UNIVERSITÀ
DEGLI STUDI
FIRENZE

DOTTORATO DI RICERCA IN
INGEGNERIA INDUSTRIALE

CICLO XXVII

Towards efficient multidisciplinary design optimization for car body structures

Settore Scientifico Disciplinare ING/IND-14

Candidato

Ramses Sala

Tutori

Prof. Marco Pierini
Prof. Niccolò Baldanzini

Controrelatori

Prof. Fabian Duddeck
Prof. Kazuhiro Saitou

Coordinatore del Dottorato

Prof. Maurizio De Lucia

Anni 2012/2015

ISBN XXX–XX–XXXX–XXX–X
D/XXXX/XXXX/XX

Through the effort and patience of my family and teachers,
with gratitude to them

Summary

Automotive transportation plays an important role in most industrialized countries. The automotive industry receives stimuli by legislators and public demand to develop cars satisfying high standards with respect to energy efficiency, emissions, and safety. At the same time, the car manufacturers and suppliers are subjected to strict cost and time constraints, due to the competitive free market system.

The developments in the fields of computer technology and numerical methods contributed to the implementation of computer-aided design and simulation tools in structural engineering, and vehicle design. Computational simulation models of systems and subsystems are widely used in the vehicle design process, and provide means to improve the "time to market", and reduce the number design iterations and prototype testing on a part, system and module level. When several or many multidisciplinary design requirements are involved, it remains however a great challenge to efficiently obtain a good design, even with the help of structural simulations. There is an industrial demand for research on the analysis, selection and development of numerical optimization methods that can aid the design of complex systems or structures.

The aim of the here presented research activity is to contribute to the identification and development of efficient strategies for multidisciplinary design optimization of vehicle structures involving, crashworthiness, vibro-acoustic and lightweight design criteria. The literature survey at the start of this activity, identified: that although a large variety of optimization strategies and methods are described in the literature, only few comparisons or guidelines are available for the selection of efficient optimization algorithms or methods for vehicle optimization related problems, involving the mentioned design criteria.

In this work, several state of the art optimization algorithms for multidisciplinary design optimization have been selected and are systematically compared for their efficiency on applications that typically occur within a car body design optimization context. Although these comparisons mainly involved existing methods, the resulting comparisons on the industrially relevant application of vehicle design related optimization problems extended the currently available literature. The results could serve as initial guidelines for practitioners in industry and as a starting point for further research.

In the optimization literature, there are many test functions/problems available that are commonly used for comparative assessments of optimization algorithms. These test problems are however difficult to relate to industrially relevant problems and vice versa. A novel Representative Surrogate Problem approach is developed in the scope of this work, which could be summarized as a strategy to construct optimization test problems, based on response characteristics of real-world problems. The approach is presented and investigated for its application to car body design problems.

Inspired by the response characterization strategies and results, a novel test function generation method based on the composition of random fields is presented. The resulting method is a contribution to the field op global optimization in general and not restricted to automotive applications. This method enables the construction of synthetic optimization problems with various interesting function features. Due to the parameterized nature of the method, these test functions enable structured investigations on the influence of particular problem features on the performance of optimization algorithms. Compared to existing test functions the method provides a further step towards systematic problem feature orientated performance analysis of meta-heuristic optimization methods, which contributes to the analysis, selection and development of optimization algorithms for non-convex optimization problems.

The overall results of the performed comparisons and case studies with the developed methods showed that significant gains in optimization efficiency can be achieved by selecting suitable optimization algorithms, and tuned parameter settings for optimization problem formulations relevant to car body design. The comparison results, stressed the need to take into account optimization efficiency, whereas in many case studies in the literature, optimization algorithms are selected without proper justification. The presented results and methods are relevant for practitioners in industry and for further research on the development of optimization algorithms for complex problems.

Table of contents

List of figures

List of tables

Acronyms List

AAO	All-at-once
ABS	Anti-lock Braking System
ATC	Analytical Target Cascading
BIP	Body in Prime
CAD	Computer-Aided Design
CAE	Computer-Aided Engineering
CAM	Computer-Aided Manufacturing
CPU	Central Processing Unit (computer processor)
CSP	Constraint Satisfaction Problem
DE	Differential Evolution
DOE	Design of Experiments
DRF	Discrete Random Field
EA	Evolutionary Algorithms
ESP	Electronic Stability Program
FEM	Finite Element Method
FFA	Fire Fly Algorithm
FFT	Fast Fourier Transform
FRF	Frequency Response Functions
GA	Genetic Algorithm
GHG	Green House Gas
GRESIMO	Green and Silent Mobility
GSA	Global Sensitivity Analysis
HEB	High-dimensional Expensive Black-box
IAAFT	Iterative Adjusted Amplitude Fourier Transform
IDF	Individual Discipline Feasible
IIHS	Insurance Institute for Highway Safety
IP	Interior Point
MAC	Modal Assurance Criterion
MATLAB	MATrix LABoratory (software program)
MBS	Multi-body Simulations
MDO	Multidisciplinary Design Optimization
MHA	Meta-heuristic algorithms
MOP	Multi Objective Problem
MP	Mathematical Programming
NCAP	New Car Assessment Program
NFL	No Free Lunch (theorem)

NHTSA	National Highway Traffic Safety Administration
NSGA	Non-dominated Sorting Genetic Algorithm
NVH	Noise Vibration and Harshness
OC	Optimality Criteria
OFAT	One-factor-at-a-time
PRM	Pseudo-Random Mapping
PRNG	Pseudo-random number generator
PSO	Particle Swarm Optimization
RBF	Radial Basis Function
RDO	Robust Design Optimization
ROI	Relative Objective Improvement
RSF	Representative Surrogate Function
RSM	Response Surface Method
RSP	Representative Surrogate Problem
RSS	Representative Surrogate System
SA	Simulated Annealing/Sensitivity Analysis
SQP	Sequential Quadratic Programming
SSAS	Single Sided Amplitude Spectrum
TFAT	Two-factor-at-a-time
TPA	Transfer Path Analysis

Preface

"I realized that no one can discover everything God is doing under the sun. Not even the wisest people discover everything, no matter what they claim"

-Solomon 970-931 B.C

This dissertation is submitted in partial fulfillment of the requirements for the degree of "Dottorato di Ricerca" at the University of Florence. All of the work presented henceforth was conducted at the Department of Industrial Engineering at the University of Florence. The majority of the research activity of the author was carried out in the scope of the Marie Curie Project "GRESIMO" (GREen and SIlent MObility), funded by the European Commission in the 7th Framework Program.

This work is to the best of my knowledge original, except where acknowledgements and references are made to previous work. Neither this nor any substantially similar dissertation has been submitted for any degree, diploma or other qualification. Parts of the research activity and this work have been submitted together with my supervisors, as separate publications:

(1) Ramses Sala, Niccolò Baldanzini, Marco Pierini. *An intuitive variance based variable screening method.* Poster session presented at the MascotNum workshop, Computer Experiments and Meta-models for uncertainty quantification; 2014 April 23-25; Zurich, Switzerland

(2) Ramses Sala, Marco Pierini, Niccolò Baldanzini. *The development and application of tailored test problems for meta-simulation of multidisciplinary optimization of vehicle structures.* Presentation at the (XI) World Congress on Computational Mechanics; 2014 July 20-25; Barcelona, Spain

(3) Ramses Sala, Marco Pierini, Niccolò Baldanzini. *Optimization efficiency in multidisciplinary vehicle design including NVH criteria.* Proceedings of the Leuven Conference on Noise and Vibration Engineering (ISMA); 2014 September 15-17; Belgium

(4) Ramses Sala, Niccolò Baldanzini, Marco Pierini. *Representative Surrogate Problems as test functions for expensive simulators in multidisciplinary design optimization of vehicle structures.* In Press, Structural Multidisciplinary Optimization, Springer 2016

(5) Ramses Sala, Niccolò Baldanzini, Marco Pierini. *On variable screening and optimization of Car Body Structures subject to multidisciplinary constraints*. Presentation at the International Conference on Engineering Vibration Ljubljana, Slovenia, 7-10 Sept. 2015.

(6) Ramses Sala, Niccolò Baldanzini, Marco Pierini. *Global optimization test problems with higher order interactions based on random field composition.* In Press, Optimization Letters, Springer 2016

1. Introduction and motivation

"Fat men cannot run as fast as thin men, but we build most of our vehicles as though dead-weight fat increased speed! Saving even a few pounds of a vehicle's weight ... could mean that they would also go faster and consume less fuel. Reducing weight involves reducing materials, which, in turn, means reducing cost as well."

"There is one rule for the industrialist and that is: Make the best quality of goods possible at the lowest cost possible, paying the highest wages possible."

-Henry Ford 1923

1.1. Transportation and its impact on health and the environment

Since these words of Mr. Ford many things have changed, but some of the principles are timeless, and still apply to the multidisciplinary challenges in the automotive industry today. Most of the earth's human population presently lives in a society where transportation has a huge influence on life and environment. The influence of transportation extends to various aspects of life: such as social and settlement habits on local and global scales. Mobility affects how we spend our time and resources, and it influences our health and climate:

- In 2013, 25.8% of the total energy use in the EU area (28 countries) was attributable to the road transportation sector [Eur15a, Eur15b]. Road transport is also the biggest contributor to transport related Greenhouse Gas (GHG) emissions and their potential future growth.
- In China in 2010 the Years of life lost due to Road traffic accidents exceeded the losses due to Lung Cancer [Yan13].
- Road traffic accidents, not AIDS, cancer, or any other disease - are the major cause of death for 15-19-year-olds worldwide [EdL07].
- "At least one million healthy life years are lost every year from traffic-related noise in the western part of Europe. Sleep disturbance and annoyance, mostly related to road traffic noise, comprise the main burden of environmental noise" [WHO12].
- In the US, the total greenhouse emissions of the transportation sector exceeded those of the Industrial sector [USE15].

Transportation can however also bring many benefits. Therefore, it is natural to strive to sustainable transportation systems that minimize negative social and environmental impacts. Following the definition articulated in [WCE87] the term ‘sustainable transport’ means transport that meets the needs of the present without sacrificing the ability of future generations to do the same. In a free market economy as currently present in most of the western world, also financial aspects have to be considered. The transportation sector has been described as a ‘*complex and porous social technical and economic system*’ that is hard to address comprehensively [Gld06]. Many of the involved challenges have been identified, and various type of targets have been set on political [WCE87, KYO97, EUP14], corporate [WBC04] and research [TRB97] board levels during the last decades. Achieving a transportation system that meets such targets requires planning, methodology and actions, by the transportation users, suppliers, and responsible legislators.

An important target is the reduction of GHG emissions caused by the transportation sector. In [Cce15] several directions to establish this have been addressed:

1. Fuel transition. By using biofuels or other low-carbon energy sources such as electricity produced from renewable sources, GHG emission can be reduced.
2. Efficient transportation technology. The development of alternative vehicle designs that are more energy efficient can reduce the energy usage and GHG emissions due to the transport sector
3. Increasing transportation system efficiency. By traffic monitoring, using modern information technology systems and mobile communication techniques, traffic congestion can be avoided, and the efficiency of the road usage can be increased, leading to savings in time, energy and emissions.
4. Reducing vehicle travel demand, by changing the travel habits and means of travel. Such as for example, the substitution of vehicle by walking, biking, or rail transport energy usage and GHG emissions can be reduced.

Since transportation is a complex sector, that is interacting with local infrastructure and customs, many different approaches [Shi13, Arf13, Gud13] are proposed, applied and evaluated for all of these directions, often from a regional perspective. For further analysis and descriptions of the challenges in sustainable transport in general, is referred here to [Nkp94, Gre97, Ric05, Gld06] and [Eli15].

Another important target is safety improvement. A general analysis regarding road transport safety was presented in [WHO04]. The proposed interventions can be roughly divided in the following categories:

A. Managing risk exposure through effective transport and land-use policy making
B. Securing compliance with safety regulations
C. Shaping the network for injury prevention
D. Technological improvement of active and passive safety of transportation vehicles
E. Providing effective post-accident care

The realization of interventions to improve safety is driven by legislation and public demand. Presently revisions to the EU General Safety Regulation 2009/661 are considered [ETSC15]. In addition, the publication of voluntary vehicle safety assessment results (such as EURO NCAP), result in increased consumer awareness for crash safety and strongly stimulate car manufacturers to develop safer cars.

Although many of the problems are identified, and targets have been set, it remains a great challenge to achieve the targets. Innovative approaches, methods, and technologies are required to reach successful transportation systems. Scientific research can partly contribute to this goal by providing paradigms, theories, methods, and technology that can aid to the realization of more sustainable transport.

From the far past until the present, transportation related topics were and are of great interest in various scientific communities. The achievements ranging from: the invention of the wheel thousands of years ago, up to the recent spacecraft landing Philae on the Comet 67P in November 2014, all involved transport related science. Besides politics and practical craftsmanship, also mathematical methods can contribute to the challenges involved with transportation. Examples range from the treatment of the “traveling salesman” (graph theory) problem by Hamilton and Kirkman [Big76] in the nineteenth century, to development of artificial intelligence programs to control driverless cars. And from Newton’s “Principia”, [New1726] describing the laws of motion, that are still the basis for state-of-the-art numerical simulation techniques for vehicle dynamics and crash simulation, to the principles of chaos theory [Pon1890] of which the application extends to among others the field of Computational Fluid Mechanics used aerodynamic for optimization of car and airplane designs.

The work presented in this thesis was partially done in the scope of the GRESIMO project funded by the European Commission, which was aimed to set research and development steps toward green and silent mobility for passenger vehicles. Therefore from the variety of different means of transportation available, this work is primarily aimed at automotive transportation and passenger vehicles in particular.

Of the different directions proposed in [Cce15] and [WHO04], to set steps towards sustainable and safe mobility, the work in this thesis is related to the selection and development of efficient numerical optimization methods for the design and development process that targets technical improvements in the transport vehicles. It should however be noted, that the essence of the presented investigations and methodologies could also be relevant to optimization of other complex structural design problems such as aircraft design, but such applications are not directly treated within this work. The following section will discuss the selected challenges in structural vehicle design optimization in more detail.

1.2. Targeted challenges in multidisciplinary design optimization of car body structures

The automotive industry receives direct and indirect stimuli by European legislation and public demand to reduce emissions, improve vehicle safety and fulfill noise regulations for production vehicles. At the same time, manufacturers strive to shorten the product development times, product costs and development costs, to be competitive in the automotive market. Computer-aided design, numerical simulation techniques, and structural optimization methods are key enablers to achieve these goals in the vehicle development process [Sai05, Hir13].

There are many engineering sub-disciplines involved with the development of a passenger vehicle [Hap01]. Examples are: aerodynamics, structural integrity, vehicle dynamics, acoustics, ergonomics, control engineering and electrical engineering to only name a few. The work of this thesis will be focused on numerical methods for applications related to structural engineering requirements of the car body design.

An important objective in structural engineering is the analysis of the mechanical resistance of structures to achieve a design that satisfies the functional requirements. The task of the engineers is to apply scientific methods and engineering knowledge to create a feasible design satisfying the structural criteria, and other design objectives and constraints. In the scope of car body design in an industrial context, not only the physical realization or manufacturability of a single component or product is to be regarded, but also the industrialization of the manufacturing and assembly process should be taken into account. From an industrial engineering point of view, an automotive manufacturer does not only produce cars, but coordinates the development of factories, production and assembly lines that produce the final consumer product.

In the first part of this section, a brief overview is provided on the targeted structural engineering requirements and design criteria for the car body design that will be considered in this work. The second half of this section gives a short overview on the selected numerical methods for Multidisciplinary Design Optimization (MDO).

1.2.1. Design criteria

The automotive car body is designed to comply with a wide variety of mechanical loadings, and related safety, quality and comfort criteria [Web14]. From creep resistance of composite structure components during quasi-static loadings, to the fatigue life of spot-welds during stochastic transient vibration loads. From vibration amplitude restrictions at the central rear mirror to energy absorption due large plastic deformations during a frontal crash, to only address a few criteria related to different engineering disciplines. Other design objectives and constraints can involve styling, economical, legal, environmental aspects.

In the research task description of the GRESIMO project (which was the main funding source for this activity), it was targeted to make a scientific contribution to industrially relevant MDO problems that deal with weight, crashworthiness and vibro-acoustic criteria of automotive vehicle structures. These car body design criteria are of high relevance to the performance regarding acceleration, energy efficiency and consumer quality perception of the vehicle. Besides the design criteria related to weight, passive safety, and vibrational comfort also other design criteria are of importance for the car body design, and no claim is made that this subset of criteria is of higher importance than other criteria. In agreement with the aims of the funding project and the consortium industry partners the

MDO problems investigated in this activity however only involve design criteria and response types from these disciplines. The following subsections provide a brief overview of the relevance of the selected criteria and the involved challenges.

Weight Reduction

The mass of automotive vehicle structures is high compared to the passenger weight. According to [Fen01] only a small fraction of fuel energy is used in hauling the driver for typical car masses. Weight reduction is one possible way to reduce the energy consumption and GHG emissions of passenger vehicles. Besides the direct effects of weight reduction of a structural component, there are also beneficial secondary effects. Significant reductions in car body mass also allow for modifications or reductions in other components such as for example the drive train, leading to additional mass reduction and increased energy efficiency while maintaining a constant power-to-weight ratio, or other dynamic elasticity targets [Pag06, Kof10].

Lightweight design can be defined as a design paradigm in which the objective is to design a structure with minimum structural mass while satisfying the structural requirements and other constraints involved. When multidisciplinary structural requirements that involve nonlinear responses are regarded (such as crashworthiness and vibrational comfort in the case of car body design), lightweight design becomes complex and challenging.

Safety and Crashworthiness

Automotive vehicle safety strategies can be roughly classified into active safety, and passive safety. The importance of the first category has increased drastically during the last decades due to the developments of electronic safety systems (such as ABS (Anti-lock Braking system) in the 70’s, and ESP (Electronic stability program) in the 90’s) [Yu08]. The European Transport Safety Council was founded in 1993 with the aim to provide objective advice on transport safety matters to governing bodies within the EU. By the year 2010, the forthcoming regulations reduced the number of road deaths since 2001 by 42% in the EU [ETSC2011]. Despite the advances in vehicle safety, motor vehicle accidents still have a significant impact on the society. During 2010, in the United States, motor vehicle crashes were responsible for 33 thousand fatalities, 3.9 million injuries, and 24 million damaged vehicles. The involved economic cost, when also “quality of life” valuations are regarded, summed to 836 billion Dollar [Bli15]. In comparison, the number of fatalities in the EU in 2014 was still 25.7 thousand, and was higher than the target intended [ECP15].

The Euro NCAP (New Car Assessment Program) is a voluntary vehicle safety assessment program that publishes safety reports on new cars regarding their passive safety performance under specific conditions. These test results are openly available for the public and are often referred to by popular automotive consumer magazines. The increased consumer awareness for crash safety, the interest of insurance companies and legislators, strongly stimulate car manufacturers to develop safer cars. Besides the need for further improvements in the field of active safety, also passive safety which embodies vehicle crashworthiness performance remain of high significance. Vehicle crashworthiness criteria, set strict structural requirements on the car body design and therefore contribute significantly to the vehicle mass [Ben14].

In the Euro NCAP [NCA15] frontal crash load cases are included that typically involve large structural deformations The vehicle responses for such load cases are generally highly nonlinear w.r.t. changes in vehicle design parameters, due to the phenomena involved such as buckling, plasticity, and contact and fracture. Experimental crash safety performance assessment is expensive, especially during the prototype stage. Although numerical simulation techniques are available to evaluate the influence of design changes to the vehicle responses, there remain many challenges due to the complex high nonlinear nature of the physical phenomena, and the high computation cost involved.

Noise and Vibration comfort

According to [WHO12] relationships exist between noise and specific health effects (cardiovascular disease, sleep disturbance, cognitive impairment, and annoyance). “The results indicate that at least one million healthy life years are lost every year from traffic-related noise in the western part of Europe.” [WHO12]. Tire rolling induced vibrations in automotive vehicle structures can resonate through the driver body and exposure may cause muscle fatigue and back injuries [Nah09].

International standards such as the ISO 2631 aim to regulate the admissible noise and vibration levels for different time durations and standardize the measurement methods. In the consumer vehicle industry, the targeted comfort criteria are however generally much stricter than the legislative criteria, since most consumers consider vibrational comfort of high importance for their quality perception. Besides on human annoyance and fatigue, structural car body vibrations are also related to structural durability. The field of NVH and related car body design criteria is wide and still receives high attention from industry and academia.

1.2.2. Computer-aided multidisciplinary design analysis and optimization methods

The vehicle design criteria from the selected disciplines (weight, crashworthiness and vibrational comfort) can be experimentally assessed on the final physical product. It is however of industrial relevance to estimate and assess, the design criteria and structural responses during the design process. This can be done by experimental testing on prototypes, or by the use of numerical simulation models.

In traditional system development paradigms: design, calculation, and testing were distinct activities that iteratively lead to the final product. The developments during the last decades in the fields of computer technology and numerical methods, contributed to the implementation of Computer-Aided Engineering (CAE) tools, in structural engineering and vehicle design [Tho95, Odn03, Hir13].

The developments of Computer-Aided Drafting (CAD), numerical simulation methods such as the Finite Element Method (FEM), and Computer-Aided Manufacturing (CAM), not only increased the effectiveness of the individual design, calculation, manufacturing and testing activities, but also the interfaces in between them. The integration of these CAE methods in the product development process aims to reduce the number of physical prototypes, the number of design iterations and shortening the development times. In order to achieve that, research and development on more accurate and computationally efficient simulations methods are currently active topics in science and engineering. In this activity several state-of-the-art numerical simulation methods are used for the evaluation of the structural performance of car body design variants. An overview of used vehicle models and numerical methods is provided in chapter 2. The challenges that accompany the application of these simulation types are the involved computational cost, and the nonlinear responses, in particular for crashworthiness simulation.

CAE also includes the application of numerical optimization and design analysis methods in the engineering process. The methodological focus of this thesis is on multidisciplinary design optimization. Other numerical methods such as FEM and meta-modeling are used and applied but not further analyzed in the scope of this work.

Multidisciplinary Design Analysis and Optimization

When the geometry, materials, and loads are appropriately modeled, numerical techniques such as the Finite Element Method can give useful estimations of the structural response and resistance. Whereas the design process flow of the computer design model to the simulation-based mechanical response can be automated straight forward, the feedback mechanism in the traditional design process involves human designers to make the design modifications based on the simulation results. When several or many multidisciplinary design requirements are involved, and the simulation responses are complex it, remains challenging to obtain a good designs, even with the help of structural simulations [Sai05].

Together with the developments in computer systems and numerical simulation methods, notable developments on computational optimization methods for structural optimization were developed in the second half of the last century. Early numerical investigations on evolutionary computation in 1950's [Bar54, Bar57] were followed by the development of evolution process based programs and algorithms to solve more general mathematical optimization problems [Bre62]. Soon thereafter, these ideas were used for the design optimization of technical systems [Schw65, Rch71]. Since then, many other nature-inspired optimization algorithms, and meta-heuristic algorithms have been developed and applied to industrial optimization problems [Hol75, Kir83, Ken95, Sto97]. Surveys on such methods are given in [Flo09, Tng09, Rio12].

From an industrial and academic perspective it is of great relevance to deal with design problems that involve design criteria from multiple disciplines simultaneously [Sbi95, Agt10]. The aim in Multidisciplinary Design Optimization (MDO) is: to optimize the design w.r.t. the objective while satisfying all other design requirements at the end of the optimization procedure, or design process. In the general case (which applies to the design of many complex structures), it is much harder to find or establish a design that deals with all requirements, than to deal with the individual requirements separately.

During the last decades various works have been published considering the application of MDO methods to car body design related problems involving lightweight design, NVH or crashworthiness responses [Yng01, Sbi01, Dud08, Lia08, Yil12, Abb14]. There remain however many challenges for the application of MDO methods for this application with the selected disciplines. Especially the crashworthiness criteria pose a challenge due to the highly nonlinear responses and high computational cost. Briefly summarized, these problems are difficult due to: the high dimensionality of the search space, the nonlinear responses, and high computational cost of the simulation responses. More details on the challenges, used vehicle models, load cases and design criteria are provided in chapter 2. These challenges are however not only relevant for the selected application, but are shared among complex product and system design optimization problems, and categorized as HEB problems (High-dimensional, Expensive (computationally), Black-box) [Sha10].

The "no free lunch theorems for search" [Wol95] implicate that averaged over all possible optimization problems search algorithms perform equally "well" or "bad". On particular problem types, some algorithms could however perform better than others. Although many different meta-heuristic optimization methods have been proposed and many applications of such methods to particular problems related to car body design have been studied, only very few significant comparisons have been made to select efficient algorithms for these particular problems. This observation has also been addressed in recent literature [Sha10, Wan13]. The literature survey in [Sha10] exposed that direct modeling and optimization strategies to address HEB problems are "*scarce and sporadic*". The review also revealed that research trends tend to focus on sampling and modeling techniques themselves and neglect to investigate the characteristics of the underlying expensive functions [Sha10]. In [Wan13] it was emphasized that there are not enough comparative assessments that could help to choose from the many available algorithms for simulation optimization problems.

All in all, these observations confirmed a statement made a decade earlier in [Fu02] that there is a gap between the "toy" test problems often used in theory and development of algorithms and the complexity of complicated real-world problems.

1.3. Scope and aim of the thesis

This thesis aims to contribute to the selection and development of effective optimization and analysis methods to improve the efficiency of the multidisciplinary car body design process involving weight, crashworthiness and vibrational comfort criteria.

The described investigations and developments are made and tested for this particular application type. Although the results are application dependent, many of the methods used are general and not limited to the selected application. Therefore the content of this thesis could also be of interest to others who deal with multidisciplinary design analysis and optimization methods involving computationally expensive black-box functions with nonlinear responses.

The main contributions of this thesis work are:

1. **A meta-model based comparative assessment on the performance of optimization algorithms for car body design problems, involving lightweight, vibrational comfort and crashworthiness criteria.** The few significant meta-model based studies available in the literature, only use a single vehicle model per problem type, while the meta-models are based on few function evaluations. The presented assessment is based on investigations using models of various vehicles, while a larger number of construction points was used for the meta-models than in previous works. Furthermore, this is to the knowledge of the author the first comparative assessment for this application type where the meta-model based benchmark performance results are compared to the corresponding direct simulation-based benchmark results.

