On the duality of Phase-based and Phase-less RSSI MUSIC algorithm for Direction of Arrival estimation

MARCO PASSAFIUME, STEFANO MADDIO, ALESSANDRO CIDRONALI and GIANFRANCO MANES
University of Florence
Dept. of Information Engineering
Via di S.Marta, 3
Florence, Italy
marco.passafiume@unifi.it, stefano.maddio@unifi.it, alessandro.cidronali@unifi.it, gianfranco.manes@unifi.it

Abstract: In last years the interest in indoor localization techniques is grown very quickly: due to huge increase of interest in domotic technologies and to fast improvement of capabilities for mobile devices, which are becoming always-connected support points for standard users, the capability of making spatial data-contextualization has become an important point of interest. Existing GPS technologies aren't capable of realizing sufficient accuracy needed for indoor positioning, so it is necessary to find alternative cost-effective methods to resolve localization problem. Excluding odometry-based tracking methods (which present a lot of uncertainty for making an absolute positioning), famous signal DoA (Direction Of Arrival) spectral-based localization algorithms like CAPON, ESPRIT, MUSIC[1],[2] provide valid localization accuracy assuming device capability to analyze in real-time physical parameters (amplitude/phase) of received radiocommunication signals to a spatial antenna array. This requirement could be achieved using ad-hoc radio equipment, which causes an increase of production costs: maybe, it is far better to develop some kind of new algorithms capable of using standard IEEE wireless protocols information (like RSS indicator, RSSI) for positioning elaboration. In this article, it will be analytically demonstrated the capability of MUSIC[1] algorithm to elaborate RSSI standard transceiver values (placing some measurement conditions), and there will be shown a comparison between DoA localizations using the classical information set (amplitude/phase) and the RSSI set (obtained simply reading transceiver standard registers).

Key–Words: indoor positioning, indoor localization, DoA, RSSI, wireless, MUSIC, tracking, mobile devices

1 Introduction

In indoor network of devices it is always possible to identify a direct path for the radio link between two nodes: localization service could be provided using a constellation of routers (anchor nodes) in known positions, each of which can perform DoA identification respect its angular reference system (giving a pair of angular coordinates as shown in fig.1).
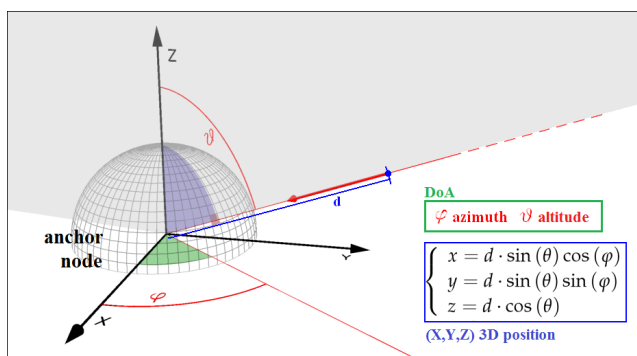
Using a GPS-like approach, the knowledge of almost two pair of angular coordinates (two DoAs, one for every anchor node) allows to determine the solution for system in fig.1, obtaining an unique 3D spatial localization of a transmitter node: adding more nodes can improve the accuracy of localization, reducing the impact of DoA identification error (fig.2).
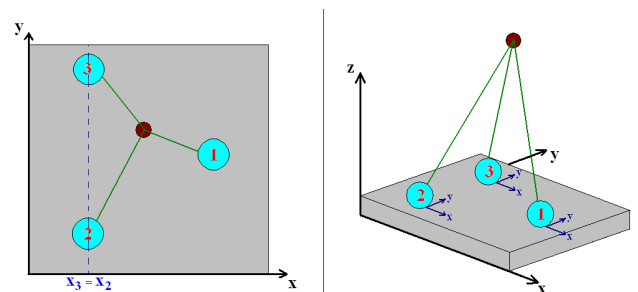

Figure 2: Multianchor localization

To achieve the DoA identification, with every type of algorithm, an anchor node must present some kind of spatial-diversity which enforces an almost biunivo-

Figure 1: Anchor Node DoA reference system

cal relationship between different DoAs and different sampled data sets (defined *steering vector*). In these terms, relative phases between antennas of array is the first type of sensible information present in a physical sampled data set (sampling at physical level also amplitude and phase of RF receiving signal, for every antenna).
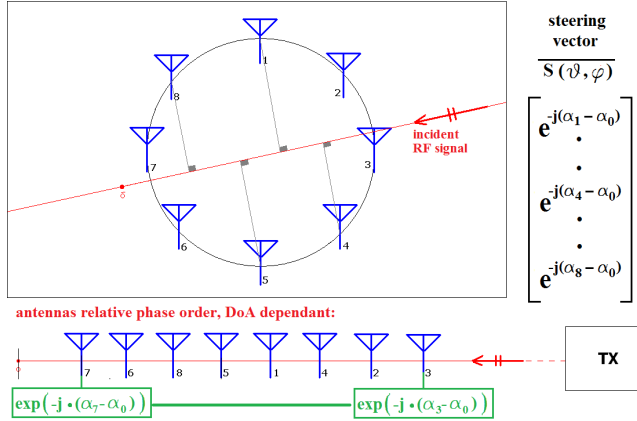

