater Sci Forum* 2012; 697–698: 71–74.
10. Wan M and Zhang WH. Systematic study on cutting force modelling methods for peripheral milling. *Int J Mach Tool Manuf* 2009; 49: 424–432.
11. Imani BM, Sadeghi MH and Elbestawi MA. An improved process simulation system for ball-end milling of sculptured surfaces. *Int J Mach Tool Manuf* 1998; 38: 1089–1107.
12. Wan M, Lu MS, Zhang WH, et al. A new ternary-mechanism model for the prediction of cutting forces in flat end milling. *Int J Mach Tool Manuf* 2012; 57: 34–45.
13. Afazov SM, Ratchev SM and Segal J. Modelling and simulation of micro-milling cutting forces. *J Mater Process Technol* 2010; 210: 2154–2162.
14. Özel T, Sima M, Srivastava AK, et al. Investigations on the effects of multi-layered coated inserts in machining Ti-6Al-4V alloy with experiments and finite element simulations. *CIRP Ann Manuf Technol* 2010; 59: 77–80.
15. Gonzalo O, Jauregi H, Uriarte LG, et al. Prediction of specific force coefficients from a FEM cutting model. *Int J Adv Manuf Technol* 2009; 43: 348–356.
16. Altintas Y and Jin X. Mechanics of micro-milling with round edge tools. *CIRP Ann Manuf Technol* 2011; 60: 77–80.
17. Kaymakci M, Kilic ZM and Altintas Y. Unified cutting force model for turning, boring, drilling and milling operations. *Int J Mach Tool Manuf* 2012; 54–55: 34–45.
18. Wan M and Altintas Y. Mechanics and dynamics of thread milling process. *Int J Mach Tool Manuf* 2014; 87: 16–26.
19. Wan M, Pan WJ, Zhang WH, et al. A unified instantaneous cutting force model for flat end mills with variable geometries. *J Mater Process Technol* 2014; 214: 641–650.
20. Wan M, Kilic ZM and Altintas Y. Mechanics and dynamics of multifunctional tools. *J Manuf Sci E Trans ASME* 2015; 137: 011019.
21. Altintas Y and Ko JH. Chatter stability of plunge milling. *CIRP Ann Manuf Technol* 2006; 55: 361–364.
22. Yussefian Z, Moetakef-Imani B and El-Mounayri H. The prediction of cutting force for boring process. *Int J Mach Tool Manuf* 2008; 48: 1387–1394.
23. Wan M, Wang YT, Zhang WH, et al. Prediction of chatter stability for multiple-delay milling system under different cutting force models. *Int J Mach Tool Manuf* 2011; 51: 281–295.
24. Nouri M, Fussell BK, Ziniti BL, et al. Real-time tool wear monitoring in milling using a cutting condition independent method. *Int J Mach Tool Manuf* 2015; 89: 1–13.
25. Sarhan A, Sayed R, Nassr AA, et al. Interrelationships between cutting force variation and tool wear in end-milling. *J Mater Process Technol* 2001; 109: 229–235.
26. Grossi N, Sallese L, Scippa A, et al. Chatter stability prediction in milling using speed-varying cutting force coefficients. *Proced CIRP* 2014; 14: 170–175.
27. Grossi N, Sallese L, Scippa A, et al. Speed-varying cutting force coefficient identification in milling. *Precis Eng*. Epub ahead of print 20 April 2015. DOI: 10.1016/j.precisioneng.2015.04.006.
28. Scippa A, Sallese L, Grossi N, et al. Improved dynamic compensation for accurate cutting force measurements in milling applications. *Mech Syst Signal Process* 2015; 54–55: 314–324.
29. Quintana G, Ciurana J, Ferrer I, et al. Sound mapping for identification of stability lobe diagrams in milling processes. *Int J Mach Tool Manuf* 2009; 49: 203–211.
30. Rahman M and Ito Y. Detection of the onset of chatter vibration. *J Sound Vib* 1986; 109: 193–205.



31. Kuljanic E, Totis G and Sortino M. Development of an intelligent multisensor chatter detection system in milling. *Mech Syst Signal Process* 2009; 23: 1704–1718.
32. Campatelli G and Scippa A. Prediction of milling cutting force coefficients for Aluminum 6082-T4. *Proced CIRP* 2012; 1: 563–568 (In: *5th CIRP conference on high performance cutting*, Zurich, 4–6 June 2012).
33. Witty M, Bergs T, Schäfer A, et al. Cutting tool geometry for plunge milling—process optimization for a stainless steel. *Proced CIRP* 2012; 1: 506–511.

$h$	chip thickness (mm)
$k_c$ and $k_e$	cutting and edge force coefficients, respectively (N/mm <sup>2</sup> ) and (N/mm)
$M$	number of $\Delta a$ elements
$R$	tool radius (mm)
$t, a, f$ pedices	tangential, radial, axial system
$x, y, z$ pedices	Cartesian system
$\alpha$	average engaging angle (rad)
$\Delta a$	finite element of chip thickness (mm)
$\varphi_{ex}$	disengaging angle (rad)
$\varphi_j$	$j$ th flute instantaneous angle (rad)
$\varphi_{pitch}$	pitch angle (rad)
$\varphi_{st}$	engaging angle (rad)

## Appendix I

### Notation

$F$	forces (N)
$f_{tooth}$	feed per tooth (mm/tooth)