2. **The development of a novel Representative Surrogate Problem (RSP) approach to construct test problems for comparative assessments based on simulation responses related to car body design problems.** Multidisciplinary car body optimization problems with crashworthiness criteria are computationally expensive. Comparative assessments are orders of magnitude more expensive and therefore such studies are very scares in the literature. Meta-model based comparative assessments use a computationally cheap "black-box" approximation model to represent a black-box simulation model providing little insight into the problem characteristics. The new approach is based on numerical analysis of the optimization problem and simulation response characteristics and enables the construction of test problems based on the observed characteristics.

3. **The development of a new method based on random field composition to construct global optimization test functions with a wide variety of function characteristics.**
Compared with existing optimization test functions in the literature, the presented method can generate optimization test functions that are parameterized with respect to dimension, modality, variance contribution distribution, and interaction order. This enables more systematic analysis of

meta-heuristic optimization algorithms, which could lead to the development of more efficient optimization algorithms for real-world applications

Related to these contributions the following specific questions have been addressed:

1. **Are the relative optimization algorithm performances on a particular vehicle design problem correlated with the relative performance on a similar vehicle design problem involving another vehicle model?** No comparative study which used different vehicle models for a similar problem formulation is available in the literature. Therefore it is not known, how optimization performance on a particular car body optimization problem, is correlated with the optimization performance of a similar problem on a different vehicle model.

2. **How representative are meta-model optimization benchmarks for vehicle design problems compared to full direct simulation-based optimization performance benchmarks?** The few comparative assessments of optimization algorithm performance on problems involving crashworthiness responses are all based on meta-models or response surface models, in order to reduce the computational cost involved. It has however not been investigated how the characteristics of such models and the approximation errors affect the optimization algorithm performance w.r.t. the optimization performance on a simulation-based vehicle model.

3. **Are the differences in performance between meta-heuristic algorithms on various problem formulations of typical car body design optimization problems involving crashworthiness responses, of practical relevance?** Based on the current state of the art it is not clear how significant the performance differences between the optimization algorithms are for the typical optimization problems related to car body design w.r.t. the selected criteria, under a tight function evaluation budget.

4. **What are the characteristics of the simulation responses of the selected design criteria w.r.t. changes in the design variables? (Are there any typical response characteristics over similar problems involving different vehicle models?)** The simulation response characterization of some of the selected design criteria is computationally expensive, and to the knowledge of the author not published in the literature. In order to develop efficient optimization methods, it is required to understand the structure of the problem involved. Also, investigations on the differences and similarities between different problem instances or vehicle models are of interest.

5. **How to formulate computationally affordable test problems which are representative for simulation-based car body design optimization problems and their response characteristics?** The only computationally affordable surrogate test problems for crashworthiness responses are meta-model or response surface based. Although as approximations such models are often the a reasonable choice, such models replace a “black-box” simulation model with a “black-box”

approximation model and provide little insight in the response structure. Furthermore, these methods are interpolation based, and the responses between the construction points are smooth, whereas underlying simulation responses could be non-smooth (particularly in the case of crashworthiness responses).

6. **How to construct optimization test functions with relevant problem features, in a way that enables systematic performance analysis w.r.t. particular response characteristics?** Many of the commonly used optimization test problems have characteristics that have been criticized for their lack of realism and complexity. The construction of optimization test functions with a complexity and function structure features that are also present in real-world problems would be a step towards bridging the gap between the complexity of real-world optimization problems and the available test functions.

In the next chapters these questions will be referred to by the abbreviations Q1, Q2, ..., Q6. In the next chapter an overview is given of the methods and models used in this thesis, combined with an overview on the state of the art. In chapter 3 a comparative assessment between different optimization algorithms is made, using several optimization formulations and different vehicle models. In chapter 4 a new approach is presented to construct representative surrogate test problems with similar function characteristics as the simulation responses. An analysis of the simulation responses was performed on two vehicle models, and characteristics have been identified and quantified. A novel approach is proposed, which provides a way to incorporate the function characteristics in computationally affordable test functions. The approach is presented and tested using the application of car body design related optimization problems. In chapter 5 a new method is presented to construct global optimization test problems with parameterized function characteristics, based on random field composition. This method generalizes some aspects of the ideas in the approach of chapter 4, to construct global optimization problems with more realistic complexity, in a systematic way. In the final chapter a general summary and discussion of the results are presented together with overall conclusions and an outlook for further related work.

2. Literature overview and description of the used methods and models

"Taking a model too seriously is really just another way of not taking it seriously at all."

-Andrew Gelman [Gel09]

In this work several investigations and methodologies relevant to simulation-based optimization of car body structures are presented. Various methods related to simulation, meta-modeling, and optimization have been used, of which an overview is provided in this chapter.

The first section provides an overview of the available literature on the performance assessment of optimization algorithms; the topic is introduced in a general context, followed by a more focused perspective in the frame of car body optimization problems.

Finite Element Method based numerical simulation models have been used to estimate the structural responses of the design variants of several vehicle models. The vehicle models have been parameterized w.r.t. the selected design variables. A workflow is programmed to automatize the pre-processing, solving, and post-processing stages, which involved various computer programs, such that a program or function is established which returns the design responses as a function of the design variables. This workflow is then used for the function evaluations of the simulation responses, which are the basis for the meta-model construction points, optimizations, and response characterization.

The other sections of this chapter provide descriptions of the used methods and models, together with a brief overview of significant references on each of the topics, in the context of car body optimization problems. The application of new approaches and methods developed in the scope of this thesis are treated in subsequent chapters.

2.1. Literature review on optimization algorithm performance analysis for car body design problems.

2.1.1. Optimization algorithm performance analysis and testing

Genetic algorithms and other meta-heuristic algorithms (MHA) and have been applied to many complex search and optimization problems. In the last decades, there has been an explosion of new and differently named search heuristics or optimization algorithms. Having the choice between many different algorithms and implementations, naturally the question arises: “which algorithm is the best suitable for the problem at hand?”

The “No Free Lunch” (NFL) theorems [Wol95, Wol97] state that the performance of all MHAs is equivalent when averaged over all possible problem types. This implies that statements in the form of “algorithm A performs better than algorithm B” are invalid under the assumptions under which the NFL class theorems hold.

The validity and implications of this statement are quite intuitive for discrete/non-continuous pseudo-random problems. For a general instance of a function that maps to a random field it is not necessary that previously visited points contain any information about the location of local or global optima. Therefore, it is unfeasible to outperform enumerative/random search, by using another heuristic approach.

After the publication of the NFL papers many discussions on the assumptions and implications on particular problem classes and practical real life optimization problems followed. In the work of [Dro99], five different types of optimization scenarios are identified in which for several scenarios specific techniques exist that are superior to general ones. For some problem classes, there are at least free “*appetizers*” [Dro99] or “*leftovers*” [Cor03] available. In [Stre03] two broad classes of functions were identified where the NFL does not hold. The work of [Ige01] showed/proved that on most subsets of all possible functions the precondition of the NFL are not fulfilled, which allows the existence of a performance measure where some algorithms have better performance than others when averaged over the considered objective functions with a probability close to one. Furthermore, it has to be noted that those particular subsets of functions are characterized exactly by the sort of properties that “real life” problems might possess [Dro02]. The characterization of problem classes where the NFL theorems hold or not hold, and its implications on performance comparisons is still an ongoing research problem.

For some function classes and particular real life optimization problems, there might be algorithms that perform better than other. Thus, statements in the form of: “Algorithm A performs better than algorithm B” can be valid if coupled with a suitable well-formulated disclaimer containing the domain of validity for the statement [Cul96]. Regarding optimization algorithm comparisons in the context of general purpose performance and performance on particular problems, [Eng96] stated: “*The preoccupation with the best optimizer should shift to an interest in finding the right optimizer for the job.*” Later in the conclusions section of that work the following comparison was made:

“*Hammers contain information about the distribution of nail-driving problems. Screwdrivers contain information about the distribution of screw-driving problems. Swiss army knives contain information about a broad distribution of survival problems. Hammers and screwdrivers do their own jobs very well, but they do each other’s jobs very poorly. Swiss army knives do many jobs, but none particularly well.”*

Thus specific algorithms can have better performance on particular problem types, than general purpose search heuristics [Dro02]. Although it may not always be practical to design an optimization algorithm for a particular problem, one could tune the optimization algorithm meta-parameters for a particular problem type, or select efficient existing algorithms based on empirical testing and benchmark performance.

In [Barr95] several criteria are proposed for meaningful or significant empirical testing and comparisons of algorithms. Experimental competitive testing of algorithms can show which algorithms are faster than others, but it has been criticized among others by [Hoo95] because: "*its failure to yield insight into the performance of algorithms*". Therefore, he emphasized the importance of more theoretical analysis approaches. A summary of such approaches is given in [Pit12]. As stated in [Mit92] "*there is much about the GA's behavior that is not well understood, even on very simple landscapes*". Just as in most other fields of science, theoretical analysis alone is not enough to address all relevant issues and questions. There are many real-world optimization problems that have an urgent need/necessity for improved performance. "*In the absence of a good, predictive theory of GA performance, unavoidably we are only left with an experimental approach*." [Bor04]. The position of empirical testing, is well placed in context by the words of [Coh95], "*It is good to demonstrate performance, but even better to explain performance*".

Since no techniques for theoretical performance analysis of meta-heuristic algorithms on car body design related problems are available, this work aims to evaluate and extend existing comparative assessments and assessment strategies for optimization of car body structures.

2.1.2. **Optimization test functions and benchmark problems**

Theoretical performance analysis is difficult and unfortunately still only restricted to simple MHAs and particular problem types. Therefore "empirical" analysis methods, based on numerical experiments are commonly used to assess the performance of different algorithms on various problem types [Bor04]. In order to compare the different MHAs many benchmark functions have been proposed, which are widely used for performance assessment by optimization algorithm developers and optimization practitioners in engineering physics and various fields. Some examples of such functions are the Rosenbrock function [Ros60] the Rastrigin function [Ras74] for single objective problems, and the ZDT functions by Zitzler, Deb, and Thiele [Zit00] for multi-objective optimization problems. In works by i.a. De Jong et al. [Jon75], Floudas et al. [Flou99] and Andrei [And08] compilations of such benchmark or test functions is were made. Many authors have used a selection of such functions to compare newly developed or existing MHAs w.r.t. performance i.a. [Yao99, Ves04, Bre06, Bis07, Bao09] or even to set up suites for performance competitions [Tang13, Liang13], such that the number of works in the literature with such comparisons is quite large.

In recent works such as for example [Bar11, Die12, Lia05] many of the commonly used test functions have however been criticized because they are not very challenging, and do not represent the difficulty[1] of real-world problems. In [Bar11] the topic of test function

[1] From a problem centered perspective, when "difficulty" or "hardness" is averaged over all possible search or optimization algorithms no problems are intrinsically harder than others. From an algorithm centered perspective however some problem classes can be more difficult than others, for a particular algorithm [Wol95].

generators for assessing the performance of meta-heuristic optimization algorithms on multimodal functions was discussed. It was highlighted that many of the currently available test functions in the specialized literature are too simple, and show regularities, such as symmetry, uniform spacing of optima, and centered optima which can easily be exploited by algorithm designers (see also [Lia05]), and which are unrealistic testing environments for the algorithm performance on real life problems. Although several different strategies to generate more complicated and realistic test functions have been proposed [Bal05, Add07, Gal06, Ahr10], none of these or other approaches (to knowledge of the author) deals actively with the topic of test function structure related to higher order interactions between the design variables, and variance contribution distributions, in a structured manner.

Such test functions, sometimes also named "artificial landscapes", are often expressed as simple closed form expressions, which require little computational effort such that millions of function evaluations can be achieved in a small amount of time on modern computers. It remains however a challenge to relate such standard analytical test functions to particular real-world problems, and vice versa.

An alternative to analytical test problems could be the use of simulation-based structural optimization benchmarks, based on standardized problem instances. The need for more complex realistic system benchmark problems is expressed in [Ali10], and a relatively recent initiative to start an open benchmark database for simulation-based multidisciplinary optimization problems with engineering relevance is presented in [Var12]. For the optimization of vehicle design problems, involving crashworthiness and NVH responses, no relevant open-source benchmark problems are available yet. Although vehicle models are made publicly available by the vehicle modeling laboratory of the National Crash Analysis Center (NCAC), none of these or other models are to the knowledge of the author used for any standardized simulation-based optimization benchmark problems. Even if standardized simulation-based benchmark optimization problems of full vehicle models would become available in the near future, the hardware and software resources required for the computationally expensive simulations remain a big hurdle to perform, the large amount of function evaluations required to obtain statistically significant performance comparisons of optimization methods and algorithms for these problem types. These difficulties also exist for other structural optimization benchmark problems that involve resource demanding simulations.

2.1.3. Comparative optimization algorithm assessment in the context of car body design related problems

The conclusions of a relatively recent review paper [Wan13] in the context of review on simulation-based optimization were expressed as follows: *"In the literature, many techniques and algorithms have been proposed. But there is not enough research on the comparisons between them."*

As discussed in the previous sections, theoretical performance analysis is difficult, and only possible for very simple problems. Empirical performance testing on test problems is quite common, but many of the common test functions have been criticized, and the available test problems are difficult to relate to real-world problems. Real-world inspired structural optimization problem-based benchmark problems exist but simulation based problems of industrial signifficance are often so resource demanding that they are hardly used by those in the optimization research community who are developing new optimization strategies.

In [Gom11, Mig12, Gho13] several optimization algorithms have been compared on structural optimization problems with frequency constraints, these problems were however composed of simple truss structures. Early investigations on structural optimization with crashworthiness responses used strongly simplified components or sub structures to overcome the computational burden [Yng94, Schr98]. Although nowadays MDO using meta-heuristic search algorithms is commonly applied in an automotive industrial context, the literature provides still nearly no significant performance comparisons, or guidelines for efficient optimization, of more than two relevant algorithms applicable for vehicle design problems involving crashworthiness. Statistically significant comparative studies on optimization algorithms for problems that involve vehicle crashworthiness constraints are relatively rare, and those available are often limited in their reproducibility, statistical significance and their comparative scope w.r.t. the number of optimization algorithms compared. Studies

An early small comparative study on stochastic optimization methods for crash and NVH problems was presented in [Dud03]. In that study, two different optimization algorithms were compared for 6 repeated optimization runs on the same problem. The results indicated that the evolutionary strategy performed 11% better than the Monte-Carlo scheme. Furthermore it was noted that these type of problems are too complex to truly find the global optimum, and that in an industrial context the optimization typically aims at a significant design improvement, within the feasible computation cost. In the outlook, further work on finding suitable and efficient algorithms for these problem types was recommended.

In [Nil04] a novel Stochastic Optimization Zooming Method was proposed for crashworthiness design, and compared with an RSM based optimization approach. The comparison was made using several problems that included crashworthiness simulation responses. The results were however based one run per algorithm per problem, therefore the statistical significance of the results is difficult to assess.

In a later work, [Dud08] presented several benchmark studies for NVH and crashworthiness related problems, together with a list of search algorithm requirements on such optimization problems. In the work several comparisons based on different problem types were presented. The comparisons that involved crashworthiness criteria however only included the results of two different optimization algorithms.

In [Gu13] a comparative study on multi-objective and robust optimization for the design of vehicle structures involving crashworthiness criteria was presented. A fairly detailed vehicle model was used, but the optimization problem formulation only included 6 design variables. The comparison results were all based on static RBF-meta-models using 36 construction points based on the simulation model.

Recently in [Kia15], a comparative study between five meta-heuristic optimization algorithms was presented. The comparison was made by optimizations on a static meta-model response based on 46 training points that were based on a full vehicle crashworthiness simulation model with 22 design variables.

Among the few studies optimization on optimization performance for car body design problems, the studies in which more than two optimization algorithms were compared on problems that involved FEM based vehicle crashworthiness simulation responses [Gu13, Kia15], were based on meta-model based function evaluations. In both cases no comparison with the optimization performance based on direct simulation based function evaluations was made. Furthermore, the number of construction points used for the meta-models was relatively low, and no clear quantifications about the accuracy or representativeness of the used meta-models were provided.

One of the reasons for the scarcity of literature with statistically significant comparisons on optimization problems which involve full vehicle optimizations, including crashworthiness and NVH simulation responses is that these problems are resource demanding. Such optimizations are generally expensive in terms of hardware and software resources (solver licenses), modeling effort and computation time. For such MDO studies in an industrial context the computational budget is often restricted to approximately 200-500 function evaluations [Dud08, Kno05, Kno09] due to the required computational cost. If several crashworthiness load cases are regarded in a car body optimization problem, each design evaluation could require several hundreds of CPU hours [Dud08], while such problems can have easily more than 20 design variables. Not only for car body design problems but for many industrial problems that involve computationally expensive simulators the results are significantly affected by the constraint on the number of function evaluations due to the computational cost [Kno05, Kno09].

For optimization problems involving crashworthiness responses and more generally for MDO involving expensive simulators with complex responses, the current state of the art could be paradoxically stated as: *the problems for which optimization performance matters the most, because they are computationally expensive and restricted to a limited evaluation budget, are also the problems for which it is too expensive to compare algorithms, tune the optimization parameters or develop specialized optimization methods.*

None of the comparative studies available in the literature that deal with crashworthiness responses of full vehicle models did consider or investigate the transferability of the comparative assessment results. Is the relative optimization performance among a set of algorithms on a particular car body optimization problem, relevant or correlated to a similar optimization problem for a different vehicle model? This important question has not been assessed in the literature yet. Also the representativeness of performance comparison results, based on meta-model function evaluations w.r.t. direct simulation based function evaluations have not been assessed yet for car body design problems.

Since theoretical analysis of algorithm performance is presently only possible for simple algorithms and problems, empirical numerical testing and benchmarking are important for finding efficient optimization algorithms for particular problem types. Many of the commonly used test functions used for optimization algorithm benchmarking are criticized for their lack of complexity and relation to real-world problems. In one review paper the authors stated: *“current modeling research tends to focus on sampling and modeling techniques themselves and neglect studying and taking the advantages of characteristics of the underlying expensive functions”* [Sha10]. In this work these open topics are addressed for the application of typical car body design optimization problems.

2.2. Simulation methods

The term “simulation” is commonly used in many contexts with various meanings. In the scope of this thesis the term simulation refers to the application of mathematical methods to study engineering problems. This process generally consists of three phases [Kry72]:

- Modeling: the formulation of the physical problem into a mathematical description referred to as a model
- Solving: the treatment and manipulation of the model in order to obtain the desired results of the physical problem
- Analysis: the translation and/or interpretation of the mathematical results into physical terms and meaning

Although some engineering problems can be treated with analytical simulation methods for which closed form solutions can be obtained, many problem types of practical relevance quickly become too complex or require an unfeasible large amount of operations to be of use. For such problems computer implementations of numerical simulation methods were developed since the 1950s, and gained importance in many fields of engineering. In the core chapters of this thesis, Finite Element Method based numerical simulation models have been used to estimate the structural responses of the design variants of several vehicle models.

2.2.1. Brief overview on the history of the Finite Element Method

Computational Mechanics is a sub-discipline of theoretical and applied mechanics that is targeting the development and implementation of computational methods to model and analyze the mechanics of systems [Odn03]. The numerical simulations used in this thesis to obtain estimates of the structural responses for different vehicle design variants are based on the Finite Element Method (FEM).

Based on early pioneering works by among others Euler [Eul1744], Schellbach [Sbh1851], Ritz [Rtz1908] and Galerkin [Glk1915] on analytical variational approaches, practical numerical approximation methods for engineering applications such as the Displacement Method, the Force method, and the Direct Stiffness Method and Matrix Structural Analysis were developed in the first half of the twentieth century [Dnc34, Ptr40, Lev53], primarily targeting aeronautic and submarine structures.

Later some of these ideas were generalized for the analysis of complex structures by, among others, the works of Argyris (summarized in [Arg60]), and Turner et al. [Tur56] and became popular and known under the name Finite Element Method [Cgh60] in the second half of the twentieth century. It was probably the combination of general applicability and the developments in computer technology after the 1950’s which exponential progressed since [Mor65] that led to the large increase in attention and popularity of the FEM and other discretization based approximation methods (such as the Boundary Element Method [Brb77], and Finite Volume Method [Pat80]). Overviews on the history of the development of the fundamentals of FEM are given in [Gnd12, Flp01]. A historic reference of the method is the textbook of [Zwz67]. An early mathematical treatment and analysis of the method is given in [Str73]. The application of FEM is not limited to structural engineering problems, a comprehensive introduction on the application of the method for different for linear problems (such as: heat transfer, fluid mechanics, electromagnetism) in engineering is given in [Brt87]. An introduction to of the method for nonlinear problems for structural engineering applications is given in [Csf96]. There are however many textbooks available with various perspectives targeting different application fields.

In this work, FEM is applied for numerical estimation of the lower frequency eigenmodes and eigenfrequencies of the car body structure, and to estimate the response of the vehicle model under crashworthiness load cases. The following sections introduce the fundamentals of the methodology and models used in this work.

2.2.2.FEM and explicit-dynamic time integration

The crashworthiness criteria investigated in the scope of this thesis are related to highly dynamic events in a small time period (100ms), with very large deformations. The typical numerical solution procedure for such problems is by explicit time integration (see [Dok89] for a review on explicit time integration methods).

For the crash simulations in this work the LSTC LS-DYNA solver (version 971) was used. This section provides a short summary of the general method based on the solver documentation, for details is referred to [Hal06, Hal07]. For dynamic deformation problems were damping is omitted, the principle of virtual work over a homogeneous bodzcontinuous volume V (with boundary surface S) including the inertial terms can be stated as:

$$\delta \mathrm{W} = \int_V \rho \boldsymbol{a} \delta \boldsymbol{u} \, \mathrm{d}V + \int_V \boldsymbol{\sigma} \delta \boldsymbol{\varepsilon} \, \mathrm{d}V - \int_V \boldsymbol{b} \delta \boldsymbol{u} \, \mathrm{d}V - \int_S \boldsymbol{\tau} \delta \boldsymbol{u} \, \mathrm{d}S \tag{2.1}$$

Where the terms on the right hand side summarize the different sources of virtual work:

- The first term: $\int_V \rho \boldsymbol{a} \delta \boldsymbol{u} \, \mathrm{d}V$ corresponds to the virtual inertial work, were $\delta \boldsymbol{u}$ are the virtual displacements that satisfies the Dirichlet (locally prescribed displacement) boundary conditions; ρ is the material density; $\boldsymbol{a}$ is the instantaneous acceleration.
- The second term: $\int_V \boldsymbol{\sigma} \delta \boldsymbol{\varepsilon} \, \mathrm{d}V$ corresponds to the virtual internal work, were $\boldsymbol{\sigma}$ refers to the instantaneous Cauchy stress, and the virtual work conjugate strain is denoted by $\delta \boldsymbol{\varepsilon}$.
- The third term: $\int_V \boldsymbol{b} \delta \boldsymbol{u} \, \mathrm{d}V$ is the work due to the body forces $\boldsymbol{b}$ acting on the volume.
- The fourth term: $\int_S \boldsymbol{\tau} \delta \boldsymbol{u} \, \mathrm{d}S$ corresponds to work due to the traction forces $\boldsymbol{\tau}$ over the surface of the volume.

The sum of the third and fourth term is often referred to as the virtual external work.

If the displacement vector function $\boldsymbol{u}$ in the continuum is approximated by local interpolation (shape) functions belonging to a superimposed mesh of finite elements between a discrete set of nodal points with finite displacement vector $\boldsymbol{u}_N$ such that $\boldsymbol{u} \approx \mathrm{N}^T (\xi, \eta, \zeta) \, \boldsymbol{u}_N (t)$,(where (ξ, η, ζ) are the parametric element coordinates), equation (2.1) can be written as the algebraic equation:

$$\delta \mathrm{W} = \delta \boldsymbol{u}_N \mathbf{M} \boldsymbol{a}_N + \delta \boldsymbol{u}_N \mathbf{K} \boldsymbol{u}_N - \delta \boldsymbol{u}_N \mathbf{f} \tag{2.2}$$

Were the following substitutions have been made

$$\mathbf{M} = \int_V \rho N^T N \, dV$$
$$\mathbf{K} = \int_V B^T \boldsymbol{C} B \, dV$$
$$\mathbf{f} = \int_V N^T \boldsymbol{b} \, dV + \int_S N^T \boldsymbol{\tau} \, dS + N^T \mathbf{f}_p \tag{2.3}$$

in which B is the strain displacement matrix formed by applying differential operator D on the local interpolation functions $B = D^T N, C$ is the elasticity tensor and $\mathbf{f}$ is the total of external body ($\boldsymbol{b}$), traction ($\boldsymbol{\tau}$) and point ($\mathbf{f}_{\boldsymbol{p}}$) Forces (introduced due to the discretization). Due to the superimposed mesh of finite elements the integration operation is now composed of a summation over element volume integrals in their local coordinate system which are approximated by numerical integration methods such as Gauss and Lobatto quadrature integration. If the trivial case $\delta \boldsymbol{u}_N = 0$ is excluded the zero virtual work statement is satisfied if:

$$\mathbf{M}\boldsymbol{a}_N + \mathbf{K}\boldsymbol{u}_N = \mathbf{f} \tag{2.4}$$

A frequently applied solution procedure for this equation is based on the implementation of diagonal element mass matrices, together with an explicit time integration scheme for displacement approximation. For a current (time) state *i* with known displacements the accelerations are determined by rewriting equation (2.4) as:

$$\boldsymbol{a}_N{}^{\boldsymbol{i}} = \mathbf{M}^{-1}(\mathbf{f}^{\mathrm{i}} - \mathbf{K}^{\mathrm{i}} \boldsymbol{u}_N{}^{\boldsymbol{i}}) \tag{2.5}$$

Where subscripts $[\cdot]_N$ have been omitted for better readability. The non-prescribed displacements at each of the nodal points for the next state (i+1) are extrapolated according to the formulas in expression (2.6), based on the assumption of constant acceleration during the small time increment Δt

$$\boldsymbol{u}^{i+1} = \boldsymbol{u}^{i} + \Delta \mathrm{t}^{i+1} \boldsymbol{v}^{i+1/2}$$
$$\boldsymbol{v}^{i+1/2} = \boldsymbol{v}^{i-1/2} + \frac{\Delta \mathrm{t}^{i+1} + \Delta \mathrm{t}^{i}}{2} \boldsymbol{a}^{i} \tag{2.6}$$

This assumption implies conditional stability on the numeric solution with the restriction that Δt is smaller than the period of the highest eigenfrequency of the discrete system. For shell elements the time step limitation is often estimated with:

$$\Delta t < \frac{L_{crit}}{\sqrt{E/\rho(1-\nu^2)}} \tag{2.7}$$

Where L_{crit} in stands for the characteristic length of the most critical (smallest/distorted) element of the system, and the denominator represents the dilatation wave speed in the shell plane (dependent on Young's modulus E, material density ρ and poisson's ratio ν). A concise (but a bit dated) treatment of relevant numerical topics in crashworthiness simulation in an automotive context is given in [Swz92]. More details and derivations of applied simulation methods in this work are given in [Hal06].

