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Angular dependence in anomalous microwave propagation: A bidimensional treatment

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The angular dependence of the advancing in the arrival time relative to microwave propagation experiments is derived from measurements previously performed. It turns out that the anticipation in the arrival time tends to be most extreme for angles around 30°. The results are then interpreted according to a model, based on fast-complex waves, suitably modified by a bidimensional treatment. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4868089]

I. INTRODUCTION

In a recent paper dealing with the arrival time of the front-edge in microwave propagation experiments,[1] some new results were reported and interpreted along the lines of a model already developed in a previous work.[2] What can be made evident from the data shown in Fig. 4 of Ref. 1 is that the angular dependence of the shortening in the delay time, expressed as the sum $\alpha + \beta$ (α being the observation angle and β the slope-angle of the wave), reaches its maximum extension when $\alpha + \beta$ is around 30°. In order to see whether some correlation exists, we have reconsidered the angular dependence of such delay in all the cases previously reported in Ref. 2. There, such a dependence was shown in only two cases (Figs. 6 and 7 in Ref. 2), while in the other cases (Figs. 1–5), the advancing time was shown as a function of the receiver-antenna displacement l. In Sec. II an analysis of the results is presented. Section III is devoted to the theoretical models, while a discussion and conclusive remarks are given in Sec. IV.

II. ANALYSIS OF THE DATA

For all the considered cases, once the angular dependence is obtained, the results are summarized in Table I. For each value of the antenna distance L, the maximum observed α_M is given by $\tan^{-1}(l/L)$ (see Fig. 4 in Ref. 1 where $L \equiv D$). As for the complex-value angle $\beta = \beta_r + i\beta_i$, which determines the pole-position in the complex-plane of the direction angle (z in Ref. 2), for all the cases taken from Ref. 2 we assume the value $\beta_r = 11°$ and $\beta_i = 11.5°$, as deduced from the data fitting of Fig. 6 in Ref. 2, while for the two cases of Ref. 1, β_r was assumed to be equal to the half fire-angle of the launcher horn (that is, $\beta_r = 20°$), and β_i was considered to be negligible.

The data reported in the other columns of Table I were obtained as follows. According to Ref. 2, the ratio of the light velocity c to the wave velocity v_{pp} is given by

$$\frac{c}{v_{pp}} = \cos(\alpha + \beta_r)\cosh\beta_i. \qquad (1)$$

Consequently, the shortening of the traversal (or delay) time is expressed as

$$\Delta\tau_{theo} = \frac{L}{c}\left(1 - \frac{c}{v_{pp}}\right), \qquad (2)$$

where L/c represents the traversal time at velocity c, and is compared with $\Delta\tau_{exp}$ as results from the measurements. The penultimate column reports the sum $\alpha_M + \beta_r$. We eliminate the first ($L = 21$ cm) case, which exhibits a disproportionate $\Delta\tau_{exp}$ that is even greater than L/c (thus making this result untrustworthy), and the last case ($L = 111$ cm), which results in a $\alpha_M + \beta_r$ value that is far from an average one, the anomaly is presumably due to the inadequacy of the theory for $L \gtrsim 1m$. In such a way, the average over the remaining seven cases turns out to be

$$\overline{\alpha_M + \beta_r} \simeq 32° \pm 2°, \qquad (3)$$

where the deviation $\bar{\epsilon}$ of the average is given by

$$\bar{\epsilon} = \sqrt{\frac{\Sigma\ \epsilon_i^2}{N(N-1)}}, \qquad (4)$$

with $N = 7$ and ϵ_i given in the last column of Table I. It is therefore evident that, in spite of some uncertainty regarding the β_r value, the resulting average behavior is quite consistent with an angular dependence of the observed phenomenon—the shortening of the traversal time—which tends to be most evident around 30°, as the results of Ref. 1 (27° and 33°) anticipated. The role played by β_i is more critical when we have to consider the attenuation of the complex wave, the amplitude of which is given by[2]

$$2\pi i \exp\left[-2\pi\ \frac{\rho}{\lambda}\sin(\beta_r + \alpha)\sinh\beta_i\right], \qquad (5)$$

where ρ and α are the polar coordinates of the observation point, the receiver horn in our case, and λ is the wavelength.

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TABLE I. Results of microwave propagation experiments once converted in order to point out the angular dependence of the advancing in the delay time.

L (cm)	α_M (°)	β_r (°)	β_i (°)	c/v_{pp}	L/c (ns)	$\Delta\tau_{theo}$ (ns)	$\Delta\tau_{exp}$ (ns)	$\alpha_M+\beta_r$ (°)	$\|\epsilon_i\|$ (°)	References
21	30	11	11.5	0.769	0.700	0.161	1.1	41	9	2
49	20	11	11.5	0.874	1.633	0.205	0.7	31	1	2
50	13	20	…	0.839	1.67	0.269	0.25	33	1	1
61	23	11	11.5	0.846	2.033	0.314	0.6	34	2	2
72	25	11	11.5	0.825	2.40	0.419	0.66	36	4	2
83	28	11	11.5	0.793	2.77	0.513	0.50	39	7	2
96	7	20	…	0.890	3.20	0.352	0.35	27	5	1
99	15	11	11.5	0.917	3.30	0.274	0.40	26	6	2
111	7	11	11.5	0.970	3.70	0.111	0.20	18	14	2

The quantity expressed by Eq. (5) can be compared with the amplitude of a "normal" wave, which is given simply by $(\lambda/\rho)^{1/2}$ for the cylindrical (unidimensional) geometry of Ref. 2. As we shall see here as follows, the capability of the model to explain the observation in the experiments is rather poor, as for the relative amplitude of the two waves is concerned. In the following, we will search for an improvement of the theoretical framework.