Figure 3: Simple TDOA approach example

For example, in fig.3, it is shown the classical TDoA approach (Time Difference of Arrival) for an L-antennas circular array, where the *steering vector* (array acquired data set) can be simply an L-dimension vector containing phase displacement terms for received signal at every antenna respect a known reference signal. More information to steering vector could be added by sampling incident power at every antenna of array, adding in this way data about relationship between DoA and different angular gains for antennas: anyway, standard algorithm formulation expects as input data L-vectors of complexes, focusing the interest over relative phase-terms.

1.1 Standard MUSIC algorithm

In this section we will make a short introduction to MUSIC algorithm in its analythical form: for a more detailed description look at [1].

MUSIC algorithm is defined as a *spectral-based* DoA localization algorithm: *spectral-based* means that effective DoA identification is achieved performing some kind of analysis of membership of a measured/received data set (the *steering vector*) into a larger reference data set, which represent a reliable model for all possible received data set associated to every expected DoA. Usually, the *membership function* is like a *likelihood function* that for every reference *steering vector* (for every DoA) gives an index of probability that obtained data set could actually be that one.

Following formal MUSIC formulation in [1], obtained *steering vector* (data set) for an L-antennas array become as an L-element vector of complex samplings of signals amplitude/phase at every antenna. In a generic M-signals case, the *steering vector* will be somewhat like in eq.1.

$$\overline{\mathbf{x}}(t) = \sum_{j=1}^{M} \underbrace{\begin{pmatrix} g_1\left(\theta_j, \varphi_j\right) e^{-\mathrm{j}\,\varphi_{1j}} \\ g_2\left(\theta_j, \varphi_j\right) e^{-\mathrm{j}\,\varphi_{2j}} \\ \vdots \\ g_L\left(\theta_j, \varphi_j\right) e^{-\mathrm{j}\,\varphi_{Lj}} \end{pmatrix}}_{\overline{\mathbf{m}}(\theta_j, \varphi_j) = \overline{\mathbf{m}}_j} s_j(t) + \overline{\mathbf{n}}(t)$$

with (θ_j, φ_j) = DoA for j-th incident signal   (1)

In eq.1 overall observation noise (measure noise plus radio noise over linkpath) is modeled by $\overline{\mathbf{n}}(t)$ L-elements vector. Like in [1], every $n_i(t)$ term can be considered as AWGN noise, with zero-mean, known variance and impulsive autocorrelation (producing $n_i(t)$ terms statistically independent among them).

Each $\overline{\mathbf{m}}_j$ vector corresponds to singular steering vector for given j-th signal (associated to j-th DoA): every antenna component is defined with a $g_i\left(\theta_j, \varphi_j\right)$ term modeling the antenna directional linear gain and a phase delay which represent relative antenna phase displacement (both terms dependent from signal DoA, like in fig.3, with improved gain-DoA dependance.

Imagining to implement localization service within a common network of devices, effective communication should be achieved using a protocol of channel multiplexing (like TDM or FDM) also for minimizing communications interferences, so considering localizing only one signal $s_1(t)$ is a realistic restriction.

Following standard MUSIC implementation [1], from $\overline{\mathbf{x}}(t)$ an $\mathbf{R}$ autocorrelation matrix is produced (eq.2).

$$\begin{aligned} \mathbf{R}_{ij} &= E\{x_i(t)x_j^*(t)\} = \\ &= m_i\left(\theta_1, \varphi_1\right) m_j^*\left(\theta_1, \varphi_1\right) \cdot E\{{s_1}^2(t)\} + \\ &\quad + E\{n_i(t)n_j^*(t)\} + \\ &\quad + m_i\left(\theta_1, \varphi_1\right) \cdot E\{s_1(t)n_i(t)\} + \\ &\quad + m_j^*\left(\theta_1, \varphi_1\right) \cdot E\{s_1(t)n_j(t)\} = \\ &= m_i m_j^* P + \begin{cases} N \leftarrow i = j \\ 0 \leftarrow i \neq j \end{cases} \end{aligned} \quad (2)$$

Overall $\mathbf{R}$ matrix can be seen as a linear combination of two principal submatrices (eq.3), presenting a maximum rank equal to L (equal to number of antennas) because adding with identity matrix generated by uncorrelated noise terms (eq.2).

$$\mathbf{R} = \underbrace{\mathbf{M}\left(\theta_1, \varphi_1\right) \cdot P}_{\substack{\mathbf{R}_S \\ rank=1}} + \underbrace{\mathbf{I} \cdot N}_{\substack{\mathbf{R}_N \\ rank=L}} \quad (3)$$

Because of $\mathbf{R}$ maximal rank, $\mathbf{R}$ matrix completely defines a $\mathbf{C}^L$ vectorial space: consequently every its base of eigenvectors defines a complete base for $\mathbf{C}^L$ space. It is easy to demonstrate (and it is demonstrated in [1]) that in eq. 3 $\mathbf{R}_S$ has unitary rank with only one eigenvector (equal to $P \cdot \overline{\mathbf{m}}(\theta_1, \varphi_1)$), that results parallel to characteristic DoA steering vector in reference data set.