2.2.3. FEM-based modal analysis

For the estimation of the selected eigenvalue and eigenfrequency criteria, Finite Element Method based modal analysis was used. FEM-based solutions for vibration problems are available since the 1960's [Daw65, Irr02]. Based on equation (2.4), resonance or eigenfrequencies are characterized by dynamic equilibrium in the absence of external forces, such as can be expressed as:

$$\mathbf{M}\boldsymbol{a}_{\boldsymbol{N}} + \mathbf{K}\boldsymbol{u}_{N} = \mathbf{0} \tag{2.8}$$

Under the assumption of harmonic motion, the nodal displacement as a function of time can be expressed as:

$$\boldsymbol{u}_N(\boldsymbol{t}) = \boldsymbol{u}_N\boldsymbol{e}^{\imath\omega t} \tag{2.9}$$

Where ω is the angular frequency natural of the system and $\imath$ is the imaginary unit in the complex plane which satisfies $\imath^2 = -1$. When the expression for the nodal displacement is two times differentiated w.r.t. the time an expression for the nodal accelerations is obtained.

$$\boldsymbol{a}_N(\boldsymbol{t}) = \ddot{\boldsymbol{u}}_N(t) = -\omega^2\boldsymbol{u}_N\boldsymbol{e}^{\imath\omega t} \tag{2.10}$$

Such that the corresponding eigenvalue problem can be expressed as:

$$\mathbf{K}\boldsymbol{u}_N - \mathbf{M}\omega^2\boldsymbol{u}_N = \mathbf{0} \tag{2.11}$$

Which can be rewritten as:

$$(\mathbf{K} - \omega^2\mathbf{M})\boldsymbol{u}_N = \mathbf{0} \tag{2.12}$$

For the problems treated in this work the low-frequency non-zero eigenfrequencies are of interest. Due to the use of diagonal mass matrices as also mentioned in the previous section, also large systems of this form can be solved using sparse eigensolver techniques, such as the Lanczos method [Lan50], for more details is referred to [Hal06].

2.3. Selected vehicle models, design variables, load cases and design criteria

2.3.1. Overview of the used vehicle models

The objectives and constraints of the example optimization problems are based on the response of numerical simulations using the Finite Element Method (FEM). The vehicle models used in this work are based on the models available from the National Crash Analysis Center (NCAC) finite element model archive [NCA12], these models been developed by The National Crash Analysis Center (NCAC) of The George Washington University under a contract with the FHWA and NHTSA of the US DOT (formal courtesy notice). The preparation of such FEM-based vehicle models requires significant effort. Due to the use of readily available vehicle models for this work, more emphasis could be placed on issues related to the optimization process. An advantage of these models is, that results can be published without restrictions on confidentiality, in addition the selected models have also been used in many other works in the literature.

For the presented investigations and case studies, the models displayed in Table 1 are used. The Metro model (A) is selected because, the low mesh resolution and forthcoming low computational cost, which enabled a larger number of function evaluations for the response characterization. The Neon model (B) has a much higher FEM mesh resolution, and a more detailed model structure. The Taurus model (C) has a computational cost between those of models A and B, and allows the large number of function evaluations required for the corroboration of the approach. The number of design variables is different for each of the vehicle models because different design car body construction concepts are used, and the vehicles used are modeled with a different level of detail. Although this work is dealing with “similar” vehicle-related optimization problems, it is rather typical in industry, that there are some differences between the different problem instances. The differences in geometry, FEM mesh resolution, number of design variables, and car body concepts, enable the assessment of the robustness of the response characterization for the different vehicle models and presented benchmark approach. Typical full vehicle crashworthiness models applied in industry today have about 1-10 million elements, and require computation times in the order of magnitude of 100 CPU hours, for a single 100ms crash event. The models used for the response characterization had significantly lower mesh resolution and required less computation time (see Table 1). These models are less accurate in representing the exact behavior of a particular vehicle model, however in this work the identification of typical response characteristics w.r.t. the design variables is prioritized over the accuracy required in a detailed analysis of a particular vehicle design. The response characterization results in section 5 did not indicate any dependency of the statistical response characteristics depending on the mesh, although the used models differed in mesh resolution for an order of magnitude.

The models have been slightly modified for the use in the simulation workflows for the presented investigations. The modifications consisted of placing additional spot-welds, small geometric changes to avoid contact penetrations, local re-meshing to avoid small elements that caused excessive small time steps in explicit time integration, modifications in the floor-bead-geometry in order to avoid local low-frequency, resonances, and sheet thickness changes. More details about the vehicle models can be found in Table 1.

As design variables for the optimization problems, the scaling factors on the sheet thickness of Body In Prime (BIP) components have been parameterized. The design variables are normalized to be in the unit hypercube domain and scale the nominal part thickness by a scaling factor varying between 0.5 and 2. Components appearing on both sides of the vehicle are scaled symmetrically. In Table 1, the parts with variable thickness are colored in the pictures of the modal analysis models, while constant parts are displayed in gray, the same design variables have been used for the crash simulation. For vehicle models A, B, and C the total nominal mass of the parameterized components accounts for 75%, 90% and 90% of the total BIP mass respectively.

Table 1 Overview of the used vehicle models

	model A	model B	model C
	Metro	Neon	Taurus
Modal analysis models (in color the parts with variable thickness)			
Crashworthiness models			
Nr. of elements Crash model	16k	271k	28k
Total CPU time[2] [hr.] for a crash simulation of 100ms	0.4	30	1.1
Nr. of design variables	32	72	50

The simulation workflow was programmed using MATLAB, VBA, and batch scripts, to execute the preprocessors (Altair Hypermesh and LS-PREPOST), solver, and post-processing programs (LS-PREPOST). The eigenfrequency simulation and crashworthiness simulation of the finite element models were performed using the implicit (direct), and explicit LS-DYNA (971) solvers respectively. The Modal Assurance Criterion (MAC) was implemented in the simulation workflow to identify the corresponding eigenmodes, for the different designs evaluations (see the next section for more information).

[2] Approximate CPU time per simulation using a single logical core of a HP Z600 with 2 Intel Xeon E5520 processors, and 24GB DDR3 Memory.

2.3.2. Description of the selected load cases and simulation responses

There are many structural requirements which a car body structure has to satisfy. Although it would be of interest to consider all or many of them in a multidisciplinary design optimization context, in this thesis a reduced set of design criteria is considered. The considered set of design criteria is composed of lightweight, vibrational comfort and crashworthiness criteria. The selected criteria set considers several design aspects that are of industrial importance, and the combination of criteria is representative for a multidisciplinary design optimization problem with different sources of complexity. Similar design criteria sets have also been used in other works in the literature [Cra02b, Sta03, Fng05a, Goe09]. The following subsection provides an overview of the selected design criteria for the optimization problem case studies:

Lightweight criteria

As already mentioned in the introduction only a fraction of the energy used by a passenger vehicle is for the transport of the passengers [Fen01]. This is partly due to the high weight of the vehicle structure compared to the passenger weight. One way to reduce the vehicle energy consumption and GHG emissions is to reduce the weight of the vehicle. If the weight of components or modules is significantly changed during the design process, also additional weight savings on other components due to secondary effects can be achieved [Kof10]. Several studies have been made that estimated the fuel consumption reduction to 0.15 l of gasoline, or 0.12 l of diesel per 100 km for a reduction of 100 kg if only the direct effects are included, and up to 0.45 l and 0.3 l if also secondary effect such as gear ratio and engine displacement are changed, (while maintaining a constant power to weight ratio, and dynamic elasticity values) [Kof10]. Another study [Ch10] estimated reductions in energy consumption of about 7% for every 10% reduction in vehicle weight. Although the exact relation between mass reduction and energy savings depends on the particular vehicle concept and driving cycles considered, these results indicate the relevance of weight reduction and lightweight design. A general overview on lightweight design in an automotive context is provided in [Fen01, Mal10]. An analysis how lightweight design can contribute to the reduction of the environmental impact of automotive vehicles was presented in [Sch14].

Although lightweight design is relevant for all vehicle parts, the focus in this work is on the weight of the car body structure. As a lightweight design criterion, the total mass of the BIP structure of the vehicle models is used. The mass for each vehicle model design variant is calculated by summation over the lumped finite element masses of a predefined set of elements representing the components of the vehicle BIP. In the proceeding of this work this design criteria is referred to as the “Mass” response.

Vibrational comfort criteria

The effect of vibrations on the loss of performance of workers, fatigue, and health problems have been investigated by various researchers in the literature [Hor61, Gret71, Ljung07]. As also mentioned in the introduction significant relationships exist between noise and specific health effects [WHO12]. Besides noise related regulations by the legislators the automotive industry sets strict targets on Noise Vibration and Harshness (NVH) related criteria, because these aspects are of high importance in the perception of quality by the consumers.

Automotive NVH requirements cover subjective and objective criteria, related to tactile vibration and audible sound. A common practice is to relate subjective criteria to objective criteria of reference vehicles and use the relative objective criteria as performance targets for a new vehicle design. NVH targets can be categorized as follows [Har04]:

- Whole vehicle exterior noise targets (e.g. drive pass noise levels for legislative approval)
- Single component exterior noise targets (e.g. engine-radiated noise, exhaust noise)
- Whole vehicle interior noise targets (e.g. A-Weighted or C-weighted sound pressure levels at the drivers ear position under full load acceleration conditions)
- Ride quality targets (e.g. low-frequency vibration levels at the seat rail at 80 km/h on a typical tarmac road)

Sources of noise can be categorized as air-borne or structure-borne. The structure-borne noise below the 125Hz region is important because most of the noise energy is present in this range. [Dun96]. Structure-borne noise sources (road, tire and powertrain induced) can often be controlled by designing for insulation. Typical subsystem performance parameters to relate the full vehicle targets to are:

- Trimmed body natural frequencies
- Acoustic body impedance such as P/F transfer functions at chassis and powertrain attachments
- Car body mobility such as A/F transfer functions at chassis and powertrain attachments
- Body in Prime[3] natural frequencies
- Body in Prime static stiffness

Although there are many NVH related design criteria relevant for car body design, in the scope of this thesis the investigations are limited to global natural eigenfrequencies of the BIP structure.

Optimization of global bending and torsion frequencies is one of the most basic challenges in automotive vehicle body design. Several approaches have been proposed in [Don09, Mih10, Mih12] to already assess these criteria in the early concept phase of the vehicle development process by using simplified structural beam representations. Although these methods are able to represent the static and dynamic behavior of detailed geometries by models of computationally reduced complexity, the reverse process to derive more prototype or production geometry from such representations is not straight forward, and will generally not replace the optimization of more detailed geometric models in later design stages. To achieve vehicle designs with efficient trade-off solutions it is important to apply multidisciplinary design optimization techniques throughout the development process. Not only during early design stages with highly simplified models, but also in later design stages

[3] The term body in white refers to the joint composition of the main load carrying car body components, before the assembly of moving parts, covers glass etc. The Body in Prime (BIP) is the body in white with in addition the front windshield and if applicable other load-carrying windows attached to the main structure of the car body.

when the simulation models become computationally expensive. Therefore, this work concentrates on the optimization problems using vehicle geometry models that are of a level of detail sufficient to simulate crashworthiness behavior, which typically receives high priority in vehicle design. The optimization of the eigenfrequencies and eigenmodes is also at the basis of other NVH criteria such as Frequency response functions (FRF) and Transfer Path Analysis (TPA).

The vibrational comfort related design criteria used in this work are the eigenfrequencies (under free-free boundary conditions) that belong to the first natural bending and torsion mode of the vehicle structure. The corresponding eigenmodes and eigenfrequencies are estimated based on FEM-based modal analysis, using LS-DYNA-implicit (version 971). Figure 1 shows a scaled deformation plot of the first natural torsion frequency of vehicle model B

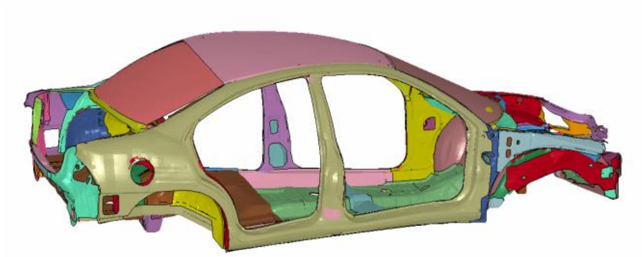


Figure 1 Simulation response: scaled deformation plot of the first natural torsional eigenfrequency of the BIP of (vehicle model B)

The eigenmodes are distinguished using the Modal Assurance Criterion (MAC) [All82] with respect to the dynamic behavior of the nominal vehicle model configuration. The MAC is defined as:

$$MAC(\psi_i, \psi_j) = \frac{|\psi_i{}' * \psi_j|^2}{(\psi_i{}' * \psi_i)(\psi_j{}' * \psi_j} \qquad (2.13)$$

Where ψ_i represents an eigenvector (or mode shape), and $\psi_i{}'$ denotes its transposed complex conjugate. In the following chapters of this work the eigenfrequencies that that can be attributed to the first bending mode, and first torsion mode will be referred to as the "NBF1" and "NTF1" responses.

Although optimization with respect to mass and NVH or vibrational comfort criteria is already a challenge, the complexity of the optimization problem drastically increases when also objectives or constraints are included from other disciplines such as for example structural crashworthiness responses.

Crashworthiness criteria

Despite significant improvements in accident avoidance and active safety technologies, motor vehicle crashes still occur and can have significant consequences. As mentioned in the introduction road traffic related incidents, cause ten thousands of human fatalities, millions of injuries [Bli15]. Besides further technological improvements in the field of active safety, also improvements and strict passive safety standards are required to improve passenger car safety.

Whereas the field of active safety deals with technologies and systems that aid to avoid accidents such as ABS, ESP and the maneuverability of the car, the field of passive safety deals with technologies that reduce the effects of an accident, such as airbags, seat belts and safe car body structures [Sei07]. The ability of a structure to protect its occupants during an impact is commonly referred to as crashworthiness. An introduction to the topic of vehicle crashworthiness is given in [Hua02] and further information can be found in [DuB04,]. Vehicle crashworthiness performance can be assessed by a posterior study of vehicle crashes from the past, by crash experiments and by numerical modeling and simulation. Many different vehicle crash test protocols are defined by several institutions (see also [Per13]) such as for example the Insurance Institute for Highway Safety (IIHS), European New Car Assessment Program (Euro NCAP), and the National Highway Traffic Safety Administration (NHTSA). The different test protocols target occupant safety and vulnerable road user (such as pedestrian) safety by various means and testing procedures.

Some of the previously mentioned test protocols include frontal impact load cases at relatively high speeds (50-64 km/h) which involve large kinetic energies and thus require significant energy absorption of the car body structure, while at the same time the integrity of the passenger compartment must be assured. Such crashworthiness load cases usually involve large structural deformations and damage the vehicle to an extent which makes reparation practically unfeasible. Crashworthiness tests are therefore expensive, especially during the design process when physical prototypes are required. Therefore numerical techniques have been developed to evaluate structural designs and design changes by computer simulations (see section 2.2).

Although many different crash types and load conditions are relevant for a vehicle design to comply with the various international test programs, in the scope of this work only a single crashworthiness load case is considered. In order to make a contribution to optimization of car body design problems that involve multidisciplinary design criteria including crashworthiness a compromise between completes and computation, and modeling effort was made. The selected load case of a frontal crash against a rigid wall, is rather academic, but it includes the important aspect of large highly nonlinear deformations, while also taking into account the mass ratio effect such that comparable results for different vehicle models can be obtained (which would not automatically be the case for impact with a deformable barrier [Lom01]). The vehicle safety during such tests is often assessed using crash dummy models, however also deformation and acceleration criteria can be used to assess the vehicle responses, and such responses exclude the influence and complications of the crash dummy kinematics during impact. Frontal impact crash load cases, and structural responses similar to those selected for this work, have also been used for other optimization related studies in i.a. [Cra02, Red04, Sun11].

The crash load case is a frontal crash configuration against a rigid wall at 64 km/h. For the crashworthiness simulations, nonlinear transient dynamic analysis by means of explicit FEM is used (see section 2.2.2). Figure 2 illustrates the typical phenomena involved in the load cases. As crashworthiness criteria the maximum acceleration values[4] at the vehicle tunnel, and the deformation between the A- and B- pillar were used. In the following chapters of this work these simulation responses will be referred to with the abbreviations "P. acc". and "ABP. Def." respectively.

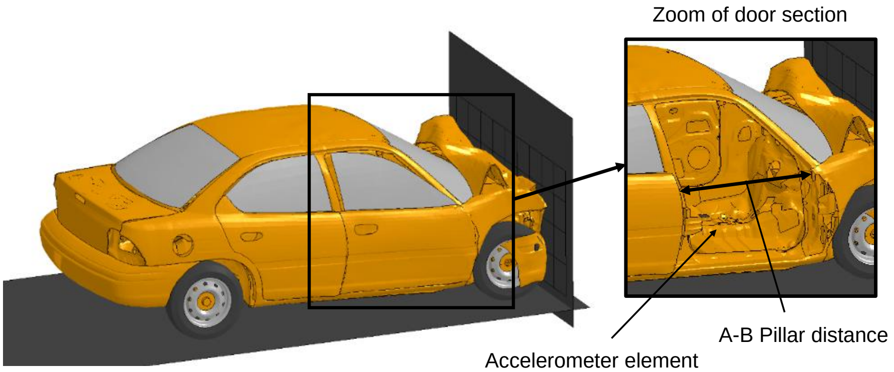


Figure 2 Deformation plot from a crashworthiness simulation of a frontal impact against a rigid wall (vehicle model B)

Summary of the design criteria

In the proceeding of this work the following design criteria and abbreviations will be used:

- Vehicle body mass (Mass)
- First (free-free) natural torsion eigenfrequency (NTF1)
- First (free-free) natural bending eigenfrequency (NBF1)
- Deformation between A- and B-Pillars during crash (ABP. Def.)
- Peak acceleration during crash (P. acc.)

[4] The peak acceleration results are based on SAE 60 Hz low pass filtered acceleration values of an accelerometer element located at the center of the vehicle on the tunnel.

2.4. Meta-modeling methods

Meta-models also often referred to as surrogate models or emulators, or response surface methods (RSM), are approximation models that relate model input and output based on provided training data, or construction points. For input values at which no output data is known, output values are estimated using different estimation and interpolation techniques depending on the type of meta-model. A loose definition of RSM is provided in [Mye89] as: *"a collection of tools in design or data analysis that enhance the exploration of a region of design variables in one or more responses*". In much of the literature, the term RSM is however often used for models based on particular sampling schemes and polynomial approximation functions. In this work we will use the word meta-model referring to a model of a model in a similar sense as the previously stated loose definition of an RSM (note that the term meta-model can however be used even more general in the context of other (non-engineering) literature).

Meta-modeling methods were used in the scope of this thesis, but no technical contributions to this field are treated in the scope of this work. Therefore, this section is restricted to a brief summary of relevant references, an overview of the meta-modeling method used, and a short critical analysis of the use of meta-models in the literature related to car body optimization problems.

The idea to represent given data with a function, or to interpolate data, is general and natural, therefore the origin of meta-modeling is not traceable to a single scientific work. The works of [Box51] and [Sac89] are however generally considered as the seminal papers in this field. Reviews on meta-modeling in general are provided in [Hil66, Mea75, Mye89]. Reviews in a structural engineering context are presented in [Bar93, Wng07] and more recently in [Via14]. The next section will give a brief overview of comparisons in the literature on meta-modeling techniques in car body design problems that involved crashworthiness simulation responses. Followed by a section, in which the applied method used in this thesis is described.

2.4.1. Meta-modeling in crashworthiness optimization problems

In [Dud15] the use of physical surrogate models for crashworthiness related robustness and reliability assessments is discussed. There is a distinction between mathematical and physical surrogate models. The latter are computational models with strong simplifications of the involved physical phenomena. These simplifications can be used to reduce computation cost, or to deal with course models in early stages of the design process when design details are not available yet. Such physical surrogate models can be used to aid the optimization process [Ham03, Ham04], or when combined with sensitivity analysis methods, they can be used in early design stages or to simplify models involving complex composite material vehicle components [Hes15]. In the scope of this work, physical surrogate models, are not further addressed, and in the following the term surrogate model refers to mathematical models.

In [Fng05a] a polynomial RSM and Radial Basis Function (RBF) based methods are compared for their performance in a crashworthiness optimization context. The comparisons were based on full crash vehicle simulation problem with 10 design variables, using 28 function evaluation based construction points. The results based on error comparisons at 8 independent design points indicated that RBF based approximations performed better than the RSM based approximations. The use of polynomial augmented RBF functions was recommended by the authors of that work.

Kriging and (classical) polynomial regression based RSM based meta-models were compared for crashworthiness related responses in [For06]. The results indicated that kriging models reduce the amplitudes of oscillations in optimization responses, but also that kriging based optimization procedures may converge earlier to a local optimum. Therefor the results based on these investigations were not conclusive enough so select a clearly preferred meta-model type for impact problems.

In [Sta04] three metamodeling methods (RSM, Kriging, Feed Forward Neural Networks (FFNN)), were compared for several problem types among wich a vehicle crashworthiness problem. The best results were obtained using a linear successive response surface technique.

Several metamodeling methods (RSM, FFNN, and Radial Basis Neural Network (RBNN)) were compared in [Ham04b] w.r.t. their approximation accuracy to model crush tubes with various section dimensions. The results indicated that RBNN had better accuracy.

In [Fng06] a comparative study between several meta-modeling methods was presented on different problem types, among which a vehicle impact problem. The use of polynomial augmented RBF functions was recommended by the authors of that work for the approximation of finite lement simulation based responses.

In [Yng05] a comparative study between 5 meta-modeling methods was presented for a vehicle crashworthiness problem with 4 design variables. From the results, no decisive conclusions could be drawn, and further research was suggested to obtain statistically significant results.

The application of various RSM, meta-modeling, or surrogate modeling techniques is widespread for the application to optimization problems involving crashworthiness responses. Nevertheless, there is no clear consensus on which methods to apply. The few comparative studies in the literature that deal with crashworthiness related simulation responses dealt only with problems of modest size (less than 20 design variables), and used a relatively low amount of construction points and function evaluations on the simulation response.

Although the use of meta-models can be useful, and justified when dealing with computationally expensive simulators, one cannot assume automatically that the meta-model is representative for simulation response behavior. Caution should be used in particular for highly nonlinear non-smooth responses such as can be the case with crashworthiness responses. The non-smoothness of the responses w.r.t. to changes in the design variables are due to the interactions between the nonlinear phenomena involved such as local buckling, plasticity, contact and fracture. Under such conditions, small changes of the design variables result in bifurcations or abrupt changes in the topological behavior of the system and the resulting structural response. In a high-dimensional design variable space, this highly non-smooth behavior is difficult to be represented by traditional meta-models. Even the use of methods that are generally considered as suitable for nonlinear responses (such as Radial Basis Functions (RBF) and kriging), results in smooth behavior between the construction points. Such methods can capture the “global” nonlinearities with a limited number of construction points, which is useful for approximation models. But to represent local non-smooth behavior (which can affect the optimization algorithm performance on the simulation-based model) these meta-models require a number of construction points proportional to the number of local “peaks” and “valleys” of the response “landscape” to be represented. For high-dimensional non-smooth crashworthiness responses this requires a large number of sampling points.

The curse of dimensionality and its challenges for meta-modeling are important topics of present research. In [Bck11] the development of high-dimensional engineering application problems for the comparison of meta-modeling techniques was suggested as a topic for future research. Also in [Via14] the need for the development of suitable meta-modeling methods for high-dimensional problems was emphasized. One of the recommendations for future work in the paper of [Agt10] was: the need to develop "*surrogate models adaptable to the complexity of the design space to strike a proper balance between the cost and the scarcity of the DOE coverage of the design space*" [Agt10].

The variety of meta-modeling methods is not so large as the variety of optimization algorithms, and more comparative studies are available in the literature. Nevertheless, the selection and development of meta-modeling algorithms for crashworthiness and other real-world problems that involve computationally expensive simulations, deals with similar difficulties as the development and selection of suitable optimization algorithms: analytical comparisons are difficult, and numerical comparisons with statistical significance are computationally expensive. In [Jin11] the need for rigorous benchmarking and test problems that reflect the major difficulties of real-world applications at a feasible computation cost was emphasized.

2.4.2. Description of the used polynomial augmented RBF method

Although the current state of the art provides no clear consensus on which meta-models are best for to represent crashworthiness responses, based on the results in [Fng05a] and [Fng06], RBF-based models were used in this work. Several researchers came up independently with ideas related to the method of RBF [Har71, Nad64]. Later a new interpretation of the method as a layered adaptive learning method [Bro88] increased the popularity of the idea.

For the brief explanation of the RBF methodology used, the analogy with a multivariate nearest neighbor interpolation with weighting functions is used. If in a multivariate space, for a set of *n* points with coordinates $\boldsymbol{\varrho}_i$ the corresponding function values $y_{\boldsymbol{i}}$ are known, an estimation of function values at any given point in the space can be made by:

$$y = \sum_{i=1}^{n} \lambda_i \phi\left(\|\boldsymbol{x} - \boldsymbol{\varrho}_i\|\right) \tag{2.14}$$

Where λ_i are the weights corresponding to each of the points with coordinates $\boldsymbol{\varrho}_i$, and $\phi(\,)$ is a radial function on the distance between the location of interpolation $\boldsymbol{x}$ and the points of the training set $\boldsymbol{\varrho}_i$. Before such an interpolation can be used it is required to calculate appropriate weights λ_i, this can be done by solving a linear system of equations that results from the following expression.

$$y_{\boldsymbol{i}} = A_{\boldsymbol{ij}} \lambda_j \tag{2.15}$$

Where matrix $A_{\boldsymbol{ij}}$ is defined as:

$$A_{\boldsymbol{ij}} = \phi\left(\left\|\boldsymbol{x}_i - \boldsymbol{\varrho}_j\right\|\right) \tag{2.16}$$

There are several different radial functions $\phi(\,)$, and distance measures $\|\cdot\|$ that can be used (see [Buh00] for an overview). If not mentioned otherwise in the scope of this work, Gaussian RBF models are used with radial functions $\phi(r) = e^{\varepsilon r^2}$ where radius $r = \|\boldsymbol{x_i} - \boldsymbol{\varrho}_j\|$ is the Euclidean distance between two points $\boldsymbol{x}_i$ and $\boldsymbol{\varrho}_j$.