III. UNIDIMENSIONAL AND BIDIMENSIONAL MODELS

Analogously to Ref. 2, we assume that the radiated field from the launcher can be expressed as that of a rectangular aperture that has the dimensions of the mouth of the horn launcher. According to the geometry of Fig. 1, the total beam in the semispace $z>0$ can be expressed, in a scalar approximation, as a superposition of plane waves in the form[3]

$$F(x,y,z) = \exp(-i\omega t)\int\int A(x,y)\exp\left[ik(\bar{\alpha}x+\bar{\beta}y+\bar{\gamma}z\right]d\bar{\alpha}d\bar{\beta}, \tag{6}$$

where $A(x, y)$ is the amplitude over the mouth at $z=0$, $k=2\pi/\lambda$, $\bar{\alpha}=\sin\varphi$, and $\bar{\beta}=\sin\psi$ and $\bar{\gamma}=\cos\varphi\cos\psi$ are the director cosines of each elementary wave. By adopting polar coordinates, where we denote by apices the angles φ and ψ, we have

$$x=\rho\sin\varphi', y=\rho\sin\psi', z=\rho\cos\varphi'\cos\psi'. \tag{7}$$

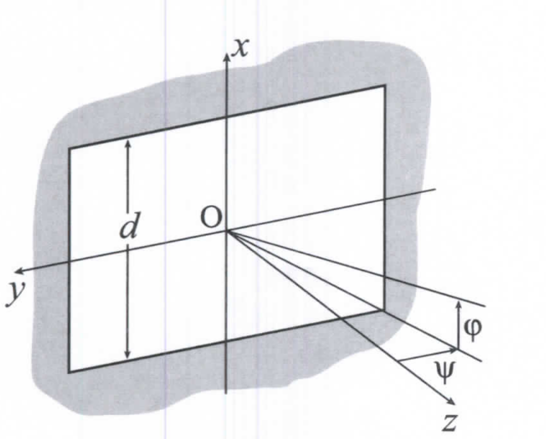

FIG. 1. Diagram of the mouth of the horn launcher as an aperture on the Σ plane with the relative coordinate axes and angular positions of the observer point.

By making a substitution in Eq. (6), we obtain the result that, apart from the temporal dependence and the factor $k\rho$, the phase is given by

$$\Phi = \sin\varphi\sin\phi' + \sin\psi\sin\psi' + \cos\varphi\cos\psi\cos\varphi'\cos\psi'. \tag{8}$$

A solution for $k\rho \gg 1$ can be obtained in the stationary-phase approximation, first by considering the zero of the phase derivative with respect to ψ', namely,

$$\frac{\partial\Phi}{\partial\psi'} = \sin\psi\cos\psi' - \cos\varphi\cos\psi\cos\varphi'\sin\psi' = 0, \text{ for } \psi,\psi'=0, \tag{9}$$

and the second derivative

$$\left.\frac{\partial^2\Phi}{\partial\psi'^2}\right|_{\psi,\psi'=0} = -\cos\varphi\cos\varphi'. \tag{10}$$

Therefore, by integrating in $d\bar{\beta}=\cos\psi d\psi$, $(-\frac{\pi}{2}\le\psi\le\frac{\pi}{2})$, the integral in $d\bar{\alpha}=\cos\varphi d\varphi$ becomes

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi\left(\frac{\lambda}{\rho}\right)^2 \frac{\cos\varphi}{(\cos\varphi\cos\varphi')^{\frac{1}{2}}}\exp\left\{i\left[k\rho\cos(\varphi-\varphi')-\frac{\pi}{4}\right]\right\}, \tag{11}$$

where we have assumed, for the sake of simplicity, $A(x, y)=1$. By applying the stationary-phase approximation to integral (11), for $\psi,\psi'=0$ we have

$$\frac{\partial\Phi}{\partial\varphi'} = \sin\varphi\cos\varphi' - \cos\varphi\sin\varphi' = \sin(\varphi-\varphi') = 0, \text{ for } \varphi=\varphi' \tag{12}$$

and

$$\frac{\partial^2\Phi}{\partial\varphi'^2} = -\cos(\varphi-\varphi') = -1, \quad \text{for } \varphi=\varphi', \tag{13}$$

so that, by reintroducing the amplitude factor, the integral (11) is found to be given simply by

$$A(P_0)\,\frac{\lambda}{\rho}\exp\left[i\left(k\rho-\frac{\pi}{2}\right)\right], \tag{14}$$

where $P_0:\psi,\psi'=0;\ \varphi=\varphi'$. Under the assumption that a pole singularity exists in the complex plane of φ at the complex

