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A DYNAMIC APPROACH TO ENVIRONMENTAL EXTERNALITIES: STRUCTURAL CHANGES, INSTITUTIONS, AND FOREIGN DIRECT INVESTMENTS

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Preface

The thesis analyses three different, yet interconnected, issues of environmental externalities: how structural changes may deteriorate natural resources and workers' welfare, how policy instruments may reduce environmental corruption, and how foreign direct invests may affect pollution level and land owners' welfare of a local economy. The study of these three issues is crucial in several developing countries characterized by ill-defined property rights on natural resources, on interaction with institutions, on protection against pollution, and high levels of inequality. The tool used is economic dynamics, more specifically, two-sector growth models and evolutionary games.

Introduction

The first paper, titled *Disequilibrium ecological dynamics, structural change and inter-sectoral mobility in a two-sector economy*, studies the dynamics of a two-sector economy (with a natural resource-dependent sector and an industrial sector) characterized by free inter-sectoral labor mobility and heterogenity of agents (workers and entrepreneurs). In such a context, we analyze the effects of the deterioration of natural resources, caused by the production activity of both sectors, on inter-sectoral movements of the labor force (structural changes), on ecological dynamics and on the revenues of workers and entrepreneurs. As in the seminal work by Matsuyama (1992), we obtain that a low productivity of labor in the resource-dependent sector can fuel the industrialization process. However, differently from Matsuyama (1992), in our model the industrialization process may give rise to a reduction in workers' revenues if the contribution to environmental depletion of the industrial sector, per unit of product, is higher than that of the resource-dependent one.

The second paper, titled *Green licenses and environmental corruption: a random matching model*, studies environmental corruption via a random matching evolutionary game between a population of firms and a population of bureaucrats in order to release a "green" license. A firm obtains the license if the bureaucrat checks that it complies with environmental regulations, otherwise it is sanctioned. In this model there are two types of bureaucrats (honest and dishonest), two types of firms (compliant and not compliant), and two types of crimes (corruption and extortion). Corruption is when a dishonest bureaucrat accepts a bribe from a not compliant firm, while extortion is when a dishonest bureaucrat extorts a bribe from a compliant firm. When there is no dominance of strategies, we obtain two bistable regimes, in which two attractive stationary states exist, and two regimes with an internal stable equilibrium, which corresponds to the mixed strategy Nash equilibrium of the one-shot static game, surrounded by closed trajectories. Moreover, from the comparative statics of the last two dynamic regimes emerges that policy instruments can help the Public Administration to reduce both corruption and extortion, though increasing sanctions, probability of being sanctioned and inspection effort do not always get the desired results.

The third paper, titled *Foreign direct investments, land rent, and pollution in a local economy*, studies the possible effects of foreign direct investments in land on the development of a local economy. To this aim, we use a two-sector model (*external* and *local*) with heterogeneous agents: *external investors* and *local land owners*. The dynamics is given by the accumulation of pollution and local physical capital, while the external physical capital accumulation is driven by foreign direct investments. We assume that both sectors are negatively affected by pollution, but only the external sector is polluting. The local government can tax its production activities to finance environmental defensive expenditures. We compute local agents revenues via numerical simulations analysis. A welfare-improving growth path may occur only if the pollution tax is high enough and the impact of the external sector on pollution is low enough, since the revenues of local land owners depend inversely on pollution level. Otherwise, a welfare-reducing growth path may occur, and foreign direct investments decrease the revenues of local land owners.

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DISEQUILIBRIUM ECOLOGICAL DYNAMICS, STRUCTURAL CHANGE, AND INTER-SECTORAL MOBILITY IN A TWO-SECTOR ECONOMY

Abstract

This paper studies the dynamics of a two-sector economy (with a *natural resource-dependent sector* and an *industrial sector*) characterized by free inter-sectoral labor mobility and heterogenity of agents (*workers* and *entrepreneurs*). In such a context, we analyze the effects of the deterioration of natural resources, caused by the production activity of both sectors, on inter-sectoral movements of the labor force (*structural changes*), on ecological dynamics and on the revenues of workers and entrepreneurs. As in the seminal work by Matsuyama (1992), we obtain that a low productivity of labor in the resource-dependent sector can fuel the industrialization process. However, differently from Matsuyama (1992), in our model the industrialization process may give rise to a reduction in workers' revenues if the contribution to environmental depletion of the industrial sector, per unit of product, is higher than that of the resource-dependent one.

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Keywords: structural change; environmental externalities; two-sector growth model.

1. Introduction

In 2011, nearly 50 percent of the population in developing countries lived in areas classified as urban, compared with less than 30 percent in the 1980s (World Bank, 2013). This means that in the last three decades there has been a significant migration from rural to urban areas. This phenomenon is, in many cases, associated to a structural change (SC) determined by a movement of labor force and production activities from natural resource-dependent sectors towards manufacturing sectors. It is often argued that SCs are cause and consequence of economic development and growth (see, e.g., Lewis, 1955; Ranis and Fei, 1961; Lucas, 2004), exactly as it happened in Europe in the nineteenth century due to the Industrial Revolution (Bade, 2008).

The main reason why many workers leave rural areas is the hope of improving their condition, escaping from a situation of poverty or unemployment, and attracted by a higher wage rate. In economic growth theory, there are two main explanations of the structural changes: the changing of consumer preferences (demand-side) and the technological innovation (supply-side). In the first case (Kongsamut et al., 2001), as income rises, the representative consumer increases manufacture and service demand, and reduces agricultural one; this modifies the production system and, hence, the composition of the labor force. In the second case (Acemoglu and Guerrieri, 2008), the technological innovation, lower in traditional sectors, rises profits and wages in the secondary and tertiary sectors, with a consequent increase of investments and labor force employment in such sectors.

Whatever is the cause of SCs, there is general agreement that SCs are an integral part of the economic growth process in developing countries and that they produce improvements in welfare of economic agents. For instance, Bretschger and Smulders (2012) argue that the process of intra-sectoral migration can lead to a level of technology that allows the sustainability of growth itself, also in an economy with exhaustible resources and low elasticity of substitution between natural resources and man-made inputs. Other economists, as Pasche

(2002), claim that it is possible having beneficial effects, thanks to technological improvements, only in the short term, while in the long term the sustainability is guaranteed only through a decrease in consumption and wealth.

However, an increasing share of literature on SCs deals with the negative effects on welfare due to the depletion of free access-natural resources which, in some cases, accompanies SCs. López (2003, 2007) and Antoci et al. (2010, 2012, 2014) argue that environmental degradation, caused by the expansion of the industrial sector, may fuel a self-enforcing process of structural change determined by a decrease in productivity in the traditional resource-based sector. In such a context, the industrialization process is often associated with growing problems of environmental degradation, declining or stagnant wages and the perpetuation of poverty. López (2007) refers to these cases as *perverse structural changes*.

Natural resources degradation is a serious problem in several developing countries characterized by ill-defined property rights on natural resources and high levels of inequality. Environmental degradation is playing a key role especially in those countries where strong growth rates have been observed in recent years, such as India and China, where many citizens are forced to change their behavior to defend themselves against the pollution effects of the industrialization process (Economy, 2004; World Bank, 2007; Dhamodharam and Swaminathan, 2010; Boopathi and Rameshkumar, 2011; Deng and Yang, 2013; Holdaway, 2013).

This paper analyses the dynamics of a two-sector economy (with a *natural resource-dependent sector* and an *industrial sector*) characterized by free inter-sectoral labor mobility and heterogeneity of agents (*workers* and *entrepreneurs*). In such a context, we study the effects of the deterioration of natural resources, caused by the activity of both sectors, on inter-sectoral movements of labour force (*structural changes*), on ecological dynamics and on workers and entrepreneurs revenues. In our model, as in the seminal work by Matsuyama (1992), a low productivity of labour in the resource-dependent sector can be the engine of the industrialization process. However, differently from Matsuyama (1992), we assume that the industrialization process generates environmental degradation and, consequently, a reduction in labour productivity in the resource-dependent sector. This may give rise to a self-enforcing process according to which the expansion of the industrial sector generates, via an increase in environmental degradation, a reduction in labour productivity in the resource-dependent sector and therefore leads workers to move from the resource-dependent sector towards the industrial one; the consequent further expansion of the industrial sector generates further environmental degradation and reduction in labour productivity in the resource-dependent sector, and so on. In such a context, the expansion of the industrial sector, at the expenses of the resource-dependent one, may be associated to a decrease in workers' revenues and an increase in entrepreneurs' revenues; that is to an increase in inequality between the two classes of economic agents. Our study starts from the framework proposed by Antoci et al. (2014), but introduces some crucial differences. More specifically, in Antoci et al. (2014), the polluting sector is the industrial sector and not the resource-dependent sector, while in this paper we assume that both sectors negatively affect environmental resources. Furthermore, we augment the model of Antoci et al. (2014) by introducing inter-sectoral dynamics from one sector to the other; labor allocation dynamics is determined by the difference between the wage rate in the industrial sector and the per capita output in the resource-dependent sector.[1] Augmenting the two-dimensional dynamic system analysed in Antoci et al. (2014), by the introduction of inter-sectoral dynamics, we obtain a dynamics which takes place in a three-dimensional box of the plane (K, N, E), where K is the capital stock, N the number of workers employed in the traditional resource-dependent sector, E the stock of an environmental resource. In this context we prove that, differently from Antoci et al. (2014), the stationary state in which both sectors coexist can be attractive only if it corresponds to a structural change which improves workers' welfare. However we also show, as in Antoci et al. (2014), that, if the contribution to environmental depletion of the natural resource-dependent sector is below a given threshold value, then there always exist trajectories converging to a stationary state where the economy becomes *specialized* in the industrial sector (that is, the variable N approaches the value 0) and a structural change occurs characterized by a reduction in workers' welfare.

The paper is organized as follows. The model is presented in Section 2. Section 3 contains local analysis, Section 4 deal with global analysis, Section 5 studies the welfare properties of the stationary states and Section 6 concludes.

2. Set up of the model

We examine a small open economy with two sectors -the E-sector and the I-sector-, free inter-sectoral labour mobility and heterogeneous agents. The production activity in the E-sector is based on a free-access environmental resource, while the production in the I-sector is based on the stock of physical capital accumulated in the economy. Economic agents belong to two different communities, one made of "workers", the other of "industrial entrepreneurs". The former are endowed only with their own working capacity and use it either in the E-sector or working as employees of the industrial entrepreneurs in the I-sector. In turn, the latter, who own physical capital and hire labour force, produce industrial goods.

The economy we consider is *small* and *open*, therefore the prices of the goods produced in both sectors can be considered as exogenously determined regardless of what happens in the economy. For simplicity, it is assumed that entrepreneurs do not invest in the E-sector, the latter being composed of small firms each of which is run by a worker.

The aggregated production functions of the E- and I-sectors

[1] In the model of Antoci et al. (2014), instantaneous adjustment of the labour market is assumed; that is, in each instant of time, the allocation of labour force between the two sectors of the economy is such that the wage rate in the industrial sector equals per capita output in the resource-dependent sector.

are given, respectively, by:

$$Y_I = (\overline{N} - N)^{\alpha} K^{1-\alpha} \qquad 1 > \alpha > 0, \overline{N} > 0 \tag{1}$$

$$Y_E = \beta N E \qquad \beta > 0 \tag{2}$$

where the variable $N \in [0, \overline{N}]$ (respectively, $\overline{N} - N$) represents the labour force employed in the E-sector (respectively, the I-sector) and the parameter $\overline{N}$ represents the size of the population of workers; E is the stock of a free-access natural resource and K is the aggregated stock of physical capital accumulated by the entrepreneurs; the parameter β is a measure of productivity in the E-sector. The production function (2) was proposed by Schaefer (1957) for fishery and is widely used in modelling production processes based on the exploitation of natural resources (Munro and Scott, 1993; Conrad, 1996; Brander and Taylor, 1998; McAusland, 2005; López, 2010).

The dynamics of the variables K, E and N is assumed to be represented by the three-dimensional dynamic system:

$$\begin{aligned} \dot{K} &= s\left[(\overline{N} - N)^{\alpha} K^{1-\alpha} - w(\overline{N} - N)\right] - dK \\ \dot{N} &= \gamma\left(\frac{Y_E}{N} - w\right) \\ \dot{E} &= E(\overline{E} - E) - \delta Y_E - \varepsilon Y_I \end{aligned} \tag{3}$$

with the non negativity constraints $E \geq 0$ and $N \geq 0$, where $Y_E/N = \beta E$ and w represent, respectively, per capita output in the E-sector and wage rate in the I-sector, which is assumed coinciding with marginal productivity of $\overline{N} - N$: $w = \alpha(\overline{N} - N)^{\alpha-1} K^{1-\alpha}$. The parameter $\overline{E} > 0$ measures the carrying capacity of the environmental resource; the parameters $\delta > 0$ and $\varepsilon > 0$ represent, respectively, the negative effects on E of the production activities of the E- and I-sectors; the parameters s, $d \in (0, 1)$ measure, respectively, the propensity to save of entrepreneurs and the depreciation rate of K.

We assume that each economic agent takes aggregate productions Y_E and Y_I as exogenously given. Consequently, both sectors produce environmental negative externalities that economic agents are not able to internalize due to coordination problems. This assumption plays a crucial role in shaping the results of our model. Environmental externalities affect economic activities especially in developing countries, where property rights tend to be ill-defined and ill-protected, environmental protection institutions and regulations are weak and natural resources are more fragile than in developed countries, which are located in temperate areas instead than in tropical and subtropical regions.

In the above-described context, we will analyse the dynamics generated by the system (3) and we will show how they depend on the relative level of carrying capacity $\overline{E}$ and the environmental pressures (measured by the parameters δ and ε) of the economic activities.

3. Local analysis

By substituting $Y_E/N = \beta E$ and $w = \alpha(\overline{N} - N)^{\alpha-1} K^{1-\alpha}$ in system (3), this can be written as

$$\begin{aligned} \dot{K} &= s(1-\alpha)(\overline{N} - N)^{\alpha} K^{1-\alpha} - dK \\ \dot{N} &= \gamma\left[\beta E - \alpha(\overline{N} - N)^{\alpha-1} K^{1-\alpha}\right] \\ \dot{E} &= E(\overline{E} - E) - \delta\beta N E - \varepsilon(\overline{N} - N)^{\alpha} K^{1-\alpha} \end{aligned} \tag{4}$$

In order to find possible stationary states of system (4) in the open box $\mathcal{B} = (0, \overline{K}) \times (0, \overline{N}) \times (0, \overline{E})$ and study their stability properties, we consider a suitable choice of the units of measurement, which leads to a rescaling of (4). Precisely, set $K = \eta K'$, $E = \mu E'$, $\overline{E} = \mu \overline{E}'$ such that $d\eta^{\alpha} = s(1-\alpha)$, $\beta\mu = \alpha\eta^{1-\alpha}$. Renaming K', E', $\overline{E}'$ as K, E, $\overline{E}$ and rescaling t by setting $t' = \mu t$, the system becomes

$$\begin{aligned} \dot{K} &= lK^{1-\alpha}\left[(\overline{N} - N)^{\alpha} - K^{\alpha}\right] \\ \dot{N} &= m\left[E - (\overline{N} - N)^{\alpha-1} K^{1-\alpha}\right] \\ \dot{E} &= E(\overline{E} - E) - pNE - q(\overline{N} - N)^{\alpha} K^{1-\alpha} \end{aligned} \tag{5}$$

where, with respect to system (4), $l = \frac{d}{\mu}$, $m = \beta\gamma$, $p = \frac{\beta\delta}{\mu}$, $q = \frac{\varepsilon\eta^{1-\alpha}}{\mu^2} = \frac{\beta\varepsilon}{\alpha\mu}$, $\mu = \frac{\alpha[s(1-\alpha)]^{\frac{1-\alpha}{\alpha}}}{\beta d^{\frac{1-\alpha}{\alpha}}}$. In fact the above rescaling of K and E amounts to choosing the unit of measurement of K as the capital stock per worker (so that $\overline{K} = \overline{N}$) and the unit of measurement of E in such a way that the wage per worker in the two sectors is measured by the same unit. Moreover, it follows from the above expression of $\dot{E}$ that p and q can be interpreted, respectively, as the contribution to the environmental depletion per unit of product of the traditional and the industrial sector. Then straightforward computations lead to the following result.

Proposition 1.

1. *There exists one and exactly one stationary state of system (5) in the box $\mathcal{B} = (0, \overline{N})^2 \times (0, \overline{E})$ if and only if $p \neq q$ and there exists λ, $0 < \lambda < 1$, such that $\overline{E} = 1 + \lambda p\overline{N} + (1-\lambda) q\overline{N}$. Then the stationary state is the point $\widetilde{P} = (K, N, E) = \left((1-\lambda)\overline{N}, \lambda\overline{N}, 1\right)$.*

2. *If $p = q$ and $\overline{E} = 1 + p\overline{N} = 1 + q\overline{N}$, then the stationary states in $\mathcal{B}$ fill the segment $\left\{K = \overline{N} - N, 0 < N < \overline{N}, E = 1\right\}$.*

3. *In all the other cases there is no stationary state of (5) in $\mathcal{B}$.*

The E-sector and the I-sector coexist in the stationary state $\widetilde{P}$, the strictly positive values λ and $1 - \lambda$ measuring the shares, in a population of size $\overline{N}$, of workers employed in the E-sector and in the I-sector, respectively. Since the rescaling of K amounts to choosing the unit of measurement of K as the capital stock per worker (so that $\overline{K} = \overline{N}$), the value of K at $\widetilde{P}$ coincides with the number of workers employed in the I-sector, that is

$K = \overline{N} - N = (1-\lambda)\overline{N}$. According to Proposition 1, $\widetilde{P}$ exists if and only if the carrying capacity $\overline{E}$ of the environmental resource is neither "too high" nor "too low", that is

$$1 + \min\left(p\overline{N}, q\overline{N}\right) < \overline{E} < 1 + \max\left(p\overline{N}, q\overline{N}\right) \quad (6)$$

Hence the stationary state $\widetilde{P}$ exists if the value of the carrying capacity $\overline{E}$ belongs to an interval whose extremes are determined by the size $\overline{N}$ of the population of workers and by the contributions to environmental depletion, per unit of product, of the E-sector (measured by p) and of the I-sector (measured by q). Notice that, given $\overline{N}$, such interval expands if the difference between the values of p and q increases; therefore the set of values of $\overline{E}$ implying the existence of $\widetilde{P}$ expands if the heterogeneity between the two sectors, with respect to the environmental impact, increases. Now, assume that one stationary state $\widetilde{P} = \left(\overline{N} - \widetilde{N}, \widetilde{N}, 1\right)$ exists in $\mathcal{B}$. Then the Jacobian matrix is easily computed to be

$$J(\widetilde{P}) = \begin{pmatrix} -l\alpha & -l\alpha & 0 \\ \frac{-m(1-\alpha)}{\overline{N}-\widetilde{N}} & \frac{-m(1-\alpha)}{\overline{N}-\widetilde{N}} & m \\ -q(1-\alpha) & -p+q\alpha & \overline{E}-2-p\widetilde{N} \end{pmatrix}$$

and the following proposition holds.

Proposition 2. *Assume system (5) has one stationary state $\widetilde{P} = \left(\overline{N} - \widetilde{N}, \widetilde{N}, 1\right) \in \mathcal{B}$. Then:*

1. *if $p < q$, $\widetilde{P}$ is either a saddle with a two-dimensional stable manifold or a source;*
2. *if $p > q$, $\widetilde{P}$ is either a sink or a saddle with a one-dimensional stable manifold.*
3. *All the previous cases can occur.*

Proof. Straightforward computations yield

$$\det J(\widetilde{P}) = lm\alpha\,(q-p)$$

i.e. $sgn\left[\det J(\widetilde{P})\right] = sgn\,(q-p)$, which proves the first two statements of the proposition. As to the third one, easy computations show that:

- Let $l = m = 1, p = 2, q = 1, \alpha = 0.5, \overline{N} = 4, \overline{E} = 5.8$. Then the stationary state $\widetilde{P} = (3.2, 0.8, 1)$ is a saddle with a one-dimensional stable manifold (in fact, $trace J(\widetilde{P}) > 0$).
- Let $l = m = 1, p = 2, q = 1, \alpha = 0.5, \overline{N} = 4, \overline{E} = 8.2$. Then the stationary state $\widetilde{P} = (0.8, 3.2, 1)$ is a sink (Routh-Hurwicz conditions imply that $J(\widetilde{P})$ has three eigenvalues with negative real part).
- Let $l = m = 1, p = 1, q = 2, \alpha = 0.1, \overline{N} = 4, \overline{E} = 5.8$. Then the stationary state $\widetilde{P} = (0.8, 3.2, 1)$ is a saddle with a two-dimensional stable manifold (in fact, $trace J(\widetilde{P}) < 0$).
- Let $l = m = 0.1, p = 1, q = 2, \alpha = 0.1, \overline{N} = 4, \overline{E} = 5.8$. Then the stationary state $\widetilde{P} = (0.8, 3.2, 1)$ is a source (Routh-Hurwicz conditions imply that $J(\widetilde{P})$ has three eigenvalues with positive real part). □

According to the above proposition, the stationary state $\widetilde{P}$ - where both sectors coexist - can be locally attractive only if the contribution to environmental depletion, per unit of product, of the E-sector (measured by p) is higher than that of the I-sector (measured by q). In order to get an intuitive idea of what mechanism generates such result, take into account that, in the context $p < q$, an increase in the share of workers employed in the I-sector (i.e. an increase of $\overline{N} - N$) produces a higher environmental degradation and, consequently, a reduction of labor productivity in the E-sector. Workers defend themselves from the reduction of labor productivity in the E-sector by increasing their labor offer to the I-sector. The consequent further expansion of the I-sector produces further environmental degradation and so on. Such a mechanism is clearly self-enforcing and does not favour the coexistence between the two sectors. The opposite holds if $p > q$: in such a case, an increase in the share of workers employed in the I-sector has the effect of reducing environmental degradation and increasing labor productivity in the E-sector. Consequently an increase in $\overline{N} - N$ supports the relative performance of the E-sector pushing workers out of the I-sector, which, obviously, favours the coexistence of the two sectors.