The here described principle can be extended by augmenting polynomials such as described in [Kri03]. In the applied method, monomial terms $[b_1, b_2 x, b_3 x^2]$, (without variable interaction) were used as augmented polynomials $P_k(\boldsymbol{x})$.

$$y = \sum_{i=1}^{n} \lambda_i \phi\left(\|\boldsymbol{x} - \boldsymbol{\varrho}_i\|\right) + \sum_{k=1}^{m} P_k(\boldsymbol{x}) \tag{2.17}$$

The additional constants can be solved by using equation 2.18 (provided that sufficient data points are available).

$$\begin{bmatrix} A_{\boldsymbol{ij}} & B_{ik} \\ B_{ki} & [0] \end{bmatrix} \begin{bmatrix} \lambda_i \\ b_k \end{bmatrix} = \begin{bmatrix} y_{\boldsymbol{i}} \\ [0] \end{bmatrix} \tag{2.18}$$

where index k is enumerated over the total number of coefficients m of the augmented polynomial functions. And matrix B_{ik} is defined as $B_{ik} = P_k(\varrho_i)$ and $B_{ki} = {B_{ik}}^T$.

2.5. Optimization methods

2.5.1. A brief overview of the history of optimization in a structural engineering context

It seems a natural desire of humans to improve tools, structures and methods. The concept of “Trial and error” is presumably one of the oldest design paradigms. When it is combined with human intuition, an individual memory, and a collective memory in the form of culture, it is a remarkably powerful approach.

For problems in which the human intuition fails to achieve a sense of logic in the interpretation of the already established trials and results, or in cases where intuition indicates that many more trials are necessary, the approach can be however tedious, unsatisfactory, or too time demanding, and a systematic approach seems desirable. Following a systematic approach to achieve a predefined goal can be considered as optimization in a loose sense. The word optimization comes from the Latin word *optimus* “best” (used as a superlative of *bonus* “good”) [Oed15]. Although in a stricter sense optimization is related to achieving the best or most favorable solution for a specific problem, the verb “optimization” is in practice however often used as a synonym for improvement. In the scope of this work the term optimization is used from an engineering perspective, referring to: the use of numerical algorithms to achieve improvement of specific objectives under specified constraints and boundary conditions.

In the documented history of science, early optimization related works are attributed to i.e.:

- Euclid of Alexandria (325-265 BC) “*Elements”* (dealing i.a. with geometry problems)
- Heron of Alexandria (10-75 BC) “*Catoptrica*” (dealing i.a. with light propagation)
- Johannes Kepler (1615 AD) “*Nova stereometria doliorum vinariorum”* (on the optimization of wine barrels)
- René Descartes (1596-1650) “*La Géometrie”* (i.a. on tangent lines or “derivatives” of certain functions)
- Pierre de Fermat (1601-1665) “*Methodus ad disquirendam maximam et minima”* (on the use of tangents for finding function extrema, see also [Str68] for a reconstruction of the approach)
- Isaac Newton (1668) “*Philosophiae naturalis principia mathematica*” (on i.a. the problem of the body of least resistance)
- Leonhard Euler (1744) “*Methodus inveniendi lineas curvas*” (on variational calculus) and “*Scientia navalis*” (1749) (on optimal ship design)
- Joseph-Louis Lagrange (1788) “*Mécanique Analytique*” (i.a. on the introduction of Lagrangian multipliers for constraint handling)

These are just some examples of well-known scientist living before the 19th century who dealt with optimization related problems, but works by many others are relevant for the current state of the art. A brief historical overview of important developments in the field of applied mathematics and optimization is given in chapter 2 of [Krn14]. Many ideas used in optimization are strongly related to mathematics, and therefore to the history of mathematics about which many books are written (i.e. [Bal60] and [Sti02]).

During the nineteenth century the first optimization algorithms were refined and developed for deterministic differentiable functions: Basic linear programming (by Joseph Fourier, see also [Gra70]). The “Newton-Raphson-Simpson” method (see [Kol92] and [Ypm95]); steepest descent [Cau1847]; least-squares minimization by Gauss and Legendre (see also [Sti81]). In the next century many more methods followed i.a: the Simplex method [Dan55], Non-linear Programming [Kar39] and Dynamic Programming [Bel54]. All of these methods would find the optimum solution for convex problems, but for multimodal non-convex problems they are however likely to converge to a local optimum, and not the global optimum.

Non-convex, multimodal global optimization problems are however relevant for many practical and industrial applications. These are typically the type of problems previously mentioned where the relation between trial and error points are unintuitive, and where most obvious systematic approaches fail to achieve satisfying results. For these type of problems meta-heuristic algorithms were developed since the second half of the twentieth century.

Pioneering work in 1950’s, by Barricelli [Bar54, Bar57] on evolutionary computation was followed by the development of evolution process based approaches and strategies to solve mathematical optimization problems [Bre62], and design optimization problems of complex technical systems [Schw65, Rch65, Rch71, Schw75], and artificial intelligence tasks [Fog62, Fog66, Hol75]. The independent development of the idea’s inspired by biological reproduction mechanisms led to slightly different approaches with various names: Cybernetic Evolution [Schw65, Rch65], Evolutionary Strategy [Rch71, Schw75], Evolutionary Programming [Fog66], Genetic Algorithms [Hol75], and later more varieties were developed [Sto97] that are now all covered by the umbrella term Evolutionary Algorithms (EA). An introduction to Evolutionary Strategies is given in [Bey02], and an overview of interesting properties of evolutionary mechanisms is given in [Schw12].

Besides evolution based algorithms, also other meta-heuristics were developed, such as: Random search [And53, Ras64], the Nelder-Mead method (another simplex based method) [Nel65], Simulated Annealing (SA) [Kir83], Particle Swarm Optimization (PSO) [Ken95], Fire Fly optimization [Yan09], and many more. For a review on meta-heuristics the author refers to [Flo09, Rio12] and for a review on nature-inspired algorithms to [Tng09]. In section 2.5.3 an brief overview is provided of the optimization algorithms used in the work of this thesis.

Structural and multidisciplinary optimization

Optimization methods can be applied to various hypothetical and practical problems in many different fields of application. The focus of this work is structural or mechanical design. Although the design and development of structures and improving their designs is a common activity throughout the human history, the term structural optimization was introduced as late as 1960 by Schmit [Schm60]. In that work the consequences of the multidisciplinary nature in structural design were demonstrated at the hand of a weight optimization of a simple truss structure with multiple load conditions. The example showed as a counter-intuitive result that the minimum weight design was not a fully stressed design in neither of the load conditions.

The goal of lightweight design is to design a structure with minimum structural mass while satisfying the structural requirements and other constraints involved. Following the introduction of [Schm60], the typical iterative structural design cycle to achieve this can be divided in three steps:

1. generate a trial design
2. evaluate the structural response of the trial design by a structural analysis
3. modify the trial design as required

Structural optimization deals with the systematic approaches or procedures to achieve an improved or optimal design. As already mentioned in the previous sections, since the late 1950’s several important advancements have been made that strongly enabled progress in this field:

- numerical solution procedures for the solution of the mechanical or structural problems
- numerical solution procedures for the design optimization of the structural problem
- computer technology to execute the numerical procedures[5].

Since the 1970’s regularly reviews have been published on the topics and advances in the structural and multidisciplinary optimization community ([Ven78], [Schm81], [Aro90], [Mar04], [Sim08]). The research and applications grew however fast during the last decades and the activities in the field of structural optimization diversified over different directions, applications, and subfields. A few examples of such sub-directions also relevant for car body optimization problems are: Topology Optimization [Bend88, Roz01, Roz09, Weh15]; Shape optimization [Haf86, Hun13], Discrete variable optimization [Cel73, Tha95]) and continuous variable optimization [Pin13]), optimization for high-dimensional design problems [Sha10]. Besides differentiation w.r.t the type or dimensionality of the design variables also reviews for particular applications or response types were published: optimization for eigenfrequency responses [Gra93], impact responses [Schr96], and composite materials [Ven99]. The large amount of research publications in this field underlines that although structural and multidisciplinary optimization involves many established techniques already used in the industry, it is also still a field of ongoing research in many directions.

An important characteristic of structural optimization is that it typically deals with multidisciplinary design criteria. Each of the relevant criteria could be described by a different model, and for complex systems it might be beneficial to organize and structure the models involved in the optimization and design process. Several different optimization architectures have been proposed in the literature such as: All-at-once (AAO), Individual Discipline Feasible (IDF), or Analytical Target Cascading (ATC). Although several case studies of ATC and other design optimization formulations for vehicle design applications are presented in the literature [Kim03, Kok02], these methods still received only modest attention in the literature and the automotive industry. Overviews of ATC and other (distributed) MDO architectures for the optimization of complex systems are given in [Sbi87,

[5] Analytical solutions for particular problems can be formulated requiring without these techniques, such cases are however rather exceptional in the set of industrially relevant problems.

Crm94, Sbi00, Ted06, Mar13]. In a recent review [Mar13] a summary and classification of optimization architectures were made. It was concluded that much work remains to be done in the area of benchmarking of different architectures, and that the development of test problems would be extremely useful. The development of distributed optimization architectures remains a topic of ongoing research. In this work only the conventional AAO Approach was applied. There are however no implicit restrictions in applying similar studies, and the developed assessment strategies to the performance assessment of distributed optimization methods. In fact the presented test problem generation methods, are developed such that extensions to applications in this field are straightforward.

The focus of this work is on car body optimization with lightweight, vibrational comfort and crashworthiness design criteria, using continuous[6] design variables scaling the material thickness (sizing). The following sections provide a brief overview on the state of the art of structural optimization for related applications.

2.5.2. State of the art of structural optimization involving eigenfrequency and crashworthiness design criteria

Optimization and vibrational comfort and eigenfrequency criteria

Optimization with frequency constraints is of practical importance for many applications and has been of interest since the early days of computer-aided structural optimization [Tur67]. Since then many advancements have been made, common solution strategies include the use of Optimality Criteria (OC) techniques and Mathematical Programming (MP) methods [Gra93]. For non-convex problems these methods are however likely to converge to local optima, therefore also the use of meta-heuristic global optimization methods is of interest for these applications [Gho08, Zuo11]. To emphasize an important observation, a brief list of several works from the recent literature on optimization methods for natural frequency responses is provided:

- Zuo W., Xu T., Zhang H., Xu T. "Fast **structural optimization with frequency constraints** by **genetic algorithm** using adaptive eigenvalue reanalysis methods." *Structural and Multidisciplinary Optimization 43.6 (2011): 799-810.*

- Gomes HM. "**A firefly meta-heuristic** for structural size and shape optimization with **natural frequency constraints**." *International Journal of Meta-heuristics 2.1 (2012): 38-55.*

- Meinhardt G., and Sengupta S. "Optimization of **Axle NVH Performance** Using **Particle Swarm Optimization**." *Proceedings of the ICAM 2014 May 28-30 (2014).*

[6] Traditionally sheet metal (steel or aluminum) are only available in discrete thicknesses. Due to modern manufacturing processes, large scale customers can order batches of sheets with custom thickness, (or produce even tailored blanks with varying thickness according to specification. Such sheets are however still subject to production tolerances, the uncertainties due to such tolerances are however not regarded in this work.

- Karakaya S., and Soykasap Ö. "**Natural frequency** and buckling optimization of laminated hybrid composite plates using genetic algorithm and **simulated annealing**." *Structural and Multidisciplinary Optimization 43.1 (2011): 61-72.*

- Luo Y., Fu J. and Zhang Y. "Robust Design for **Vehicle Ride Comfort** and Handling with Multi-Objective **Evolutionary Algorithm**." *SAE Technical Paper No. 2013-01-0415., 2013.*

- Kaveh A., and Zolghadr A. "**Democratic PSO** for truss layout and size optimization with **frequency constraints**. ." *Computers and Structures, 130, (2014): 10-21.*

In the list, the different optimization methods and applications are written in bold. The list shows that eigenfrequency related design criteria are relevant for many applications and that many different optimization algorithms are used. Several benchmark problems have been developed for shape and sizing optimization problems, based on them a several small comparative assessments have been made recently [Gom11, Mig12, Gho13]. These benchmarks were however based on the optimization of relatively simple truss structures. Comparative optimization performance assessments on typical car body design optimization problems using full vehicle models are of industrial relevance, but such studies are relatively scarce in the literature, especially for problems were also crashworthiness design criteria are involved.

Optimization and crashworthiness criteria

Before the availability of FEM with explicit time integration for transient impact problems, vehicle crash-optimization was restricted to the optimization of simplified mass and spring models such as for example in the work of [Ben77]. About a decade after the first numerical crashworthiness simulation of a vehicle structure by Haug et al. [Haug86] early feasibility studies of design optimization methods applied to automotive structures involving crashworthiness analysis on sub-structures such as those by Yang et al. [Yng94] and Schramm et al. [Schr98] were published. These feasibility studies were later followed by basic studies of MDO of full vehicle structures w.r.t. crash, NVH and lightweight criteria in the works of, for example, Yang et al. [Yng01] and Sobieszczanski-Sobieski et al. [Sbi01]. Since then the investigations and showcase studies applying various types of optimization methods on vehicle design problems have increased strongly in quantity (e.g. [Cra02, Yng02, Dud03, Kod04, Nil04, Yng05]) and continued to be of interest during the recent years (e.g. [Dud08, Hor09, Yil12, Gu13, Hes15, Kia15, Rak15]).

Vehicle crashworthiness responses depend on complex interactions between the involved vehicle components. Nevertheless, there is also attention in the literature for optimization of separate components. Investigations on design optimization of thin rectangular thin-walled crash beams were presented in [Liu08]. In [Hou08] different optimization formulations for the design of crash absorbing multi-cell beam structures were investigated. Investigations on graded foam-filled structures, where the foam has varying density throughout the crash-beam are presented in [Sun10]. Investigations on multidisciplinary optimization of composite absorber crash boxes have been presented in [Lnz04]. A combination of experimental and numerical crashworthiness studies on crash boxes is described in [Zar08]. The study presented in [Rus08] indicated the importance of the modeling of strain-rate sensitivity and boundary conditions on the location of the collapse of the crash-beam/box structures. In the work of [Mar11] genetic algorithms combined with neural networks were used for the MDO of crash tubes.

Although the previous studies indicate that also the optimization of separate components are of interest, the challenge of higher industrial interest is the optimization of component assemblies and vehicle structures. In [Etm96] a multipoint Sequential Quadratic Programming (SQP) optimization approach is applied to a crashworthiness problem-based on multi-body simulation. In the presented study, the "noisiness" and nonlinear nature of the simulation results w.r.t. changes in the design variables were identified, and multipoint approximations were used to tackle the non-smoothness of the responses. Although at the time multi-body simulations (MBS) were computationally expensive, MBS makes coarse approximations w.r.t. local stiffness of the components involved in a crash, and is typically much less computationally expensive as FEM-based crash simulation. The study in [Sbi01] (which was a cooperation between the NASA, SGI High-performance computing and the Ford Motor company) demonstrated the industrial application of MDO on a car body structure for minimum mass, under frequency and crashworthiness constraints, using FEM-based simulation techniques. Later in the study of [Cra02] a similar vehicle optimization problem was solved using a response surface method combined with a multi-start variant of the leap-frog dynamic trajectory method. Many studies using different algorithms on crashworthiness problems followed. In [Red04] the application of a stochastic optimization approach was investigated for several analytical test functions, and two simplified crashworthiness problems. In [Lia08] a two stage multi-objective optimization on a vehicle optimization problem was presented. The use of a stepwise regression model in an optimization context for the design of a car body structure was presented in [Lia08B]. In [Dud08] several optimization algorithms were compared using different benchmark studies, and a list of requirements for optimization algorithms for application to simulation-based car body optimization problems was presented. A new particle swarm based optimization approach was used in [Yil12] for a vehicle crashworthiness problem. In [Abb14] concurrent usage of a hybrid Neuro-fuzzy model and the Taguchi method were applied to an automotive crashworthiness optimization problem.

Similarly as was the case with the optimization with eigenfrequency criteria, also many different optimization approaches and algorithms are used for several very similar problems that involve crashworthiness criteria. In many of these and other works that include crashworthiness criteria, response surface modeling, surrogate modeling or meta-modeling techniques are used. It should be noted that in some of the presented works the optimization was only performed based on a static meta-model response. In that case the choice of the optimization algorithm, and the optimization efficiency is almost trivial, since the computation cost required for function evaluations on the meta-model is very low. For industrial applications the aim is however often not to run an optimization on a static meta-model but to optimize and explore the search space of the simulation responses. Although meta-models can be used to guide the optimization procedure on the simulation responses, the meta-model responses should not be confused with the simulation responses, because especially for the highly nonlinear, non-smooth responses that are involved with crashworthiness, significant deviations and errors between the two can exist. The topic of meta-modeling is discussed in section 2.4 in further detail. An overview on the state of the art on comparative assessments of optimization algorithms and car body design applications was given in section 2.1.

2.5.3. Used optimization algorithms

In the literature, both gradient based as well as meta-heuristic algorithms are used for problems with lightweight, eigenfrequency and/or crashworthiness criteria. For the comparative assessment and other case studies several different optimization algorithms were used. For each of the optimization algorithms selected a short overview of properties and references is provided.

Interior point (IP) algorithm
The group of Interior point algorithms (or barrier algorithms) are generally used to solve nonlinear convex problems. According to [Boy09] interior point methods can solve convex problems typically within 10-100 steps, where in each of the steps first and second-order derivatives of the constraint and objective functions are required. If the derivatives are not implicitly provided by the objective and constraint functions, they can be established by finite differences at the cost of additional function evaluations (this cost increases then proportionally to the number of design variables). For a description of the algorithm it is referred here to the textbook of [Boy09] chapter 11. The implementation in MATLAB 2013a embedded in the “fmincon” function (option 1) was used in this work.

Sequential quadratic programming (SQP)
The Sequential quadratic programming approach is generally used to solve smooth nonlinear problems. SQP solves the optimization problem by sequential steps of the Newton method. The Newton method successively updates its search points on the location of the estimated minimum according to the assumed quadratic model. To the description of the algorithm a book chapter is dedicated in [Flt10]. The implementation in MATLAB 2013a embedded in the “fmincon” function (option3) is used in this work.

Genetic algorithm (GA)
Genetic algorithms are a class within the evolutionary algorithms that mimic the genetic process of the reproduction of biological life, in an iterative optimization procedure. Starting from a given or random initial population, genetic operators such as crossover and mutation are used to generate a new offspring population of search points. The optimization procedure is based on the principle of “survival of the fittest” by selecting parent (search points) based on fitness criteria that correspond to the objective function evaluation, to generate the offspring (new search points) for the next generation, and iteration step. A detailed description of genetic algorithms can be found in [Gld88]. The implementation used in this work was the “ALGA” implementation included in MATLAB 2013a.

Non-dominated sorting genetic algorithm (NSGA2)
NSGA-2 is a Multi-objective evolutionary algorithm proposed in [Deb00], as an improvement over the original NSGA presented in [Sri94]. It can handle any number of objectives and strives to find designs close to the Pareto front. The application of this algorithm is unconventional for single objective problems, preliminary investigations showed reasonable performances for the type of single objective problems of interest, and therefore the algorithm was included in the comparison. The variation of the algorithm used in this work is Reference-point based NSGA-II implemented by [Lin11].

Differential Evolution (DE)
Differential Evolution (DE) is another evolutionary algorithm used for optimization. The main distinction of Differential Evolution algorithms with respect to genetic algorithms is the use of parameter vectors. Where for GAs the objective improvement is dependent on selection, of improvements by quasi-random changes, DE can make targeted steps by adding "gradient like" weighted difference vectors between two points to a third point. A comprehensive overview and a MATLAB implementation is provided in [Sto97]. The implementation used was an adaptation of that code by [Buh13], combined with a penalty approach to enforce nonlinear constraint handling.

Particle Swarm optimization (PSO)
Particle swarm optimization algorithms are inspired by the movement of groups of organisms in for example a bird flock or fish school. A group or population of particles, changes their position in each step of the algorithm, based on its local best position, the global best known position, its velocity vector, and particle inertia. A description of the algorithm principles can be found in chapter 8 of [Yan10a]. The implementation of [Bir06] was used combined with an additional penalty factor approach to enforce nonlinear constraints.

Simulated Annealing (SA)
Simulated annealing is a meta-heuristic search algorithm, developed in the early 80's by [Kir83] and [Cer85] inspired by the thermodynamic process involved in the metallurgic annealing heat treatment. The principle behind the algorithm is that starting from an initial set of search locations (equivalent to a population), a change of location is induced in each time step, which corresponds to the kinetic movement of atoms in the annealing analogy. Changes to lower energy states are admitted but higher energy states are admissible according to a probability function depending on the temperature. At increasing time and number of algorithm iterations, the temperature decreases according to a cooling scheme, and thus convergence to lower energy states is enforced. A description of the algorithm can be found in [Yan10b], together with an implementation of the algorithm, that was used in this assessment.

Fire Fly Algorithm (FFA)
Fire Fly inspired optimization algorithms are population-based and follow the analogy of fireflies attracted to surrounding fireflies by light intensity (a fitness equivalent) to reproduce offspring (new function evaluation samples), to explore and exploit the search space. An algorithmic description of is provided in chapter 10 of [Yan10b], together with the MATLAB implementation that was used in this work.

3. Meta-model based comparative assessments of optimization algorithms for various multidisciplinary car body design problem formulations

“All models are wrong, but some are useful.”

-George E. P. Box [Box87]

3.1. Motivation and aim of the comparative assessment study

Based on the literature review chapter 2, it could be concluded that although many different optimization algorithms are available and used for these type of problems, there are no clear guidelines on which algorithms to choose. Comparative assessments of optimization algorithms for problems related to this application type are very rare in the literature, although they are of industrial interest [Wan13].

Of the few comparative studies available in the literature, most do not perform the comparative assessment by means of the simulation responses, but on static meta-models of the simulation responses, to reduce the involved computation cost [Gu13, Kia15]. In none of those works, the validity of the obtained results based on meta-models, is verified with similar optimizations based on the simulation responses. Moreover, in all of these works the number of construction points for the meta-models was rather low. In the two available studies that were not meta-model based, either only a few algorithms were compared [Dud08], or insufficient repetitions were performed to obtain statistically significant results [Nil04]. Furthermore in all of these works only a single vehicle model for each problem type was investigated.

The here presented comparative assessment aims to extend the available work in the literature on the following points:

- Use of two different vehicle models per optimization formulation
- Verification of the optimization performance of the meta-model based comparison, with a simulation workflow based comparison
- The performance of 8 state-of-the-art optimization algorithms is compared
- The comparisons are based on several optimization problem formulations
- The meta-models are based on a number of construction points that exceeds previous studies by at least an order of magnitude (1000 simulation-based function evaluations)

Because the set-up of this comparative assessment includes features not available in previous studies, the following research questions (as introduced in the introduction) can be addressed:

Q1 Are the relative optimization algorithm performances on a particular vehicle design problem correlated with the relative performance on a similar vehicle design problem involving another vehicle model?

Q2 How representative are meta-model optimization benchmarks for vehicle design problems compared to full direct simulation-based optimization performance benchmarks?

Q3 Are the differences in performance between meta-heuristic algorithms on various problem formulations of typical car body design optimization problems involving crashworthiness responses, of practical relevance?

In the next section, a description of the assessment set-up will be given, some notes on statistical consideration in optimization performance are discussed in section 3.3. The results of the meta-model based comparative assessment and a corroboration using an independent vehicle model are provided in sections 3.3 and 3.4 respectively, followed by the conclusions, discussion, and outlook.

3.2. Description of the comparative assessment study

In this study, a selection of optimization algorithms is compared with respect to their performance efficiency for problems involving basic NVH criteria. The comparative assessment is made for several different optimization formulations that include multidisciplinary objectives and constraints related to: global vehicle eigenfrequencies, vehicle mass, and nonlinear crashworthiness responses. The significance of the meta-model based assessment results are corroborated using results based on direct simulation workflow results of a third vehicle model.

To make statistically significant comparisons a large number of optimization runs, and thus function evaluations are required. In order to achieve a high number of function evaluations the optimization runs are performed on meta-models of 2 vehicle models, constructed from 1000 quasi-random function evaluations per vehicle model. The meta-models used are polynomial augmented Radial Basis Function (RBF) models following the recommendations in [Fng06] (see also section 2.4).

The meta-model based assessment is made using results from two distinct vehicle models (Vehicle models A and B). A detailed description of the vehicle models, design variables and simulation responses is given in section 2.3.

The optimization algorithms compared are:

1. Interior Point (IP) algorithm
2. Sequential quadratic programming(SQP)
3. Genetic algorithm (GA)
4. Non-dominated sorting genetic algorithm (NSGA2)
5. Differential Evolution (DE)
6. Particle Swarm Optimization (PSO)
7. Simulated Annealing (SA)
8. Fire Fly algorithm (FFA)

More details about the implementations are provided in section 2.5.3. For each of the algorithms and each problem formulation, the performance statistics of 100 algorithm runs are compared for a budget of 250 and 500 function evaluations. The results are expressed in terms of the Relative Objective Improvement (ROI) denoted by the symbol ξ.

The ROI for a number of *i* function evaluations is defined as:

$$\xi_i = \frac{F_{nom} - F_{min_{\mathrm{i}}}}{F_{nom} - F_{min_{REF}}} \tag{3.1}$$

where $F_{min_{\mathrm{i}}}$ is the minimum feasible objective after *i* function evaluations, F_{nom} is the objective value of the nominal design, and $F_{min_{REF}}$ is the minimum objective value found for the given problem formulation. Such that the ROI expresses the ratio of the objective improvement at a given number of iterations, relative to the maximum achievable improvement known for the problem.