As $\mathbf{R}_N$ matrix term has maximum rank, extracting a full base of eigenvectors with releated eigenvalues and observing that $\mathbf{R}_N$ will present a set of L equal eigenvalues (equal to N, as in eq.2), it is clear that the signal eigenvector will be that one associated to the maximum extracted eigenvalue. So, making a Singular Value Decomposition [3] of $\mathbf{R}$, the (L-1) eigenvectors with associated smaller eigenvalues will be generators of $\mathbf{C}^L$ *subspace of absent signals DoA*, while others eigenvectors (with one signal, only one) will generate the complementary *subspace of received signals DoA*.

It is possible to define a *projection function* for a steering vector over the *space of absent signals DoA*. Defining the $\mathbf{U}_N$ matrix containing all the eigenvectors for the *absent space* (with dimension $N \times (N-1)$) and $\mathbf{U}_N{}^H$ its hermitian, the normalized projection function of every reference *steering vector* $\overline{\mathbf{m}}(\theta, \varphi)$ over the *absent space* could be defined as in eq.4.

$$p_m(\theta, \varphi) = \underbrace{\frac{||\mathbf{U}_N{}^H \cdot \overline{\mathbf{m}}(\theta, \varphi)||}{||\overline{\mathbf{m}}(\theta, \varphi)||}}_{normalized\ projection} \qquad (4)$$

The MUSIC-spectrum it is defined with maximum corresponding to best estimated DoA. The *projection function* is defined over the space of DoAs with precomputated *steering vectors* in reference data set (on an $\mathbf{R}^2$ space), and has the minimum value for the best estimated DoA: so, the effective MUSIC-spectrum function is shown in eq.5, which is an equivalent form for function shown in [1].

$$P_m(\theta, \varphi) = \frac{1}{p_m(\theta, \varphi)} = \frac{||\overline{\mathbf{m}}(\theta, \varphi)||}{||\mathbf{U}_N{}^H \cdot \overline{\mathbf{m}}(\theta, \varphi)||} \qquad (5)$$

2 Applying RSSI measurements to MUSIC

Until now the theorical MUSIC algorithm has been repeated. In literature localization accuracy analysis (CRB-like) are performed only for direct physical signal analysis [2], because following standard definition in [1] added measure-noise term follow AWGN noise definition, which it is reliable only for physical signal observation. Also, the projection capability of reject valid steering vector over *absent signal subspace* is stronger the more stronger is the condition of orthonormality between two different DoA steering vectors (minimizing their scalar product), and this condition is helped with complex steering vectors.

a meno di una costante Below it will be suggested an analitycal application model for RSSI values MUSIC application, and through that there will be presented some conditions for assuring right MUSIC execution with simple RSSI measurements.

2.1 Analytical application model

In eq.6 the standard projection operator between obtained steering vector $\overline{\mathbf{v}}$ and the reference one $\overline{\mathbf{m}}(\theta, \varphi)$ is reproposed.

$$||\overline{\mathbf{v}}^* \cdot \overline{\mathbf{m}}(\theta, \varphi)|| = \left| \sum_{i=1}^{L} v_i^* m_i(\theta, \varphi) \right| = \qquad (6)$$

$$= \left| \sum_{i=1}^{L} \underbrace{\left(g_i(\theta_1, \varphi_1) e^{j\,\varphi_{1i}(\theta_1,\varphi_1)} \right)}_{meas.\ steering\ vector} \underbrace{\left(g_i(\theta, \varphi) e^{-j\,\varphi_{1i}(\theta,\varphi)} \right)}_{ref.\ steering\ vector} \right|$$

From eq.6 it is clear that with same antenna gains $(g_i(\theta_1, \varphi_1) = g_i(\theta, \varphi))$ the projection is maximized (and so MUSIC-spectrum is maximized) if results

$$\forall i, \varphi_{1i}(\theta_1, \varphi_1) = \varphi_{1i}(\theta, \varphi) \qquad (7)$$

Without using phase displacement information, the point is if the projection maximization is achieved only for right reference steering vector, associated to right DoA. This analysis is placed considering the acquisition of RSSI values.