Remark 3. *In the bifurcation case $p = q$, when $\overline{E} = 1 + p\overline{N}$, as we have said, all the points of the segment $\left\{K = \overline{N} - N,\ 0 < N < \overline{N},\ E = 1\right\}$ are stationary states. Then it is easily checked that there exists a value $\widehat{N} \in \left(0, \overline{N}\right)$ such that, when $N^* \in \left(\widehat{N}, \overline{N}\right)$, the stationary state $P^* = \left(\overline{N} - N^*, N^*, 1\right)$ is endowed with a two-dimensional stable manifold. Hence for $N^* \in \left(\widehat{N}, \overline{N}\right)$ the phase portrait in the box is stratified.*

Remark 4. *It is easily observed that the definition of the dynamic system (equivalently, by (4) or (5)) implies, for any choice of the above parameters, the existence of trajectories, in the box $\mathcal{B}$, reaching either the side $E = 0$ or the side $N = 0$ within a finite time. In fact, take, for example, a point $P_0 = (K_0, N_0, 0)$, with $K_0 > 0$ and $0 < N_0 < \overline{N}$. Consider the negative trajectory through P_0 defined, say, by (5), i.e. $\Gamma(t)$ with $t \leq 0$ and $\Gamma(0) = P_0$. Then, for $0 > t \geq -\varepsilon$, ε being sufficiently small, $\Gamma(t) \in \mathcal{B}$. Called $\Gamma(-\varepsilon) = P_\varepsilon$, it follows that the positive trajectory from P_ε reaches the side $E = 0$ at the time ε. Clearly an analogous argument applies if we consider a point $Q_0 = (K_0, 0, E_0)$, such that $K_0, E_0 > 0, E_0 < \left(K_0\overline{N}\right)^{1-\alpha}$ (implying $\dot{N}(Q_0) < 0$). We also observe that some trajectories leaving the box through $N = 0$ may get back into the box, which cannot occur for those reaching (in a finite time) $E = 0$.*

4. Global analysis: one stationary state

4.1. The case $p < q$

Let us assume that one stationary state $\widetilde{P}$ in $\mathcal{B}$ exists. As observed in Remark 4, even if $\widetilde{P}$ is a sink (implying $p > q$), there exist trajectories reaching in a finite time the boundary of the box $\mathcal{B}$ (i.e. the sides $N = 0$ or $E = 0$). So the question arises: when $p < q$, there still exists a sub-region (i.e. an open connected subset) whose trajectories remain in the box for all $t > 0$, that is a *positively invariant region*?

In fact the following results provide a positive answer. Assume

$$(A1)\quad \overline{E} = 1 + [\lambda p + (1-\lambda) q]\,\overline{N} \text{ (i.e. } \widetilde{P} \text{ exists)},\ p < q$$

Then we prove

Lemma 5. *Suppose that* $(A1)$ *holds and that a trajectory lying in* $\mathcal{B}$*, say* $\Gamma(t) = (K(t), N(t), E(t))$*, is such that there exists an increasing sequence of times* $t_n > 0$ *for which* $\lim\limits_{t_n \to T} K(t_n) = 0$*. Then* $T = +\infty$ *and there exists* $\bar{t} > 0$ *such that* $\dot{K}(t) < 0$ *as* $t \in (\bar{t}, +\infty)$*.*

Proof. Assume that along a trajectory $\Gamma(t) = (K(t), N(t), E(t))$ lying in $\mathcal{B}$ there exists an increasing sequence of times $t_n > 0$ such that $\lim\limits_{t_n \to T} K(t_n) = 0$, $T \le +\infty$. First of all we show that $K(t_n)$ cannot keep oscillating. In fact, in such a case there should be a sequence of maxima, say $K_n' = K(t_n')$, and a sequence of minima, say $K_n'' = K(t_n'')$, with $t_n', t_n'' \to T$ and $K_n', K_n'' \to 0$. Pose $\Gamma(t_n') = (K_n', N_n', E_n')$, $\Gamma(t_n'') = (K_n'', N_n'', E_n'')$. Then $N_n' = \overline{N} - K_n'$, $N_n'' = \overline{N} - K_n''$, $E_n'' < 1 < E_n'$. Therefore we can assume $t_n' < t_n''$, in such a way that at a time $\overline{t_n} \in (t_n', t_n'')$ $E(\overline{t_n}) = 1, \dot{E}(\overline{t_n}) < 0$, whereas $\dot{K}(t) < 0$ (i.e. $\overline{N} - N(t) < K(t)$) as $t \in (t_n', \overline{t_n})$. Hence $\overline{N} - N(\overline{t_n}) < K(\overline{t_n}) < K_n'$. It follows that, for a sufficiently high n, $\dot{E}(\overline{t_n}) = \overline{E} - 1 - p\overline{N} + \varepsilon_n = (1-\lambda)(q-p)\overline{N} + \varepsilon_n > 0$, since $\varepsilon_n \to 0$ as $n \to +\infty$, thus leading to a contradiction. Hence, if a trajectory $\Gamma(t)$ lying in $\mathcal{B}$ satisfies the assumption of the Lemma, then there exists T, $0 < T \le +\infty$, such that along $\Gamma(t)$ $\lim\limits_{t \to T} K(t) = 0$ and $\dot{K}(t) < 0$ as $t \in (\bar{t}; T)$, $\bar{t} \ge 0$, implying $\lim\limits_{t \to T} N(t) = \overline{N}$. Pose $[K(t)]^\alpha = H(t)$, $\left[\overline{N} - N(t)\right]^\alpha = \nu(t)$. Then

$$\dot{H}(t) = \alpha l\,[\nu(t) - H(t)]$$

so that, for $t > 0$, $H(t) = H(0)e^{-\alpha l t} + \alpha l e^{-\alpha l t}\int_0^t e^{\alpha l s}\nu(s)\,ds$, $H(0) > 0$. Therefore $\lim\limits_{t \to T} K(t) = 0$ clearly implies $T = +\infty$. □

Theorem 6. *Given assumption* $(A1)$*, consider the segment*

$$\Sigma = \left\{K = \overline{N} - N,\ \lambda\overline{N} < N < \overline{N},\ E = 1\right\} \tag{7}$$

Then all the trajectories starting from points of Σ *tend, as* $t \to +\infty$*, to the boundary point* $\widehat{P} = \left(0, \overline{N}, \widehat{E}\right)$*, where* $\widehat{E} = \overline{E} - p\overline{N} = 1 + (1-\lambda)(q-p)\overline{N}$*.*

Proof. First of all, consider a trajectory $\Gamma(t)$ starting from a point of Σ, that is $\Gamma(0) = P_0 = \left(\overline{N} - N_0, N_0, 1\right)$, $\lambda\overline{N} < N_0 < \overline{N}$. Then it is easily computed that $\dot{K}(0) = \ddot{K}(0) = \dot{N}(0) = 0$ and $\dot{E}(0) > 0$, implying $\ddot{N}(0) > 0$ and $\dddot{K}(0) < 0$. Hence, in a right neighborhood of $t = 0$, $\dot{K}(t) < 0$, that is $\overline{N} - N(t) < K(t)$.

So, set $K_0 = \overline{N} - N_0$, consider first the case $0 < K_0 < \frac{\overline{E} - 1 - p\overline{N}}{q} = \frac{(1-\lambda)(q-p)}{q}\overline{N}$. If the trajectory from such a P_0 should cross again the plane $E = 1$, say at $P_1 = (K_1, N_1, 1)$, then we would have $\overline{N} - N_1 < K_1 < K_0$. Hence it is easily checked that, at P_1, $\dot{E} > \overline{E} - 1 - p\overline{N} - qK_1 > \overline{E} - 1 - p\overline{N} - qK_0 > 0$, leading to a contradiction. It follows that along the trajectory $K(t)$ keeps decreasing, with $E(t) > 1$ and $\overline{N} - N(t) < K(t)$. Hence, by applying the Lemma, it is easily checked that $\lim\limits_{t \to +\infty} K(t) = 0$, $\lim\limits_{t \to +\infty} N(t) = \overline{N}$ and $\lim\limits_{t \to +\infty} E(t) = \widehat{E} = 1 + (1-\lambda)(q-p)\overline{N}$.

Now, assume by contradiction that among the trajectories starting from Σ only those from a sub-segment $\widetilde{\Sigma} = \left\{K = \overline{N} - N, \widetilde{N} < N < \overline{N}, E = 1\right\}$, $\lambda\overline{N} < \widetilde{N}$, tend to $\widehat{P}$. Hence, by continuity, also the trajectory from $\widetilde{P} = \left(\overline{N} - \widetilde{N}, \widetilde{N}, 1\right)$ tends to $\widehat{P}$, as it is easily checked. Then, after a sufficiently long time $T > 0$, this trajectory will reach a point, say, $P_1 = (K_1, N_1, E_1)$, where $0 < K_1 < \frac{(1-\lambda)(q-p)}{2q}\overline{N}$, $\overline{N} - N_1 < K_1$, $E_1 > 1 + \frac{(1-\lambda)(q-p)\overline{N}}{2}$. Therefore consider a point of Σ $P_0 = \left(\overline{N} - N_0, N_0, 1\right)$, where $\lambda\overline{N} < N_0 < \widetilde{N}$. If $\widetilde{N} - N_0 < \varepsilon$, ε being sufficiently small, then the trajectory from P_0 will reach at time T, by Gronwall's Lemma, a point $P_2 = (K_2, N_2, E_2)$, where $0 < K_2 < \frac{(1-\lambda)(q-p)}{q}\overline{N}$, $\overline{N} - N_2 < K_2$, $E_2 > 1 + \frac{(1-\lambda)(q-p)\overline{N}}{4}$, with $\dot{K}(P_2) < 0$. Hence it follows from the above arguments that such a trajectory will also tend to $\widehat{P}$ as $t \to +\infty$. This concludes the proof of the theorem. □

It follows from the proof itself of the above theorem that there exists a sub-region of $\mathcal{B}$ (containing a *tubular* neighborhood of Σ) whose trajectories stay in $\mathcal{B}$ and converge, as $t \to +\infty$, to $\widehat{P}$. Moreover the equilibrium $\widetilde{P}$ belongs to the boundary of such a region. These facts motivate the following

Conjecture 7. *If* $(A1)$ *holds and* $\widetilde{P}$ *is a saddle, then the two-dimensional stable manifold of* $\widetilde{P}$ *separates the trajectories of* $\mathcal{B}$ *leaving in a finite time the box from those which tend, as* $t \to +\infty$*, to the boundary point* $\widehat{P}$*.*

In fact the previous arguments justify a further conjecture, namely

Conjecture 8. *Suppose* $(A1)$ *holds and* $\widetilde{P}$ *undergoes a Hopf bifurcation, changing from a saddle into a source. Then such a bifurcation is supercritical, i.e. a limit cycle arises, surrounding* $\widetilde{P}$*, endowed with a two-dimensional stable manifold.*

What motivates this second conjecture is the fact that, after the bifurcation takes place, the source $\widetilde{P}$ continues to belong to the boundary of the trajectories tending to $\widehat{P}$, i.e. to the separatrix between trajectories staying in, as $t \to +\infty$, and leaving the box. Hence it is reasonable to suppose that such a separatrix is now the stable manifold of a cycle arisen through the Hopf bifurcation.

According to the above results, if $p < q$ (i.e. the contribution to environmental depletion of the I-sector, per unit of product, is higher than that of the E-sector) and the stationary state $\widetilde{P}$ (in such a context never attractive, see Proposition 2) exists, then the dynamics is path-dependent, in that more regimes can occur, depending on the initial conditions of the dynamics in the box $\mathcal{B}$. In particular:

1. There always exist initial conditions of the variables K, N, E from which the trajectories reach in a finite time the side $E = 0$ of the box $\mathcal{B}$ (i.e., the ecological-economic dynamics leads to total depletion of the environmental resource). When this happens, we can expect that the economy will become specialized in the I-sector (that is, no economic agent works in the E-sector), since labor productivity in the E-sector is equal to zero when $E = 0$.

2. Furthermore, there are initial conditions from which the ecological-economic dynamics leads asymptotically to a point $\widehat{P} = \left(0, \overline{N}, \widehat{E}\right)$ lying on the boundary of the box $\mathcal{B}$, where the I-sector disappears ($K = 0$) and therefore the economy becomes specialized in the E-sector. The trajectories approaching asymptotically the boundary point $\widehat{P}$ start from a sub-region of B containing a tubular neighborhood of the segment Σ (see (7)).

Note that the limit boundary point $\widehat{P} = \left(0, \overline{N}, \widehat{E}\right)$, where $\widehat{E} = \overline{E} - p\overline{N}$, coincides with the unique stationary state with $E > 0$ of the one-sector dynamics that would be observed in absence of the I-sector. In such a context, set $K = 0$ and $N = \overline{N}$, the evolution of E would be described by the equation

$$\dot{E} = E(\overline{E} - E) - p\overline{N}E \tag{8}$$

admitting the two stationary states: $E = 0$ (a source) and $E = \widehat{E} = \overline{E} - p\overline{N}$ (a sink).

The above situation is illustrated in Fig. 1, which refers to the case in which the stationary state $\widetilde{P}$ is a saddle with a two-dimensional stable manifold, showing trajectories tending (as $t \to +\infty$) to the boundary point $\widehat{P}$ or reaching in a finite time the side $E = 0$ of the box $\mathcal{B}$. This result is not surprising, since, when $p < q$, the dynamics is conditioned by the self-enforcing mechanism outlined in Section 3, according to which the expansion of the I-sector generates, via an increase in environmental degradation, a reduction in labour productivity in the E-sector and therefore leads workers to move from the resource-dependent sector towards the industrial one. The consequent expansion of the I-sector generates further environmental degradation and reduction in labor productivity in the E-sector, and so on.

4.2. *The case $p > q$*

Assume

$$(A2) \quad \overline{E} = 1 + [\lambda p + (1 - \lambda) q]\,\overline{N}, \; p > q$$

Then the unique stationary state $\widetilde{P} = \left(\overline{N} - \widetilde{N}, \widetilde{N}, 1\right) \in \mathcal{B}$ is (generically) either a sink or a saddle with a one-dimensional

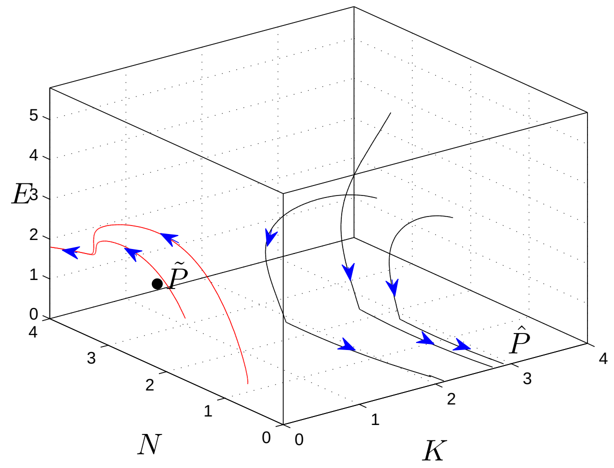

Fig. 1. Case $p < q$: $l = m = 1, p = 1, q = 2, \alpha = 0.1, \overline{N} = 4, \overline{E} = 5.8$. Trajectories approaching the boundary point $\widehat{P}$ and trajectories reaching, in a finite time, the side $E = 0$ of the box $\mathcal{B}$, in the context in which the stationary state $\widetilde{P} = (0.8, 3.2, 1)$ is a saddle with a two-dimensional stable manifold.

stable manifold and both cases can occur, as showed in Section 3. In fact, the question arises: when, by suitably varying one parameter of the system (e.g., $\alpha, l, \overline{E}$), $\widetilde{P}$ changes stability and thus, generically, a Hopf bifurcation occurs, what is the nature - supercritical or subcritical - of such bifurcation? Actually we don't have an analytic demonstration, but several numerical experiments suggest that the Hopf bifurcation is always subcritical, that is, when $\widetilde{P}$ changes from a saddle into a sink, then, generically, a limit cycle Γ arises, surrounding $\widetilde{P}$. In such a case (see, e.g., Guckenheimer and Holmes, 1997) the euclidean space $\mathbf{E}^3_Q$ pointed at every $Q \in \Gamma$ can be seen as generated by three *directions*: l_1, tangent in Q to Γ; l_2, corresponding to the contracting direction of a suitable Poincaré return map in Q; l_3, corresponding to the expanding direction of the same map. Then the two-dimensional manifold which at any $Q \in \Gamma$ is tangent to the plane generated by l_1 and l_2 constitutes (part of) the boundary of the basin of attraction of $\widetilde{P}$. Fig. 2 illustrates, by a numerical example, the basin of attraction (exhibiting a conic shape) of the sink $\widetilde{P}$ surrounded by a limit cycle. Fig. 3 shows two trajectories belonging to the basin of $\widetilde{P}$ and two trajectories reaching in a finite time the side $E = 0$ of the box $\mathcal{B}$ (the parameter values are the same as in Fig. 2). The next Fig. 4 shows the basin of the sink $\widetilde{P}$ when the limit cycle is no more contained in $\mathcal{B}$ (as it may cross the plane $N = 0$). Finally Fig. 5 suggests that, when $\widetilde{P}$ becomes a saddle with a one-dimensional stable manifold, then all the trajectories in $\mathcal{B}-\left\{\widetilde{P}\right\}$, except two, leave the box within a finite time.

It is worth to stress that in the context $p > q$ (i.e. the contribution to environmental depletion of the I-sector, per unit of product, is lower than that of the E-sector), when the stationary state $\widetilde{P}$ is a sink, the dynamics is path-dependent, as at least two regimes (generically) occur, depending on the initial conditions of the dynamics in the box $\mathcal{B}$. In particular:

1. There exist initial conditions of the variables K N, E

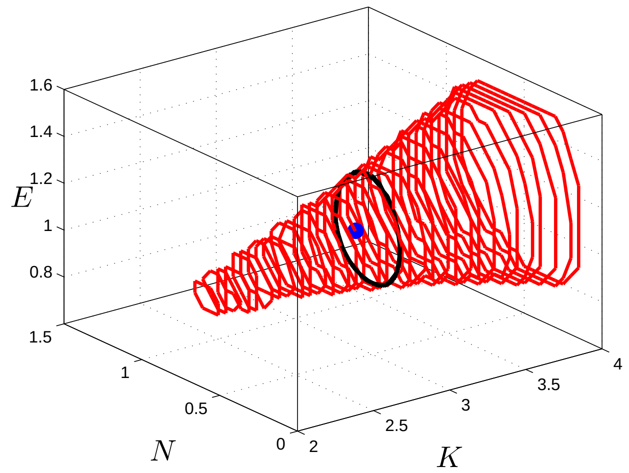

Fig. 2. Case $p > q$: $l = m = 1, p = 1.2, q = 0.4, \alpha = 0.15, \overline{N} = 4, \overline{E} = 3.24$. Basin of the sink $\widetilde{P} = (3.2, 0.8, 1)$ surrounded by a limit cycle.

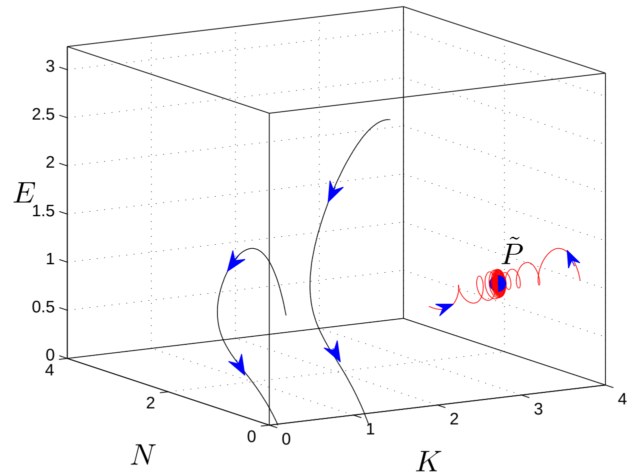

Fig. 3. Case $p > q$: $l = m = 1, p = 1.2, q = 0.4, \alpha = 0.15, \overline{N} = 4, \overline{E} = 3.24$. Two trajectories approaching the sink $\widetilde{P} = (3.2, 0.8, 1)$ and two trajectories reaching in a finite time the side $E = 0$ of the box $\mathcal{B}$.

from which the trajectories reach in a finite time the sides $\{E = 0\} \cup \{N = 0\}$ of the box and therefore we expect the economy to get specialized in the I-sector.