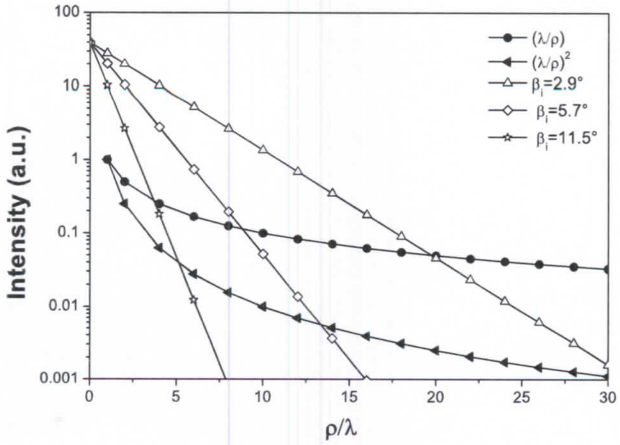

FIG. 2. Comparison of the intensity relative to the complex wave, as computed for different values of β_i, and the intensity relative to the "normal" wave for the case of unidimensional (λ/ρ) and bidimensional $(\lambda/\rho)^2$ treatments.

angle $\chi = \chi_r + i\chi_i$, the integral (11) can be conveniently evaluated using the saddle-point method, i.e., by deforming the original integration path $(-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2})$ into the steepest-descent path in the complex plane of φ, in order to capture the pole singularity by means of the same procedure as in Ref. 2 for the unidimensional case. In this way, we recover the contribution relative to a fast complex wave with an amplitude, to be added to the "normal" wave (14), given by

$$2\pi i \,\mathrm{res}\left[A(\varphi \to \chi)\right] \exp(ik\rho \cos(\varphi + \chi)), \qquad (15)$$

which, for $\mathrm{res}\left[A(\varphi \to \chi)\right] = 1$,[4] and $\varphi \equiv \alpha$ and $\chi \equiv \beta$, has the same characteristics in velocity and in attenuation as Eqs. (1) and (5), respectively. The only difference with respect to the unidimensional case of Ref. 2 is that, in the present bidimensional case, the "normal" wave is given by Eq. (14), which properly represents a spherical wave that attenuates like λ/ρ and not like $(\lambda/\rho)^{\frac{1}{2}}$ as previously anticipated for a cylindrical wave.[5]

IV. DISCUSSION AND CONCLUSIONS

We are now in a position to compare the contribution of the "normal" wave and the complex wave in order to test the validity of the computational scheme over distances of the order of those considered in Table I (that is, for $\lambda \approx 3$ cm, over some tens of ρ/λ).

In Fig. 2, we report the wave intensity which, for the complex wave, is given by the square of Eq. (5) computed for $\alpha + \beta_r = 32°$ (that is, the average value of $\alpha_M + \beta_r$ of Table I) and for $\beta_i = 11.5°, 5.7°$, and $2.9°$, which is represented in a logarithmic scale by straight lines. In the same graph, we also report the intensity of the "normal" wave for the case of cylindrical wave λ/ρ and for the spherical wave (λ/ρ),[2] while still assuming that $A(P_0) \approx 1$. We note that, in order to have a prevalence of the complex wave up to a distance of the order of 1 m, that is up to $\rho/\lambda \approx 30$, we have to consider, in even the more realistic case of a bidimensional (spherical) model, the smaller value of $\beta_i = 2.9°$. Instead, for the other values, an attenuation of the complex wave is not acceptable.

We can therefore conclude that the computational model adopted gives a plausible description of the experimental behavior, provided that the improved version depicted here is adopted.

When this work was nearly finished, we became aware of the issue of some experiments, performed over metal bars stimulated by ultrasounds.[6] In this situation, the emission of nuclear particles (alpha and neutrons) was evidenced with an angular dependence of the exciting system, with respect to the longitudinal axis of the bars, which privileged angles around 30°.[7] If we can exclude the possibility that this was nothing more than a mere coincidence with regard to our results, this matter deserves to be examined more thoroughly.

[1]A. Ranfagni and D. Mugnai, J. Appl. Phys. **114**, 014905 (2013).
[2]A. Ranfagni, P. Fabeni, G. P. Pazzi, and D. Mugnai, Phys. Rev. E **48**, 1453 (1993).
[3]G. Toraldo di Francia, *Electromagnetic Waves* (Interscience, New York, 1955), Chap. 4.

[4]This implies that $A(\varphi) = \frac{1}{\varphi - \chi}$ so that $\mathrm{res}\left[A(\varphi \to \chi)\right] = 1$, but $A(P_0)$ is not necessarily equal to unity.
[5]The approximation of cylindrical waves is acceptable only if the dimension of the mouth along the y axis is much greater than the dimension d along the x axis in Fig. 1.

[6]F. Cardone and R. Mignani, *Deformed Spacetime* (Springer, Dordrecht, the Netherlands, 2007), Chap. III.
[7]R. Mignani and A. Petrucci, private communication (2013).