For IEEE wireless standards, the transceiver RSSI parameter is a number that is computed during the demodulation process, and it is given by an average over a variable-length window of symbols of cross-correlation peaks level between demodulated RF base-band signal and symbols reference signal sequences (for spread-spectrum modulated signals). It can be roughly correlated with dB level of power signal detected at radio interface in input to transceiver, weighted for a calibration constant (not necessarily known). Due to its evaluation nature, the RSSI parameter itself presents a measure error that could be modeled in first approximation like an added AWGN noise: so, for the i-th antenna the RSSI detected value is something like $X_i = RSSI_i + \Delta R_i$, with ΔR_i the AWGN measure noise.

Applying MUSIC[1], for a given steering vector the autocorrelation matrix $\mathbf{R}$ has to be created: correlation value must tend to 0 when a i-th good signal received from antenna i and a j-th null signal received

from blind antenna j are correlated, so it is necessary to transform acquired value into linear equivalent ones. So, with measure noise (within signal noise can also be modeled, which causes an alteration of RSSI readings), every term of steering vector $\overline{\mathbf{x}}$ becomes like that in eq.8.

$$x_i(\theta_1, \varphi_1) = K_0 \cdot 10^{\overbrace{\frac{(RSSI_i + \Delta R_i)}{10}}^{x_i}} = \quad (8)$$
$$= K_0 [P \cdot N_i \cdot g_i(\theta_1, \varphi_1)]$$

with $P = E\{s_1^2(t)\}$ = incident signal power

Note that in comparison with eq.1 data sampling over time lost its natural time-dependency: now, with a statical signal DoA, antenna sampling over time will result in a constant data set with minor changes due to measure noise, but not releated to intrinsic signal shape.

Reverting RSSI units to linear ones, a catastrophic effect is caused by N_i i-th coefficient generation due to linear conversion from RSSI noise ΔR_i: within projection of acquired steering vector over reference data set, it will change eq.6 to eq.9, causing dramatics alterations of projection norms over possible DoA steering vectors subspace, with consequently dramatics mismatches over DoA identification, so some conditions over RSSI measurements have to be done.

$$\|\overline{\mathbf{v}}^* \cdot \overline{\mathbf{m}}(\theta, \varphi)\| = \sum_{i=1}^{L} N_i [g_i(\theta_1, \varphi_1) \cdot g_i(\theta, \varphi)] \quad (9)$$

It will be investigated how the MUSIC $\mathbf{R}$ autocorrelation matrix will be altered, with related eigenvectors. Due to time-variant nature of RSSI measurement noise term $\Delta R_i(t)$, necessarily steering vector terms will have the form in eq.10, removing K_0 coefficient of eq.8 thanks to MUSIC-spectrum projection normalization (eq.5).

$$x_{i(\theta_1, \varphi_1)}(t) = P \cdot N_i(t) \cdot g_i(\theta_1, \varphi_1)$$
$$\text{con } N_i(t) = 10^{\frac{\Delta R_i(t)}{10}} \quad (10)$$

Using eq.10 directly into eq.2 does not lead to valid results, because into new $x_{i(\theta_1, \varphi_1)}$ terms does not exist a linear separation between signal terms and noise terms.

For leading a result comparable with eq.2 and then to eq.3, with capability of making a *absent DoA signal vectorial subspace*, some evaluation over $N_i(t)$ term must be done. Considering $\Delta R_i(t)$ as an unknown statistical variable that belong from an analog-digital process, it will be assigned an uniform zero-mean statistic: making a worst case assumption, so linear

units $N_i(t)$ term will be an uniform distributed variable which limits belongs from $\Delta R_i(t)$ ones. Respective properties are shown in eq.11.

$$\Delta R_i(t) \in [-\epsilon_{MAX}, \epsilon_{MAX}] \Rightarrow$$
$$\Rightarrow N_i(t) \in \left[10^{\frac{-\epsilon_{MAX}}{10}}, 10^{\frac{\epsilon_{MAX}}{10}}\right]$$

$$\Delta R_i(t) \begin{cases} \mu_R = 0 \\ \sigma_R^2 = \frac{\epsilon_{MAX}^2}{3} \end{cases} \quad (11)$$

$$N_i(t) \begin{cases} \mu_N = \frac{10^{\frac{\epsilon_{MAX}}{10}} + 10^{\frac{-\epsilon_{MAX}}{10}}}{2} \\ \sigma_N^2 = \frac{1}{12}\left(10^{\frac{\epsilon_{MAX}}{10}} - 10^{\frac{-\epsilon_{MAX}}{10}}\right)^2 \end{cases}$$