2. There exist also trajectories tending asymptotically to the stationary state $\widetilde{P}$ where both sectors coexist (such trajectories fill an open region, which may exhibit a conic shape, see Fig. 2).

The context $p > q$ "favours"the coexistence of the two sectors, since (see Section 3), when $p > q$ holds, an increase in the share of workers employed in the I-sector reduces the overall negative impact of economic activity on the environmental resource. The consequent increase in the stock E causes an increase of labor productivity in the E-sector and, therefore, stimulates workers to work in such a sector. According to this mechanism, an initial increase in the number $\overline{N} - N$ of workers employed in the I-sector exerts an upward pressure on N, which may counterbalance the initial increase in $\overline{N} - N$. Such process is obviously stabilizing. However, as underlined above, also in this coexistence-favouring context the economy may follow trajectories leading towards a specialization in industrial production.

5. Welfare analysis

In this Section, we analyze welfare properties of the possible limit points of the dynamic system (5). Remember that all the trajectories reaching in a finite time the boundary $E = 0$ of the box $\mathcal{B}$ cannot get back into the box, since $\dot{E} < 0$ always holds for $E = 0$. Taking into account the non negativity constraint $E \geq 0$, we can imagine that, when $E = 0$, the condition $\dot{E} < 0$ might be replaced by $\dot{E} = 0$, so that a dynamics may take place on the side $E = 0$. Such a fact can be modelled by introducing a discontinuity in the vector field (see, for example, Utkin, 1978). It follows, as it is easily checked, that all the

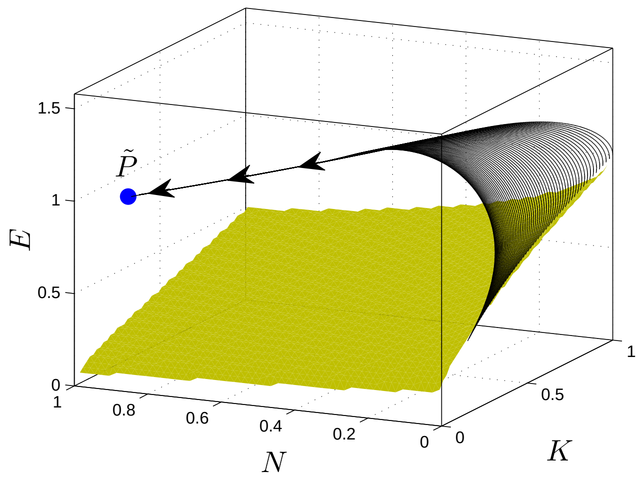

Fig. 4. Case $p > q$: $l = m = 1, p = 0.6, q = 0.4, \alpha = 0.15, \overline{N} = 1, \overline{E} = 1.58$. Basin of the sink $\widetilde{P} = (0.1, 0.9, 1)$ without limit cycle.

trajectories reaching in a finite time the side $E = 0$ approach the boundary point $\overline{P} = (K, N, E) = \left(\overline{N}, 0, 0\right)$, where physical capital accumulation reaches its maximum possible value (at a stationary state) and the E-sector disappears from the economy. We will evaluate entrepreneurs and workers' revenues at the points

$\widetilde{P} = (K, N, E) = \left(\overline{N} - \widetilde{N}, \widetilde{N}, 1\right)$, with $\overline{N} > \widetilde{N} > 0$ (coexistence of the two sectors)

$\widehat{P} = (K, N, E) = \left(0, \overline{N}, \widehat{E}\right)$, with $\widehat{E} > 0$ (specialization in the E-sector)

$\overline{P} = (K, N, E) = \left(\overline{N}, 0, 0\right)$ (specialization in the I-sector)

We will use as benchmark the revenues evaluated at the state $\widehat{P}$, which is a possible limit point of the dynamics (5), but also

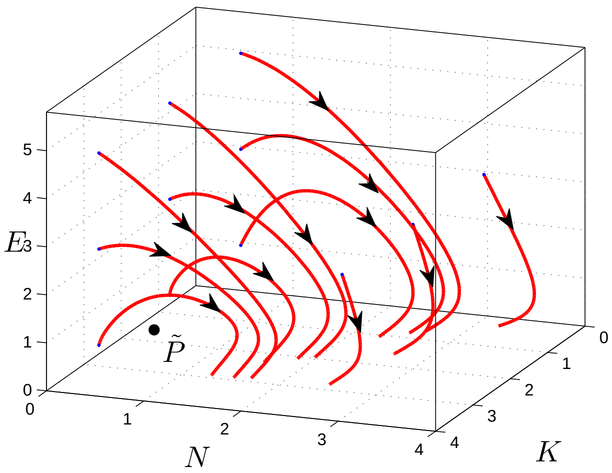

Fig. 5. Case $p > q$: $l = m = 1, p = 2, q = 1, \alpha = 0.5, \overline{N} = 4, \overline{E} = 5.8$. $\widetilde{P} = (3.2, 0.8, 1)$ is a saddle with a one-dimensional stable manifold. Almost all the trajectories leave the box $\mathcal{B}$.

corresponds to the stationary state $E = \widehat{E}$ that the economy would reach in absence of the I-sector, according to Eq.(8). If workers' revenues in $\widehat{P}$ are higher than in $\widetilde{P}$ or in $\overline{P}$, then the stationary states $\widetilde{P}$ and $\overline{P}$ represent contexts in which the structural change observed after the introduction of the I-sector is welfare reducing (from the point of view of workers).

Per capita revenues of workers evaluated at $\widehat{P}$, $R_w^{\widehat{P}}$, are determined by the value of E at $\widehat{P}$, that is

$$R_w^{\widehat{P}} = \widehat{E} = \overline{E} - p\overline{N}$$

Let us now compare $R_w^{\widehat{P}}$ with per capita revenues of workers evaluated at the stationary states $\widetilde{P}$ and $\overline{P}$, belonging to the isocline $\dot{K} = 0$ (along which $K = \overline{N} - N$ holds). Workers' revenues are given by $(\overline{N} - N)^{\alpha-1}K^{1-\alpha}$ (at $\widetilde{P}$, workers' revenues in the two sectors are the same, since the equilibrium condition $(\overline{N} - N)^{\alpha-1}K^{1-\alpha} = E$ holds). Posing $K = \overline{N} - N$ in $(\overline{N} - N)^{\alpha-1}K^{1-\alpha}$, we obtain that workers' revenues at $\widetilde{P}$ and $\overline{P}$, respectively $R_w^{\widetilde{P}}$ and $R_w^{\overline{P}}$, are $R_w^{\widetilde{P}} = R_w^{\overline{P}} = 1$. We have that $R_w^{\widehat{P}} < R_w^{\widetilde{P}} = R_w^{\overline{P}} = 1$ holds if $\widehat{E} = \overline{E} - p\overline{N} < 1$, that is

$$p > \frac{\overline{E} - 1}{\overline{N}} \tag{9}$$

Condition (9) is satisfied if the stock E of the environmental resource at the point $\widehat{P}$ is lower than 1, the value assumed by E at the stationary state $\widetilde{P}$, and therefore the revenues of workers employed in the E-sector are higher in $\widetilde{P}$ than in $\widehat{P}$.

It is easy to check that, when (9) holds, a necessary condition for the existence of the internal stationary state $\widetilde{P} = \left(\overline{N} - \widetilde{N}, \widetilde{N}, 1\right)$ is $q < \left(\overline{E} - 1\right)/\overline{N}$. This implies that, when $\widetilde{P}$ exists, then workers' revenues in $\widetilde{P}$ can be higher than in $\widehat{P}$ only in the context $p > q$; that is, if the contribution to environmental depletion, per unit of product, of the E-sector is higher than that of the I-sector. So, according to the above result and to stability analysis of Section 3, the stationary state $\widetilde{P}$ can be attractive only if the structural change observed when the economy leaves the stationary state $\widehat{P}$ and reaches $\widetilde{P}$ (where the traditional and the industrial sector coexist) generates an increase in revenues of workers, otherwise $\widetilde{P}$ is a saddle or a source. Differently from the stationary state $\widetilde{P}$, the state $\overline{P} = (K, N, E) = \left(\overline{N}, 0, 0\right)$ is always attractive for the trajectories belonging to an open region of the box $\mathcal{B}$, even if the inequality (9) is reversed (that is, if workers' revenues in $\overline{P}$ are lower than in $\widehat{P}$).

Finally, notice that entrepreneurs' revenues, evaluated along the isocline $\dot{K} = 0$, are proportional to

$$K^{1-\alpha}(\overline{N} - N)^{\alpha} = K^{1-\alpha}K^{\alpha} = K$$

This implies that their revenues in $\overline{P}$ are higher than in $\widetilde{P}$ (and, obviously, higher than in $\widehat{P}$, where $K = 0$). So, if condition (9) holds, then entrepreneurs and workers' revenues in $\widetilde{P}$ (coexistence of the two sectors) and $\overline{P}$ (specialization in the I-sector) are higher than in $\widehat{P}$ (specialization in the E-sector) and $\overline{P}$ Pareto-dominates $\widetilde{P}$ and $\widehat{P}$. If inequality (9) is reversed (in this case, the point $\widetilde{P}$ cannot be attractive), then workers' revenues in $\widehat{P}$ are higher than in $\overline{P}$, while the opposite holds for entrepreneurs' revenues.

6. Conclusions

The economic and ecological dynamics we analyzed takes place in a three-dimensional box $\mathcal{B} = (0, \overline{N})^2 \times (0, \overline{E})$ of the plane (K, N, E), where K is the capital stock, N the number of workers employed in the E-sector, E the stock of the environmental resource. The main results we reached, through local and global analysis techniques, can be summarized as follows.

1. One (and only one) stationary state $\widetilde{P} = (K, N, E) = \left(\overline{N} - \widetilde{N}, \widetilde{N}, 1\right) \in \mathcal{B}$ exists if and only if the carrying capacity $\overline{E}$ of the environmental resource lies in an interval whose extremes are determined by the size $\overline{N}$ of the population of workers and by the contribution to the environmental depletion, per unit of product, of the traditional and the industrial sector, measured respectively by the parameters p and q (that is $1 + \min\left(p\overline{N}, q\overline{N}\right) < \overline{E} < 1 + \max\left(p\overline{N}, q\overline{N}\right)$). In $\widetilde{P}$, both sectors of the economy coexist.

2. The stability properties of the stationary state $\widetilde{P}$ depend on whether the contribution to environmental depletion, per unit of product, is higher in the I-sector (in which case the stationary state is either a saddle with a two-dimensional stable manifold or a repeller), or in the E-sector (in which case the stationary state is either an attractor or a saddle with a one-dimensional stable manifold).

3. In case $p < q$ (the contribution to environmental depletion is higher in the I-sector), we proved the possible existence of three *regimes*, depending on the initial conditions of the dynamics in the box $\mathcal{B}$. Precisely: *(a)* there exist initial conditions of the economy (i.e. of the variables K, N, E), belonging to an open subset of the box $\mathcal{B}$, from which the trajectories reach in a finite time the side $E = 0$ of the box $\mathcal{B}$ (that is, the ecological-economic dynamics leads to total depletion of the environmental resources); *(b)* there exists an open connected subset of the box $\mathcal{B}$ whose trajectories tend, as $t \to +\infty$, to the boundary point $\widehat{P} = (K, N, E) = \left(0, \overline{N}, \widehat{E}\right)$, where $\widehat{E} = \overline{E} - p\overline{N}$ and the I-sector disappears ($K = 0$); *(c)* there exist initial conditions of the variables K, N, E, belonging to an open subset of the box $\mathcal{B}$, from which the trajectories converge to the specialized stationary state $P_2 = (K, N, E) = \left(\overline{N}, 0, E_2\right)$, with $0 < E_2 < 1$. If $2 > 1 + q\overline{N} > \overline{E} > 1 + p\overline{N}$ holds, then all the regimes *(a)*, *(b)* and *(c)* are simultaneously present.

4. In case $p > q$ (the contribution to environmental depletion is higher in the E-sector), two regimes can be observed: there exist initial conditions of the economy, belonging to an open subset of the box $\mathcal{B}$, from which the trajectories reach in a finite time the side $E = 0$ of the box $\mathcal{B}$ (regime *(a)*, in point 3); furthermore, there exist trajectories in the box $\mathcal{B}$ tending (asymptotically) to the stationary state $\widetilde{P}$, where both sectors coexist, and such trajectories fill an open region if $\widetilde{P}$ is attracting. The coexistence of the two sectors of the economy, in an attractive stationary state, is possible only in this context (i.e., if $p > q$).

The limit boundary point $\widehat{P} = (K, N, E) = \left(0, \overline{N}, \widehat{E}\right)$, where $\widehat{E} = \overline{E} - p\overline{N}$ (see point 3), corresponds to the unique (attractive) stationary state $E = \widehat{E}$ of the one-sector dynamics (with $K = 0$ and $N = \overline{N}$), described by the Eq.(8), that would be observed in absence of the I-sector. The trajectories reaching in a finite time the side $E = 0$ of the box $\mathcal{B}$ (see the above points 3 and 4) approach eventually the point $\overline{P} = (K, N, E) = \left(\overline{N}, 0, 0\right)$, where the economy gets specialized in the I-sector.

Welfare analysis showed that:

(i) If $p > \left(\overline{E} - 1\right)/\overline{N}$ (i.e. if condition (9) is satisfied), then workers' revenues in $\widehat{P}$ are lower than in $\widetilde{P}$ and $\overline{P}$, vice-versa if $p < \left(\overline{E} - 1\right)/\overline{N}$. Under the assumption $p > \left(\overline{E} - 1\right)/\overline{N}$, the stationary state $\widetilde{P}$ exists only if $q < \left(\overline{E} - 1\right)/\overline{N}$, and therefore $p > q$. This implies that the stationary state $\widetilde{P}$, where both sectors coexist, can be attractive only if workers' revenues in $\widetilde{P}$ are higher than in $\widehat{P}$ (see the above point 4). Differently from the stationary state $\widetilde{P}$, the state $\overline{P}$ is always attractive for the trajectories belonging to an open region of the box $\mathcal{B}$ even if $p < \left(\overline{E} - 1\right)/\overline{N}$ (i.e. if worker' revenues in $\overline{P}$ are lower than in $\widehat{P}$).

(ii) Entrepreneurs' revenues, evaluated at the possible limit points, are positively proportional to capital accumulation K, and then their revenues in $\overline{P}$ are higher than in $\widetilde{P}$ (and, obviously, higher that in $\widehat{P}$, where $K = 0$).

According to points (i) and (ii), if condition (9) holds, then entrepreneurs and workers' revenues in $\widetilde{P}$ (coexistence of the two sectors) and $\overline{P}$ (specialization in the I-sector) are higher than in $\widehat{P}$ (specialization in the E-sector). Furthermore, $\overline{P}$ Pareto-dominates both $\widetilde{P}$ and $\widehat{P}$. If the reverse of condition (9) holds (remember that, in this case, the point $\widetilde{P}$ cannot be attractive), then workers' revenues in $\widehat{P}$ are higher than in $\overline{P}$, while the opposite holds for entrepreneurs' revenues. In this zero-sum game scenario, the structural change driving the economy towards $\overline{P}$ (where a complete specialization in the I-sector takes place) is welfare reducing for workers and welfare increasing for entrepreneurs.

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GREEN LICENSES AND ENVIRONMENTAL CORRUPTION: A RANDOM MATCHING MODEL

Abstract

This paper studies environmental corruption via a random matching evolutionary game between a population of firms and a population of bureaucrats in order to release a "green" license. A firm obtains the license if the bureaucrat checks that it complies with environmental regulations, otherwise it is sanctioned. In this model there are two types of bureaucrats (honest and dishonest), two types of firms (compliant and not compliant), and two types of crimes (corruption and extortion). Corruption is when a dishonest bureaucrat accepts a bribe from a not compliant firm, while extortion is when a dishonest bureaucrat extorts a bribe from a compliant firm. When there is no dominance of strategies, we obtain two bistable regimes, in which two attractive stationary states exist, and two regimes with an internal stable equilibrium, which corresponds to the mixed strategy Nash equilibrium of the one-shot static game, surrounded by closed trajectories. Moreover, from the comparative statics of the last two dynamic regimes emerges that policy instruments can help the Public Administration to reduce both corruption and extortion, though increasing sanctions, probability of being sanctioned and inspection effort do not always get the desired results.

JEL classification: C73, D21, D73, K42, Q52.
Keywords: bureaucratic corruption, evolutionary game, environmental regulations, economics of crime.

1. Introduction

Recently, the media have focused a lot of attention on compliance with environmental regulations by industrial enterprises and on inspection by bureaucrats after the so called *Volkswagen scandal*[1]. However, the attention of economists on these issues is not new. Several studies have analysed the negative effects of corruption on environmental policy and on environmental degradation. The first strand has examined the effects of bureaucracy and lobbying groups on environmental policy (Lopez and Mitra, 2000; Damania et al., 2003; Fredriksson et al., 2003; Fredriksson and Svensson, 2003; Cole et al., 2006). The second strand, instead, has investigated the effects of corruption on the shape of the *Environmental Kuznets Curve*[2] (Welsch, 2004; Cole, 2007; Leitao, 2010).

Generally, not only in the specific case of environmental corruption, bribery is considered as an evil, particularly for economic development[3], and, not surprisingly, the most corrupt

[1]In September 2015, the German automaker Volkswagen Group has received a notice of violation of the Clean Air Act from the United States Environmental Protection Agency (EPA). It was found that the enterprise had used a software applied to some diesel engines to activate certain emissions controls only during laboratory emissions testing. The software caused the compliance with U.S. environmental standards for nitrogen oxide (NO_x) output during laboratory tests, but produce up to 40 times higher NO_x output in real-world driving. See, for further details, both links:
`http://www.ft.com/intl/vw-emissions-scandal`;
`http://www.nytimes.com/interactive/2015/business/international/vw-diesel-emissions-scandal-explained.html?_r=0`.

[2]The EKC is an inverted U-shape relationship between environmental degradation and income. This means that environmental degradation increases at early stages of economic development and decreases when income exceeds a certain level. For further details, see Grossman and Krueger (1991), Shafik and Bandyopadhyay (1992), Panayotou (1993), and Borghesi (2001). The name was coined due to its similarity to the work of Kuznets (1955).

[3]Although some authors argue that bribes may lead firms to allocate their

countries have low income level (Svensson, 2005). This link is usually explained by the literature via the role of institutions, since corruption is considered as an example of bad institutions (Lipset, 1960; Demsetz, 1967; Treisman, 2000; Glaeser et al., 2004; Acemoglu et al., 2012).

In *game theory*[4] corruption, and crime deterrence in general, is modelled as a strategic interaction between at least two players. An example of this kind of models[5] are the *inspection games*, where one player, a policemen, must decide whether to inspect the other player, who in turn must decide whether to infringe a regulation (Tsebelis, 1989, 1990). According to Holler (1993), the inspection games have no Nash equilibrium in pure strategies since both players have the possibility to improve their payoff values by choosing an alternative strategy, given the strategy of the other player. Therefore, there is a single mixed strategy Nash equilibrium (see Nash, 1951) which has counter-intuitive comparative statics properties (Andreozzi, 2004). In fact, these models show how increasing sanctions and probabilities of being discovered, for the player who must decide whether infringe a norm, have no effects on the law enforcement. The only way to reduce crimes is to increase the inspection incentives for the policemen.

In this paper we adopt the framework of the inspection games using an evolutionary context (see, e.g., Andreozzi, 2002). The *evolutionary game theory* supposes that large populations of players with bounded rationality learn, imitate, and adopt the relatively more rewarding strategies. We believe, according to Cressman et al. (1998), that this context seems particularly appealing for the study of crime, where, for example, the influence of good role models in society is often stressed as an important factor for reducing crime.

We propose a random-matching evolutionary game between a population of firms and a population of bureaucrats[6]. In each instant of time, there is a large number of random pairwise encounters between firms and bureaucrats. In each encounter a bureaucrat checks the compliance with environmental regulations by a firm. When the environmental laws are respected, the firm obtains a "green" license, like a sticker. Otherwise, the firm receives a penalty. There are two kinds of firms, compliant and not compliant, and two kinds of bureaucrats, honest and dishonest. Moreover, we suppose the existence of two crimes: corruption and extortion. Corruption is when a dishonest bureaucrat accepts a bribe from a not compliant firm, while extortion is when a dishonest bureaucrat extorts a bribe from a compliant firm. Finally, we introduce the existence of an anti-corruption agency that monitors the behaviour of bureaucrats and firms.

When there is no dominance of strategies, we obtain four dynamic regimes, two are bistable and two with an internal stable equilibrium. In the first two dynamic regimes, similar economies (same rules, same sanctions, etc.) can converge to different stationary states, it depends on the initial shares of the strategies in the two populations. In the other two dynamic regimes, instead, the shares of the two populations oscillate around an internal stable equilibrium. This implies that similar economies can lie on different trajectories and, therefore, can have different long-run behaviours.

The analysis of comparative statics of the dynamic regimes with an internal stable equilibrium shows that policy instruments (sanctions, probability of being discovered by the anti-corruption agency and inspection effort) can reduce both corruption and extortion. The effectiveness of policy instruments depends on initial shares of strategies in the two populations and if adopting a strategy represents an evolutionary advantage.