3.2.1. Optimization formulations

A general description for an optimization problem with k nonlinear constraints is:

$$\min f(\boldsymbol{x}) \text{ Subject to: } g_k(\boldsymbol{x}) \leq 0 \quad (3.1)$$

Where objective function $f(\boldsymbol{x})$ and constraint functions $g_k(\boldsymbol{x})$ are a function of the design variable vector $\boldsymbol{x}$: $f: R^n \to R$ and $\boldsymbol{x} \in R^n$.

In this study depending on the formulation investigated, the result of objective function $f(\boldsymbol{x})$ corresponds to the either, the vehicle BIP mass, or the negate of the first torsion eigenfrequency (minimizing the negate is maximizing the original). The constraint functions $g_k(\boldsymbol{x})$ are either an upper bound on the vehicle BIP mass, a lower bound on the natural frequencies, an upper bound for the maximum acceleration, and/or a lower bound on the deformation between the A- and B-pillars. Details regarding the combinations of objectives and constraints for the investigated formulations are described in the following list:

1. Unconstrained mass optimization with design variable bounds (reference 1)
2. Mass optimization with design variable bounds and with natural frequency constraints
3. Mass optimization with design variable bounds and with natural frequency, and crashworthiness constraints
4. Unconstrained natural frequency optimization with design variable bounds (reference 2)
5. Natural frequency optimization with design variable bounds and mass constraints
6. Natural frequency optimization with design variable bounds with mass, and crashworthiness constraints

The unconstraint mass optimization with variable bounds (formulation 1) is only included in the formulation selection as a reference. The mass estimation of the vehicle structure with sheet thickness parameters is a trivial problem by itself for two reasons: 1 it is computationally cheap to estimate; 2 the optimal solution is intuitive (minimum thickness throughout the structure). The formulation is however useful as a reference, because provided a maximum computation budget, based on the typical budget of the other formulations; it can provide an estimation of the upper bound for the optimization algorithm performance.

3.2.2. Statistical considerations in optimization performance

Search methods such as the IP algorithm and SQP generally exploit the search space according to deterministic operators such that the “search path” is dependent on the initial starting point. A common way to overcome “optimum” solutions restricted to a single local optimum is the application of multi-start approaches (repeatedly starting the method from another starting point).

Most meta-heuristic algorithms or nature-inspired optimization algorithms have stochastic operators to enforce diversification in order to avoid getting trapped in local optima. Examples of such operators are mutation or crossover operators that are applied to create new search points/populations based on previous search points subjected to a pseudo-random change or combination of properties. The progress and history of a search of an algorithm with such operators is therefore also dependent on the initial starting points and the “seed” that was used to generate the pseudo-random state for the stochastic operators.

The search performance of a single algorithm run, on a particular problem is thus a probabilistic quantity since it depends on the starting point(s) and random seed of the stochastic operators. Therefore, the performance assessment is made, based on the obtained statistics of several repetitions of algorithm runs with different starting points and random seeds. As an example, Figure 3 plots the best feasible objective value during a series of 10 optimization runs on a meta-model based problem. The objective of the optimization runs is weight reduction of body in prime design, constrained with natural bending and torsion frequencies, combined with constraints on A-B pillar deformation, and maximum acceleration at the tunnel during a frontal crash against a rigid wall (see also the definitions of the design criteria “Mass”, “NTF1”, “ABP. Def”, and “P. Acc” in section 2.3).

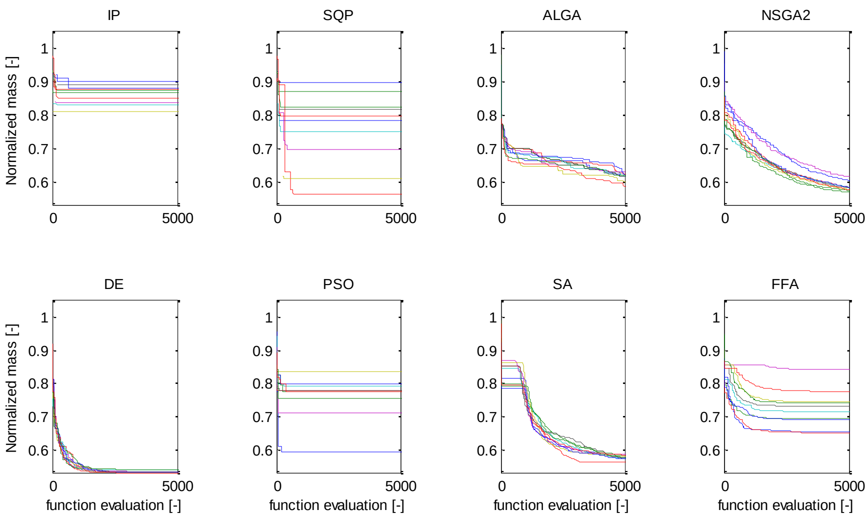


Figure 3 Best feasible objective history plots for 10 repeated optimization runs for 8 different algorithms.

The diagrams in Figure 3 show the optimization characteristics of the repeated runs of the investigated algorithms, on a single optimization problem instance. These diagrams are established based on optimization runs with a maximum of 5000 function evaluations. Due to the high computation cost for each design evaluation, optimizations are in industrial practice often limited to a single run, with a strictly limited function evaluation budget. According to the examples and suggestions in [Dud08, Kno05, Kno09], typical optimization budgets for these applications are between 200 and 500 function evaluations.

Although significant design improvements can even be achieved with such small sampling budgets, the result of such optimization runs are restricted to a preliminary “local optima” or a “lucky pick”, since this is a function evaluation range where the meta-heuristic algorithms generally don’t reach global convergence yet. In this early stage of the optimization trajectory also the stochastic aspects can have a dominant influence on the resulting optimization performance (see Figure 3 and Figure 4). The distributions of the optimization performance are unsymmetrical, thus an algorithm (A) could outperform another algorithm (B) on average, whereas the upper 90% performance quantile of algorithm A could be worse than that of algorithm B.

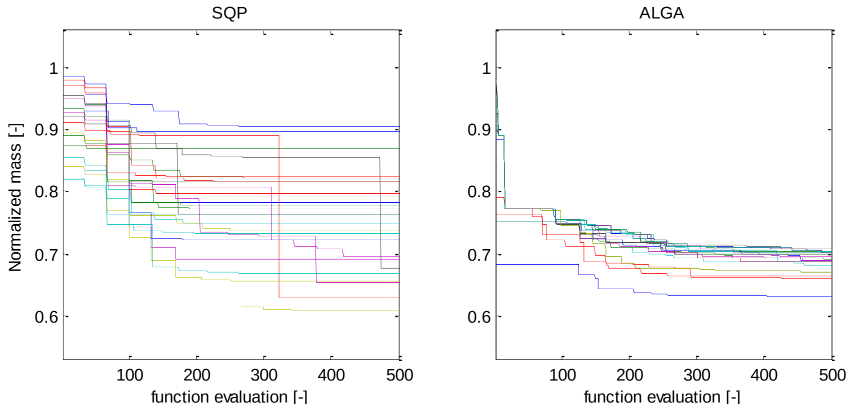


Figure 4 Best feasible objective history plots for 25 repeated optimization runs for an optimization budget of 500 function evaluations for: left SQP and right the GA algorithms

The statistical quantities (averages and quantiles) that are used to summarize algorithm performance, in the proceeding of this work are based on 100 optimization repetitions per investigation.

3.3. Optimization efficiency assessment for six optimization formulation types

For each optimization formulation, and each vehicle model the ROI results of the optimization algorithms are compared in bar diagrams. In Figure 5 the optimization performance results based on the meta-model of vehicle model A for formulations 1, 2 and 3 are displayed. In Figure 6 the corresponding results are displayed based on the meta-model of vehicle model B.

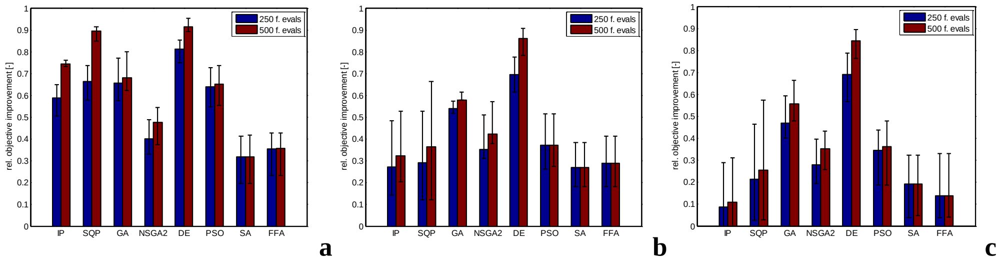


Figure 5 A comparison of relative objective improvement for the selected optimization algorithms after 250 and 500 function evaluations, for optimization formulations 1, 2 and 3 on vehicle model A.

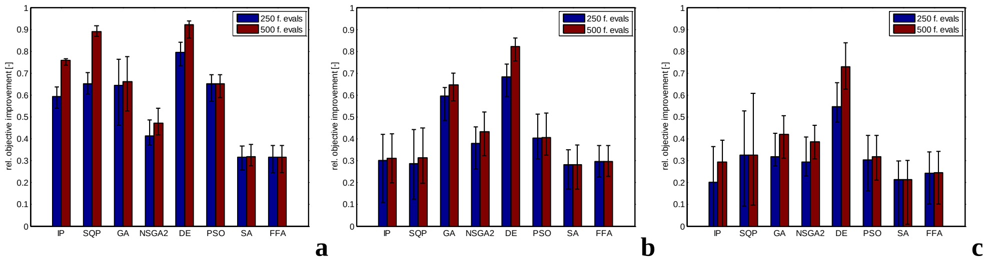
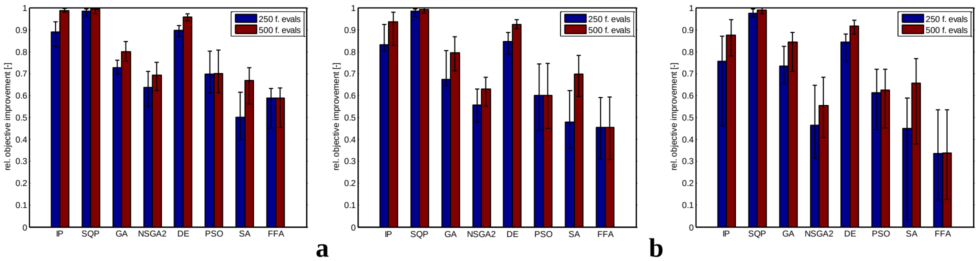


Figure 6 A comparison of relative objective improvement for the selected optimization algorithms after 250 and 500 function evaluations, for optimization formulations 1, 2 and 3 on vehicle model B.

As already mentioned in section 3.2, formulation 1 (unconstrained mass minimization) is only included as a reference. None of the selected optimization algorithms finds the minimum value (within a 1% tolerance) within the given function evaluation budget. The respective ROI-values show how the function evaluation budget limits the optimization performance of each of the algorithms for the (trivial) unconstraint problem. The differences between ROI-values between figures a and b, b and c indicate how the additional constraints affect the optimization performance. As might be expected the results show that additional constraints decrease the ROI-values.

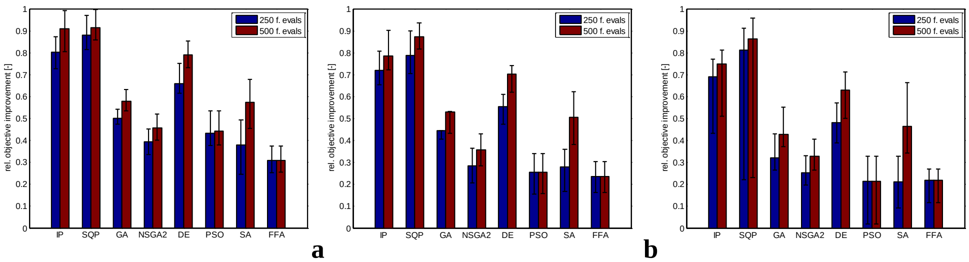
Although the additional constraints affect the efficiency of all algorithms, the IP method and SQP algorithms have a large performance decrease when constraints are added. Besides decreasing optimization efficiency also the variance and 10% and 90% percentiles over the optimization results increase when adding constraints. Comparing the results "vertically" (5a with 6a, 5b with 6b and 5c with 6c) the results indicate not identical but very similar relative optimization performance for corresponding problem formulations on different vehicle models.

In Figure 7 the optimization performance results based on the meta-model of vehicle model A for formulations 4, 5 and 6 are displayed. In Figure 8 the corresponding results are displayed based on the meta-model of vehicle model B.



a **b** **c**

Figure 7 A comparison of relative objective improvement for the selected optimization algorithms after 250 and 500 function evaluations, for optimization formulations 4, 5 and 6 on vehicle model A.



a **b** **c**

Figure 8 A comparison of relative objective improvement for the selected optimization algorithms after 250 and 500 function evaluations, for optimization formulations 4, 5 and 6 on vehicle model B.

Comparing the ROI-diagrams in Figure 7 and Figure 8 from left to right, the similar observation descriptions as before apply:

1) The results of the unconstrained frequency optimization (formulation 4) are a reasonable upper bound estimate of the expected optimization efficiency.
2) The efficiency decreases with increasing constraints (formulations 5 and 6).
3) The optimization efficiency distributions are similar for both vehicle models.

Comparing the corresponding formulation pairs (1 and 4, 2 and 5, 3 and 6) “vertically”, it can be observed that the optimization efficiency decreases less for the frequency response optimization as for the mass response optimization when constraints are added.

Correlations in optimization performance for similar problems of different vehicle models

An important point of interest (See Q1) is the correlation of the optimization algorithm performance between the investigations on vehicle models A and B. In Table 2 an overview of the (Pearson) correlation coefficients (CC) and the corresponding p-values (p) is given (see [Rod88] for a discussion on different interpretations of the correlation coefficient).

Table 2 Overview of the correlations between the algorithm optimization performance between the dataset of vehicle model A, and vehicle model B.

	250 f. evals		500 f. evals	
formulation	**CC**	**p**	**CC**	**p**
1	0.995	3.1E-07	0.998	2.1E-08
2	0.990	2.7E-06	0.981	1.7E-05
3	0.910	1.7E-03	0.937	6.1E-04
4	0.938	5.8E-04	0.965	1.1E-04
5	0.950	2.9E-04	0.972	5.5E-05
6	0.837	9.5E-03	0.869	5.1E-03
mean	**0.937**	**2.0E-03**	**0.954**	**9.8E-04**

The p-value which lies in the domain between 0 and 1 expresses the probability that the null hypothesis is true [Fis50]. In this context, the null hypothesis refers to the statement that there is no relationship between the two data sets. Thus a low p-value ($p<0.01$) indicates that the relation between the predictor and the validation data is significant, or conversely that there is a probability of p that the obtained results are either obtained by random chance or that the null hypothesis is true.

The correlation coefficient is a scale-independent measure, therefore relative optimization performance distributions are compared by this measure. The results indicate that for all of the optimization formulations (1-6), the correlations are significant. This indicates that the meta-model based optimization performance distribution of vehicle model are A is strongly correlated to the distribution for vehicle model B. This was the case for the ROI at 250 function evaluations (f. evals) as well as for 500. Also the results from the corroboration study in section 3.4 with a third vehicle model support the thesis that correlations between similar problems on different vehicle models are significant.

Relevance of the optimization algorithm performance

A point of interest is that for all formulations the mean of the difference between the ROI at 250 and 500 function evaluations ($\Delta\mathrm{ROI}_{250-500}$) is smaller than the average absolute difference (AAD) of the ROI among the different algorithms. Table 3 provides an overview of the statistics for the six problem formulations and both vehicle models. The results show that the change in ROI by doubling the maximum number of function evaluations from 250 to 500, has less influence on the ROI than the average absolute difference between the performance of a particular optimization algorithm w.r.t. the mean performance of the investigated algorithms (the AAD is thus equal to the standard deviation of the ROI-values over the different optimization algorithms).

Table 3 A comparison of the influence between increasing the number of function evaluations and algorithm selection in terms of average change of ROI.

	Vehicle Model A			Vehicle Model B		
formulation	$\Delta\mathrm{ROI}_{250-500}$	AAD_{250}	AAD_{500}	$\Delta\mathrm{ROI}_{250-500}$	AAD_{250}	AAD_{500}
1	0.08	0.15	0.19	0.08	0.15	0.19
2	0.05	0.12	0.14	0.03	0.12	0.15
3	0.05	0.15	0.18	0.06	0.07	0.11
4	0.06	0.14	0.14	0.08	0.18	0.19
5	0.08	0.16	0.16	0.09	0.18	0.19
6	0.08	0.18	0.18	0.09	0.20	0.20
mean	**0.06**	**0.15**	**0.16**	**0.07**	**0.15**	**0.17**

This implies that improvement of ROI, by performing more function evaluations in this range is low compared to the differences between the different algorithms. Thus selecting the appropriate optimization algorithms is more effective than applying a "randomly" chosen optimization algorithm and doubling the computational effort for these problems within the investigated computational budget range.

Table 4 contains a summary of the assessment by listing the best-performing optimization algorithms for each of the investigated optimization formulations. The summarized results are expressed in ROI-values corresponding to the worst case 90% percentiles averaged over vehicles A and B for 250 function evaluations.

Table 4 A summary of the best-performing algorithms per optimization formulation. The results are ROI-values corresponding to the worst case 90% quantiles averaged over vehicles A and B for 250 function evaluations.

	constraints							
objective	**None (unconstraint optimization)**		**mass, feasibility fraction 60%**		**eigenfreq. (1st. Nat. Bend, 1st. Nat. Tors) feasibility fraction 40%**		**crashworthiness (A-B Pillar deformation, tunnel peak acceleration) feasibility fraction 60%**	
mass (BIP)	**formulation 1**				**formulation 2**		**formulation 3**	
	DE	**0.74**			**DE**	**0.60**	**DE**	**0.52**
	SQP	**0.59**			**ALGA**	**0.50**	**ALGA**	**0.34**
	PSO	**0.56**						
eigen-frequency (BIP)	**formulation 4**		**formulation 5**				**formulation 6**	
	SQP	**0.89**	**SQP**	**0.83**			**SQP**	**0.59**
	IP	**0.77**	**IP**	**0.72**			**DE**	**0.57**
	DE	**0.74**	**DE**	**0.63**				

Besides these relative figures, the results presented in the following section also provide absolute figures in mass savings and eigenfrequency improvements, which additionally emphasize the importance of proper algorithm selection.

3.4. Result validation

The presented comparative assessment of optimization algorithm performance was based on function evaluations on meta-models representing the simulation responses of two vehicle models. A few meta-model based comparative assessment studies for optimization algorithms on car body design problems were presented in the literature [Gu13, Kia15]. The aim of such assessments is to investigate which algorithms perform well on the problems of interest. The meta-models are used to reduce the computational cost that are involved when doing a similar study in which each of the function evaluations in the optimization process is based on the simulation response. None of the available studies, did however compare the meta-model based optimization performance results with, simulation-based performance results, for these problem types. In this work, validation[7] performance assessments have been performed directly on a full vehicle simulation workflow for some of the problem formulations (2, 3 and 5). Since the required computation cost on a the simulation workflow is orders of magnitude larger than on the meta-models, the validation examples are based on 5 repetitions per optimization algorithm, and restricted to 250 function evaluations per optimization run. Figure 9 and Figure 10 contain the diagrams comparing the results for formulations 2 and 5 respectively.

[7] Because in this work the investigations are limited to simulations and examples of vehicle models no general statements can be proven, the term “validation” should be considered in its proper context. As stated in [Ore94] validation does not necessarily denote an establishment of truth, but it establishes legitimacy. Similar to the validity of a contract (“A valid contract is one that has not been nullified by action or inaction”) a valid assumption or model is one that has not been nullified by observation or logical flaws. In this context additional independent observations (based on numerical simulations) can support/validate, refute or cast doubt on a model or assumption.

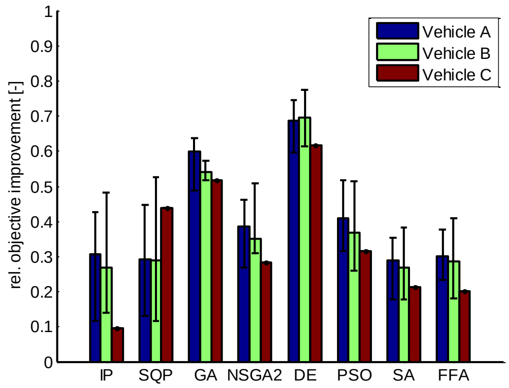


Figure 9 A comparison of the optimization algorithm performance between the meta-model based assessment (vehicles A and B) and the validation assessment (vehicle C) for formulation 2

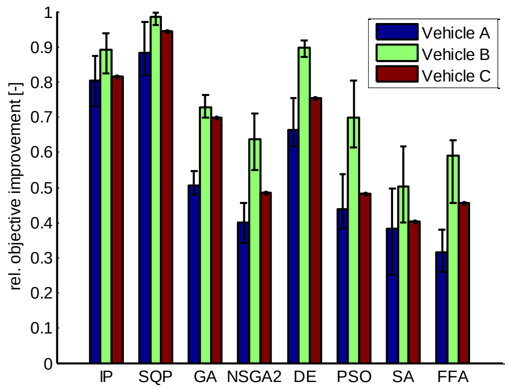


Figure 10 A comparison of the optimization algorithm performance between the meta-model based assessment (vehicles A and B) and the validation assessment (vehicle C) for formulation 5.

The optimization algorithm performance in the validation assessment is qualitatively similar to the results obtained in the previous assessment. The average ROI-values (of vehicle C) are quantitatively not always strictly within the quantile bounds of the original assessment, but follow a very similar distribution. The correlation coefficients between the averaged ROI vectors (vehicles A and B) and validation ROI vector (vehicle C) are 0.84 ($p=0.0082$), and 0.97 ($p=0.00006$) for formulation 2 and 4 respectively, and thus confirm the statistical significance and correlation between the meta-model based assessment, and simulation-based assessment results of the independent vehicle model. It should however be noted that such significant correlations between the meta-model based optimization efficiency, and the validation on the vehicle model simulation-based optimization efficiency were only obtained for optimization problems with mass, or eigenfrequency responses (formulations 1,2,4 and 5). For the optimization formulation (3) that included the crashworthiness response, the statistical test indicated a correlation between the meta-model based benchmark results and the validation examples but the significance was rather marginal (CC=0.75 and p>0.05). The results indicate that the meta-model based optimization performance results are representative for similar simulation-based comparisons for the investigated problem formulations that do not include crashworthiness responses. For the investigated problem formulation that included crashworthiness responses the results were of marginal significance in this investigation, and a larger study would be required to quantify any existing correlation with sufficient statistical significance.

The cause for the difference in correlation significance between the problem formulations with and without crashworthiness, remains unclear at this point. A possible explanation could be the difference in local smoothness between the simulation-based responses and the meta-model based responses. Although the used amount of construction points for the meta-models for the crash responses is unprecedented in the literature, it seems that the number of construction points for the interpolation is still insufficient to sufficiently model the high degree of nonlinearity that is characteristic for these responses. Further work, which quantifies the relation between the correlations and the number of construction points for the meta-models could investigate the influence of the local smoothness.

For the validated assessments, dimensionless quantities of comparison (the ROI-values) were used. In the diagrams in Figure 11 and Figure 12 the assessment results of the validation case (vehicle C) are expressed in absolute objective improvements, sorted by their optimization efficiency performance. Note that the optimization algorithms are ordered w.r.t. increasing efficiency and that this order is different for each of the graphs.

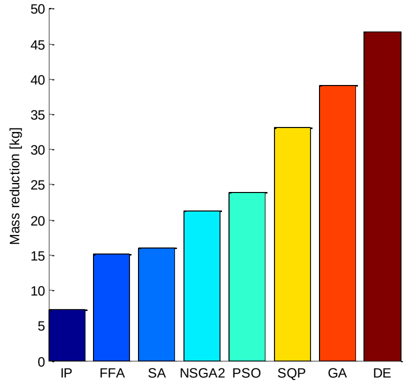


Figure 11 A Comparison of average weight savings for different optimization algorithms (problem formulation 2)

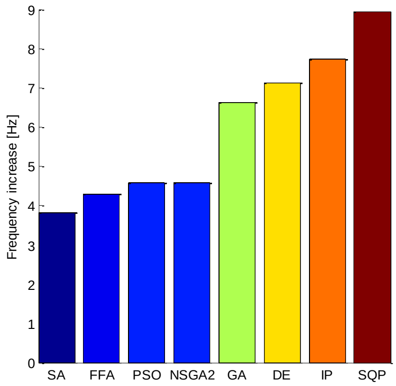


Figure 12 A Comparison of averaged frequency shift for different optimization algorithms (problem formulation 5)

The comparison, in terms of absolute results, underlines the importance of finding and selecting the right optimization algorithm for the right problem. Choosing the best performing algorithm, leads on average to more than 50% additional objective improvement than, choosing randomly one of these optimization algorithms. The efficiency increase in terms of function evaluations is even higher.

3.5. Discussion and outlook

Whereas previous studies only used one vehicle model per comparative assessment study, in this work two different vehicle models were used (and a third as a validation model). Although two or three vehicle models are too few to make detailed general claims, a few trends could be identified. The corroboration with the third vehicle model, confirmed the qualitative significance of the assessment. Furthermore, no similar investigations are available in the literature with more than a single vehicle model, and sufficient repetitions on full vehicle model simulation-based problems to achieve statistically significant results.

The demonstrated efficiency gains by appropriate optimization algorithm selection, enabled by application oriented benchmarking motivates to establish publicly available benchmark problems that are representative for industrial problems, and contribute to the reproducibility and comparability between optimization performance comparisons such as these. Especially for more complex and computationally expensive NVH criteria, similar studies can aid to increase the optimization efficiency in industrial applications and thus lead to improved design quality.

The similarities between the results, despite the differences in the vehicle models used, indicate robustness of the assessment results. Nevertheless it should be emphasized that the assessment results are only relevant for problems that are similar to the benchmark problems in terms of: design variables, response types, and vehicle concepts. The algorithms that did not perform well in this benchmark could, still be suited for different problem types as tested here, or optimizations with a larger function evaluation budget. Further aspects that could influence the optimization performance such as problem dimensionality, effective dimensionality, degree of nonlinearity should be investigated. The presented comparison was made using the (general purpose) settings coded by default in each of the algorithms. The influence of the optimization meta-parameter settings on the example problems should be further investigated. Also extensions by additional disciplines, load cases and further design criteria are industrially relevant.