Following eq.11, $N_i(t)$ factor can be written as

$$N_i(t) = \mu_N + n_i(t) \leftarrow \begin{cases} \mu = 0 \\ \sigma = \sigma_N \end{cases} \quad (12)$$

with $\begin{cases} E\{n_i^2(t)\} = \sigma_N^2 & \text{equivalent noise power} \\ E\{n_i(t) \cdot n_i(t - \tau)\} = 0 & \text{if } \tau \neq 0 \end{cases}$

where $n_i(t)$ term is not an AWGN noise, but share with it its impulsive autocorrelation (meaning that $n_i(t)$ and $n_j(t)$ terms are statistically independent). The μ_N and σ_N terms are releated respect ϵ_{MAX} (equals to maximum transceiver given RSSI deviation) following plots in fig.4.
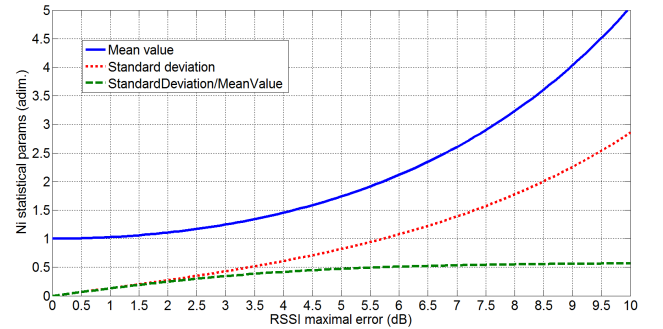

Figure 4: Ni(t) mean and variance, respect Emax

With new linear formulation for $N_i(t)$, now it is possible to rearrange eq.2: single $x_{i(\theta_1, \varphi_1)}(t)$ become as shown in eq.13.

$$x_{i(\theta_1, \varphi_1)}(t) = P \cdot \overbrace{[\mu_N + n_i(t)]}^{N_i(t)} \cdot g_i(\theta_1, \varphi_1) = \quad (13)$$
$$= \mu_N P \cdot g_i(\theta_1, \varphi_1) + n_i(t)P \cdot g_i(\theta_1, \varphi_1)$$

So, using noise properties as shown in eq.12, generic $\mathbf{R}_{ij}$ term results as in eq.14, where $\overline{\mathbf{g}}(\theta_1, \varphi_1) = \overline{\mathbf{m}}(\theta_1, \varphi_1)$ because the lack of phase displacement

complex terms of eq.1.

$$\mathbf{R}_{ij} = E\{x_i(t)x_j^*(t)\} = \qquad (14)$$

$$= g_i(\theta_1, \varphi_1) g_j(\theta_1, \varphi_1) P^2 \cdot \left(\mu_N^2 + E\{n_i(t)n_j^*(t)\}\right) +$$

$$+ g_i(\theta_1, \varphi_1) P \cdot E\{n_i(t)\} +$$

$$+ g_j(\theta_1, \varphi_1) P \cdot E\{n_j(t)\} =$$

$$= \underbrace{(\mu_N P)^2 g_{i1}g_{j1}}_{\mathbf{R}_S} + \underbrace{\begin{cases} (\sigma_N P)^2 g_{i1}^2 \leftarrow i = j \\ 0 \leftarrow i \neq j \end{cases}}_{\mathbf{R}_N}$$

Comparing eq.14 with eq.2, while correspondence between $\mathbf{R}_S$ terms is clear, the structure of $\mathbf{R}_N$ submatrix is heavily changed, becoming directly dependent to DoA because the presence of $g_i^2(\theta_1, \varphi_1)$ terms. The distortion effect over DoA subspaces is directly releated with $\mathbf{R}_N$ relative weight over linear combination (as in eq.3), so the ratio between $\mathbf{R}_S$ and $\mathbf{R}_N$ matrices will be directly releated to MUSIC estimation error. Applying SVD decomposition [3], submatrix weights are defined as in eq.15.

$$\mathbf{M}_{\text{weight}} = \det(\mathbf{M}) = \sum_{i=1}^{L} \lambda_i \cdot ||\overline{\mathbf{v}}_i||^2 \qquad (15)$$

where λ_i is the i-th eigenvalue

$\overline{\mathbf{v}}_i$ is the i-th eigenvector

L is the matrix order

For standard MUSIC, results are below:

$$\mathbf{R}_S \text{ weight} = P \cdot ||\overline{\mathbf{m}}(\theta_1, \varphi_1)||^4 = P \cdot \left(\sum_{i=1}^{L} g_{i1}^2\right)^2$$

$$\mathbf{R}_N = \underbrace{N \cdot \mathbf{I}}_{rank=L} \Rightarrow \boxed{\mathbf{R}_N \text{ weight} = N \cdot L} \qquad (16)$$

For RSSI MUSIC:

$$\mathbf{R}_S \text{ weight} = (\mu_N P)^2 ||\overline{\mathbf{m}}||^4 = (\mu_N P)^2 \cdot \left(\sum_{i=1}^{L} g_{i1}^2\right)^2$$

$$\mathbf{R}_N = (\sigma_N P)^2 \cdot \begin{pmatrix} g_{11}^2 & 0 & 0 \\ & \ddots & \\ 0 & 0 & g_{L1}^2 \end{pmatrix}$$

$$\boxed{\mathbf{R}_N \text{ weight} = (\sigma_N P)^2 \cdot \left(\sum_{i=1}^{L} g_{i1}^2\right)} \qquad (17)$$

Quality of MUSIC estimation (interpreted as proximity between hypothesized and real signal DoA) is directly releated to submatrices weights ratio. Between

implementations, these "'quality factors'" are those shown in eq.18.