The wealth of dynamics and the results of comparative statics make our model different from other inspection games both static and evolutionary. In fact, our game has Nash equilibria both in pure and in mixed strategies. The inspection games, instead, as described above, have a single mixed strategy Nash equilibrium, and, therefore, only dynamic regimes with oscillating trajectories. Moreover, with regard to the properties of comparative statics, in our model also sanctions and probability of being discovered can reduce crimes, not only the effort of policeman as in the inspection games.

The paper is organized as follows. Sections 2 and 3 describe the model, Section 4 shows the basic results, Section 5 deals with the dominance relationship between strategies, Section 6 analyses the dynamic regimes, Section 7 contains the effectiveness of policy instruments, and Section 8 concludes.

2. The model

Let us assume that in each instant of time $t \in [0, +\infty)$, a randomly-chosen firm plays a game with a randomly-chosen bureaucrat. Each firm has to choose *ex ante* between two possible strategies: (C) to comply with environmental regulations and to support the compliance cost C_C, or (NC) not to comply with environmental laws. Each bureaucrat has to choose *ex ante* between two possible strategies: (H) to be honest and to do her job properly, or (D) to be dishonest and to accept a bribe from a not compliant firm or to claim a bribe from a compliant firm. Tables 1 and 2 describe the firm's and the bureaucrat's payoff matrix, respectively.

resources more efficiently, in an economy afflicted by slow bureaucracy and rigid laws (see, e.g., Leff, 1964; Huntington, 1968).

[4] A not game theoretic approach to study crime deterrence is the *decision theory* that involves only one actor (see, for further details, Becker, 1968; Garoupa, 1997; Polinsky and Shavell, 2000). We do not adopt this approach since corruption is an agreement between at least two actors: a player that decides to infringe a regulation and, for not being sanctioned, offers a bribe to another player who, at same time, accepts the bribe and decides to not sanction the other player.

[5] Another way to model corruption adopting a game theoretic framework is the *principal-agent theory*, where the crime occurs due to the asymmetry between the principal, usually the Public Administration, and the agent, a public official (see, for further details, Bardhan, 1997; Di Gioacchino and Franzini, 2008). We do not use this setting since in our model the analysis is focused on the encounters, and not in the asymmetry, between players.

[6] Differently from other evolutionary models on corruption, as in Antoci and Sacco (1995, 2002), that study the dynamics of only one population.

Table 1
Payoffs of strategies C and NC

	H	D
C	$\pi_C^H = -C_C$	$\pi_C^D = -C_C - b_e + \theta\eta$
NC	$\pi_{NC}^H = -ps_1$	$\pi_{NC}^D = -b_c - \theta s_2$

If a Compliant firm encounters an Honest bureaucrat, it obtains the green licence; while, if it encounters a Dishonest bureaucrat it may be victim of extortion, and, therefore, the Compliant firm has to pay an extortion bribe (b_e). However, this crime could be discovered by the anti-corruption agency with probability θ, in that case, the public administration will compensate the extortion bribe (η). If a Not Compliant firm encounters an Honest bureaucrat, it will be sanctioned (s_1) for not being compliant with a probability p, that depends if the honest bureaucrat checks well. Otherwise, if a Not Compliant firm encounters a Dishonest bureaucrat, it will pay a corruption bribe (b_c) and will take the risk of being sanctioned (s_2) for the corruption crime and for not being compliant by the anti-corruption agency with probability θ. We suppose that $C_C > 0$, $b_c > b_e \geq 0$, $\eta \geq 0$, $s_2 \geq s_1 > 0$, $1 > \theta > 0$, $1 > p > 0$.

Table 2
Payoffs of strategies H and D

	C	NC
H	$\pi_H^C = w$	$\pi_H^{NC} = w$
D	$\pi_D^C = w + b_e - \theta\sigma_1$	$\pi_D^{NC} = w + b_c - \theta\sigma_2$

If a bureaucrat is Honest, she will obtain the wage (w), regardless of what kind of firms she will encounter. While, if a Dishonest bureaucrat encounters a Compliant firm, she will obtain, in addition to wage, an extortion bribe and will take the risk of being sanctioned (σ_1) by the anti-corruption agency with probability θ. Otherwise, if a Dishonest bureaucrat encounters a Not Compliant firm, she will obtain, in addition to wage, the corruption bribe and will take the risk of being sanctioned (σ_2) by the anti-corruption agency with probability θ. We suppose that $w > 0$, $b_c > b_e \geq 0$, $\sigma_1 \geq 0$, $\sigma_2 > 0$, $1 > \theta > 0$.

3. The Dynamics of the game

Let $c(t) \in [0, 1]$ represent the share of firms adopting strategy C and let $h(t) \in [0, 1]$ represent the share of bureaucrats adopting strategy H, at time t. Consequently, $1-c(t)$ and $1-h(t)$ represent, respectively, the shares of firms playing strategy NC and of bureaucrats playing strategy D.

The firms' expected payoffs from playing strategies C and NC are:

$$\Pi_C(h) = \pi_C^H \cdot h + \pi_C^D \cdot (1 - h)$$

$$\Pi_{NC}(h) = \pi_{NC}^H \cdot h + \pi_{NC}^D \cdot (1 - h)$$

where h and $1 - h$ represent the probabilities that a firm is matched with a bureaucrat who plays, respectively, strategy H or D.

The bureaucrats' expected payoffs from playing strategies H and D are:

$$\Pi_H(c) = \pi_H^C \cdot c + \pi_H^{NC} \cdot (1 - c)$$

$$\Pi_D(c) = \pi_D^C \cdot c + \pi_D^{NC} \cdot (1 - c)$$

where c and $1-c$ represent the probabilities that a bureaucrat is matched with a firm who plays, respectively, strategy C or NC.

The average payoffs in the population of firms and of bureaucrats are:

$$\overline{\Pi}_F = c \cdot \Pi_C(h) + (1 - c) \cdot \Pi_{NC}(h)$$

$$\overline{\Pi}_B = h \cdot \Pi_H(c) + (1 - h) \cdot \Pi_D(c)$$

We assume that the time evolution of c and h is described by the standard replicator dynamics, a learning-by-imitation model of evolution widely used in economics (see, among others, Hofbauer and Sigmund, 1988; Björnerstedt and Weibull, 1993; Weibull, 1997; Schlag, 1998). The replicator dynamics postulates that players are bundled rational and update their choices by adopting the relatively more rewarding behaviour that emerges from available observations of others' behaviours. The shares c and h will increase (decrease) the more, the higher (lower) their payoff differential with respect to the population average payoff. Accordingly, in our two-strategy context the dynamic system is:

$$\begin{aligned} \dot{c} &= c[\Pi_C(h) - \overline{\Pi}_F] = c(1 - c)[\Pi_C(h) - \Pi_{NC}(h)] \\ \dot{h} &= h[\Pi_H(c) - \overline{\Pi}_B] = h(1 - h)[\Pi_H(c) - \Pi_D(c)] \end{aligned} \tag{1}$$

where $\dot{c}$ and $\dot{h}$ represent the time derivatives dc/dt and dh/dt of the shares c and h, respectively. The factors $c(1 - c)$ and $h(1 - h)$ are always non-negative, so the signs of $\dot{c}$ and $\dot{h}$ will depend respectively on the signs of the payoff differentials.

4. Basic results

The system (1) is defined in the unit square S:

$$S = \left\{(c, h) \in R^2 : 0 \leq c \leq 1, 0 \leq h \leq 1\right\}$$

The graphs of the payoff differentials $\Pi_C(h) - \Pi_{NC}(h)$ and $\Pi_H(c) - \Pi_D(c)$ are shown in Figs. 1(a) and 1(b). Strategy C (H) is dominant when the graph of $\Pi_C(h) - \Pi_{NC}(h)$ ($\Pi_H(c) - \Pi_D(c)$) lies entirely above the c-axis (h-axis) in the interval $[0, 1]$. Conversely, strategy NC (D) is dominant when it lies entirely below the c-axis (h-axis) in the interval $[0, 1]$. Finally, if it intersects the interior of the interval $[0,1]$, then no dominant strategy exists. Figs. 1(a) and 1(b) show the possible cases that can be observed.

The payoff differentials can be written as follows:

$$\begin{aligned}&\Pi_C(h)-\Pi_{NC}(h)=\\&b_c-b_e+\theta\,(\eta+s_2)-C_C\\&-[b_c-b_e+\theta\,(\eta+s_2)-ps_1]\,h\end{aligned}\tag{2}$$

$$\begin{aligned}&\Pi_H(c)-\Pi_D(c)=\\&\theta\sigma_2-b_c+[b_c-b_e+\theta\,(\sigma_1-\sigma_2)]\,c\end{aligned}\tag{3}$$

According to the dynamic system (1), $\dot{c}=0$ holds if either $c=0,1$ or if the value of the share h is such that $\Pi_C(h)-\Pi_{NC}(h)=0$, that is:

$$h=\bar{h}:=\frac{b_c-b_e+\theta\,(\eta+s_2)-C_C}{b_c-b_e+\theta\,(\eta+s_2)-ps_1}\tag{4}$$

Considering Eq. (2), we can distinguish between two cases.

Case (a):

$$b_c-b_e+\theta\,(\eta+s_2)-ps_1<0,\ \textit{that is},\ s_1>\bar{s}_1\tag{5}$$

where $\bar{s}_1:=\frac{b_c-b_e+\theta\,(\eta+s_2)}{p}$.

Case (b):

$$b_c-b_e+\theta\,(\eta+s_2)-ps_1>0,\textit{that is},\ s_1<\bar{s}_1\tag{6}$$

The graph of the payoff differential $\Pi_C(h)-\Pi_{NC}(h)$ is a line with positive slope (i.e., $\Pi_C(h)-\Pi_{NC}(h)$ is an increasing function of h) in Case (a), while it is a line with negative slope (i.e., $\Pi_C(h)-\Pi_{NC}(h)$ is a decreasing function of h) in Case (b). This implies that, in the context of Case (a), the strategy C becomes relatively more remunerative (compared to the strategy NC) when the share of Honest bureaucrats h increases; the opposite occurs in Case (b).

Analogously, according to the dynamic system (1), $\dot{h}=0$ holds if either $h=0,1$ or if the value of the share c is such that $\Pi_H(c)-\Pi_D(c)=0$, that is:

$$c=\bar{c}:=\frac{b_c-\theta\sigma_2}{b_c-b_e+\theta\,(\sigma_1-\sigma_2)}\tag{7}$$

Taking into account Eq. (3), we can distinguish between two cases.

Case (c):

$$b_c-b_e+\theta\,(\sigma_1-\sigma_2)>0,\ \textit{that is},\ \sigma_1>\bar{\sigma}_1\tag{8}$$

where $\bar{\sigma}_1:=\sigma_2-\frac{b_c-b_e}{\theta}$.

Case (d):

$$b_c-b_e+\theta\,(\sigma_1-\sigma_2)<0,\ \textit{that is},\ \sigma_1<\bar{\sigma}_1\tag{9}$$

The graph of the payoff differential $\Pi_H(c)-\Pi_D(c)$ is a line with positive slope (i.e., $\Pi_H(c)-\Pi_D(c)$ is an increasing function of c) in Case (c), while it is a line with negative slope (i.e., $\Pi_H(c)-\Pi_D(c)$ is a decreasing function of c) in Case (d). This implies that, in the context of Case (c), the strategy H becomes relatively more remunerative (compared to the strategy D) when the share of Compliant firms c increases; the opposite occurs in Case (d).

The four vertices of S, that is $(c,h)=(0,0)(1,0)(0,1)(1,1)$, are always stationary states of the dynamic system (1). In these stationary states, the populations of firms and bureaucrats play only one strategy. In $(1,1)$ all firms play C and all bureaucrats play H; in $(0,0)$ all firms play NC and all bureaucrats play D, and so on.

Another stationary state of the system (1) is the intersection point $(\bar{c},\bar{h})$ of the straight lines (4) and (7), when it belongs to the interior of the square S, that is when $0<\bar{c}<1$ and $0<\bar{h}<1$. At the stationary state $(\bar{c},\bar{h})$ all the strategies C, NC, H and D coexist.

Finally, all the points belonging to the side of S with $h=0$ (respectively, $h=1$) are stationary states in the case in which $\bar{h}=0$ (respectively, $\bar{h}=1$) holds. Analogously, all the points belonging to the side of S with $c=0$ (respectively, $c=1$) are stationary states if $\bar{c}=0$ (respectively, $\bar{c}=1$) holds.

5. Dominance relationship

In this section, we give the conditions under which a given strategy does not dominate the alternative one, in each population of players. Proposition 1 refers to Case (a) and Case (b), while Proposition 2 refers to Case (c) and Case (d).

Proposition 1. *In Case* (a) *(see (5)),* $\Pi_C(h)-\Pi_{NC}(h)$ *is strictly increasing in h (see Fig. 1(a)), and there is no dominance of strategies if:*

$$s_2<\bar{s}_2:=\frac{C_C+b_e-b_c}{\theta}-\eta\quad\textit{and}\quad s_1>\frac{C_C}{p}\tag{10}$$

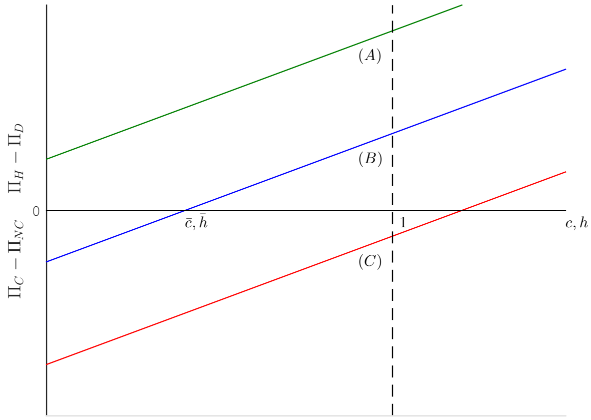
In Case (b) *(see (6)),* $\Pi_C(h)-\Pi_{NC}(h)$ *is strictly decreasing in h (see Fig. 1(b)), and there is no dominance of strategies if:*

$$s_2>\bar{s}_2\quad\textit{and}\quad s_1<\frac{C_C}{p}\tag{11}$$

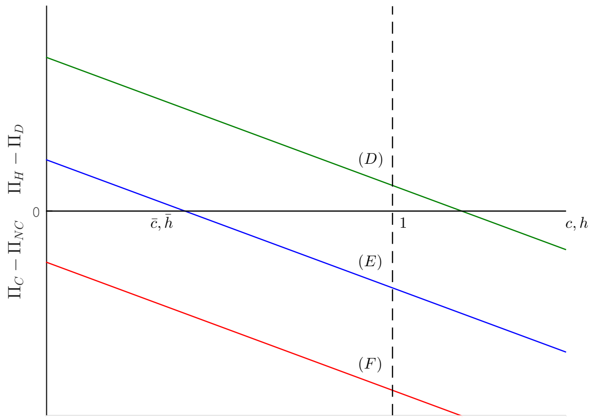
Proof. See Appendix. □

In Case (a), if condition (10) holds, then no strategy dominates the other one, and the graph of the payoff differential $\Pi_C(h)-\Pi_{NC}(h)$ intersects the h-axis at $h=\bar{h}\in(0,1)$ (see (4)): for $h>\bar{h}$ (respectively, $h<\bar{h}$), it holds $\Pi_C(h)-\Pi_{NC}(h)>0$ (respectively, $\Pi_C(h)-\Pi_{NC}(h)<0$).

On the contrary, in Case (b), if condition (11) holds, then no strategy dominates the other one, and the graph of the payoff differential $\Pi_C(h)-\Pi_{NC}(h)$ intersects the h-axis at $h=\bar{h}\in(0,1)$ (see (4)): for $h>\bar{h}$ (respectively, $h<\bar{h}$), it holds $\Pi_C(h)-\Pi_{NC}(h)<0$ (respectively, $\Pi_C(h)-\Pi_{NC}(h)>0$).



(a) Increasing payoff differentials.



(b) Decreasing payoff differentials.

Fig. 1. Dominance of strategies.
Legend: line (A) dominant strategy C or H, line (B) no dominant strategy, line (C) dominant strategy NC or D; line (D) dominant strategy C or H, line (E) no dominant strategy, line (F) dominant strategy NC or D.

Proposition 2. *In Case* (c) *(see (8)),* $\Pi_C(h) - \Pi_D(h)$ *is strictly increasing (see Fig. 1(a)), and there is no dominance of strategies if:*

$$\sigma_1 > \frac{b_e}{\theta} \quad \text{and} \quad \sigma_2 < \frac{b_c}{\theta} \tag{12}$$

In Case (d) *(see (9)),* $\Pi_C(h) - \Pi_D(h)$ *is strictly decreasing (see Fig. 1(b)), and there is no dominance of strategies if:*

$$\sigma_1 < \frac{b_e}{\theta} \quad \text{and} \quad \sigma_2 > \frac{b_c}{\theta} \tag{13}$$

Proof. See Appendix. □

In Case (c) if condition (12) holds, then no strategy dominates the other one, and the graph of the payoff differential $\Pi_C(h) - \Pi_D(h)$ intersects the c-axis at $c = \bar{c} \in (0, 1)$ (see (7)): for $c > \bar{c}$ (respectively, $c < \bar{c}$), it holds $\Pi_C(h) - \Pi_D(h) > 0$ (respectively, $\Pi_C(h) - \Pi_D(h) < 0$).

On the contrary, in Case (d), if condition (13) holds, then no strategy dominates the other one, and the graph of the payoff differential $\Pi_C(h) - \Pi_D(h)$ intersects the c-axis at $c = \bar{c} \in (0, 1)$ (see (7)): for $c > \bar{c}$ (respectively, $c < \bar{c}$), it holds $\Pi_C(h) - \Pi_D(h) < 0$ (respectively, $\Pi_C(h) - \Pi_D(h) > 0$).

6. Dynamic regimes

When a dominated strategy exists, then the share of agents adopting it decreases monotonically over time and approaches (asymptotically) the value 0; therefore, in such a context, the dynamics is very simple. The most interesting dynamic regimes that may be observed correspond to the cases in which no strategy dominates the other one, in each population of agents. In such cases, the internal stationary state $(\bar{c}, \bar{h})$ exists. The following subsections illustrate such dynamic regimes. The classification of regimes we are going to give refers to all the possible contexts that can occur[7]:

1) the context in which the parameter values satisfy the conditions (5) and (8) characterizing, respectively, the Case (a) (relatively to the population of firms) and the Case (c) (relatively to the population of bureaucrats);
2) the context in which the parameter values satisfy the conditions (6) and (9) characterizing, respectively, the Case (b) and the Case (d);
3) the context in which the parameter values satisfy the conditions (6) and (8) characterizing, respectively, the Case (b) and the Case (c);
4) the context in which the parameter values satisfy the conditions (5) and (9) characterizing, respectively, the Case (a) and the Case (d).

The proofs of the following propositions are straightforward, since the dynamic regimes that may be observed under replicator equations, in a context with two populations and two strategies, have been completely classified (see Hofbauer and Sigmund, 1988; Weibull, 1997).

6.1. Dynamic regime in the context of Cases (a) *and* (c)

This context is characterized by the conditions (see (5) and (8)):

$$s_1 > \frac{b_c - b_e + \theta\,(\eta + s_2)}{p} \tag{14}$$

$$\sigma_1 > \sigma_2 - \frac{b_c - b_e}{\theta} \tag{15}$$

[7] For simplicity, we do not consider the non-robust cases with $s_1 = \bar{s}_1$ and/or $\sigma_1 = \bar{\sigma}_1$

which, as shown in Section 5, imply that the payoff differentials $\Pi_C(h) - \Pi_{NC}(h)$ and $\Pi_H(c) - \Pi_D(c)$ are strictly increasing in h and c, respectively. Furthermore, in such a context, no dominance relationship (between the two available strategies) exists, in each population, if the following conditions are satified (see (10) and (12)):

$$s_2 < \frac{C_C + b_e - b_c}{\theta} - \eta \quad \textit{and} \quad s_1 > \frac{C_C}{p} \tag{16}$$

$$\sigma_1 > \frac{b_e}{\theta} \quad \textit{and} \quad \sigma_2 < \frac{b_c}{\theta} \tag{17}$$

Notice that if condition (17) holds, then also condition (15) holds. If conditions (14), (16) and (17) are satisfied, then a "bistable" dynamic regime is observed, described by the following proposition:

Proposition 3. *If conditions (14), (16) and (17) are satisfied, then the stationary states* $(c, h) = (0, 0)$ *and* $(c, h) = (1, 1)$ *are sinks (i.e., locally attractive), the stationary states* $(c, h) = (1, 0)$ *and* $(c, h) = (0, 1)$ *are sources (i.e., repulsive) and the stationary state* $(c, h) = (\bar{c}, \bar{h})$*, in the interior of the square S, is a saddle point. The basins of attraction of* $(0, 0)$ *and* $(1, 1)$ *are separated by the stable branch of* $(\bar{c}, \bar{h})$ *(see Fig. 2(a)).*

In such a context, strategy C is the best reply when the share of Honest bureaucrats is high, while strategy NC is the best reply when the share of Honest bureaucrats is low. With regard to bureaucrats' behaviour, instead, strategy H is the best reply when the share of Compliant firms is high, while strategy D is the best reply when the share of the Compliant firms is low. This occurs because:

- if the share of Honest bureaucrats is high, for firms is more rewarding to adopt strategy C, since s_1 and p are relatively high, while C_C is relatively low ($s_1 > C_C/p$);
- if the share of Honest bureaucrats is low, for firms is more rewarding to adopt strategy NC, since s_2, b_c, θ and η are relatively low, while C_C and b_e are relatively high ($s_2 < (C_C + b_e - b_c)/\theta - \eta$);
- if the share of Compliant firms is high, for bureaucrats is more rewarding to adopt strategy H, since σ_1 and θ are relatively high, while b_e is relatively low ($\sigma_1 > b_e/\theta$);
- if the share of Compliant firms is low, for bureaucrats is more rewarding to adopt strategy D, since σ_2 and θ are relatively low, while b_c is relatively high ($\sigma_2 < b_c/\theta$).