Although a selection of 8 different optimization algorithms is compared, a wide variety of other optimization algorithms and implementations are available. Not this work neither a following work can contain a comparison of all available algorithms. Therefore, the presented assessment results can only be considered an initial guideline which could be extended and refined by future work.

3.6. Summary and conclusions

Based on a performance comparison of 8 publicly available optimization algorithms, a selection is made of recommended algorithms for several different of optimization problem formulations. For the meta-model based performance assessment simulation responses of two vehicle models were used. The significance of the assessment results was compared with a validation example, using optimization studies on a third vehicle model. This comparison is the first benchmark study in which more than a single vehicle model is used.

The results indicated that the correlations between similar optimization problems of different vehicle models are significant for the investigated vehicle models (Q1).

The comparison with the validation example vehicle model indicates that for the problem formulations that do not include crashworthiness responses the correlations between meta-model based and simulation model based optimization performance are significant. While for optimization formulations that included crashworthiness responses, the results indicated a lower correlation and marginal significance (Q2).

In general the results showed that there was a large variety in optimization algorithm performance. For each of the algorithm formulations the mean absolute difference in performance between a particular algorithm and the mean performance over all algorithms was larger than the average increase performance obtained when doubling the function evaluation budget from 250 evaluations to 500. Furthermore, the results indicated that the algorithm performance was highly dependent on the problem formulation, which emphasized the importance of proper algorithm selection for each problem type (Q3).

Since there is few available literature containing guidelines or comparisons of optimization algorithms for full vehicle design optimization involving NVH and crashworthiness requirements, even the modest investigations presented here contribute to the state of the art. To reach more general conclusions the work could be extended, by using additional vehicle models, a larger collection of optimization algorithms, and adding additional objective and constraint criteria. The assumed requirements in terms of computer resources, software resources, and manpower seemed to have prevented such studies from being established and documented in the literature.

The work presented shows that appropriate optimization algorithm selection can contribute significantly to the optimization efficiency and thus the optimization results achieved in industrial practice. The author encourage readers dealing with similar problems to apply the assessment results as presented here as an initial guideline. If however the reader has data available from previous similar problems, it is recommended to create their own benchmark problems and tailor them to any particular needs. For readers with different NVH related optimization problems, the presented results could motivate to make performance assessments for different problem types.

The following chapter deals with the characterization of simulation responses in order to derive representative response test functions, which can be made publicly available to increase the reproducibility of benchmark studies. Furthermore such representative response test functions can be used for meta-simulation of the optimization process in order to increase the optimization efficiency, by selecting and adapting optimization algorithms under consideration of available hardware and software (solver licensing) resources.

4. A representative surrogate problem approach and its application in a car body design context

"...when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the state of science, whatever the matter may be."

-William Thomson (Lord Kelvin) [Tho1883]

4.1. Introduction and motivation

The use of optimization algorithms to solve structural engineering problems has gained much interest in the last decades. As already mentioned in the previous chapters a great variety of optimization algorithms has been developed which can be applied to MDO problems (see [Ven78, Sbi97, Sim08, Zan10, Rio12] for reviews). But which of these algorithms to choose for a particular problem? In general there is no "magic bullet" or universal algorithm that is efficient for all problem types [Wol95]. However, particular algorithms can perform well on particular problems [Eng96]. The challenge is to identify the corresponding efficient algorithms for the problems of interest.

There are many analytical test functions proposed (compilations can be found in [Jon75, Flou99, And08]) which are commonly used to compare optimization algorithms [Yao99, Ves04, Bre06, Bao09]. Many of these functions are however criticized for their lack of complexity and representativeness for real-world industrially relevant problems [Bar11, Die12]. Besides the lack of complexity it is also difficult to relate such test functions to engineering and other real-world problems.

In case, an engineering or structural optimization problem can be expressed as a closed form solution, or its numerical solution is not computationally expensive, the optimization efficiency is trivial. When non-convex structural optimization problems involve computationally expensive function evaluations, the optimizations problem is typically several orders of magnitude more expensive as a single function evaluation. In many of such industrially relevant problems, the optimization procedure is constrained by a tight function evaluation budget, and thus optimization efficiency is important [Kno05, Kno09]. Unfortunately it is difficult to determine the performance or optimization algorithm efficiency on such problems. Theoretical analysis is currently still restricted to particular very simple problems [Bor04] and, “empirical” performance comparisons by repeated numerical experiments are exactly burdensome on such problems because of the involved computational cost. Therefore comparative assessments on optimization problems that involve computationally expensive function evaluations are rare in the literature. This leads to the unsatisfactory situation that problems for which optimization efficiency matters the most, are also the problems on which there is only few information available on optimization algorithm efficiency.

The results of the previous chapter indicated that meta-model based comparisons can be used for some car body optimization related problems, but that this strategy is not suitable for problems which included highly nonlinear crashworthiness responses. Is the only alternative then to perform the optimization algorithm comparison on the expensive simulation-based problem? In the review paper [Sha10] it was noted that presently research trends tend to focus on sampling and modeling techniques themselves and neglect to investigate the characteristics of the underlying expensive functions.

In this chapter a new approach is presented to construct test functions (Representative Surrogate Problems) which are based on the characteristics of the simulation responses. An extensive analysis of the simulation responses w.r.t. changes in the design variables is made in order to identify and quantify typical characteristics and response features. At the hand of this response characterization results the test problem formulation is explained, and formulated. The results are evaluated using two case studies using an independent validation vehicle model.

This chapter aims to answer the questions:

Q4 What are the characteristics of the simulation responses of the selected design criteria w.r.t. changes in the design variables? (Are there any typical response characteristics over similar problems involving different vehicle models?)

Q5 How to formulate computationally affordable test problems which are representative for simulation-based car body design optimization problems and their response characteristics?

The work presented in this chapter contains the following contributions to the state of the art:

- An extensive simulation response characterization is performed which is unprecedented for the application of car body design problems involving crashworthiness simulation responses.
- A novel approach to construct representative surrogate problems based on function characteristics is presented.

These points are of practical and theoretical relevance, because in order to select and develop efficient optimization algorithms it would be beneficial to avoid brute force comparisons running many repeated optimizations on computationally expensive problems. As long as there are no alternatives, comparative assessment can be of practical relevance to select efficient algorithms; they are however not intellectually satisfactory, because they provide no insight into the problem type or characteristics, and as such the results only have value for particular similar problems. The simulation response characterization, is a step towards the analysis of optimization problems in a more systematic way. The general aim is that optimization problems of practical and industrial interest can be related to particular function characteristics, and that in its turn such function characteristics can be related to optimization algorithm performance.

Presently industrial optimization problems that involve computationally expensive simulations are often limited to a subset of members in the optimization community, because the simulation-based function evaluations often require particular computer resources, software resources (solver licenses) and specialistic modeling competences. Due to these burdens, it is still common in the optimization community to use simple test functions even though they are often criticized for their simplicity. A methodology to construct test problems which are representative for real-world optimization problems, can overcome this burden, by providing accessible test functions that can be shared in the optimization community.

Furthermore, the selection of efficient optimization algorithms for such problems is not only dependent on the problem type but also on the available function evaluation budget, which is related to the available resources and time, that could be different among practitioners dealing with similar problems. Parameterized, scalable test problems can provide a mean to perform a meta-optimization of the optimization process tailored to the specific needs of the problem instance.

4.2. Description of the RSP approach

4.2.1. The concept of representative surrogate problems

In scientific literature, there is much attention for the development of new meta-heuristics, while there is relatively few attention for the analysis of the problems, and their characteristics (see also [Sha10]). The general idea of the presented approach is to construct synthetic and computationally affordable test problems based on characteristics of real-world complex structural optimization problems. In the proceeding of this work, these synthetic test problems will be called Representative Surrogate Problems (RSP). Note that (unlike conventional meta-model or surrogate modeling methods) the involved surrogate models and responses in this context are not intended to be used as an interpolation or approximation model of the targeted simulation responses, rather they aim to serve as a representative artificial response landscape with similar typical characteristics as the simulation-based response in a statistical sense. A RSP does not fit particular problem data, but is constructed to fit or satisfy selected characteristics of a problem type or class. The RSP approach can also be regarded as an adaptation and extension of surrogate data generation methods for time series such as proposed by Prichard and Theiler [Pri94] for applications with multiple correlated multivariate responses. Apart from an oral conference presentation by the author [Sal14a], in which preliminary results of this work is discussed, this or similar approaches to construct synthetic test problems based on particular real-world problems did not receive attention yet in the optimization literature.

Representative surrogate functions

In this thesis, a surrogate function for an individual response is denoted as a Representative Surrogate Function (RSF). An RSF is intended as a representative relationship between a model response w.r.t. it's design variables. In the general case this could also be a parameterized meta-model (e.g. Kriging or RBF based meta-models), in this work the author however used a function representation for the RSFs, which is inspired by the Sobol-Hoeffding function decomposition ([Hfd48; Sbl90]).

$$T(\boldsymbol{x}) \sim \theta(\boldsymbol{x}) = \theta_0 + \sum_{i=1}^{d} \theta_i(x_i) + \sum_{1 \leq i < j \leq d}^{i..d} \theta_{i,j}(x_i, x_j) + \cdots + \theta_{i,j,\ldots,d}(x_1, x_2, \ldots, x_d) \quad (4.1)$$

In equation 4.1 multi-index notation is used[8]. The terms: $T(\boldsymbol{x})$ and $\theta(\boldsymbol{x})$ refer to functions of dimension d, which can be decomposed in a series of summands of increasing interaction order. The design variable vector is denoted by the symbol $\boldsymbol{x}$, and it has elements x_i in the normalized domain of the d-dimensional unit hypercube $K^d = \{\mathrm{x} | 0 \leq x_i \leq 1; i = 1, \ldots, d\}$. The expression $T(\boldsymbol{x})$ refers to the targeted simulation-based response function, and $\theta(\boldsymbol{x})$ is refers to the surrogate function. In the scope of this work, the symbol ~ refers to similarity according to criteria to be defined by the modeler (in the example in this chapter, a particular set of such criteria will be defined and enforced as constraint expressions). When orthogonal summands are chosen, an unique and exact decomposition $T(\boldsymbol{x})$ exists, but in the case of expensive black-box functions, and approximate function decomposition based on a limited number of samples, this is of little practical relevance. The aim is to find a parameterized truncated series expansion or another computationally affordable expression that can represent the characteristic behavior of the individual simulation responses, which is not necessarily limited to an approximation of the particular response. Depending on the response type, the summands that are part of the decomposition of equation 4.1 (truncated in "interaction order") could be either represented by simple analytical functions or by series expansions over the corresponding variable subset. These "second" series expansions can again be truncated in "resolution", according to the data obtained from the response characterization. The choice for the truncation, basis functions and resulting representativeness, of such an expansion is dependent on the information obtained from the response characterization. The characteristic behavior or similarity criteria of the response output w.r.t. the design variables could involve for example the degree of nonlinearity, and the variance decomposition distribution of first and higher order interactions. The function series representation enables parameterized control over such response characteristics, whereas in data-fitting based meta-models such as RBF and Kriging surrogates, the response characteristics can only be controlled indirectly.

Representative Surrogate Systems

When more responses are involved in the optimization problems, such as in the case of MDO, the solution of the problem is not only dependent on the individual response characteristics but also on the relationship and structure between the different responses. A

[8] In particular, the expression $\sum_{1 \leq i < j \leq d}^{i..d} \theta_{i,j}(x_i, x_j)$ indicates a sum over all function decomposition terms with pairs two variables for which the index inequalities: $1 \leq i < j \leq d$ are valid. The right hand side last terms indicate the corresponding sums over variable subsets of more than two variables $(i, j, \ldots, d)$. For each interaction order k there are $\binom{k}{d}$ distinct variable subset combinations.

set of RSFs (index r in equation 4.2) combined with defined structure or relation between the involved responses is denoted as an RSF-Set or Representative Surrogate System (RSS).

$$T^r(\boldsymbol{x}) \sim \theta^r(\boldsymbol{x}) = \theta_0^r + \sum_{i=1}^{d} \theta_i^r(x_i) + \sum_{1 \le i < j \le d}^{i..d} \theta_{i,j}^r(x_i, x_j) + \cdots + \theta_{i,j,\ldots,d}^r(x_1, x_2, \ldots, x_d) \quad (4.2)$$

Representative Surrogate Problems

A RSP can be defined by choosing an optimization formulation involving objectives and constraints that are depending on RSS responses. An example of a single objective optimization problem subjected to nonlinear inequality constraints could be expressed as:

$$\min f(\theta^r(\boldsymbol{x})) \text{ subject to: } g_w(\theta^r(\boldsymbol{x})) \le 0 \quad (4.3)$$

Where index w refers to the number of constraints. Once an RSS is established it is straight forward to test different optimization formulations on a given set of responses.

RSP construction

As can be seen, from the previous definitions the most challenging part of the RSP approach is to obtain RSFs and an RSS that is representative for the responses of interest. The general structure of the approach is: to apply parameter-study and other existing sensitivity analysis methods (see section 4.2.2) to identify and quantify characteristics of the involved simulation-based responses that are common over a set of problem instances (different vehicle models in the application example). These characteristics such as nonlinearity types, sensitivity index distributions, and inter-response correlations can be used to define a constraint satisfaction problem (CSP) based on the combination of suitable basis functions with free parameters, the domain of the parameters and the constraint set that enforce the selected function characteristics (see section 4.2.3). Using the solutions of this CSP problem as a parameter set for the given basis functions will result in an RSS with a selection of similar response characteristics as the simulation-based calibration responses. The responses of the resulting RSS can be used to define a synthetic optimization problem.

The activities to construct an RSP could be summarized by the following steps:

1. response characterization
2. construction of the RSFs and the RSS by defining and solving a CSP
3. combining the optimization formulation with the resulting RSS to define an RSP
4. corroboration of the RSP.

Since the RSS and RSP are not approximative surrogates, the validation or corroboration of them can only be done indirectly by comparing the characteristics, or the performance of operators such as optimization algorithms between them, and an independent model or optimization problem instance.

Applications and general remarks

The resulting synthetic problem or RSPs could be used as a test or “toy function” to compare, select, tune and develop efficient performing optimization algorithms and optimization frameworks for the related class of real-world optimization problems. Once established they have a computational cost, orders of magnitude less than the real problem instance. In addition, they also improve the accessibility of problem types, which are normally only available to a limited community because simulation-based function evaluations often require modeling expertise, solver licenses, and considerable computation resources to be used in an optimization. Furthermore such RSPs could be made publicly available to serve as standardized benchmark problems, enabling an increased comparability, and reproducibility between performance studies on particular type of applications. In this chapter a schematic overview of the approach for the example case study on a multidisciplinary car body design application is provided.

Although the function characterization necessary for the formulation of an RSP requires much more function evaluations than a typical optimization run of a single problem instance in an industrial design environment, the cost of such investigations can be seen as an investment that provides an increased insight into the typical response structure for similar problem types. The investment to apply the approach could pay off for practitioners that deal with many similar optimization problem instances that involve expensive simulators (such as vehicle design problems), in particular for those who aim to select or develop specialized algorithms for particular complex problem types. In the case where conventional meta-models are able to represent the response characteristics, they can replace computationally expensive “black-box” simulation responses, with computationally affordable “black-box” meta-models. Although this can be practical, the additional insight for a systematic analysis of the problem is rather limited. For the systematic development of optimization strategies for difficult problems, it would be useful to analyze problems by their characteristics. The nature of the proposed approach enables the investigation on the influence of different response characteristics on the performance of optimization algorithms or strategies. Such additional insight could be a further justification for the required investment in the response characterization.

4.2.2. Simulation response characterization

Based on the vehicle models, parameterization, design space and design responses described in section 2.3, the selected simulation responses are analyzed w.r.t. changes in the design variables. The results presented in this section, are specific for the selected response types, design variable types and design space. Different results could be obtained for other choices, nevertheless the response characterization applied methods are not specific to any of the responses or design variables, and could also be applied to other problem variations or even completely different problem types.

Local one-factor-at-a-time (OFAT) and two-factor-at-a-time (TFAT), parameter studies have been performed to investigate and quantify the degree and type of nonlinearity of the response functions, as a function of the design variables. For these parameter studies one or two variables have been changed in fixed steps over the entire domain[9], while all other design variables are fixed to their nominal value, hence only first and second-order effects are investigated. The term local in this context refers to the fact that these parameter studies have only been performed at a single location w.r.t. the other design variables. It has to be noted that for other responses or design variable types (such as parameterized ply orientation in the case of composite materials), or other design variable ranges the

[9] The design variables are normalized to be in the unit hypercube domain and scale the nominal part thickness by a scaling factor varying between 0.5 and 2

relationship type between the design variables and response could be different. Figure 13 shows a representative sample of the response characterization with respect to change of a single design variable, while keeping the others fixed to the nominal value. In the present work a characterization of the responses upon first-order changes of all the design variables is performed for vehicle models A and B.

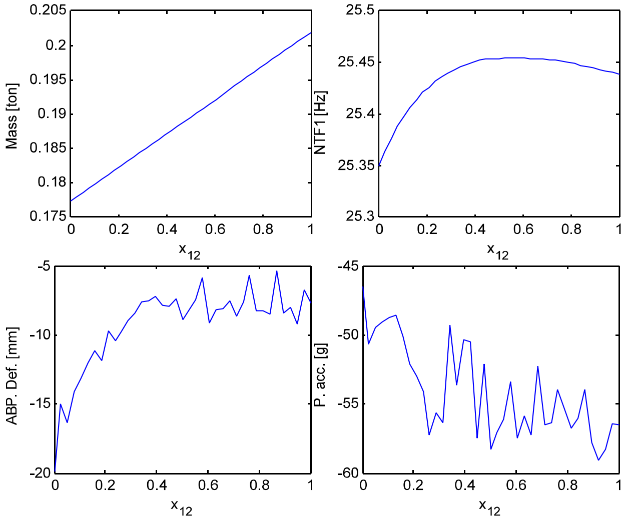


Figure 13 Overview of different types of nonlinearities in OFAT parameter studies, for four different response types, w.r.t the variation of one design variable

In the scope of this thesis, only a few results are displayed. However, the full set of parameter study results indicate very similar nonlinearity characteristics w.r.t. the design variables for each response type. The relative importance or amplitude of the first and second-order effects varied over the different design variables, but the "shape" of the relation between the responses and the design variables identified characteristic types of nonlinearity for each response type. The results indicated linear behavior for the vehicle mass response, mildly nonlinear behavior for the natural modes, and highly nonlinear behavior of the deformation and peak acceleration responses during the crashworthiness load cases.

To investigate the type of interactions, similar investigations are performed using TFAT parameter studies on a subset of the design variables. The subset is defined based on the global sensitivity analysis results described later in this section. Figure 14 shows an example of the results.

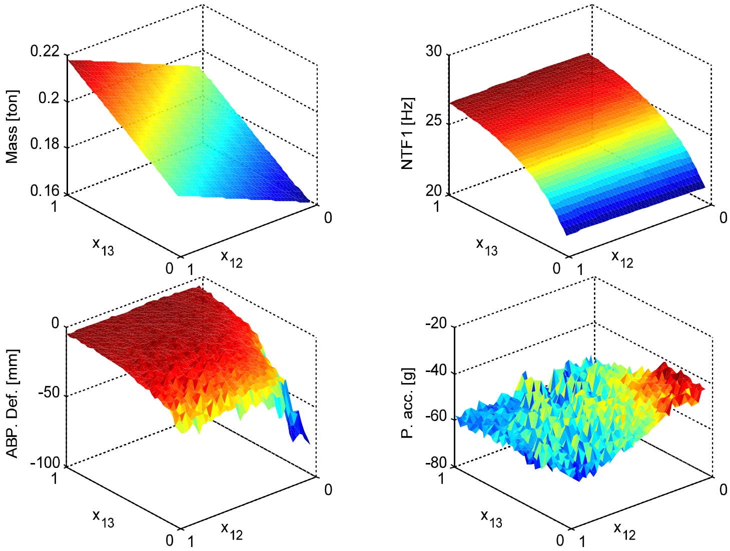


Figure 14 Overview of different types of nonlinearities in TFAT parametric studies

Further analysis is performed to quantify the nonlinearity and variations among parameters. For the responses with nonlinear and non-smooth first-order and second-order effects, the results are analyzed using one- and two-dimensional spectral wavenumber decomposition using the Fast Fourier Transform (FFT). Figure 15 and Figure 16 show examples of wavenumber decomposition analysis results for the peak acceleration response, of vehicle model A.

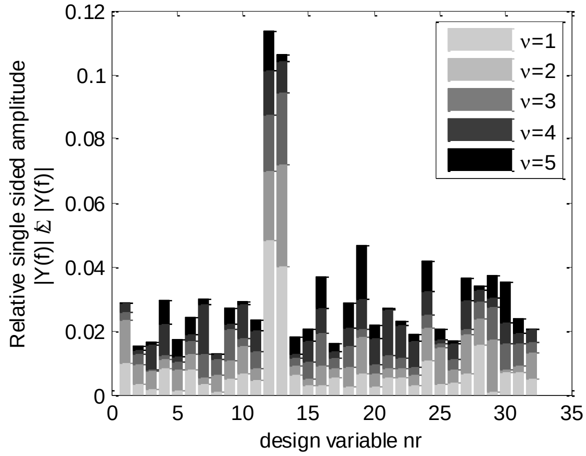


Figure 15 Fourier analysis on the OFAT parameter study results of the "P. acc." response: the 1D SSAS of low wavenumbers (ν) for all design variables

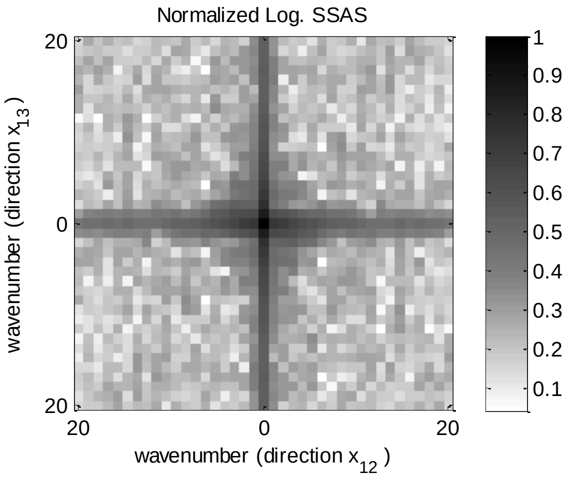


Figure 16 Fourier analysis on the TFAT parameter study results of the “P. acc.” response: the normalized single sided amplitude spectrum (SSAS) for 2d wavenumber decomposition

For all of the investigated design variables and vehicle models the results indicate that the low wavenumber “trends” are of predominant importance. Although it is difficult to discover common trends in the distribution of individual amplitude contributions per wavenumber, the amplitude contribution averaged over all design variable combinations is decreasing with increasing wavenumber.

Global sensitivity analysis and variance decomposition

Using existing global sensitivity analysis (GSA) methods, and variance based variable screening methods, the first and second-order variance contributions and/or sensitivity indices of the model output with respect to the optimization design variables are estimated for the two vehicle models[10]. First-order sensitivity indices are defined as: $S_i = V_i / VAR(Y)$ where $V_i = VAR(E_{X\sim i}(Y|X_i))$ which represents the variance (VAR) of the expected value (E) of response or model output Y conditioned w.r.t. design variable X_i. Analogously second-order indices can be defined as: $S_{ij} = VAR(E_{X\sim ij}(Y|X_{ij}))$. For an introduction and further theory of GSA methods the reader is referred to [Sbl01] and [Slt10]. The used implementations for the sensitivity index estimation and variable screening are described in [Rat10] and [Pli10]. For GSA of the response model output w.r.t. the model input, 2000 pseudo-random samples of design variable combinations are used for vehicle model A, and 1000 for vehicle model B. Figure 17 shows the sensitivity distributions[11] for the 4 different response types of two vehicle models.

[10] The design variables and the range of the design variables are as described in section 2.3

[11] The distribution of the first order sensitivity indices S_i are expressed in terms of $\sqrt{S_i}$ since this is in the opinion of the author more intuitive for visualization (in a similar manner as standard deviation can be preferred over variance in particular diagrams).

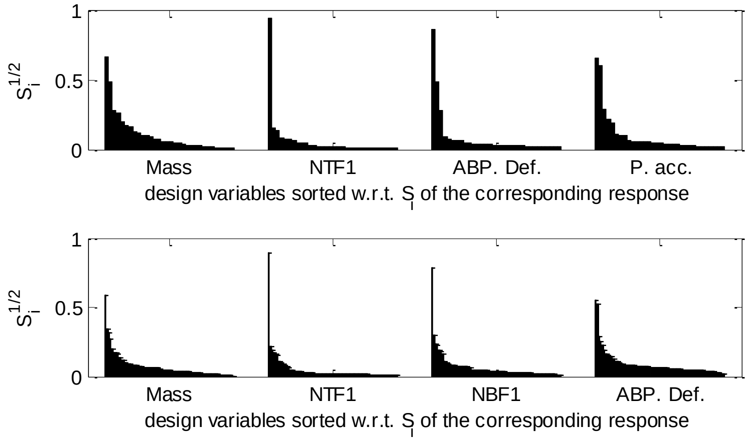


Figure 17 Sensitivity distributions for responses and 2 vehicle models, for vehicle model A (top) and B (bottom). The variables are independently sorted in descending order of relevance, within each sub-figure

The resulting estimates of the first-order sensitivity indices show characteristic distributions for all of the investigated simulation responses. For the mass, NTF1 and "ABP. Def." responses, a small fraction of the design variables have a high contribution to the total response variance. Similar results are obtained for both investigated vehicle models (A and B). It should be noted that in Figure 17 the variables are sorted in descending order of relevance according to variance contributions. The ordering for the different response types is however different, such that variables important for one response are not necessarily important for another response. This is visualized in Figure 18, where for each vehicle model a unique ordering according to the mass response is used. The relation of the variable importance between the different simulation responses is further dealt with in section 4.3.