Quality factor: $W = \left(\dfrac{\mathbf{R}_S \text{ weight}}{\mathbf{R}_N \text{ weight}}\right) \underbrace{\left(\sum_{i=1}^{L} g_{i1}^2\right)^{-1}}_{(*)\ \text{DoA weight}}$

std. MUSIC $\Rightarrow \boxed{Q_{STD} = \dfrac{P}{NL}\left(\sum_{i=1}^{L} g_{i1}^2\right) \leq \dfrac{P}{N}}$

RSSI MUSIC $\Rightarrow Q_{RSSI} = \left(\dfrac{\mu_N}{\sigma_N}\right)^2 \approx \dfrac{1}{\sigma_N^2} \qquad (18)$

Note that in eq.16-17 $\mathbf{R}_S$ weights are multiplied with a DoA releated-term that can be normalized (*) (it represents DoA accuracy dependence releated to array structure, and it is normalized by *MUSIC spectrum normalization* as in eq.5).

Comparing eq.18 ratios, it can be observed that for RSSI implementation quality factor depends directly from RSSI evaluation quality, while for standard implementation remains a DoA dependency due to effective physical SNR alteration when different DoAs lead to different signal antenna gains. For RSSI this behaviour is mended by RSSI evaluation correlation mechanism.

Using eq.11, quality factor for RSSI implementation can be directly releated to ϵ_{MAX} maximal RSSI evaluation error, as below.

$$\boxed{Q_{RSSI} = 3 \cdot \left(\dfrac{10^{\frac{\epsilon_{MAX}}{10}} + 10^{\frac{-\epsilon_{MAX}}{10}}}{10^{\frac{\epsilon_{MAX}}{10}} - 10^{\frac{-\epsilon_{MAX}}{10}}}\right)^2} \qquad (19)$$

Because for standard implementation *quality factor* is directly releated with physical SNR, it is possible to place under direct comparison RSSI transceiver uncertainty with equivalent physical signal state. In fig.5 is shown eq.19 trend.
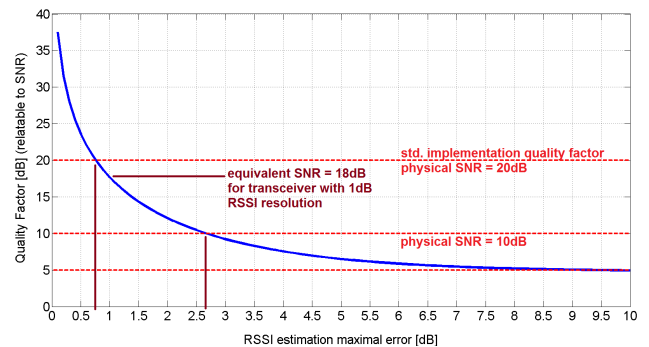

Figure 5: MUSIC RSSI implementation Q-Factor

2.2 Antenna array structural requirements

Before evaluating MUSIC RSSI vs standard implementation, some further considerations over physical constraints for antenna arrays must be performed.

A simple condition for a good phase-less RSSI MUSIC implementation is a direct consequence of vectorial projection structure. In eq.9 was shown standard RSSI projection norm of received steering vector $\overline{\mathbf{g}}(\theta_1, \varphi_1)$ over reference steering vector $\overline{\mathbf{g}}(\theta, \varphi)$: ignoring N_i noise term, projection norm is given by eq.20.

$$\|\overline{\mathbf{v}}^* \cdot \overline{\mathbf{m}}(\theta, \varphi)\| = \sum_{i=1}^{L} \left[g_i(\theta_1, \varphi_1) \cdot g_i(\theta, \varphi)\right] \quad (20)$$

A DoA mismatch happens when exists a DoA $(\theta_2, \varphi_2) \neq (\theta_1, \varphi_1)$ for which

$$\|\overline{\mathbf{v}}^* \cdot \overline{\mathbf{m}}(\theta_2, \varphi_2)\| \geq \|\overline{\mathbf{v}}^* \cdot \overline{\mathbf{m}}(\theta_1, \varphi_1)\| \quad \text{or rather}$$

$$\sum_{i=1}^{L} \left[g_i(\theta_1, \varphi_1) g_i(\theta_2, \varphi_2)\right] \geq \sum_{i=1}^{L} \left[g_i(\theta_1, \varphi_1) g_i(\theta_1, \varphi_1)\right]$$