An economy can converge to the "vicious" stationary state $(0, 0)$ if both initial shares of Compliant firms and Honest bureaucrats are relatively low. In point $(0, 0)$ corruption is the only existing crime. On the contrary, an economy can converge to the "virtuous" stationary state $(1, 1)$ if both initial shares of Compliant firms and Honest bureaucrats are relatively high. In point $(1, 1)$ there are no crimes.

6.2. *Dynamic regime in the context of Cases* (b) *and* (d)

This context is characterized by the conditions (see (6) and (9)):

$$s_1 < \frac{b_c - b_e + \theta\,(\eta + s_2)}{p} \tag{18}$$

$$\sigma_1 < \sigma_2 - \frac{b_c - b_e}{\theta} \tag{19}$$

which imply that the payoff differentials $\Pi_C(h) - \Pi_{NC}(h)$ and $\Pi_H(c) - \Pi_D(c)$ are strictly decreasing in h and c, respectively. Furthermore, in such a context, no dominance relationship exists, in each population, if the following conditions are satified (see (11)) and (13)):

$$s_2 > \frac{C_C + b_e - b_c}{\theta} - \eta \quad \textit{and} \quad s_1 < \frac{C_C}{p} \tag{20}$$

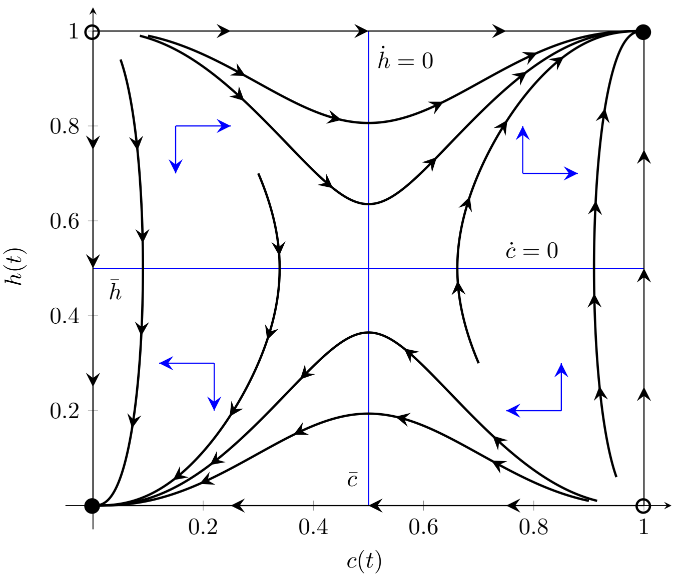
$$\sigma_1 < \frac{b_e}{\theta} \quad \textit{and} \quad \sigma_2 > \frac{b_c}{\theta} \tag{21}$$

Notice that if condition (21) holds, then also condition (19) holds. If conditions (18), (20) and (21) are satisfied, the bistable regime described by the following proposition occurs:

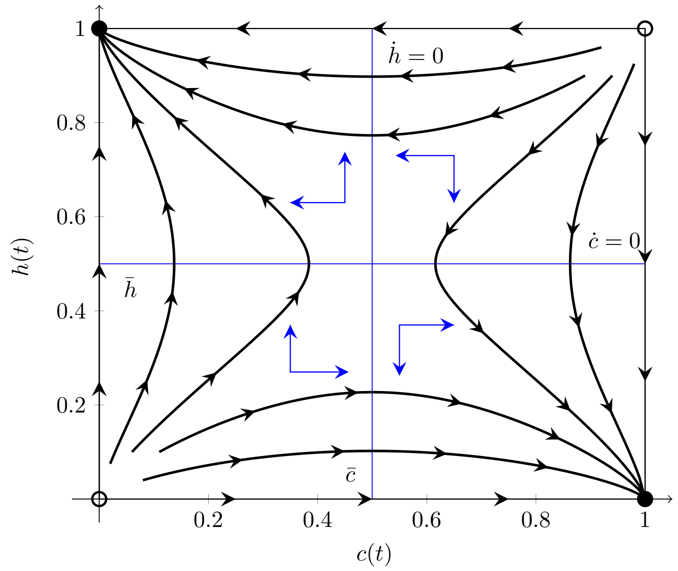
Proposition 4. *If the condition (18), (20) and (21) are satisfied, then the stationary states* $(c, h) = (0, 1)$ *and* $(c, h) = (1, 0)$ *are sinks, the stationary states* $(c, h) = (0, 0)$ *and* $(c, h) = (1, 1)$ *are sources and the stationary state* $(c, h) = (\bar{c}, \bar{h})$*, in the interior of the square S, is a saddle point. The basins of attraction of* $(0, 1)$ *and* $(1, 0)$ *are separated by the stable branch of* $(\bar{c}, \bar{h})$ *(see Fig. 2(b)).*

In such a context, strategy C is the best reply when the share of Honest bureaucrats is low, while strategy NC is the best reply when the share of Honest bureaucrats is high. With regard to bureaucrats' behaviour, instead, strategy H is the best reply when the share of Compliant firms is low, while strategy D is the best reply when the share of the Compliant firms is high. This occurs because:

- if the share of Honest bureaucrats is high, for firms is more rewarding to adopt strategy NC, since s_1 and p are relatively low, while C_C is relatively high ($s_1 < C_C/p$);
- if the share of Honest bureaucrats is low, for firms is more rewarding to adopt strategy C, since s_2, b_c, θ and η are relatively high, while C_C and b_e are relatively low ($s_2 > (C_C + b_e - b_c)/\theta - \eta$);
- if the share of Compliant firms is high, for bureaucrats is more rewarding to adopt strategy D, since σ_1 and θ are relatively low, while b_e is relatively high ($\sigma_1 < b_e/\theta$);
- if the share of Compliant firms is low, for bureaucrats is more rewarding adopt strategy H, since σ_2 and θ are relatively high, while b_c is relatively low ($\sigma_2 > b_c/\theta$).



(a) Cases (a) and (c).



(b) Cases (b) and (d).

Fig. 2. Path-dependent dynamics.
Legend: • *attractors,* ∘ *repellors.*

An economy can converge to the stationary state $(0, 1)$ if the initial share of Compliant firms is relatively low and the initial share of Honest bureaucrats is relatively high. In point $(0, 1)$ there are no crimes. On the contrary, an economy can converge to the stationary state $(1, 0)$ if the initial share of Compliant firms is relatively high and the initial share of Honest bureaucrats is relatively low. In point $(1, 0)$ extortion is the only existing crime.

The states $(c, h) = (0, 0),\ (0, 1),\ (1, 0),\ (1, 1)$, when locally attractive, as in Proposition 3 and in Proposition 4, are Nash equilibria. This finding follows from standard results in evolutionary game theory (see, e.g., Weibull, 1997). We can interpret Nash equilibria as social conventions, that is, as customary and expected states of things in which no single individual has an incentive to modify her choices if the others do not modify theirs.

6.3. Dynamic regime in the context of Cases (b) *and* (c)

This context is characterized by the following conditions (see (6) and (8)):

$$s_1 < \frac{b_c - b_e + \theta\,(\eta + s_2)}{p} \tag{22}$$

$$\sigma_1 > \sigma_2 - \frac{b_c - b_e}{\theta} \tag{23}$$

which imply that the payoff differentials $\Pi_C(h) - \Pi_{NC}(h)$ and $\Pi_H(c) - \Pi_D(c)$ are, respectively, strictly decreasing in h and strictly increasing in c. In such a context, no dominance relationship exists, in each population, if the following conditions are satified (see (11) and (12)):

$$s_2 > \frac{C_C + b_e - b_c}{\theta} - \eta \quad \textit{and} \quad s_1 < \frac{C_C}{p} \tag{24}$$

$$\sigma_1 > \frac{b_e}{\theta} \quad \textit{and} \quad \sigma_2 < \frac{b_c}{\theta} \tag{25}$$

Notice that if condition (25) holds, then also condition (23) holds. The following proposition illustrates the basic properties of the dynamic regime observed if conditions (22), (24) and (25) are satisfied:

Proposition 5. *If conditions (22), (24) and (25) are satisfied, then the stationary states* $(c, h) = (0, 1), (1, 0), (0, 0)$ *and* $(1, 1)$ *are saddle points, while the internal stationary state* $(c, h) = (\bar{c}, \bar{h})$ *is a (Lyapunov) stable stationary state surrounded by closed trajectories turning counter-clockwise (see Fig. 3(a)).*

In such a context, strategy C is the best reply when the share of Honest bureaucrats is low, while strategy NC is the best reply when the share of Honest bureaucrats is high. With regard to bureaucrats' behaviour, instead, strategy H is the best reply when the share of Compliant firms is high, while strategy D is the best reply when the share of the Compliant firms is low. This occurs because:

- if the share of Honest bureaucrats is high, for firms is more rewarding to adopt strategy NC, since s_1 and p are relatively low, while C_C is relatively high ($s_1 < C_C/p$);
- if the share of Honest bureaucrats is low, for firms is more rewarding to adopt strategy C, since s_2, b_c, θ and η are relatively high, while C_C and b_e are relatively low ($s_2 > (C_C + b_e - b_c)/\theta - \eta$);
- if the share of Compliant firms is high, for bureaucrats is more rewarding to adopt strategy H, since σ_1 and θ are relatively high, while b_e is relatively low ($\sigma_1 > b_e/\theta$);
- if the share of Compliant firms is low, for bureaucrats is more rewarding to adopt strategy D, since σ_2 and θ are relatively low, while b_c is relatively high ($\sigma_2 < b_c/\theta$).

This oscillatory dynamics could be explained using the *predator-prey* conceptual framework, where Dishonest bureaucrats are the predators and Not Compliant firms are the preys. In fact, starting from an initial condition in which the share of Not Complaint firms is high (many preys), for bureaucrats is more rewarding to adopt strategy D (the share of predators increases). However, the increase of Dishonest bureaucrats decreases the share of preys: the best reply for firms is to adopt strategy C. Few preys decreases the share of predators, since the best reply for bureaucrats is to adopt strategy H if the share of Compliant firms is high. A less share of predators allows the proliferation of the preys: for firms is more rewarding to adopt strategy NC if the share of Honest bureaucrats is high. And so on.

6.4. *Dynamic regime in the context of Cases* (a) *and* (d)

This context is characterized by the following conditions (see (5) and (9)):

$$s_1 > \frac{b_c - b_e + \theta\,(\eta + s_2)}{p} \tag{26}$$

$$\sigma_1 < \sigma_2 - \frac{b_c - b_e}{\theta} \tag{27}$$

which imply that the payoff differentials $\Pi_C(h) - \Pi_{NC}(h)$ and $\Pi_H(c) - \Pi_D(c)$ are, respectively, strictly increasing in h and strictly decreasing in c. Furthermore, in such a context, no dominance relationship exists, in each population, if the following conditions are satified (see (10) and (13)):

$$s_2 < \frac{C_C + b_e - b_c}{\theta} - \eta \quad \textit{and} \quad s_1 > \frac{C_C}{p} \tag{28}$$

$$\sigma_1 < \frac{b_e}{\theta} \quad \textit{and} \quad \sigma_2 > \frac{b_c}{\theta} \tag{29}$$

Notice that if condition (29) holds, then also condition (27) holds. The following proposition illustrates the basic properties of the dynamic regime observed if conditions (26), (28) and (29) are satisfied:

Proposition 6. *If conditions (26), (28) and (29) are satisfied, then the stationary states* $(c, h) = (0, 1), (1, 0), (0, 0)$ *and* $(1, 1)$ *are saddle points, while the internal stationary state* $(c, h) = (\bar{c}, \bar{h})$ *is a (Lyapunov) stable stationary state surrounded by closed trajectories turning clockwise (see Fig. 3(b)).*

In such a context, strategy C is the best reply when the share of Honest bureaucrats is high, while strategy NC is the best reply when the share of Honest bureaucrats is low. With regard to bureaucrats' behaviour, instead, strategy H is the best reply when the share of Compliant firms is low, while strategy D is the best reply when the share of the Compliant firms is high. This occurs because:

- if the share of Honest bureaucrats is high, for firms is more rewarding to adopt strategy C, since s_1 and p are relatively high, while C_C is relatively low ($s_1 > C_C/p$);
- if the share of Honest bureaucrats is low, for firms is more rewarding to adopt strategy NC, since s_2, b_c, θ and η are relatively low, while C_C and b_e are relatively high ($s_2 < (C_C + b_e - b_c)/\theta - \eta$);
- if the share of Compliant firms is high, for bureaucrats is more rewarding to adopt strategy D, since σ_1 and θ are relatively low, while b_e is relatively high ($\sigma_1 < b_e/\theta$);
- if the share of Compliant firms is low, for bureaucrats is more rewarding to adopt strategy H, since σ_2 and θ are relatively high , while b_c is relatively low($\sigma_2 > b_c/\theta$).

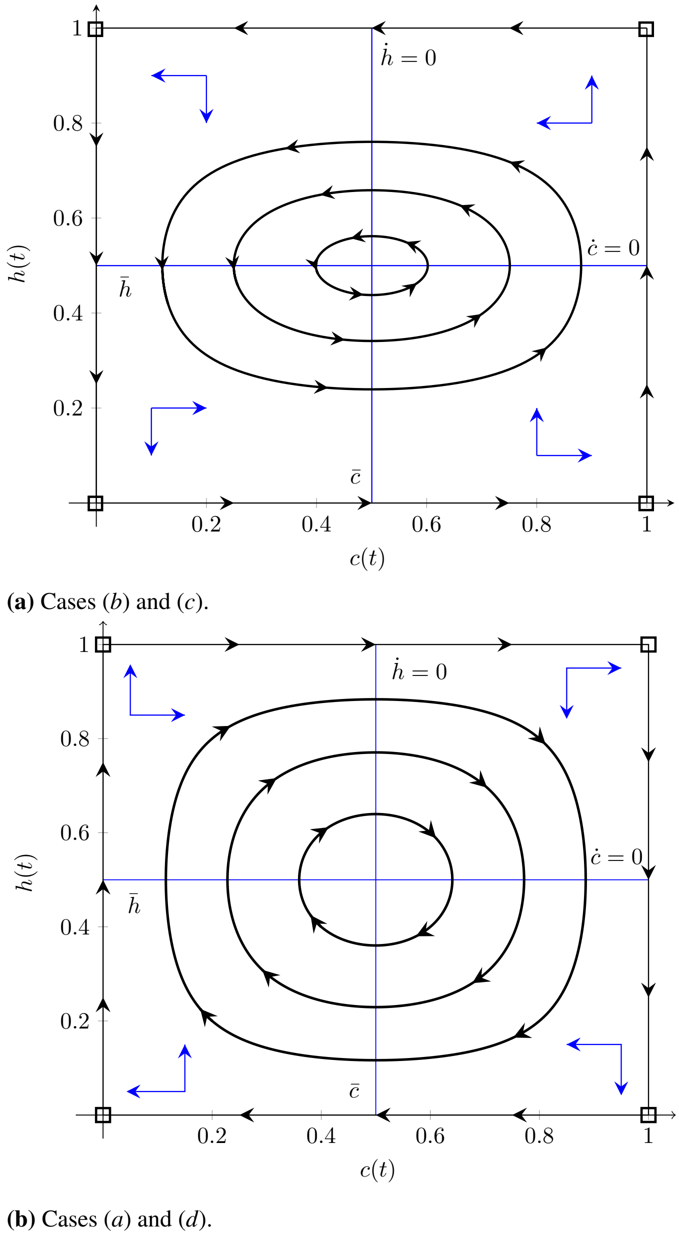


Fig. 3. Dynamics with a stable internal equilibrium.
Legend: □ *saddle points.*

This oscillatory dynamics could be explained using the *predator-prey* conceptual framework, where Dishonest bureaucrats are the predators and Compliant firms are the preys. In fact, starting from an initial condition in which the share of Compliant firms is high (many preys), for bureaucrats is more rewarding to adopt strategy D (the share of predators increases). However, the increase of Dishonest bureaucrats decreases the share of preys: the best reply for firms is to adopt strategy NC. Few preys decreases the share of predators, since the best reply for bureaucrats is to adopt strategy H if the share of Not Compliant is high. A less share of predators allows the proliferation of the preys: for firms is more rewarding to adopt strategy C if the share of Honest bureaucrats is high. And so on.

The state $(c, h) = (\bar{c}, \bar{h})$, in Propositions 5 and 6, corresponds to the mixed-strategy Nash equilibrium of the one shot (static) game defined by the payoff matrices shown in Tables 1 and 2. Accordingly, the firm chooses the strategy C with probability $\bar{c}$ and the bureaucrat chooses the strategy H with probability $\bar{h}$; therefore $(\bar{c}, \bar{h})$ would represent the equilibrium if all individuals were perfectly rational. We can also interpret $(\bar{c}, \bar{h})$ as the time-average values of the shares of compliant firms and honest bureaucrats, evaluated over the closed trajectories in Figs. 3(a) and 3(b). In this sense $(\bar{c}, \bar{h})$ can estimate the behaviour of economic agents in random observations over long time periods (see Weibull, 1997).

7. Comparative statics

This section studies the effects of variations in parameter values on the coordinates of the internal equilibrium $(\bar{c}, \bar{h})$. We focus our analysis on the two dynamic regimes where the internal equilibrium $(\bar{c}, \bar{h})$ is stable and, therefore, where the values of c and h represent also the average values along the closed trajectories (i.e., the context of Cases (*b*) and (*c*), and that of Cases (*a*) and (*d*)). In the other cases, the internal equilibrium is a saddle point and, therefore, is not stable. The following Proposition 7 gives the general comparative statics results concerning variations in the parameters of the model whose values can be influenced by the Public Administration's choices:

- p: probability that a honest bureaucrat check well if a firm is compliant;
- θ: probability of being discovered by the anti-corruption agency;
- s_1: firm's sanction for not being compliant;
- s_2: firm's sanction for corruption crime and for not being compliant;
- σ_1: bureaucrat's sanction for extortion crime;
- σ_2: bureaucrat's sanction for corruption crime.

The symbols $x \uparrow$, $x \downarrow$, and $x-$ mean, respectively, that the value of x increases, decreases or remains constant, where x may represent either $\bar{c}$ or $\bar{h}$, or a parameter of the model.

Proposition 7.

1) If $p \uparrow$, then $\bar{c} -$ always, while $\bar{h} \uparrow$ if and only if (iff) $s_2 > (C_C + b_e - b_c)/\theta - \eta$.

2) If $\theta \uparrow$, then $\bar{c} \uparrow$ iff $\sigma_2 b_e > \sigma_1 b_c$, while $\bar{h} \uparrow$ iff $s_1 < C_C/p$.

3) If $s_1 \uparrow$, then $\bar{c} -$ always, while $\bar{h} \uparrow$ iff $s_2 > (C_C + b_e - b_c)/\theta - \eta$.

4) If $s_2 \uparrow$, then $\bar{c} -$ always, while $\bar{h} \uparrow$ iff $s_1 < C_C/p$.

5) If $\sigma_1 \uparrow$, then $\bar{c} \uparrow$ iff $\sigma_2 > b_c/\theta$, while $\bar{h} -$ always.

6) If $\sigma_2 \uparrow$, then $\bar{c} \uparrow$ iff $\sigma_1 < b_e/\theta$, while $\bar{h} -$ always.

Proof. Signs of the partial derivatives of functions (4) and (7). □

7.1. Comparative statics in the context of Cases (b) *and* (c)

When occurs Cases *(b)* and *(c)*, remember that:

- The payoff differential $\Pi_C(h) - \Pi_{NC}(h)$ is a decreasing function of h, and, therefore, the more is the share of Honest bureaucrats h, the less is the payoff of strategy C relatively to strategy NC. This implies (as far as comparative statics is concerned) that, for enough low values of h, it holds $\Pi_C(h) - \Pi_{NC}(h) > 0$ (therefore, $\dot{c} > 0$), while the opposite occurs for enough high values of h.
- The payoff differential $\Pi_H(c) - \Pi_D(c)$ is an increasing function of c, and, therefore, the more is the share of Compliant firms c, the more is the payoff of strategy H relatively to strategy D. This imply (as far as the comparative statics is concerned) that, for enough low values of c, it holds $\Pi_H(c) - \Pi_D(c) < 0$ (therefore, $\dot{h} < 0$), while the opposite occurs for enough high values of c.