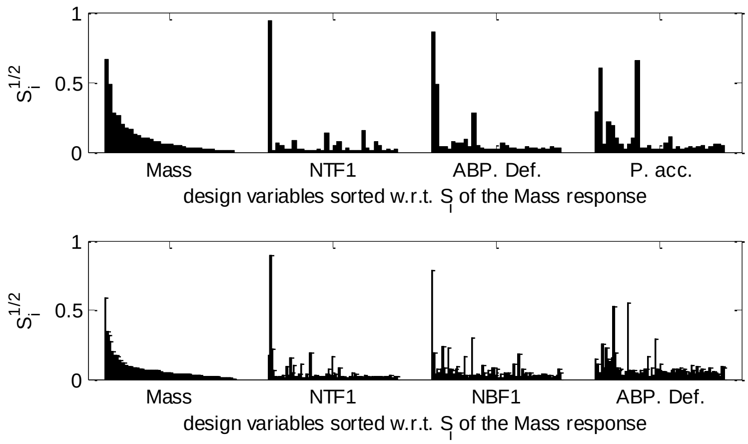


Figure 18 First-order sensitivities sorted by mass influence, for vehicle model A (top) and B (bottom)

A natural property of the sensitivity indices or Sobol indices resulting from a GSA is that the variance contributions should sum up to unity. Combining the explained variance of a linear regression model together with the previously mentioned sensitivity analysis methods for the estimation of first and second-order sensitivity indices, an overall estimation of the variance decomposition can be obtained for each of the simulation responses (Figure 19).

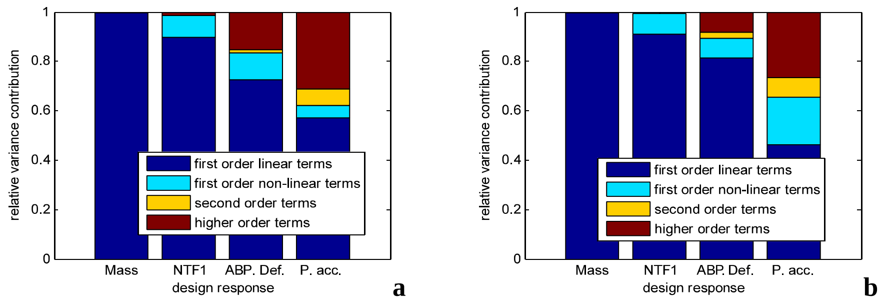


Figure 19 Variance decomposition per response for: a: vehicle model A; b: vehicle model B

Simulation response correlations

Previous sections dealt with the analysis of the individual simulation responses with respect to the design variables. In this section, a basic analysis of the structure between the different simulation responses of the system is presented. The structure between the different simulation responses and between the sensitivity distributions of the responses is investigated using the normalized covariance (correlation coefficients) (see equation 4.10).

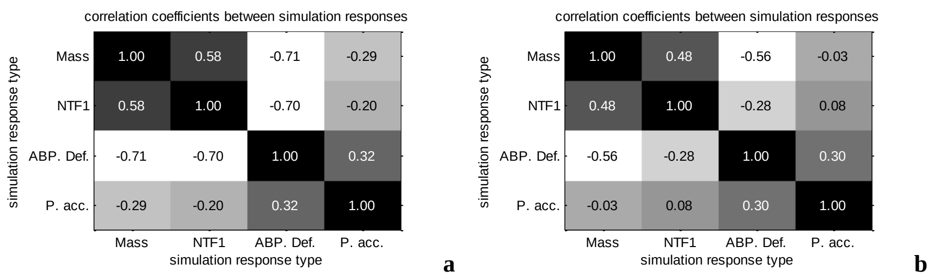


Figure 20 Linear correlation coefficients between the simulation responses for: a: vehicle model A; and b vehicle model B

Figure 20 shows the matrix of normalized covariances (also called correlation coefficients [Rod88]) between the simulation responses, based on quasi-random sampled design variable values for each of the vehicle models (A and B). Besides correlations among the design responses also the correlations between the linear first-order effects of the different simulation responses are assessed.

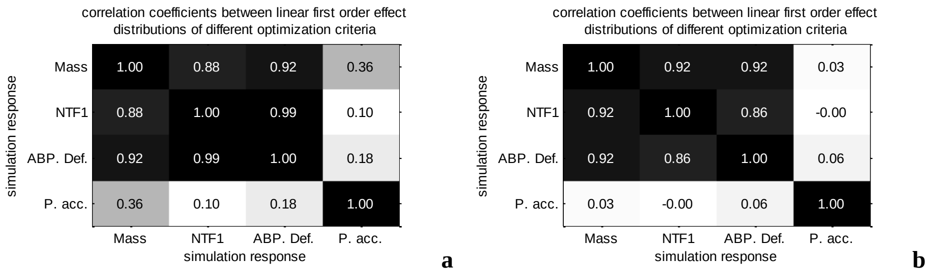


Figure 21 Linear correlation coefficients between linear first-order effect distributions of different simulation responses for: a: vehicle model A; b: vehicle model B

As an example Figure 21 shows the correlation coefficient matrix between the distributions of the linear first-order sensitivity index estimates (based on linear regression models) for each of the simulation responses.

Discussion and summary of the response characterization

No general validity can be claimed by investigating only two (or three) vehicle models with these approximate response characterization techniques. Nonetheless comparing the results between the two vehicle models, common trends, and a band of mutual differences between the investigated response characteristics can be qualitatively estimated. Although, for other applications and response types, possibly more problem instances and other characterization techniques might be required for a useful estimate, the applied characterization methods are by no means specific to the presented application, and could be used to investigate other response types.

The response characterization performed here, required a large investment in terms of computational effort. This investment could however be worthwhile if a significant increase in optimization efficiency for other instances of related problems can be obtained with the RSP approach. The computational investment for this particular example is done in scope of a proof of concept. The main goals of this section were however to show the application of different analysis techniques that can be used for the response characterization, and to provide an overview of similarities and differences in response characteristics of different simulation model instances of similar type.

4.2.3. Construction of representative surrogate problems for selected responses in vehicle design optimization

In this section an example implementation of the RSP approach is presented. As stated in the general description of the approach, the aim of an RSP is to mimic selected function characteristics of the simulation-based vehicle model responses of interest in a statistical sense, and not to approximate a particular response or dataset such as is the usual context of meta-models or surrogate models. Figure 22 displays a schematic workflow of the steps used to construct the RSP in this application example.

The results of the simulation response characterization gave an indication of common features and differences between the corresponding simulation responses of the different vehicle models. For this example the selected characteristics for a single response are:

- The type or “shape” of the response nonlinearity w.r.t. the design variables
- The distribution of the first and second-order sensitivity indices w.r.t. the design variables
- The distribution of the total variance contribution of the first, second and higher order effects

The selected characteristics of the different responses are:

- The correlation coefficients (or normalized covariance) of the responses
- The correlation coefficients of the first-order sensitivity indices between the responses

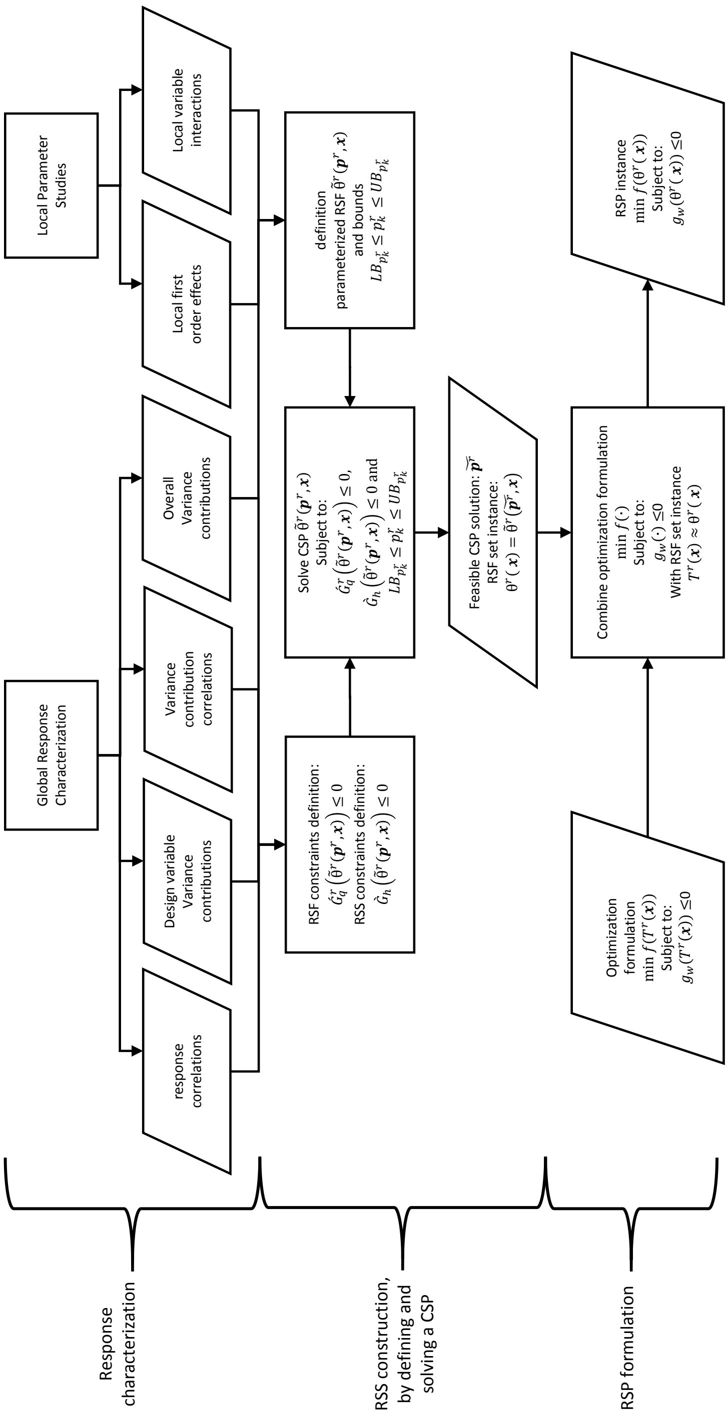


Figure 22 Schematic flow diagram for the construction of the RSP for the car body design application case study

For this example an RSF formulation for each response type (index *r*), composed of a series expansion truncated to include interaction effects up to the second-order followed by a combined higher order is used to represent the behavior of the characteristic responses.

$$\theta^r(\boldsymbol{x}) = \sum_{i=1}^{d} \theta_i^r(x_i) + \sum_{1\le i<j\le d}^{i..d} \theta_{i,j}^r(x_i, x_j) + \theta_{i,j,\dots,d}^r(x_1, x_2, \dots, x_d) \tag{4.4}$$

Superscript r is the index over the different types of outputs or pseudo responses (in this example: 1 Mass, 2 NTF1, 3 "ABP. Def.", 4 "P acc."). All operations considered are invariant to addition with a constant, which is therefore omitted at this stage. For each of the responses the choices for the summands of the representing basis functions, and the parameter bounds are summarized in Table 5. Each of the basis function summands has free parameters which are the variables for the constructed CSP. For the RSS with the general set of free parameters $\boldsymbol{p}^r$ the following notation is used: $\tilde{\theta}^r(\boldsymbol{p}^r, \boldsymbol{x})$, whereas the notation $\theta^r(\cdot)$ for the same RSS with a parameter set which satisfies all similarity criteria enforced by the constraints and simple bounds. The general set of parameter bounds is expressed as $UB_{p_k^r} \geq p_k^r \geq LB_{p_k^r}$. The particular free parameters for each summand in the RSF of each response type are listed in column 5 of table 2.

The general set of constraints on the CSP that relate to the separate RSFs is expressed as:

$$\acute{G}_q^r\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\, \boldsymbol{x}\right)\right) \leq 0 \tag{4.5}$$

And the general set of constraints working on the combined set of RSFs is expressed as:

$$\grave{G}_h\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\, \boldsymbol{x}\right)\right) \leq 0 \tag{4.6}$$

The CSP problem can be relaxed by defining tolerances for each of the constraints, or by using lower and upper bounds for the quantities of interest, in the presented example lower and upper bounds are used instead of tolerances, this is however at the cost of doubling the number of constraints in the CSP. These nonlinear constraint functions will be defined later in this section.

Selection of the basis functions

As mentioned in section 4.2.1 the choice for basis functions and the series truncation is dependent on the respective response characterization results. For the provided example the choice for the basis function types is based on the local OFAT and TFAT parameter studies on a subset of the design variables. To represent the "Mass" and "NTF1" responses w.r.t. the design variables (see Figure 13, Figure 14), linear basis functions and a subset of quadratic polynomials are selected respectively, because these functions match the "shape-characteristics" observed in the response analysis. For these response types second and higher order interaction terms are omitted, since nearly all the variance of the responses can be explained without these (see also Figure 19). Based on the parameter study results (see for example Figure 13, 14, 15, and 16) a composition of linear functions and harmonic series (expressed as complex exponentials in Table 5) was selected by the author, to represent the first and second-order characteristic nonlinear relation between the design variables and the

"ABP. Def" and "P. acc" simulation responses. The parameter bounds were chosen to roughly match the observed range of function behavior in the OFAT parameter studies.

An analysis of higher than second-order effects of the simulation responses requires a high amount of function evaluations and was computationally infeasible to the author. The performed response characterization could however provide an indication of the total magnitude of the variance contribution of unexplained third and higher order effects (see Figure 19). Based on the unsmooth behavior observed in the local sensitivity analysis, the assumption is made that, these higher order effects can be represented by a single non-smooth field with higher order interactions. To represent such a non-smooth field, functions that generate reproducible isotropic uniform distributed noise are used. These functions denoted by operator $W(\boldsymbol{x})$: $\mathbb{R}^n \rightarrow M \subseteq \mathbb{R}$ serve as a multivariate random map to pseudo-random but reproducible values in interval M, where M is a uniform distribution in the open interval $(-1,1]$**.** The magnitude of this uniform noise field is scaled by a factor q which is chosen such that variance contribution of this term matches the "explained" variance by higher order terms in the response characterization (in the example $q^3 = 0.12$ and $q^4 = 0.28,$ see also Figure 19)

Table 5 Overview on the summands for the RSFs

Response type	r	int.	representing summand formulation	function/parameter constraints
Mass	1	1	$\tilde{\theta}_i^r(a_i^1, x_i) = a_i^1 x_i$	$0 \le a_i^1 \le 1$
		2	$\tilde{\theta}_{ij}^r(x_i, x_j) = 0$	
NTF1	2	1	$\tilde{\theta}_i^r(a_i^2, b_i^2, x_i) = a_i^2(x_i - b_i^2)^2 - a_i^2(b_i^2)^2$	$-1 \le a_i^2 \le 0$ $0.7 \le b_i^2 \le 1.5$
		2	$\tilde{\theta}_{ij}^r(x_i, x_j) = 0$	
ABP. Def.	3	1	$\tilde{\theta}_i^r(b_i^r, u_i^r, x_i) = a_i^r x_i + u_i^r \sum_{n=1}^{m} c_{in}^r e^{2\pi\, inx_i}$	$0 \le a_i^3 \le 1$, $0 \le u_i^3 \le 1$ $\theta_i^r(.) \mapsto \mathbb{R}$ $c_{in}^r \sim c_{kn}^{ref\,r}$, $m = 10$
		2	$\tilde{\theta}_{ij}^r(u_{ij}^r, x_i, x_j) = u^r v_{ij}^r \sum_{n_1}^{m_1} \sum_{n_2}^{m_2} c_{i,j,n_1,n_2}^r e^{2\pi i n_1 x_i} e^{2\pi i n_2 x_j}$	$0 \le u^r \le 1$ $\theta_{i,j}^r(.) \mapsto \mathbb{R}$, $v_{ij}^r = S_i^r \otimes S_j^r$ $c_{i,j,n_1,n_2}^r \sim c_{k,l,n_1,n_2}^{ref\,r}$, $m_1, m_2 = 10$
		d	$\tilde{\theta}_{i,j,\dots,d}^r(q^r, x_1, x_2, \dots, x_d) = q^r * W(x_1, x_2, \dots, x_d)$	$q^3 \ge 0$
P. acc.	4	1	$\tilde{\theta}_i^r(b_i^r, u_i^r, x_i) = a_i^r x_i + u_i^r \sum_{n=1}^{m} c_{in}^r e^{2\pi\, inx_i}$	$0 \le a_i^4 \le 1$, $0 \le u_i^4 \le 1$ $\theta_i^r(.) \mapsto \mathbb{R}$ $c_{in}^r \sim c_{kn}^{ref\,r}$, $m = 10$
		2	$\tilde{\theta}_{ij}^r(u_{ij}^r, x_i, x_j) = u^r v_{ij}^r \sum_{n_1}^{m_1} \sum_{n_2}^{m_2} c_{i,j,n_1,n_2}^r e^{2\pi i n_1 x_i} e^{2\pi i n_2 x_j}$	$0 \le u^r \le 1$ $\theta_{i,j}^r(.) \mapsto \mathbb{R}$, $v_{ij}^r = S_i^r \otimes S_j^r$ $c_{i,j,n_1,n_2}^r \sim c_{k,l,n_1,n_2}^{ref\,r}$, $m_1, m_2 = 10$
		d	$\tilde{\theta}_{i,j,\dots,d}^r(q^r, x_1, x_2, \dots, x_d) = q^r * W(x_1, x_2, \dots, x_d)$	$q^4 \ge 0$

The Fourier series coefficients of the RSFs for responses 3 and 4 (referred to by the symbol c with the corresponding sub- and superscripts) are not part of the set of free parameters of the CSP. For the one-dimensional case for each dimension i the complex Fourier series coefficients c_{in}^{r} with index n over the frequencies are of similar structure $c_{in}^{r} \sim c_{kn}^{ref\,r}$ to the coefficients of a reference set $c_{kn}^{ref\,r}$. The coefficients of the reference set can be obtained by performing the discrete Fourier transform on gridded data points on the design variables after subtracting the linear trend (such as done in section 4). For the presented case study "similar" Fourier coefficient structures are obtained using the Iterative Adjusted Amplitude Fourier Transform (IAAFT) algorithm described by Schreiber and Schmitz [Srb96] and implemented by [Vnm03]. The IAAFT method (denoted by operator H) can generate various discrete series or fields (depending on the random seed "z" that have the same amplitude distribution and autocorrelations, as the provided input data (the various calibration fields), up to a specified tolerance "t" (in the example 0.005).

$$c_{in}^{sur\,r} = H(c_{kn}^{ref\,r}, t, z) \tag{4.7}$$

The resulting series and fields, are later scaled by the factors u_i^r which are part of the variables set of the CSP. In this context the selected similarity criteria are: amplitude distribution and autocorrelation.

For the responses with considerable nonlinear second-order interactions (responses 3 and 4), the correlation coefficient between the inner product of the first-order sensitivity indices, and the second-order sensitivity index estimate is high and significant for the calibration vehicle models. This indicates that for the application example the variables with high first-order effects are also the variables involved in the most important second-order interactions in terms of variance contribution. In order to reduce the number of free variables in the CSP the relative second-order sensitivity index distribution controlled by variable v_{ij}^r (see also table 2) is defined such that it is dependent on the first-order sensitivity distribution as: $v_{ij}^r = S_i^r \otimes S_j^r$ where $\otimes$ denotes the inner vector product. The amplitudes of the resulting nonlinear fields are scaled by variable u^rwhich is constrained such that the total variance contribution of the fields corresponds to the second-order contributions estimated in the response characterization (see Figure 19).

Besides the selection of the basic functions and parameter bounds, also function and additional parameter constraints are defined to enforce response characteristics.

RSF constraints

The choice for the targeted sensitivity index distributions is made using the global sensitivity results as presented in section 4.2.2. The sensitivity indices were sorted in descending order, for each response obtained (according to the function characterization results), such that, a fit for the sensitivity index distribution could be made. The distributions of all of the responses in this case study could be approximately described by a two-term exponential fit model (See also Figure 17). The related function constraints are defined as upper ($Z_j^{\mathrm{UB}\,r}$) and lower bounds ($Z_j^{\mathrm{LB}\,r}$) on the ordered set first-order sensitivity indices. The set of upper and lower bounds is based on the fit model on the sorted set of sensitivity indices from the calibration models. This can be expressed for the general case as:

$$\acute{G}_q^r\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}\right)\right)=Z_j^r - Z_j^{UB\,r} \quad \text{for q=1:d and j=d and}$$
$$\acute{G}_q^r\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}\right)\right)=Z_j^{LB\,r} - Z_j^r \quad \text{for q=d+1:2d and j=q-d} \tag{4.8}$$

Where Z_j^r contains the first order sensitivity index estimates S_i^r for each response r in descending order over index i using the sorting transformation $Z_j^r = \boldsymbol{\beta}(S_i^r)$. The sensitivity indices for each RSF $S_i^r = \mathbf{Q}\left(\theta^r(\boldsymbol{p}^r,\ \boldsymbol{x}_w)\right)$ are estimated using the method described in [Pli10]denoted by operator $\mathbf{Q}$ based on a set of pseudo-random samples $\boldsymbol{x}_w$.

RSS constraints

Following the described approach up to this point for each of the design responses (mass, frequency, deformation, peak acceleration) would lead to function formulations that could be representative for each simulation responses individually, but would not take into account the coupling structure between the responses. In the applied approach, the coupling between the responses is accounted for by applying constraints on the correlations between the function responses, and the correlations between the sensitivity distributions for each of the responses.

For a set of w design evaluation vectors the matrix of results (Y) for each design is defined as:

$$Y_{wr} = \theta^r(\boldsymbol{x}_w) \tag{4.9}$$

The normalized covariance or correlation coefficients between the column vectors of the responses are given by:

$$\rho_{tv}^{\mathrm{Y}} = \mathbf{R}(Y_{wt}, Y_{wv}) \tag{4.10}$$

Where $\mathbf{R}()$ is the operator that results in the correlation coefficient between two vectors defined as:

$$\rho = \mathbf{R}(A,B) = \frac{\mathrm{cov}(A,B)}{[\mathrm{cov}(A,A)\mathrm{cov}(B,B)]^{1/2}} \tag{4.11}$$

The similarity of the obtained correlation coefficient matrices of the test function can be defined by choosing lower ($\rho_{tv}^{\mathrm{S_iLB}}$) and upper bounds ($\rho_{tv}^{\mathrm{S_iUB}}$) for each of the upper diagonal matrix entries. The upper and lower bounds are based on the values obtained in the response characterization of the calibration models.

$$\grave{G}_h\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}_w\right)\right) = \rho_{tv}^{Y} - \rho_{tv}^{Y\,\mathrm{UB}} \quad \textit{for t=1:(N-1), v=(t+1):N \& h=t+N(v-1)-v(v-1)/2}$$

$$\grave{G}_h\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}_w\right)\right) = \rho_{tv}^{\mathrm{Y\,LB}} - \rho_{tv}^{Y} \quad \textit{for t=1:(N-1), v=(t+1):N \& h= t+N(v-1)-v(v-1)/2+N(N-1)/2} \tag{4.12}$$

Where N is the number of responses (4 in this example). A similar approach is used for the correlation between the first-order sensitivity indices S_i^r for all response combinations.

$$\rho_{tv}^{\mathrm{S_i}} = \mathbf{R}(S_i^t, S_i^v) \tag{4.13}$$

Also here lower and upper bounds for the correlation coefficients are defined based on the results of the response characterization of the calibration models. The corresponding constraints are defined as:

$$\grave{G}_h\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}_w\right)\right) = \rho_{tv}^{\mathrm{S_i}} - \rho_{tv}^{\mathrm{S_i UB}} \quad \textit{for } t=1:(N-1),\ v=(t+1):N\ \&\ h=t+N(v-1)-v(v-1)/2+N(N-1)$$

$$\grave{G}_h\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}_w\right)\right) = \rho_{tv}^{\mathrm{S_i LB}} - \rho_{tv}^{\mathrm{S_i}} \quad \textit{for } t=1:(N-1),\ v=(t+1):N\ \&\ h=t+N(v-1)-v(v-1)/2+3N(N-1)/2 \tag{4.14}$$

These function and problem constraints are selected to achieve representativeness of the surrogate problem to the calibration problems. The selection of the function formulation for each of the responses, and the corresponding free parameters, combined with the parameter constraints, function constraints, sensitivity index constraints, and correlation constraints define a CSP.

CSP solution

The general notation used previously for the set of parameterized basis functions to represent the responses can now be linked to the corresponding parameter values:

$$\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}\right) = \theta^r(a_i^1, a_i^2, b_i^2, a_i^3, u_i^3, u^3, a_i^4, u_i^4, u^4, x_i) \tag{4.15}$$

This expression represents the concatenation of all free parameters of the RSS (in the example $a_i^1, a_i^2, b_i^2, a_i^3, u_i^3, u^3, a_i^4, u_i^4, u^4$, see also Table 5) , into a single parameter structure $\boldsymbol{p}^r$. At this stage the RSS is thus represented by the function $\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}\right)$ with free function parameters $\boldsymbol{p}^r$, and the design variables $\boldsymbol{x}$. At this point the total number of elements in $\boldsymbol{p}^r$is still dependent and parameterized, w.r.t. to the number of design variables *d* wich determines the size of the function parameters with index *i*. Once the dimensionality of the target problem is set, the size of $\boldsymbol{p}^r$is fixed, and it can be treated as an ordinary vector variable. For the case studies that will be presented in section 4.3 and 4.4, the number of design variables was set to 50. The selection of appropriate values for the elements of $\boldsymbol{p}^r$ wich contains all free function variables, can be done by means of solving the following CSP problem:

- Variables: structure $\boldsymbol{p}^r$ with all free parameter values as its elements denoted as p_k^r
- Domain: $LB_{p_k^r} \leq p_k^r \leq UB_{p_k^r}$
- Constraints: $\acute{G}_q^r\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}\right)\right) \leq 0$ and $\grave{G}_h\left(\tilde{\theta}^r\left(\boldsymbol{p}^r,\ \boldsymbol{x}\right)\right) \leq 0$

where $LB_{p_k^r}$ and $UB_{p_k^r}$ are the collections of lower and upper bounds for each of the corresponding free function parameters respectively (see also column 5 of table 5). Besides the parameter bounds also the constraints from equations 4.12 and 4.14 are applied in the formulation of the CSP in order to enforce the selected similarity criteria. For the example application with relatively few problem instances in the training set, the upper and lower bounds for the constraints are based on a simple “averaging approach”, where the minimum and maximum values from the calibration model characterization results are set as the lower and upper bounds of the respective constraint values. The total number of constraints *K* for the CSP criteria in this example scales with d according to $K=2Nd+2N(N-1)$ where N is the number of responses.