To minimize this kind of mismatches, for working with phase-less RSSI MUSIC it is suggested to use arrays with strong spatial diversity of antenna gains, if possible with using directive antennas differently oriented as shown in fig.6. A good solution is presented in [4],[5]: in [6] array geometry impact over RSSI DoA estimation is investigated.
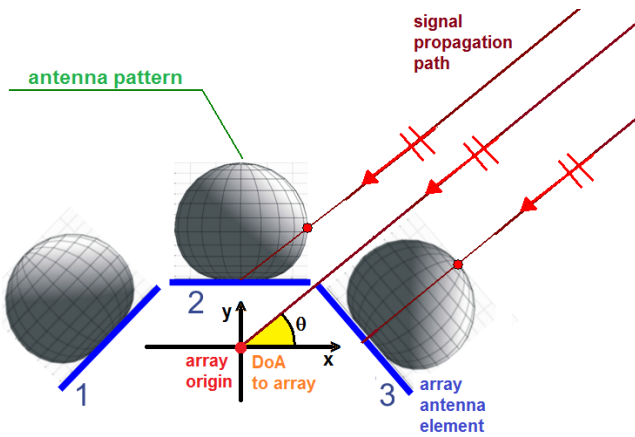

Figure 6: Example of array structure for RSSI MUSIC

3 Results comparison

Approximated CRB comparison between implementations will be shown. It is simulated a 1D DoA identification placing a planar array with an incient RF 2.45GHz ($\lambda \approx 12$ cm) signal coming from its frontal horizon (DoAs in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$): CRB index is approximated evaluating mean of standard deviations of DoA estimations, for every DoA in dominium, over 50 MUSIC executions for each implementation over a noised obtained data set of 50 samples.

Following requirement proposed in 2.2, structures for arrays will be different between phase and phase-less RSSI MUSIC implementation to optimize functional conditions. For phased MUSIC an Uniform Linear Array (ULA) is configured, maximizing center-of-phase interdistances for maximizing phase-displacement differentiation between DoAs. Instead for phase-less MUSIC an Uniform Circular Array (UCA) is placed (similar to 7) to guarantee maximal DoA antenna gains diversification (as in [6]): for both cases, antenna gains follow standard cardioid directive shape (fig.8).
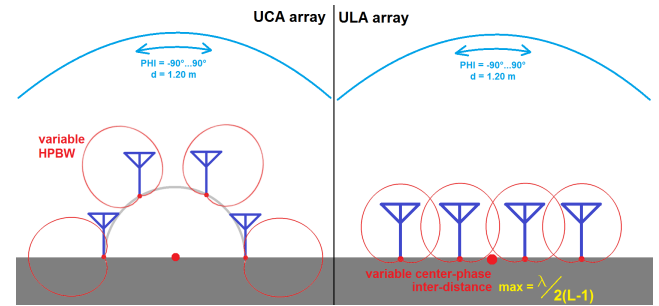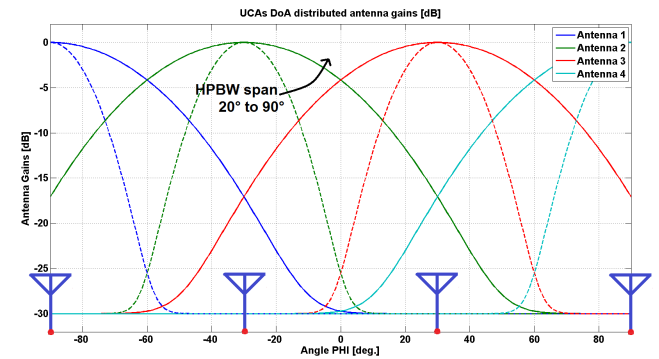

Figure 7: Simulated array structures

Figure 8: UCAs antenna gains versus DoA

Note that for ULAs configurations array lengths in range to $d \in \left(0, \frac{\lambda}{2}\right] \Rightarrow \Delta d_{max} = \frac{\lambda}{2(L-1)}$ allow to preserve coherent phase informations in front of every possible incident DoA, also with real phase-detector detection ambiguity between phases in $[-\pi, 0]$ and $[0, \pi]$ ranges.

CRBs are evaluated for different array configurations (varying antennas HPBW for UCA, array interdistance for ULA) and for different Q-Factors. In order to make a valid comparison, for both implementations x-axis plot is releated to *maximum RSSI deviation*: for