In such a context, as described in Section 6, in an *environment* characterized by a low share of Honest bureaucrats and a low share of Compliant firms, then C is the best reply for firms, while D is the best reply for bureaucrats. Therefore, following conditions (24) and (25), firms that adopt strategy C and bureaucrats that adopt strategy D show an *evolutionary advantage*. So, if the Public Administration decides to increase the probabilities (p and θ) and the sanctions (s_1, s_2, σ_1, and σ_2) due to the high shares of Not Compliant firms and Dishonest bureaucrats, the effects will be mixed (see Table 3). In fact, an increase in the policy parameters (p, θ, s_1, s_2, σ_1, σ_2) has the effect of increase the share of Compliant firms (i.e., $\bar{h}$), but, at same time, has the effect of decrease the share of Honest bureaucrats (i.e., $\bar{c}$).

Table 3
Cases (b) and (c): monotonic relations between equilibrium shares ($\bar{c}$, $\bar{h}$) and their parameters

	$\bar{c}$	$\bar{h}$
p	–	↑
θ	↓	↑
s_1	–	↑
s_2	–	↑
σ_1	↓	–
σ_2	↓	–

Legend: ↑ Increasing, ↓ Decreasing, – Independent.

Table 4
Cases (a) and (d): monotonic relations between equilibrium shares ($\bar{c}$, $\bar{h}$) and their parameters

	$\bar{c}$	$\bar{h}$
p	–	↓
θ	↑	↓
s_1	–	↓
s_2	–	↓
σ_1	↑	–
σ_2	↑	–

Legend: ↑ Increasing, ↓ Decreasing, – Independent.

7.2. *Comparative statics in the context of Cases* (a) *and* (d)

When occurs Cases *(a)* and *(d)*, remember that:

- The payoff differential $\Pi_C(h) - \Pi_{NC}(h)$ is an increasing function of h, and, therefore, the more is the share of Honest bureaucrats h, the more is the payoff of strategy C relatively to strategy NC. This implies (as far as comparative statics is concerned) that, for enough low values of h, it holds $\Pi_C(h) - \Pi_{NC}(h) < 0$ (therefore, $\dot{c} < 0$), while the opposite occurs for enough high values of h.
- The payoff differential $\Pi_H(c) - \Pi_D(c)$ is a decreasing function of c, and, therefore, the more is the share of Compliant firms c, the less is the payoff of strategy H relatively to strategy D. This imply (as far as the comparative statics is concerned) that, for enough low values of c, it holds $\Pi_H(c) - \Pi_D(c) > 0$ (therefore, $\dot{h} > 0$), while the opposite occurs for enough high values of c.

In such a context, as described in Section 6, in an *environment* characterized by a low share of Honest bureaucrats and a low share of Compliant firms, then NC is the best reply for firms, while H is the best reply for bureaucrats. Therefore, following conditions (28) and (29), firms that adopt strategy NC and bureaucrats that adopt strategy H show an *evolutionary advantage*. So, if the Public Administration decides to increase the probabilities (p and θ) and the sanctions (s_1, s_2, σ_1, and σ_2) due to the high shares of Not Compliant firms and Dishonest bureaucrats, the effects will be mixed (see Table 4). In fact, an increase in the policy parameters (p, θ, s_1, s_2, σ_1, σ_2) has the effect of increase the share of Honest bureaucrats (i.e., $\bar{c}$), but, at same time, has the effect of decrease the share of Compliant firms (i.e., $\bar{h}$).

8. Conclusions

This paper has investigated the environmental corruption via a random matching evolutionary game between a population of firms and a population of bureaucrats. In each encounter a bureaucrat checks the compliance with environmental regulations by a firm. When the environmental laws are respected, the firm obtains a “green” license, like a sticker; otherwise, it receives a penalty. We assume the existence of two types of firms, Compliant and Not Compliant, two types of bureaucrats, Honest and Dishonest, and also two types of crimes, corruption and extortion. Corruption is when a Dishonest bureaucrat accepts a bribe from a Not Compliant firm, while extortion is when a Dishonest bureaucrat extorts a bribe from a Compliant firm.

From our analysis we obtain four dynamic regimes, two are bistable, and two with an internal stable equilibrium, which corresponds to the mixed-strategy Nash equilibrium of the one-shot static game, surrounded by closed trajectories. In the two path-dependent regimes, the dynamics depends on the initial conditions, i.e., the share of Compliant firms and the share of Honest bureaucrats. In the first regime one equilibrium is “virtuous”, all firms are Compliant and all bureaucrats are Honest, while the other is “vicious”, all firms are Not Compliant and all bureaucrats are Dishonest. In the second regime, instead, in one equilibrium all firms are Compliant and all bureaucrats are Dishonest, while in the other all firms are Not Compliant and all bureaucrats are Honest.

With regard to the two regimes with an internal stable equilibrium that is surrounded by closed trajectories, in one the trajectories turn counter-clockwise, while in the other the trajectories turn clockwise. These oscillatory dynamics could be explained using the *predator-prey* conceptual framework. When the trajectories oscillate counter-clockwise, Honest bureaucrats are the predators, while Not Compliant are the preys. Otherwise, when the trajectories oscillate clockwise, Dishonest bureaucrats are the predators and Compliant firms are the preys. From comparative statics of these two dynamic regimes emerges that policy instruments can help the Public Administration to reduce both corruption and extortion. However, in an environment characterized by high shares of Not Compliant firms and Dishonest bureaucrats, an increase of policy parameters has positive effects on crime deterrence only if adopting strategies C and H represents an evolutionary advantage.

In the regime with trajectories that turn counter-clockwise, firms that adopt strategy Compliant and bureaucrats that adopt strategy Dishonest have an evolutionary advantage, in an environment characterized by high shares of Not Compliant firms and Dishonest bureaucrats. Therefore, all policy instruments increase the share of Compliant firms, but decrease the share of Dishonest bureaucrats. Conversely, in the regime with trajectories that turn clockwise, firms that adopt strategy Not Compliant and bureaucrats that adopt strategy Honest have an evolutionary advantage, in an environment characterized by high shares of Not Compliant firms and Dishonest bureaucrats. Therefore, all policy instruments increase the share of Honest bureaucrats, but decrease the share of Compliant firms.

The model can be extended and adapted to the different concepts of corruption in literature, as well as to future researches. For instance, it would be interesting to consider the possibility that the compliant firm could denounce the dishonest bureaucrat who claims a bribe. This possibility, since it complicates the actual model, could be the topic of a future model.

Appendix

Proof of Proposition 1

1) In Case (a) (see (5)), $\Pi_C(h)-\Pi_{NC}(h)$ is strictly increasing in h (see Fig. 1(a)) and the following sub-cases can occur:

   i) If:
   $$\Pi_C(0) - \Pi_{NC}(0) =$$
   $$= b_c - b_e + \theta(\eta + s_2) - C_C \geq 0$$
   that is, if:
   $$s_2 \geq \bar{s}_2 := \frac{C_c + b_e - b_c}{\theta} - \eta \tag{30}$$
   then $\Pi_C(h) - \Pi_{NC}(h) > 0$ holds for every $h \in (0, 1)$ and, consequently, the strategy C dominates the strategy NC.

   ii) If:
   $$\Pi_C(1) - \Pi_{NC}(1) = ps_1 - C_C \leq 0$$
   that is, if:
   $$s_1 \leq \frac{C_C}{p} \tag{31}$$
   then $\Pi_C(h) - \Pi_{NC}(h) < 0$ holds for every $h \in (0, 1)$ and, consequently, the strategy NC dominates the strategy C.

   iii) If neither condition (30) nor (31) hold, that is if:
   $$s_2 < \bar{s}_2 \quad \textit{and} \quad s_1 > \frac{C_C}{p} \tag{32}$$
   then no strategy dominates the other one, and the graph of the payoff differential $\Pi_C(h) - \Pi_{NC}(h)$ intersects the h-axis at $h = \bar{h} \in (0, 1)$ (see (4)): for $h > \bar{h}$ (respectively, $h < \bar{h}$), it holds $\Pi_C(h) - \Pi_{NC}(h) > 0$ (respectively, $\Pi_C(h) - \Pi_{NC}(h) < 0$).

2) In Case (b) (see (6)), $\Pi_C(h)-\Pi_{NC}(h)$ is strictly decreasing in h (see Fig. 1(b)) and the following sub-cases can be observed:

   i) If:
   $$\Pi_C(0) - \Pi_{NC}(0) =$$
   $$= b_c - b_e + \theta(\eta + s_2) - C_C \leq 0$$
   that is, if:
   $$s_2 \leq \bar{s}_2 \tag{33}$$
   then $\Pi_C(h) - \Pi_{NC}(h) < 0$ holds for every $h \in (0, 1)$ and, therefore, the strategy NC dominates the strategy C.

   ii) If:
   $$\Pi_C(1) - \Pi_{NC}(1) = ps_1 - C_C \geq 0$$
   that is, if:
   $$s_1 \geq \frac{C_C}{p} \tag{34}$$
   then $\Pi_C(h) - \Pi_{NC}(h) > 0$ holds for every $h \in (0, 1)$ and, therefore, the strategy C dominates the strategy NC.

   iii) If neither condition (33) nor condition (34) hold, that is, if:
   $$s_2 > \bar{s}_2 \quad \textit{and} \quad s_1 < \frac{C_C}{p} \tag{35}$$
   then no strategy dominates the other one, and the graph of the payoff differential $\Pi_C(h) - \Pi_{NC}(h)$ intersects the h-axis at $h = \bar{h} \in (0, 1)$ (see (4)): for $h > \bar{h}$ (respectively, $h < \bar{h}$), it holds $\Pi_C(h) - \Pi_{NC}(h) < 0$ (respectively, $\Pi_C(h) - \Pi_{NC}(h) > 0$).

Proof of Proposition 2

1) In Case (c) (see (8)), $\Pi_H(c) - \Pi_D(c)$ is strictly increasing (see Fig. 1(a)) and the following sub-cases can occur:

   i) If:
   $$\Pi_H(0) - \Pi_D(0) = \theta\sigma_2 - b_c \geq 0$$
   that is, if:
   $$\sigma_2 \geq \frac{b_c}{\theta} \tag{36}$$
   then $\Pi_H(c) - \Pi_D(c) > 0$ holds for every $c \in (0, 1)$ and, consequently, the strategy H dominates the strategy D.

ii) If:

$$\Pi_H(1) - \Pi_D(1) = -b_e + \theta\sigma_1 \leq 0$$

that is, if:

$$\sigma_1 \leq \frac{b_e}{\theta} \tag{37}$$

then $\Pi_H(c) - \Pi_D(c) < 0$ holds for every $c \in (0, 1)$ and, consequently, the strategy D dominates the strategy H.

iii) If neither condition (36) nor condition (37) hold, that is if:

$$\sigma_1 > \frac{b_e}{\theta} \quad \mathit{and} \quad \sigma_2 < \frac{b_c}{\theta} \tag{38}$$

then no strategy dominates the other one, and the graph of the payoff differential $\Pi_H(c) - \Pi_D(c)$ intersects the c-axis at $c = \bar{c} \in (0, 1)$ (see (7)): for $c > \bar{c}$ (respectively, $c < \bar{c}$), it holds $\Pi_H(c) - \Pi_D(c) > 0$ (respectively, $\Pi_H(c) - \Pi_D(c) < 0$).

2) In Case (d) (see (9)), $\Pi_H(c) - \Pi_D(c)$ is strictly decreasing (see Fig. 1(b)) and the following sub-cases can be observed:

i) If:

$$\Pi_H(0) - \Pi_D(0) = \theta\sigma_2 - b_c \leq 0$$

that is, if:

$$\sigma_2 \leq \frac{b_c}{\theta} \tag{39}$$

then $\Pi_H(c) - \Pi_D(c) < 0$ holds for every $c \in (0, 1)$ and, therefore, the strategy D dominates the strategy H.

ii) If:

$$\Pi_H(1) - \Pi_D(1) = -b_e + \theta\sigma_1 \geq 0$$

that is, if:

$$\sigma_1 \geq \frac{b_e}{\theta} \tag{40}$$

then $\Pi_H(c) - \Pi_D(c) > 0$ holds for every $c \in (0, 1)$ and, therefore, the strategy H dominates the strategy D.

iii) If neither condition (39) nor condition (40) hold, that is, if:

$$\sigma_1 < \frac{b_e}{\theta} \quad \mathit{and} \quad \sigma_2 > \frac{b_c}{\theta} \tag{41}$$

then no strategy dominates the other one, and the graph of the payoff differential $\Pi_H(c) - \Pi_D(c)$ intersects the c-axis at $c = \bar{c} \in (0, 1)$ (see (7)): for $c > \bar{c}$ (respectively, $c < \bar{c}$), it holds $\Pi_H(c) - \Pi_D(c) < 0$ (respectively, $\Pi_H(c) - \Pi_D(c) > 0$).

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UNIVERSITÀ DEGLI STUDI FIRENZE
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FOREIGN DIRECT INVESTMENTS, LAND RENT, AND POLLUTION IN A LOCAL ECONOMY

Abstract

This paper studies the possible effects of foreign direct investments in land on the development of a local economy. To this aim, we use a two-sector model (*external* and *local*) with heterogeneous agents: *external investors* and *local land owners*. The dynamics is given by the accumulation of pollution and local physical capital, while the external physical capital accumulation is driven by foreign direct investments. We assume that both sectors are negatively affected by pollution, but only the external sector is polluting. The local government can tax its production activities to finance environmental defensive expenditures. We compute local agents revenues via numerical simulations analysis. A welfare-improving growth path may occur only if the pollution tax is high enough and the impact of the external sector on pollution is low enough, since the revenues of local land owners depend inversely on pollution level. Otherwise, a welfare-reducing growth path may occur, and foreign direct investments decrease the revenues of local land owners.

JEL classification: D62, F21, O15, O41, Q50.
Keywords: two-sector model, land grabbing, environmental negative externalities, pollution.

1. Introduction

In the last 10 years there was a significant increase of studies on foreign direct investments (FDI) in land, a phenomenon often referred to *land grabbing*[1]. Some authors consider this phenomenon as an opportunity to improve local physical capital for agricultural production, while others highlight the negative long-term implications for food security (Arezki et al., 2015).

The land rented to foreign investors is used mainly to produce two agricultural goods, food and bio-fuel. In the first case, the land grabbing is an outsourcing of domestic food production of those countries in which there are a limited availability of water and arable land, such as the Gulf States (Zoomers, 2010). With regard to bio-fuel production, instead, the biggest players are high-income OECD countries and emerging economies, which include some important bio-fuel producers, such as China and South Korea (Cotula et al., 2009). In a recent article on FDI for bio-fuel in Sub-Saharan Africa, Giovannetti and Ticci (2016) have shown that capital is attracted by water abundance, weak institutional framework and ill-defined land property rights. Indeed, the investments in this activity need water, political stability of the local government, and countries where individual land rights do not represent a guarantee against large-scale acquisition.

The debate on the effects of "land grabbing" in developing countries is part of the general debate on the effects of FDI on economic development and on environmental degradation. Some authors emphasize the positive role played by FDI on economic development, while others are more critical. Authors emphasizing the "pros" highlight positive effects of FDI on physical capital accumulation in the host economy due to the introduction of innovative technologies and inputs (Borensztein et al., 1998, Kemeny, 2010; Cipollina et al., 2012), of knowl-

[1] By this term, we refer the FDI in land acquisition to produce agricultural goods in developing countries (Saturnino et al., 2011).

edge and skills through labour and manager training (Liu et al., 2001; Hansen and Rand, 2006), and of industrial competition by overcoming entry barriers and reducing the market power of exiting firms (Chung, 2001; Bitzer and Görg, 2009; Nicolini and Resmini, 2010; Damijan et al., 2013). On the contrary, some authors stress the negative effects on the development of the local economy generated by FDI via the crowding out of local firms (Aitken and Harrison, 1999; Agosin and Machado, 2005; Herzer et al., 2008; Waldkirch and Ofosu, 2010).

There is also no agreement in empirical literature on the effects of FDI on environmental degradation, despite the increasing number of studies mainly on a special case of the Environmental Kuznets Curve[2] (see for a survey of the literature, Dinda, 2004; Kijima et al., 2010; Pasten et al., 2012), i.e., the so called *pollution haven hypothesis*. The basic idea is that the polluting firms from developed countries relocate part of their production activities in developing countries, where the environmental regulations are less stringent (Grether and De Melo, 2003). Therefore, some authors argue that the more lenient environmental standards attract polluted FDI (see, e.g., Cole, 2004; He, 2006; Cole and Fredriksson, 2009). However, other economists argue that there is no relationship between FDI and environmental regulations (see, e.g., Millimet and List, 2004; Levinson and Taylor, 2008).

The debate so far has focused on empirical controversies and the modelling of the land grabbing phenomenon has been, to our knowledge, very limited. This paper proposes a two-sector model (an *external* and a *local* sector) with heterogeneous agents (*external investors* and *local land owners*) to analyse the effects of FDI in land acquisition on economic development and environmental degradation. It investigates the dynamics characterizing a small open economy, in which both sectors are negatively affected by pollution level. Both sectors produce agricultural goods and use as inputs the land endowment of the host economy and the physical capital. We exclude the labour input in both sectors, assuming that each local agent inelastically employs a unit of her labour in the local production process, as well as each external agent in the external production process. Consider, for instance, several developing countries in which land and physical capital are scant factors, while unskilled labour is relatively abundant.

In such a context, the local land owner can rent her land to the external investors or use it to the local production process. The rent price is set by the land rental market and we assume, for simplicity, instantaneous adjustments. We suppose that only the external sector has negative effects on pollution level, and, for this reason, we introduce the possibility for the local government to tax its production activity. The revenues coming from the pollution tax are used to finance environmental defensive expenditures. Our model differs from other similar frameworks proposed by López (2010) and Antoci et al. (2014, 2015a,b) who adopt two-sector models with environmental externalities and heterogeneous agents. In our model, we study two agricultural sectors, analysing the allocation of land endowment and the welfare of local land owners, and the environmental degradation is treated as pollution level. Otherwise, López (2010) and Antoci et al. (2014, 2015a,b) study an industrial sector and a resource-dependent sector, analysing the allocation of labour endowment and the welfare of local workers, and the environmental degradation is the depletion of natural resources. Moreover, in López (2010) and Antoci et al. (2014, 2015a,b) the local agents can defend themselves from environmental degradation only via working for the polluting sector. In our model, instead, in addition to rent their land to the polluting sector, also the government can defend local agents from environmental degradation via using the pollution tax.

Numerical simulations show that the dynamics may be bistable: there are two locally attractive stationary states, one in which the economy is specialized in the local sector and one in which there is coexistence of the external and the local sector, and the basins of attraction of the two attractive stationary states are separated by the stable branch of a saddle point. Moreover, the dynamics with or without specialization in the local sector is determined by the parameter values of the pollution tax and the impact of the external sector on pollution level, with respect to each other.

With regard to welfare analysis of local agents, it emerges that the revenues of land owners may be greater at stationary state in which the economy is specialized in the local sector than at stationary state in which there is coexistence between sectors. However, a welfare-improving growth path may occur if the pollution tax is high enough and the impact of the external sector on pollution is low enough, respectively. This occurs because the revenues of local agents depend inversely on pollution level. An increase of the pollution tax and a decrease of the impact of the external sector on pollution decrease the pollution level, and, therefore, increase the welfare of land owners.

The paper is organized as follows. The model is presented in Section 2. Section 3 defines the dynamics with and without specialization, Section 4 contains some basic results about the dynamics of the model, Section 5 illustrates, via numerical simulations, some possible dynamic regimes, Section 6 deals with welfare analysis, and Section 7 concludes.

2. The model

Let us consider a small open economy with two production factors (land and physical capital) and two groups of agents: "Local land owners" (L-agents) and "External investor" (E-agents). In this context, we will analyse the accumulation of local physical capital and the evolution of pollution, which depends on production activities. We exclude the labour input, since we suppose that each agent inelastically employs a unit of her labour endowment to the production process.

We assume that the production functions of the two sectors satisfy *Inada* conditions, i.e., are concave, increasing and homogeneous of degree 1 in their inputs. We assume that the populations of local and external agents are both constituted by a continuum of identical individuals, and, therefore, we consider

[2] The name was coined by Panayotou (1993) due to its similarity to the work of Kuznets (1955)

the choice processes of the representative agents. The production function of the representative L-agent is given by:

$$Y_L = \widetilde{A} K_L^{\alpha} L^{1-\alpha} \tag{1}$$

where $\widetilde{A} := A/(1 + aP)$ is a measure of productivity of the local sector, which negatively depends on the stock of pollution P; K_L is the physical capital accumulated by the representative L-agent; L is the land used in the local sector production; $0 < \alpha < 1$ and A, $a > 0$.

The L-agent's total endowment of land is normalized to 1 and the representative Land owner allocates her land endowment between the two sectors; so $1 - L$ represents the land that the local agent rents to the representative External investor. The production function of the representative External investor is given by:

$$Y_E = \widetilde{B} K_E^{\beta} (1 - L)^{1-\beta} \tag{2}$$

where K_E denotes the stock of physical capital invested by the representative E-agent in the economy; $\widetilde{B} := B/(1 + bP)$ is a measure of productivity of external sector; $0 < \beta < 1$ and B, $b > 0$. The representative E-agent chooses her land demand $1 - L$ and the stock of physical capital K_E in order to maximize her profits, i.e.:

$$\max_{1-L,\, K_E} \left[(1 - \tau)\widetilde{B} K_E^{\beta} (1 - L)^{1-\beta} - r_L(1 - L) - r_K K_E\right] \tag{3}$$

where $\tau \in (0, 1)$ is a parameter that measures the environmental taxation, r_L and r_K are, respectively, the land rental price and the cost of capital[3]. We assume that r_K is an exogenous parameter, while r_L is endogenously determined by the land rental market equilibrium condition. We suppose that K_E inflow is potentially unlimited.

Differently, in each instant of time the representative Local agent chooses the allocation of her land between the two sectors. The maximization problem is the following:

$$\max_{L} \left[\widetilde{A} K_L^{\alpha} L^{1-\alpha} + r_L(1 - L)\right] \tag{4}$$

Furthermore, we assume that the dynamics of accumulation of K_L is described by the equation

$$\dot{K}_L = s\left[\widetilde{A} K_L^{\alpha} L^{1-\alpha} + r_L(1 - L)\right] - \gamma K_L \tag{5}$$

where, $\dot{K}_L$ is the time derivative dK_L/dt of K_L, $s \in (0, 1)$ is the constant saving rate, and $\gamma > 0$ represents the depreciation of K_L. To simplify, we assume that the prices of the goods produced in the local and in the external sectors are both equal to unity; moreover, the land rental price r_L is expressed in terms of the output of the external sector. Finally, the dynamics of pollution is described by:

$$\dot{P} = \delta \bar{Y}_E - \varepsilon P - \eta D$$

where, $\dot{P}$ is the time derivative dP/dt of P, $\bar{Y}_E$ represents the economy-wide average value of Y_E, $\delta > 0$ is a parameter that measures the impact of the external sector on pollution, $\varepsilon > 0$ represents the decay rate of pollution P, D are the pollution abatement expenditures financed by taxation of external economic activities ($D = \tau \bar{Y}_E$), and $\eta > 0$ is a parameter that measures the effectiveness of pollution abatement expenditures. Therefore, the dynamics of pollution can be rewritten as:

$$\dot{P} = (\delta - \eta\tau)\bar{Y}_E - \varepsilon P \tag{6}$$

We assume that each economic agent considers as negligible the impact of her choices on $\bar{Y}_E$ and on the time evolution of P (that is, $\bar{Y}_E$ is considered as exogenously determined). Since E-agents are identical, the average output Y_E ex post coincides with the per capita value Y_E.

3. Dynamics

The dynamics is obtained by solving the maximization problems (3)-(4); the solutions of these problems allow to determine the equilibrium values of L and K_E. In particular, the maximization problem of the representative L-agent determines the following first order condition:

$$r_L = (1 - \alpha)\, \widetilde{A} K_L^{\alpha} L^{-\alpha} \tag{7}$$

Similarly, the maximization problem of the representative E-agent gives rise to the following first order conditions:

$$r_L = (1 - \beta)\, (1 - \tau)\, \widetilde{B} K_E^{\beta} (1 - L)^{-\beta} \tag{8}$$

$$r_K = \beta\, (1 - \tau)\, \widetilde{B} K_E^{\beta-1} (1 - L)^{1-\beta} \tag{9}$$

We assume that land rental market is perfectly competitive and land rental prices are flexible. E- and L- agents take r_L as given, but land rental price and land allocation between the two sectors continue to change until land rental demand is equal to land rental supply. The land rental market equilibrium condition is given by:

$$(1 - \beta)\, (1 - \tau)\, \widetilde{B} K_E^{\beta} (1 - L)^{-\beta} = (1 - \alpha)\, \widetilde{A} K_L^{\alpha} L^{-\alpha} \tag{10}$$

From Eq. (9), we have:

$$K_E = \left(\frac{\beta}{r_K}\, (1 - \tau)\, \widetilde{B}\right)^{\frac{1}{1-\beta}} (1 - L) \tag{11}$$

Substituting Eq. (11) in Eq. (10), we obtain:

$$L = \Gamma \left(\widetilde{A} K_L^{\alpha}\right)^{\frac{1}{\alpha}} \tag{12}$$

where:

$$\Gamma := \left[\frac{1 - \alpha}{(1 - \beta)\left(\widetilde{B}\, (1 - \tau)\right)^{\frac{1}{1-\beta}} \left(\frac{\beta}{r_K}\right)^{\frac{\beta}{1-\beta}}}\right]^{\frac{1}{\alpha}}$$

Function (12) identifies the land rental market equilibrium value $\widetilde{L}$ of L if the right side of Eq. (12) is lower than 1; otherwise, $\widetilde{L} = 1$, that is:

$$\widetilde{L} = \min \left\{1,\ \Gamma\, (\widetilde{A} K_L^{\alpha})^{\frac{1}{\alpha}}\right\} \tag{13}$$

[3] We can consider r_K as opportunity cost.

Consequently, from Eq. (11), the equilibrium value $\widetilde{K_E}$ of K_E is determined by:

$$\widetilde{K}_E = \left(\frac{\beta}{r_K}\,(1-\tau)\,\widetilde{B}\right)^{\frac{1}{1-\beta}}(1-\widetilde{L}) \tag{14}$$

The economy is specialized in the production of the L-sector if $\widetilde{L} = 1$ (and, consequently, $K_E = 0$). The graph of the function

$$K_L := \bar{K}_L = \frac{1}{\Gamma\,(\widetilde{A})^{\frac{1}{\alpha}}} \tag{15}$$

separates the region of the plane (P, K_L) where $\widetilde{L} = 1$ (above it) from the region where $\widetilde{L} < 1$ (below it) [4].

From condition (13) we can distinguish two possible cases: (a) if $K_L = 0$, then the economy specializes in the production of the external sector (that is, $\widetilde{L} = 0$ and $K_E = \left(\frac{\beta}{r_K}\,(1-\tau)\,\widetilde{B}\right)^{\frac{1}{1-\beta}}$ are chosen); and (b) if $K_L > 0$, instead, condition (13) excludes the specialization in the external sector (i.e., $\widetilde{L} > 0$ always holds for $K_L > 0$). In this case, we can distinguish two sub-cases, that is: (i) the case without specialization in the local sector (i.e., $\widetilde{L} \in (0, 1)$) and (ii) the case with specialization (i.e., $\widetilde{L} = 1$). When $K_L > 0$, the external sector never completely replaces the local sector since the productivity of land used in the local activities tends to infinity as $L \to 0$. On the contrary, when $K_L > 0$ the economy can fully specialize in the local sector though also the productivity of land in the external sector tends to infinity as $(1-L) \to 0$. In this case, the land rent price becomes increasingly high, therefore, External investors move their capital outside the economy and reduce K_E, which eventually goes to zero, so that the economy ends up fully specializing in the local sector.

3.1. Dynamics without specialization

If $\Gamma\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} < 1$ (see function (12)), then the representative L-agent rents a positive fraction of her total land endowment to be used by the representative E-agent. Moreover, the following proposition holds:

Proposition 1. *The equilibrium land rental price is equal to* $r_L = (1-\beta)\left(\widetilde{B}\,(1-\tau)\right)^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\right)^{\frac{\beta}{1-\beta}}$

Proof. In the context $\Gamma\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} < 1$, the equilibrium land rental price is given by:

$$\begin{aligned} r_L &= (1-\alpha)\,\widetilde{A}K_L^{\alpha}L^{-\alpha} = \\ &= (1-\alpha)\,\widetilde{A}K_L^{\alpha}\left[\Gamma\,(\widetilde{A}K_L^{\alpha})^{\frac{1}{\alpha}}\right]^{-\alpha} = \\ &= (1-\beta)\left(\widetilde{B}\,(1-\tau)\right)^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\right)^{\frac{\beta}{1-\beta}} \end{aligned}$$

□

[4] Remember that $\widetilde{A} = A/(1 + aP)$

When $\Gamma\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} < 1$, the dynamics of the capital invested in the L-sector is given by:

$$\begin{aligned} \dot{K}_L &= s\left[\widetilde{A}K_L^{\alpha}L^{1-\alpha} + r_L(1-L)\right] - \gamma K_L = \\ &= s\left[\alpha\,\Gamma^{1-\alpha}\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} + (1-\beta)\left(\widetilde{B}\,(1-\tau)\right)^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\right)^{\frac{\beta}{1-\beta}}\right] \\ &\quad - \gamma K_L \end{aligned} \tag{16}$$

while the time evolution of P is represented by:

$$\begin{aligned} \dot{P} &= (\delta - \eta\tau)\bar{Y}_E - \varepsilon P = \\ &= (\delta - \eta\tau)\,\widetilde{B}^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\,(1-\tau)\right)^{\frac{\beta}{1-\beta}}\left(1 - \Gamma\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}}\right) - \varepsilon P \end{aligned} \tag{17}$$

The system of Eqs. (16)-(17), therefore, represents the dynamics of the economy in the case without specialization.

3.2. Dynamics with specialization

If $\Gamma\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} \geq 1$ (that is, above the curve (15) in the plane $(P,\ K_L)$), the representative L-agent uses all her land endowment to the production activity of the L-sector, that is $\widetilde{L} = 1$. The dynamics of the economy in the case with specialization is described by the equations:

$$\dot{K}_L = s\left(\widetilde{A}K_L^{\alpha}\right) - \gamma K_L \tag{18}$$

$$\dot{P} = -\varepsilon P \tag{19}$$

4. Stationary states

Since a stationary state in which the economy is specialized in the external sector does not exist, two types of stationary states may be observed:

- the stationary state $A^1 = (P, K_L) = \left(0, (\frac{sA}{\gamma})^{\frac{1}{1-\alpha}}\right)$, in which the economy is specialized in the local sector, and the pollution level is equal to zero;
- stationary states in which both sectors coexist[5].

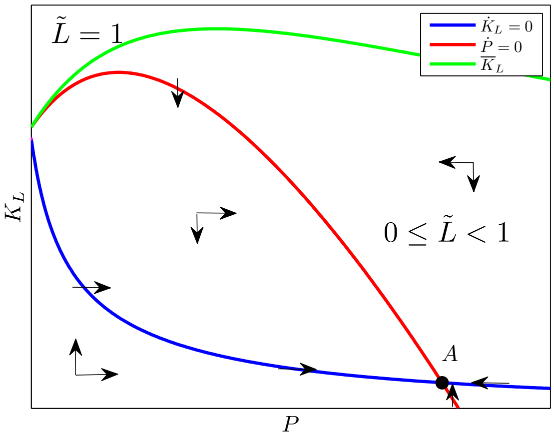
The following proposition illustrates the conditions under which the stationary state when the economy is specialized in the local sector exists.

Proposition 2. *The state* $A^1 = \left(0,\ (\frac{sA}{\gamma})^{\frac{1}{1-\alpha}}\right)$ *is a stationary state of the system (18)-(19) if and only if*

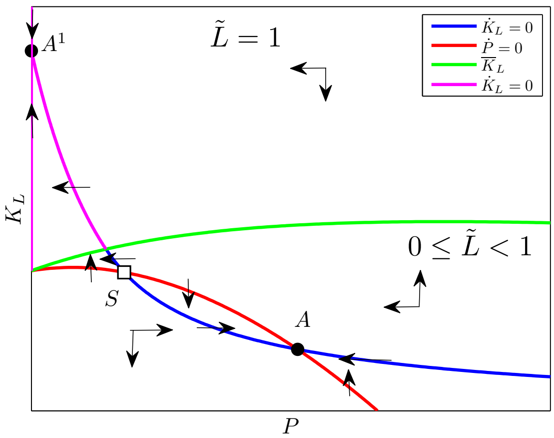
$$A \geq \left(\frac{\gamma}{s}\right)^{\alpha}\left[\frac{1-\alpha}{(1-\beta)(B(1-\tau))^{\frac{1}{1-\beta}}(\frac{\beta}{r_K})^{\frac{\beta}{1-\beta}}}\right]^{\alpha-1} \tag{20}$$

When existing, it is always locally attractive (see Fig. 1(b)).

[5] Numerical simulations, presented in Section 5, have shown that two stationary states of this type may be observed: A (attractive) and S (saddle point).



(a) $\tau = 0.12$.



(b) $\tau = 0.42$.

Fig. 1. Isoclines.
Parameter values: $A = 1$, $B = 2$, $a = 5$, $b = 2$, $\alpha = 0.65$, $\beta = 0.35$, $\delta = 0.5$, $r = 0.1$, $s = 0.6$, $\eta = 1$, $\varepsilon = 0.55$, $\gamma = 0.19$.

Proof. According to the system (18)-(19), it holds that $\dot{K}_L = 0$ for:

$$K_L = \left[\frac{sA}{\gamma(1+aP)}\right]^{\frac{1}{1-\alpha}}$$

The dynamics (18)-(19) admits an unique stationary state $A^1 = (P, K_L) = \left(0,\ \left(\frac{sA}{\gamma}\right)^{\frac{1}{1-\alpha}}\right)$ if and only if A^1 lies above the separatrix $K_L = \bar{K}_L$ (see function (15)), i.e, if $\bar{K}_L(0) \leq \left(\frac{sA}{\gamma}\right)^{\frac{1}{1-\alpha}}$, that is:

$$A \geq \frac{(\frac{\gamma}{s})^{\alpha}}{\Gamma^{\alpha(1-\alpha)}} = \left(\frac{\gamma}{s}\right)^{\alpha}\left[\frac{1-\alpha}{(1-\beta)(B(1-\tau))^{\frac{1}{1-\beta}}(\frac{\beta}{r_K})^{\frac{\beta}{1-\beta}}}\right]^{\alpha-1}$$

While the Jacobian matrix of the system (18)-(19), calculated at the stationary state A^1 is:

$$J(A^1) = \begin{pmatrix} -(1-\alpha)s\widetilde{A}K_L^{\alpha-1} & -\alpha sAK_L^{\alpha} \\ 0 & -\varepsilon \end{pmatrix}$$

with strictly negative eigenvalues: $-(1-\alpha)s\widetilde{A}K_L^{\alpha-1} < 0$ and $\varepsilon < 0$. Therefore, when the stationary state A^1 exists, it is always locally attractive. $\square$

Proposition 2 affirms that A^1, when existing, is always locally attractive, and it lies always above the separatrix $\bar{K}_L$ (where, $\widetilde{L} = 1$). From numerical simulations it emerges that if the pollution tax is low enough with respect to the impact of the external sector on pollution, the economy cannot fully specialize in the local sector (see Fig. 1(a)). This result is intuitive: the pollution tax enters as a cost in the maximization problem (3) of External investors.

Otherwise, if the economy is not specialized in the local sector, the stationary states of the system (16)-(17) are given by the solutions of the system of equations:

$$\begin{aligned} 0 &= s\left[\alpha\, \Gamma^{1-\alpha}\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} + (1-\beta)\left(\widetilde{B}\,(1-\tau)\right)^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\right)^{\frac{\beta}{1-\beta}}\right] \\ &\quad -\gamma K_L \\ 0 &= (\delta-\eta\tau)\,\widetilde{B}^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\,(1-\tau)\right)^{\frac{\beta}{1-\beta}}\left(1-\Gamma\left(\widetilde{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}}\right) - \varepsilon P \end{aligned} \tag{21}$$

From system (21), we obtain that $\dot{K}_L = 0$ for:

$$K_L = F(P) := \frac{s(1-\alpha)\bar{K}_L}{\gamma\bar{K}_L\Gamma^{\alpha} - \alpha s} \tag{22}$$

and $\dot{P} = 0$ for:

$$K_L = G(P) := \bar{K}_L - \frac{\varepsilon P(1+bP)^{\frac{1}{1-\beta}}\bar{K}_L}{(\delta-\eta)B^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}(1-\tau)\right)^{\frac{\beta}{1-\beta}}} \tag{23}$$

Two cases can occur:

i) if $\delta - \eta\tau \leq 0$, i.e., the pollution tax τ is sufficiently high with respect to the impact of the external sector on pollution (measured by the parameter δ) and the positive impact of environmental defensive expenditures (measured by the parameter η), then from Eq. (17) it holds that $\dot{P} < 0$ for every $P > 0$, and, therefore, there are no stationary states with $P > 0$ and the trajectories tend toward the axis $P = 0$.

ii) if $\delta - \eta\tau > 0$, then the curve $K_L = G(P)$ lies always below the separatrix $\bar{K}_L(P)$ and stationary states, in which both sectors coexist in a context with $P > 0$, can exist.

Moreover, $G(0) = \bar{K}_L(0)$, i.e., the curve $K_L = G(P)$ and the separatrix $K_L = \bar{K}_L(P)$ have the same intercept on the axis $P = 0$. It is not possible to demonstrate analytically how many intersection points may be observed; however, from numerical simulations it emerges that at most two stationary states with $P > 0$ exist, i.e., points A and S (see Figs. 1(a) and 1(b) and Figs. 2(a) to 2(c)).

Global dynamics of system (5)-(6) is characterized by the following result.

Proposition 3. *The set:*

$$\Omega = \{(P, K_L) : 0 \le P \le P^* \text{ and } 0 \le K_L \le K_L^*\}$$

where

$$P^* := \frac{\delta - \eta\tau}{\varepsilon} B^{\frac{1}{1-\beta}} \Big[\frac{\beta}{r_K}(1-\tau)\Big]^{\frac{\beta}{1-\beta}} \quad ,$$

$$K_L^* > \max\Big[\Big(\frac{sA}{\gamma}\Big)^{\frac{1}{1-\alpha}},\ \widehat{K}_L\Big] \quad \text{and}$$

$\widehat{K}_L$ is the maximum of the function (see (15)) $K_L = K_L(P)$, is positively invariant under the dynamics (5)-(6); every trajectory starting outside Ω and enters it in finite time. When the stationary state with specialization $A^1 = (P, K_L) = \Big(0,\ (\frac{sA}{\gamma})^{\frac{1}{1-\alpha}}\Big)$ does not exist, then no sector definitively disappears from the economy (both sectors coexist).

Proof. Considering equation (17), we can write:

$$\dot{P} = (\delta - \eta\tau)\widetilde{B}^{\frac{1}{1-\beta}} \Big[\frac{\beta}{r}(1-\tau)\Big]^{\frac{\beta}{1-\beta}} \Big[1 - \Gamma\Big(\widetilde{A}K_L^{\alpha}\Big)^{\frac{1}{\alpha}}\Big] <$$
$$< (\delta - \eta\tau)\widetilde{B}^{\frac{1}{1-\beta}} \Big[\frac{\beta}{r}(1-\tau)\Big] - \varepsilon P \le$$
$$\le (\delta - \eta\tau)B^{\frac{1}{1-\beta}} \Big[\frac{\beta}{r}(1-\tau)\Big]^{\frac{\beta}{1-\beta}} - \varepsilon P$$

Since the maximum value that $\widetilde{B}$ can assume is B, then it holds $\dot{P} > 0$ for:

$$P \ge P^* := \frac{\delta - \eta\tau}{\varepsilon} B^{\frac{1}{1-\beta}} \Big[\frac{\beta}{r_K}(1-\tau)\Big]^{\frac{\beta}{1-\beta}}$$

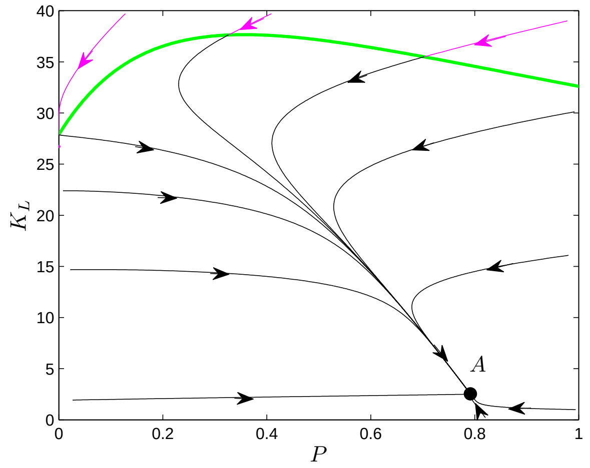
Indicating with $\widehat{K}_L$ the maximum of the function (see (15)) $K_L = \bar{K}_L(P) := \frac{1}{\Gamma(\bar{A})^{\frac{1}{\alpha}}}$ (that always exists), and remembering that the value of K_L in the stationary state A^1 is given by $\Big(\frac{sA}{\gamma}\Big)^{\frac{1}{1-\alpha}}$, it holds $\dot{K}_L < 0$ for every $K_L > \max\Big[\Big(\frac{sA}{\gamma}\Big)^{\frac{1}{1-\alpha}},\ \widehat{K}_L\Big]$. $\square$

According to Proposition 3, the coexistence between sectors is possible. Numerical simulations show that, if the pollution tax is high enough with respect to the impact of the external sector on pollution, two stationary states may be observed, A, locally attractive, and S, saddle point (see Fig. 1(b)). Otherwise, if the pollution tax is low enough with respect to the impact of the external sector on pollution, then the stationary state A^1 does not exists, and a unique stationary state A globally attractive exists (see Fig. 1(a)).