UCAs Q-Factor is known (eq.18), ans for ULAs SNR is directly set equal to UCAs Q-Factors (calculated with eq.19) for every case.
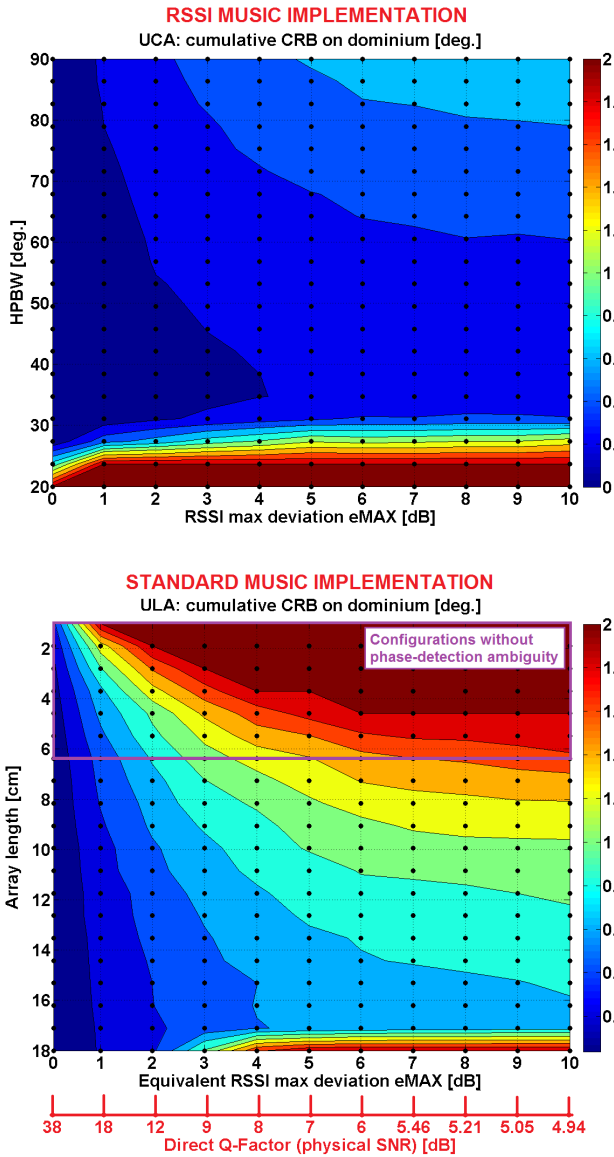

Figure 9: CRB-like estimation for UCAs(a) and ULAs(b) configs, versus equivalent RSSI max error

As shown in fig.9, it is an important fact that MUSIC RSSI implementation shows a more reliable behaviour: this is due to the presence in amplitude/phase acquisitions of noise contributions over both measured variables, increasing obtained steering vector distortion.

Obviously, a fundamental hypothesis is that RSSI measurements shouldn't be altered over ϵ_{MAX} from noise effects at lower SNRs. Digital modulations protects RSSI measurements from being skewed by channel noise (by using spread spectrum and ad-

vanced decoding correlation techniques), so in a clear environment is right to consider RSSI measurements more reliable than direct signal evaluation approach: consequently, RSSI MUSIC implementation is preferable for implementing low-cost DoA identification systems, as in [5].
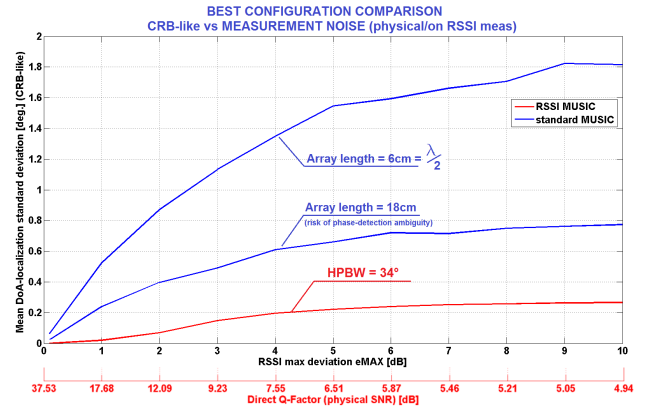

Figure 10: Cumulative CRB-like over DoAs comparison between RSSI/standard MUSIC implementations, vs maximal RSSI error/physical SNR

References:

[1] Ralph O. Schmidt, *Multiple Emitter Location and Signal Parameter Estimation*, *IEEE transaction on Antennas and propagation* 1986, n.3, vol.AP-34, pp.276–280

[2] P. Stoica and A. Nehorai, *MUSIC, Maximum Likelihood, and Cramer-Rao Bound*, *IEEE transaction on Acoustics, speech and signal processing* 1989, n.5, vol.37, pp.720–741

[3] A.K. Cline and I.S. Dhillon, *Computation of the Singular Value Decomposition*, *Handbook of Linear Algebra* 2006

[4] A. Cidronali, S. Maddio, G. Giorgetti and G. Manes, *Analysis and Performance of a Smart Antenna for 2.45-GHz Single-Anchor Indoor Positioning*, *IEEE transaction on Microwave Theory Techniques* 2010, n.1, vol.58, pp.21–31

[5] S. Maddio, M. Passafiume, A. Cidronali and G. Manes, *A Scalable Distributed Positioning System Augmenting WiFi Technology*, *Proceedings of International Conference on Indoor Positioning and Indoor Navigation (IPIN)* 2013, pp.733–742

[6] S. Maddio, M. Passafiume, A. Cidronali and G. Manes, *Impact of the Dihedral Angle of Switched Beam Antennas in Indoor Positioning based on RSSI*, *Wireless Technology Conference, EuMC* 2014, in press