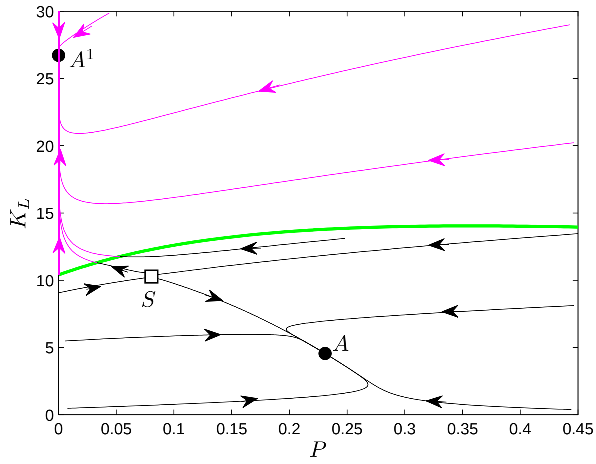
5. Simulations

This section presents the results of some numerical simulations of the dynamics of our model. Three types of dynamic regimes may be observed from numerical simulations:

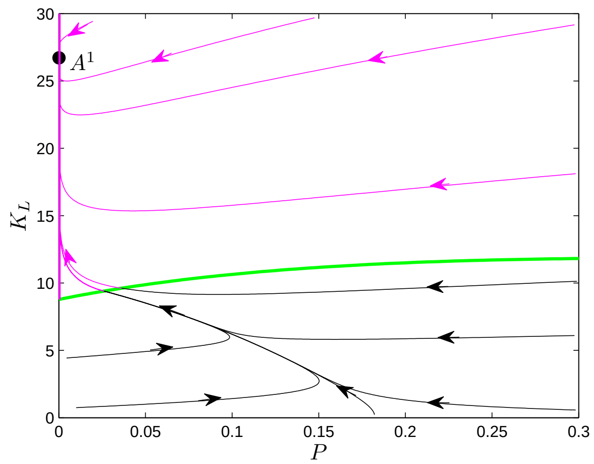
a) the regime illustrated in Fig. 2(a) characterized by the existence of a unique globally attractive stationary state A in which both sectors coexist;



(a) $\tau = 0.12$.



(b) $\tau = 0.42$.



(c) $\tau = 0.46$.

Fig. 2. Phase diagrams.
Parameter values: $A = 1$, $B = 2$, $a = 5$, $b = 2$, $\alpha = 0.65$, $\beta = 0.35$, $\delta = 0.5$, $r = 0.1$, $s = 0.6$, $\eta = 1$, $\varepsilon = 0.55$, $\gamma = 0.19$.

b) the regime illustrated in Fig. 2(b) characterized by the existence of two locally attractive stationary states: A^1 (where the economy is specialized in the local sector) and A (where both sectors coexist), and the basins of attraction of such states are separated by the stable branch of the saddle point S;

c) the regime illustrated in Fig. 2(c) where the stationary state A^1 is globally attractive and, consequently, the economy always specializes in the local sector;

Figs. 3(a) and 3(b) show bifurcation diagrams obtained, respectively, varying the parameters τ and δ; the continuous lines represent the stationary state A (except the black line $\widetilde{L}$ that represent the stationary state A_1), while the dashed lines represent the stationary state S. Moreover, from numerical simulations (shown in Figs. 2(a) to 2(c), and Figs. 3(a) and 3(b)), we can infer the following main results:

i) if the pollution tax is low enough with respect to the impact of the external sector on pollution, then a unique globally attractive stationary state A exists in which both sectors coexist (see Fig. 2(a));

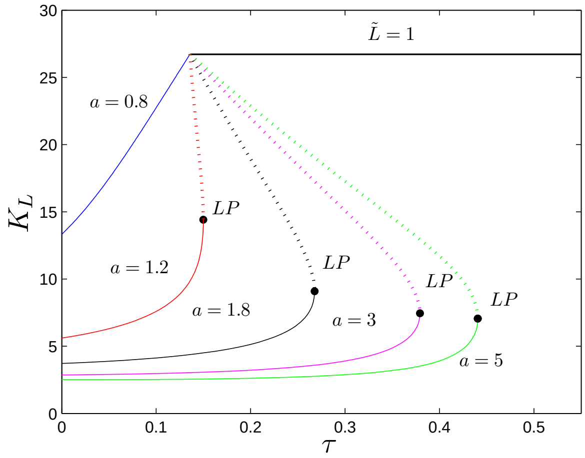
ii) for intermediate values of pollution tax, with respect to the impact of the external sector on pollution, the dynamics is bi-stable: there are two locally attractive stationary states, A^1, in which the economy is specialized in the local sector, and A, where both sectors coexist (see Fig. 2(b)); the basins of attraction of A_1 and A are separated by the stable branch of S;

iii) if the pollution tax is too high with respect to the impact of the external sector on pollution, then the stationary state A^1 becomes globally attractive, and the external sector tends to disappear (see Fig. 2(c));

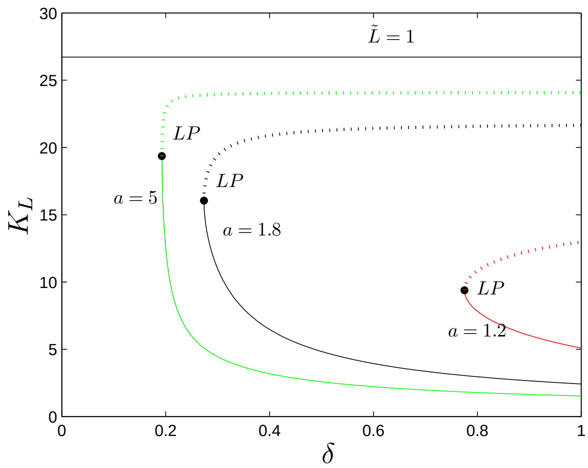
iv) two threshold values of the pollution tax exist, one such that the economy specializes in the local sector, and the other such that the external sector disappears (see Fig. 3(a));

v) a threshold value of the impact of the external sector on pollution exists, such that both sectors coexist (see Fig. 3(b)).

These results are intuitive, and can be explained by the roles of the pollution tax and the impact of production activities of the external sector on pollution. A pollution tax low enough with respect to the impact of the external sector on pollution attracts foreign direct investments, since it enters as a cost in the maximization problem (3), and the stationary state with specialization does not exist (see Fig. 3(a)). On the contrary, when it is high enough with respect to the impact of the external sector on pollution, then for the External investors is more rewarding to move their capital outside the local economy and reduce K_E, which eventually goes to zero, so that the economy ends up fully specializing in the local sector (see Fig. 2(c) and Fig. 3(a)). Finally, the revenues coming from the pollution tax are used to finance environmental defensive expenditures (see Eq. (6)). In



(a) $\delta = 0.5$.



(b) $\tau = 0.18$.

Fig. 3. Bifurcation diagrams.
Parameter values: $A = 1$, $B = 2$, $b = 2$, $\alpha = 0.65$, $\beta = 0.35$, $r = 0.1$, $s = 0.6$, $\eta = 1$, $\varepsilon = 0.55$, $\gamma = 0.19$.

fact, if the pollution tax is low enough with respect to the impact of the external sector on pollution, then the pollution level is relatively high (see Fig. 2(a)). However, if it is high enough with respect to the impact of the external sector on pollution, then the pollution level is relatively low (see Fig. 2(b)).

Otherwise, a high enough impact of the external sector on pollution with respect to the pollution tax attracts foreign direct investments (see Fig. 3(b)). This result may confirm the pollution haven hypothesis, i.e., the external firms relocate part of their production to the countries where the environmental standards are less stringent, and, therefore, with the possibility of increasing the pollution intensity.

6. Welfare of local land owners

In this section we compare the revenues of L-agents at A^1 and at stationary states in which both sectors coexist. The remuneration of capital K_E invested by the representative E-agent is $r_K K_E$ while the revenues of the representative L-agent are given by:

$$\Pi_L(P, K_L) = \bar{A} K_L^{\alpha} L^{1-\alpha} + r_K(1-L) =$$

$$= \begin{cases} \alpha\Gamma^{1-\alpha}\left(\bar{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} + (1-\beta)\left(\bar{B}(1-\tau)\right)^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\right)^{\frac{\beta}{1-\beta}} & if\ K_L < \bar{K}_L(P) \\ \bar{A}K_L^{\alpha} & if\ K_L \geq \bar{K}_L(P) \end{cases}$$

Therefore, the revenues of the representative L-agent in $A^1 = (P, K_L) = \left(0,\ \left(\frac{sA}{\gamma}\right)^{\frac{1}{1-\alpha}}\right)$ are equal to:

$$\Pi_L(A^1) = \Pi_L\left(0,\ \left(\frac{sA}{\gamma}\right)^{\frac{1}{1-\alpha}}\right) = A^{\frac{1}{1-\alpha}}\left(\frac{s}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}$$

The effects generated by the external investments on welfare of L-agents can be better understood by comparing the dynamics generated by the two-sector model considered in this paper with the one-sector dynamics that would be observed in absence of External investors:

$$\dot{K}_L = sAK_L^{\alpha} - \gamma K_L \tag{24}$$

According to the one-sector dynamics (24), the state $K_L = \left(\frac{sA}{\gamma}\right)^{\frac{1}{1-\alpha}}$ is always a globally attractive stationary state and corresponds to the stationary state A^1 of the two-sector model, when existing. We shall compare the revenues of L-agents obtained at the stationary state A^1 with those obtained at a generic state (P, K_L) where both sectors coexist. Observe that $\Pi_L(A^1) < \Pi_L(P, K_L)$ holds if and only if the P and the K_L satisfy the condition:

$$A^{\frac{\alpha}{1-\alpha}}\left(\frac{s}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} < \alpha\Gamma^{1-\alpha}\left(\bar{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} + (1+\beta)\left(\bar{B}(1-\tau)\right)^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\right) \tag{25}$$

Setting:

$$A^{\frac{\alpha}{1-\alpha}}\left(\frac{s}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} = \alpha\Gamma^{1-\alpha}\left(\bar{A}K_L^{\alpha}\right)^{\frac{1}{\alpha}} + (1+\beta)\left(\bar{B}(1-\tau)\right)^{\frac{1}{1-\beta}}\left(\frac{\beta}{r_K}\right)$$

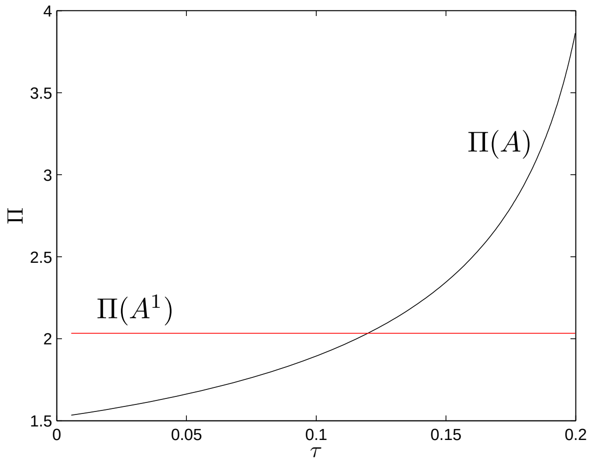
we obtain the indifference curve (IC):

$$\alpha\Gamma^{1-\alpha}(\widetilde{A}K_L^{\alpha})^{\alpha} + (1-\beta)(\widetilde{B}(1-\tau))^{\frac{1}{1-\beta}}\left(\frac{\beta}{1-\beta}\right)^{\frac{\beta}{1-\beta}} - AK_L^{\alpha} = 0 \tag{26}$$

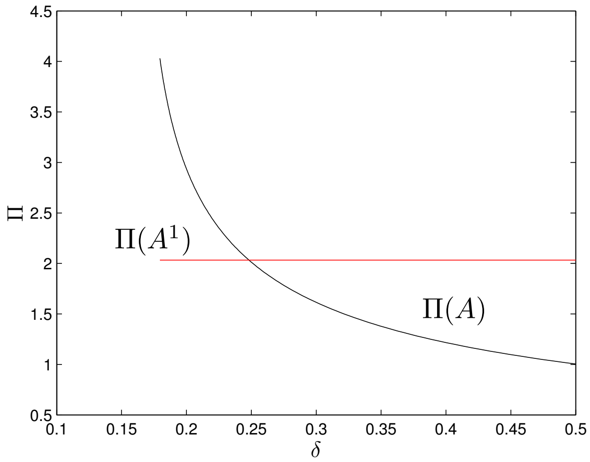
with $\Pi_L(A^1) < \Pi_L(P, K_L)$ (respectively, $\Pi_L(A^1) > \Pi_L(P, K_L)$) if the state (P, K_L) lies above (below) it, in the plane (P, K_L). The following proposition holds.

Proposition 4. *The revenues of L-agents, evaluated at a generic point (P, K_L) where both sectors coexist, are greater than in A^1 (i.e., $\Pi_L(A^1) < \Pi_L(P, K_L)$), if the point (P, K_L) lies above the indifference curve (26). Conversely, if the point A lies below the indifference curve (26), then $\Pi_L(A^1) > \Pi_L(P, K_L)$.*

According to Proposition 4, numerical simulations show that if the pollution tax is high enough with respect to the impact of the external sector on pollution, then a welfare-improving growth path (i.e., $\Pi(A^1) < \Pi(A)$) may occur (see Fig. 4(a)). On the contrary, if the pollution tax is low enough with respect to the impact of the external sector on pollution, then a welfare-reducing growth path (i.e., $\Pi(A^1) > \Pi(A)$) may occur (see Fig. 4(a)). Moreover, a welfare-improving growth path may also occur if the impact of the external sector on pollution is high enough with respect to the pollution tax (see Fig. 4(b)).



(a) $\delta = 0.2$.



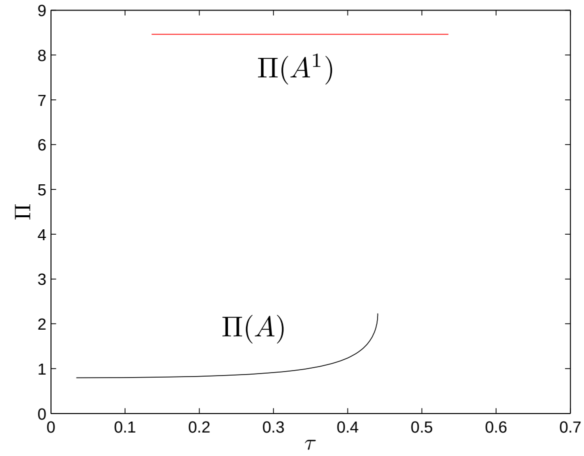
(b) $\tau = 0.18$.

Fig. 4. Welfare analysis: relationship between revenues of local land owners and pollution tax **(a)**, and impact of the external sector on pollution **(b)**. *Parameter values:* $A = 1$, $B = 2$, $a = 5$, $b = 2$, $\alpha = 0.65$, $\beta = 0.35$, $r = 0.1$, $s = 0.6$, $\eta = 1$, $\varepsilon = 0.55$, $\gamma = 0.19$.

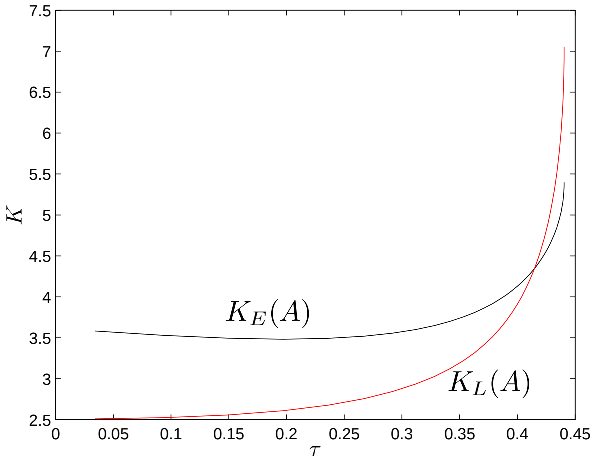
From numerical simulations shown in Figs. 5(a) and 5(b), and Figs. 6(a) and 6(b), we can infer to the following main results[6]:

i) an increase of the pollution tax may have positive effects on $\Pi(A)$ (see Fig. 5(a)) and on both local and external ca-

[6] Notice that the revenues of L-agents at stationary state in which the economy is specialized in the local sector ($\Pi(A^1)$) are invariant to an increase of the pollution tax or of the impact of the external sector on pollution (see Fig. 5(a) and Fig. 6(a)).

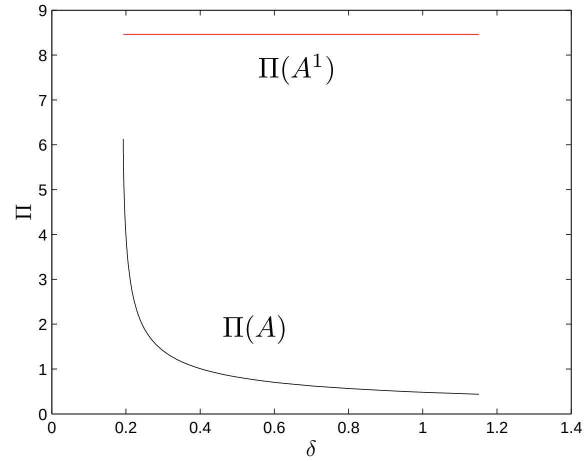


(a) $\delta = 0.5$.

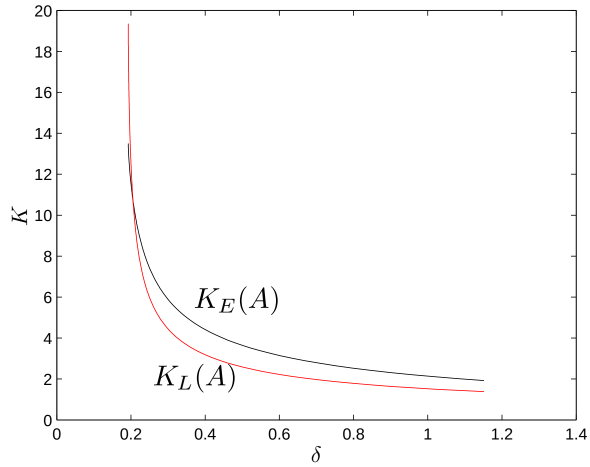


(b) $\delta = 0.5$.

Fig. 5. Welfare analysis: relationship between pollution tax and revenues of local land owners **(a)**, and both capitals **(b)**.
Parameter values: $A = 1$, $B = 2$, $a = 5$, $b = 2$, $\alpha = 0.65$, $\beta = 0.35$, $r = 0.1$, $s = 0.6$, $\eta = 1$, $\varepsilon = 0.55$, $\gamma = 0.19$.



(a) $\tau = 0.18$.



(b) $\tau = 0.18$.

Fig. 6. Welfare analysis: relationship between impact of the external sector on pollution and revenues of local land owners **(a)**, and both capitals **(b)**.
Parameter values: $A = 1$, $B = 2$, $a = 5$, $b = 2$, $\alpha = 0.65$, $\beta = 0.35$, $r = 0.1$, $s = 0.6$, $\eta = 1$, $\varepsilon = 0.55$, $\gamma = 0.19$.

pitals (see Fig. 5(b)), since decreases the pollution level, and, therefore, increases the productivity of both sectors;

ii) an increase of the impact of the external sector on pollution may have negative effects on $\Pi(A)$ (see Fig. 6(a)) and on both local and external capitals (see Fig. 6(b)), since increases the pollution level, and, therefore, decreases the productivities of both sectors;

In summary, if the pollution level is relatively high, then the productivity of both sectors decreases. Therefore, the revenues of Local land owners decrease and External investors move their capital outside the economy. This does not occur only if the pollution tax is high enough and the impact of the external sector on pollution is low enough, respectively (see Figs. 4(a) and 4(b)).

7. Conclusions

The foreign direct investments in land have increased substantially in the last ten years, and have recently been object of several empirical studies. However, to our knowledge, there is not yet a satisfactory theoretical model to explain them. The paper has investigated the possible effects of FDI in land on a small open economy with two sectors, external and local, and heterogeneity of agents, external investors and local land owners. Both sectors are negatively affected by pollution, but only the external sector is polluting. Hence, we assume the possibility for the local government to tax the production activities of the external sector to finance environmental defensive expenditures.

Numerical simulations show that the dynamics of the model may be bi-stable. The stationary states in which there is specialization in the local sector and in which both sectors coexist are locally attractive. The basins of attraction of such states are separated by the stable branch of a saddle point. However, from numerical simulations it emerges that if the pollution tax is low enough with respect to the impact of the external sector on pollution, then the specialization in the local sector does not occur and local agents have to support relatively high values of pollution. On the contrary, if the pollution tax is high enough with respect to the impact of the external sector on pollution, then the specialization in the local sector may occur and local agents have to support relatively low values of pollution.

A welfare-improving growth path may occur only if the pollution tax is high enough and the impact of the external sector on pollution is low enough, respectively. In such a context, the revenues of local agents at the stationary state in which there is coexistence between sectors are greater than at the stationary state in which the economy is specialized in the local sector. However, on the contrary, if the pollution tax is low enough and the impact of the external sector on pollution is high enough, respectively, then a welfare-reducing growth path may occur.

Foreign direct investments are attracted by a low enough pollution tax and a high enough pollution intensity, respectively. However, these parameters have positive effects on pollution level and, therefore, negative effects on productivity of both sectors. Therefore, a policy oriented to attract polluting FDI decreases the welfare of local agents and for external investors is more rewarding to move their capital outside the local economy in the long run.

On the contrary, a high enough pollution tax and a low enough impact of the external sector on pollution, respectively, can protect local capital accumulation and defend local agents from environmental degradation. Moreover, due to the relative low negative effects on both sector, for external investors is more rewarding to keep their capital inside the local economy. Therefore, an environmental policy implemented with a high enough pollution tax can exploit the FDI to increase the welfare of local agents.

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