In the previous CSP formulation, each of the constraint equations of the CSP is still dependent on $\boldsymbol{x}$. To obtain a computable solution, the realizations of $\boldsymbol{x}$ in its domain, are approximated in this example, by using fixed set of 10^5 pseudo-random samples $\boldsymbol{x}_w$ in the domain of the design variables (the unit hyper cube). Using a fixed set $\boldsymbol{x}_w$, the constraint equations can be treated as a function of the free function parameters $\boldsymbol{p}^r$ only. The validity of this approximation can be assessed by a posteriori analysis of the constraint violations with another large set of pseudo-random samples for $\boldsymbol{x}_w$.
Solutions to the formulated flexible CSP problem could be obtained using various methods. For the presented example a standard Interior point method that handles nonlinear constraints (MATLAB 2013a fmincon) is used, with the full set of constraints as separate nonlinear constraints. An auxiliary objective function defined as:

$$\vartheta\left(\boldsymbol{p}^r\right) = \sum_q \delta\left(\acute{G}_q^r(\boldsymbol{p}^r)\right) + \sum_h \delta\left(\grave{G}_h\left(\boldsymbol{p}^r\right)\right) \tag{4.16}$$

where operator $\delta(\cdot)$ is an indicator function defined as:

$$\delta(X) \coloneqq \begin{cases} X \; if \; X > 0 \\ 0 \; if \; X \leq 0 \end{cases} \tag{4.17}$$

Combining this auxiliary objective function, the parameter bounds and constraint sets from equations 4.8, 4.12 and 4.14 the CSP can be solved. For the example this is done using successive optimization runs with decreasing constraint violation tolerances ranging from 1 at the start, to 1E-6 in the final run. For the successive optimizations, the final value of the previous run is used as the initial value of the next optimization. Each feasible solution $\breve{\boldsymbol{p}}^r$ to the CSP represents a parameter set, which when combined with the basis functions forms a response set with representative characteristics with respect to the selected criteria.

Up to this point the constraints are all based on relative measures (sensitivity indices, and correlation coefficients, which are invariant with respect to addition of constants and scaling by multiplication). The absolute range of the response of the surrogate functions can be controlled by applying the corresponding offset χ^r and scaling factors ψ^r to the resulting RSS from the CSP solution.

$$\theta^r(\boldsymbol{x}) = \chi^r + \psi^r * \tilde{\theta}^r\left(\breve{\boldsymbol{p}}^r, \boldsymbol{x}_w\right) \tag{4.18}$$

Optimization algorithms are however typically programmed to be scale-independent, therefore this last step is not necessary to obtain results, and the results are not affected by the choice of the offset and scaling factors.

4.3. Application 1- a car body design case study: optimization efficiency assessment

A first application example of the RSP approach was its use in benchmarking optimization performance for particular problem types. The performance of several optimization algorithms was estimated on two RSP formulations, after which the results were compared with performance results based on simulation workflow based problems. The two different optimization problem formulations for the comparisons are:

- Objective: Minimization of the vehicle mass, subjected to crashworthiness constraints (max peak acceleration at the tunnel, and A-B-pillar deformation).
- Objective: Maximize 1st natural torsion frequency, subjected to mass constraints.

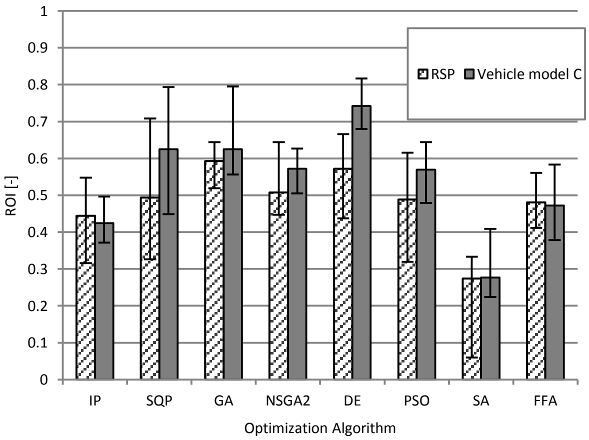
Both RSPs are based on a single RSS (obtained as described in section 5), and the results are compared with the optimization performances on the corresponding problem formulations of a full vehicle simulation workflow (vehicle model C) which was not part of the original calibration data set. The number of design variables RSS was set to 50 according to the targeted validation vehicle model. The comparison for the optimization efficiency is made for the following algorithms (see also chapter 2):

1. Interior point (IP) algorithm
2. Sequential quadratic programming (SQP)
3. Genetic algorithm (GA)
4. Non-dominated Sorting Genetic Algorithm, (NSGA2)
5. Differential Evolution (DE)
6. Particle Swarm Optimization (PSO)
7. Simulated Annealing (SA)
8. Fire Fly Algorithm (FFA)

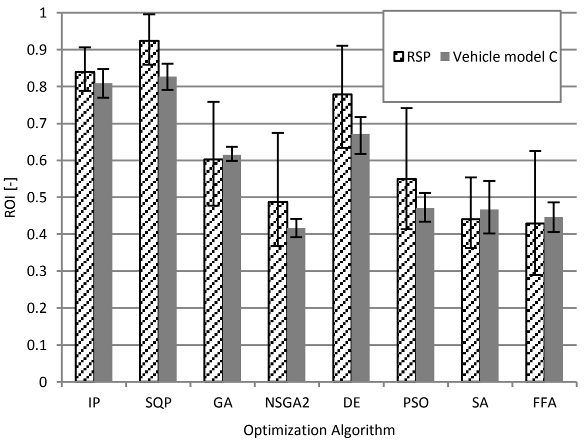
For further information on the methods and implementations is referred to the corresponding references in section 2.5. Algorithms 3, 5, 6, 7 and 8 are meta-heuristic search algorithms which are commonly used for problem types involving non-convex nonlinear responses, whereas the IP and SQP algorithms are typically used for nonlinear convex problems, and NSGA2 is a multi-objective optimization algorithm. Although the application of NSGA2 is unconventional for single objective problems, preliminary investigations showed reasonable performances for the type problems of interest. Since optimization formulation 2 does not include the highly nonlinear crashworthiness responses, also algorithms 1 and 2 were included in the comparison.

For each formulation, the optimization algorithm repeatedly runs on the same optimization problem with default optimization algorithm parameters, except for the random seed, and or initial population, such that performance statistics can be obtained. To compare the optimization efficiency for each problem, the results can be expressed in terms of Relative Objective Improvement (ROI, See equation 3.1).

Figure 23 shows the performance expressed in averaged relative objective improvement for optimization runs up to 250 function evaluations per optimization run, 100 repetitions per optimization for the corresponding RSP, and 20 repetitions per optimization on the independent full vehicle C simulation model. A higher number of repetitions would allow more accurate estimates, especially of the distribution or percentiles but, this was unfeasible due to the involved computational cost. Also, the total number of function evaluations per optimization run is limited to 250, due to the high computational cost for the validation runs. As indicated in [Dud08] it is however common in an industrial environment to apriori limit the number of function evaluations to a number too small to reach convergence, a true optimization up to convergence is rather exceptional when dealing with problems that involve computationally expensive simulations (see also [Kno05] and [Kno09]).



a



b

Figure 23 Comparison of average optimization efficiency for 8 optimization algorithms on 2 different optimization problem formulations, a: formulation 1; b: formulation 2

The results in Figure 23 show a similar trend in relative algorithm performances between the optimizations run on the RSP and the optimization runs on the simulation workflow of vehicle model C. The error bars are estimates of the 20% and 80% percentiles. The similarity between the performance prediction and results can be quantified by the correlation coefficients R and the corresponding significance by the p-values between the vectors of optimization algorithm performance results obtained with the RSP and simulation workflow, which are R=0.910, p=0.0012 and R=0.964, p=0.0001 respectively. Thus it can be concluded that in this corroboration example, the RSP approach offers a statistically significant prediction of the optimization efficiency of the tested algorithms applied to both problem formulations using the independent corroboration vehicle model. Application of the RSP approach to benchmark the algorithms and selecting the most efficient algorithms leads to optimization efficiency increases of 32% and 16% in terms of ROI for the respective optimization formulations (1 and 2) with respect to “the average” performance over the investigated algorithms. The computation cost of such a benchmark study without the RSP approach, comparing 8 algorithms, 100 algorithm run repetitions, of 250 function evaluations, each requiring about 1 CPU hour (if a computationally cheap model is used) would require $2 * 10^5$ CPU hours. Whereas the RSP approach for the same study would take about 5 CPU hours[12] (including optimization algorithm overhead), thus saving several orders of magnitude in computation time. Even including the total function evaluation cost for the formulation of the RSP requiring about $1.5 * 10^4$ function evaluations, and a total of about $1.8 * 10^5$ CPU hours, the application of the RSP approach would already be worthwhile the computational investment, if a benchmark study was to be made. To justify the computational effort and endeavor of such a comparison, the computation cost of the comparison or RSP calibration (using reduced resolution simulation models) should be compared against the cost the industrial size problem which can be about $2 * 10^5$ CPU hours for a single optimization run. For the investigated examples the difference in efficiency between the algorithms in terms of CPU time is larger than the difference in terms of ROI. If CPU time savings in the order of 20% can be made by selecting a suitable optimization algorithm, the investment of the comparison pays off after about 5 industrial scale optimizations problems. For this particular example in the field of automotive engineering the increase in efficiency can however also translate in the improved mechanical performance due to the tight time constraints between design freezes in the vehicle development process.

Although the corroboration shows a significant correlation between the relative performances, such a resemblance cannot be guaranteed for any arbitrary vehicle. Nevertheless, it seems reasonable to assume that the results can be relevant for vehicle models with a similar structural concept, optimization parameters, and response criteria as the two calibration vehicle models and the third vehicle model used for the proof of concept. Furthermore, a single RSS can be used to construct several RSPs for different optimization formulations, and thus provides information and flexibility beyond single benchmark comparison results. For application-oriented practitioners the RSS and the derived RSP approach can answer more detailed questions than published benchmarks, whereas for the community interested in optimization method development and comparisons, several

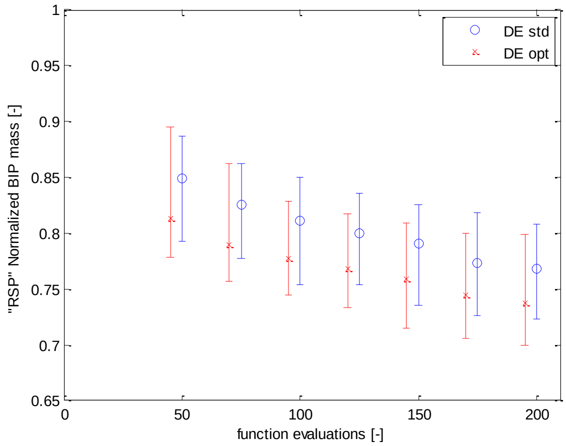
[12] CPU time for a RSP function evaluation is about 2.5E-2 [s] for the four responses in the example, using a MATLAB 2013a implementation on a Dell T3500 workstation with an Intel Xeon X5650 processor and 12 GB of DDR3 RAM. The runtime of the optimizations using the RSP is dominated by the overhead of the optimization algorithm and optimization history saving.

standardized problems can be defined, and made available in order to provide access to reproducible representative surrogate problems of problem types which would be otherwise difficult to assess.

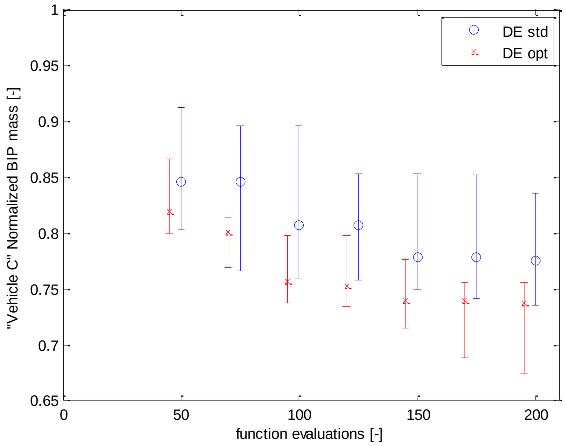
4.4. Application 2 a car body design case study: meta-optimization of the car body design optimization process

A further example application of the RSP approach regards the tuning of the parameters of an optimization algorithm to increase the optimization efficiency for problem types of interest. An optimization of the optimization parameters (or meta-optimization) is performed for the DE algorithm on an RSP. In the inner loop of the optimization, DE optimization runs with a maximum of 200 iterations are performed on the RSP. The objective in the inner loop is the minimization of the RSP mass response, with nonlinear constraints on the first natural mode and test function peak acceleration of the RSS. The design variables for the outer optimization are the optimization algorithm meta-parameters of the inner optimization (3 DE parameters: population size (π), crossover probability (μ), and step size (φ)). In the outer loop (for each parameter-setting-vector -evaluation) a set of 50 inner loop optimization run repetitions is executed on the RSP: The 80% percentile of the minimum feasible pseudo-mass determined after 200 function evaluations is set as the objective for the outer optimization. In the outer optimization loop a GA algorithm is used (with default settings) for 500 iterations to minimize objective thus finding statistically efficient performing optimization parameters for the inner optimization. A total of $5 * 10^6$ function evaluations on the RSP are performed for this case study.

The increase in optimization efficiency due the optimization meta-parameter tuning based on the RSP approach can be visualized by comparing the difference in optimization efficiency between the DE algorithm with “default” settings (π=30, $\mu = 0.7$ and $\varphi = 0.8$) and “optimized” (π=10, $\mu = 0.72$ and $\varphi = 0.87$) settings. Figure 24 shows the plots of the best feasible objective history for standard parameter settings, and optimized parameter settings, on both the RSP and the full vehicle simulation workflow based optimization problem during respectively 50 and 15 optimization runs. The error bars indicate the 20% and 80% percentiles.



a



b

Figure 24 Comparison of best feasible objective history for standard and optimized optimization parameter settings for: a: the RSP; b: the validation vehicle model C

For both the RSP as well as for the full vehicle simulation workflow based problems the optimization performance is significantly improved by the tuning of the optimization meta-parameters. Since the full vehicle optimization problem (vehicle model C) was not part of the calibration set for the RSP, these results confirm the usefulness of the RSP approach for this problem type. In the corroboration example the RSP approach based parameter tuning leads on average to additional performance gains of about 4% in terms of normalized BIP mass, for the fixed function evaluation budget.

Since optimizations of the full vehicle simulation workflow are orders of magnitude more computationally expensive than on the test functions, the number of repeated optimization runs on the vehicle simulation workflow is limited to 12 and hence, the resulting statistics are estimates only. The results have a significant common trend regarding the means, but the percentile statistics between optimization on the RSP and real problem are not quantitatively identical. Surprisingly the performance of the optimized settings is even better on the real problem than predicted. Although further refinements of the approach could possibly increase the general accuracy of the efficiency predictions, this accuracy is at the same time also capped by the nature of the approach. A surrogate problem representative for a class of problems inherently has variability in efficiency prediction accuracy similar to the efficiency variation within the class of problems targeted. If additional information on the specific target problem is available prior to the simulation run, such data could be augmented to the RSP for increased performance estimation accuracy.

4.5. Discussion and outlook

The interpretation of performance assessment results based on the RSP approach is straightforward, if for a particular problem type the optimization performance on different instances is sufficiently similar. This was the case in the presented examples, but for other applications or other problem types, this may be different. In a more general context, for a particular problem, investigations on different problem instances could possibly result in very different optimization algorithm performances. For such cases it would be of interest to investigate further, which problem feature causes such differences. Currently many engineering problems are described and classified by engineering features (such as vehicle model, simulation type, load case, design variable type). To tackle the challenge in finding and developing efficient optimization strategies for such problems, a systematic analysis and classification of the problem types and optimization algorithm operators is required. This work presented a new approach in that direction, from an engineering perspective, but many challenges remain, and further work is required. In the author's opinion it would be good to shift a part of the focus for further research from the development of new optimization algorithms, towards problem analysis and characterization. Since theoretic analysis of complex problems is presently still unfeasible, also systematic empirical studies on the influence of various problem features could contribute. In the next chapter a method towards such analysis is presented.

The comparative studies on optimization efficiency presented in this and the previous chapter are unprecedented in the literature in terms of tested algorithms, and number of simulation-based function evaluations for problems involving crashworthiness of full vehicle models. Although the results can be of practical significance, the main message of the presented results is not that algorithm A is "X" percent better than algorithms B, C and D, but that such relative optimization performances for this particular type and problem formulation can be estimated with significant accuracy using the RSP approach, based on calibration data from similar problem types. The particular benchmark results should be relativized by the fact that many different implementations and variations of the compared optimization algorithms exist, which could perform different as the implementations used. Besides that the response characterization of computational expensive problems leads to more insight of the problem structure, The main message should be that RSP-based performance benchmarks were useful for the selection of and tuning of optimization algorithms, to increase the efficiency for the investigated car body design problems, and the presented ideas could possibly also be of use for other computationally expensive simulation-based optimization problems.

The author furthermore would like to highlight: that suitable parameterized benchmark problems are of greater general value than published benchmark results. This is underlined by the fact that the optimization efficiency of algorithms on particular multidisciplinary problem instances can be dependent on problem and formulation properties such as the number of design variables, number and type of design responses, the choice for the objective and constraints, constraint limits (feasibility fraction), and the available function evaluation budget. Flexible parameterized benchmark problems, such as those constructed with the RSP approach, could be useful for the "practitioner" audience since the RSP problem instance parameters can be adapted to resemble the particular problem of interest. For the "developer" audience standardized RSP instances could be defined for industrially relevant problems, in order to make complex problem types (in terms of simulation expertise, hardware and software resources) easily accessible.

Likewise as many other works in the literature dealing with vehicle optimization, the presented study deals only with a subset of all relevant vehicle design objectives and criteria. It should be noted that to design a car suitable for production, more crash scenarios, NVH criteria, as well as structural requirements from other disciplines such as drive dynamics, and structural durability should be considered.

The vehicle models used for the response characterization are of lower mesh resolution than typical industrial models. Although lower mesh resolution models have a much lower accuracy to represent the response of a particular vehicle model, it is assumed that the most general crashworthiness response features can be represented by the models. The vehicle models used for the characterization for the RSP calibration differed in mesh resolution in about an order of magnitude while still significantly consistent response characteristics could be identified, this observation supports the previous assumption and emphasizes the robustness of the approach.

The "unsmooth" parameter study results obtained from the crashworthiness simulations could have a stochastic nature. Small design variable perturbations can trigger such chaotic dynamic response behavior. Further investigations on the application of deterministic chaos related analysis techniques to the application of crashworthiness problems could be of interest for future studies(surprisingly no such studies were found in the literature yet).. Depending on the crash simulation solver settings even non-unique solutions can be obtained for the same crash event, only by using another CPU configuration during the numerical solution such as also indicated by [Blu01] and [Dud08]. Such sensitivities to small configuration changes, are not merely simulation artifacts, also in physical testing reproducibility is an issue. Indeed for this application type further work is necessary to representatively take into account aspects regarding the robustness of the design. Investigations on the application of the RSP approach to compare different Robust Design Optimization (RDO) strategies (such as e.g. in [Yng04]) are therefore of interest.

In the presented example case studies the application of the RSP approach is calibrated and corroborated using similar problems (similar in terms of response types, design variable types, design variable range and optimization formulation), on different vehicle models. For other design variables, design criteria, or other applications, the relationship between the design variables and responses might be different, and other basis functions might be more suitable. Investigations on the application of the RSP approach on other design variables (such as implicit shape optimization [Dud13]), load-cases and design criteria [Crn02, Cra03, Lng05] are of interest for further research. Although the approach is developed for the presented vehicle design application, the general idea of the approach, the presented response characterization techniques, and the concepts to construct and solve a CSP to incorporate response characteristics for an RSP are however not limited to this particular problem type, and could of interest to be applied and tested on a wider range of problems and applications.

For a true generalization there remain however still open issues. The approach is developed based on empirical investigations, further supporting theory on which response characteristics affect the representativeness of surrogates on the performance of optimization algorithms would have to be developed. Although the application example is rather complex, the approach should be tested on other problem types, to investigate the limitations and applicability in a more general scope. The involved response characterization requires many function evaluations and can be of considerable computation cost, but the results could provide valuable insights. If the response characteristics can be represented by meta-models with sufficient accuracy, the approach could be applied indirectly by means of meta-models. Its additional value would be the characterization of the meta-models responses. Other future work regarding the development of the approach could involve an extension to accurately represent the shape, type and distribution of the Pareto-fronts between different responses, for RSP problems targeting multi-objective optimization problems. Due to the background of the author, this text is written and formulated with an “engineering” mindset. The author would like to invite practitioners and researchers from other fields to test, develop, generalize or criticize the approach, and in particularly to establish a more theoretical basis in addition to empirical experience on which it is leaning in its present form.

Further investigations on the RSP approach in the context of vehicle design problems could be: to extend or modify the approach to deal with problems with more different design variable types, and to widen the considered design response types and load cases.

As a general outlook on the applications of the RSP approach for the optimization of systems with expensive simulators, future work can involve investigations of additional aspects or response characteristics that influence the optimization efficiency. A first point of interest for future investigations is the comparison of different distributed optimization frameworks (such as for example Collaborative Optimization [Brn95], or Analytical Target Cascading [Kim03]). A second point is taking into account the available computational resources to find an efficient optimization strategy. Simulation solvers can be constrained in the number of available parallel licenses, or by the available hardware infrastructure (number and type of nodes, processors memory, etc.). Aspects such as the parallelization and scalability of a single function evaluation, combined with the ability of different optimization algorithms, to use parallel function evaluations (using for example a population-based approach) can be explored. Therefore the RSP approach could aid to find efficient optimization strategies for a particular problem, by enabling a meta-simulation of the optimization process which could take into account a particular resource environment.

4.6. Summary and conclusions

An approach is presented that could be used to construct computationally affordable synthetic test problems (RSPs) based on response characteristics of computationally expensive real-world industrial optimization problems. The approach is developed, and tested for the application of multidisciplinary vehicle design problems, involving vibrational comfort and crashworthiness responses, but the applied strategy and used methods are not limited or specific to the application example. The approach is presented in a general way to facilitate the use and testing of the concept to other application fields.

A composition of existing analysis methods (parameter studies, sensitivity analysis, Fourier analysis and correlation analysis) is used to identify and quantify typical response characteristics of the simulation responses, with respect to the design variables. Based on the response characterization results, basis functions to represent the responses are selected. The combination of: the basis functions, the function parameters, parameter bounds and the formulation of constraints that enforce selected response structure characteristics, formulate a CSP. Each feasible solution of the CSP provides a set of parameters for which the set of basis functions has response characteristics which are "representative" w.r.t. the selected criteria. These surrogate response functions can be used to formulate surrogate response based optimization problems.

The results of the simulation response analysis indicated that although the vehicle models were substantially different, several characteristic features could be identified (Q4). The distribution of the first-order effects was exponential for all responses (a small subset of design variables have a dominating effect). The distribution of the total variance contributions of first, second and higher order effects was similar for corresponding response types, over different vehicle models. Each of the simulation responses had a characteristic behavior (linear for the mass related responses; mildly nonlinear for the eigenfrequency responses; and highly nonlinear for both crashworthiness responses). Also, the normalized covariance matrix between the different simulation response sets for the vehicle models had similar distributions.

The proof of concept of the RSP approach and corroboration with an independent vehicle model indicated that for this relatively complex application such RSP-based problems can be used as benchmarks to compare optimization efficiency of different optimization algorithms (Q5), and to improve the efficiency of an optimization algorithm by tuning of the optimization meta-parameters. The response characterization required for the RSP construction is computationally expensive, but once made it can provide valuable insights in the problem structure, which could pay off for applications in which many instances of similar problems. No general theory of this novel approach has been formulated yet, neither are the limits of applicability to other problems known. The presented results on are however remarkable and encourage further investigations of the concept. The approach is a step towards systematic analysis of industrially relevant complex black-box optimization problems. The author encourages creative interpretation, application, and critiques of the approach, such that further improvements in the optimization of complex industrially relevant problems can be achieved.

5. Optimization test functions based on the systematic composition of random fields

"To find out what happens to a system when you interfere with it you have to interfere with it (not just passively observe it)."

-George E.P. Box [Box66]

5.1. Introduction and motivation

The selection and analysis of the many different meta-heuristic algorithms which have been developed remains a great challenge in the field of global optimization. For the development and selection of optimization algorithms, many analytical optimization test functions, or artificial landscapes are available i.a. [Him72, Ros60, Ras74, Ack87, Back96] (see [Jon75, Flou99, And08] for collections of such test functions). As mentioned in the previous chapters, these functions are often difficult to relate to real-world problems that occur in practice.

Furthermore many of these functions have been criticized in the literature for their lack of complexity and representativeness w.r.t. real-world problems [Lia05, Bar11, Die12]. In [Bar11] the topic of test function generators for assessing the performance of meta-heuristic optimization algorithms on multimodal functions is discussed. It is highlighted that many of the currently available test functions in the specialized literature are too simple, and show regularities, such as symmetry, uniform spacing of optima, and centered optima which can easily be exploited by algorithm designers (see also [Lia05]), and which are unrealistic testing environments for the algorithm performance on real life problems. Several new strategies to create test problems with more realistic complexity have been presented since [Bal05, Add07, Gal06, Ahr10]. However, none of these methods enables the construction of test functions, with particular variance contribution distributions, and variable order interactions[13] in a systematic way.

---

[13] Recently the issue of separability and non-separability has been addressed in a survey [Mah15] on MHAs in large-scale global optimization problems (LSGO). The survey concluded that much more effort is needed to develop de problem decomposition methods such as Cooperative Coevolution methods (see e.g. [Cao15]) with high performance on non-separable and separable subcomponents, and that their performance on imbalanced problems should be further investigated. Although the specific application to LSGO problems is not further addressed here, the proposed method generates test problems with properties of high relevance to this issue, since the Interaction order (degree of non-separability), number of interaction subgroups and dimension for a single

In the previous chapter an approach was proposed to construct optimization test functions using features obtained by the characterization of the simulation responses. The results indicated that the approach could be used to construct test problems with similar structure and characteristics as the targeted application problem. The application of this approach could be of interest to different types of applications and optimization problems. After a response analysis, the response characteristics could be used to classify and compare problems. While it is of industrial and practical relevance to estimate optimization performance using approaches such as the RSP approach, it would be of more general value for the global optimization community to also have a tool available to investigate the influence of variations of particular problem characteristics in systematic way. Therefore in this chapter the following question will be addressed (Q6): How to construct optimization test functions with relevant problem features, in a way that enables systematic performance analysis w.r.t. particular response characteristics?

In this chapter a new method is presented that enables the construction of optimization test problems with several important function characteristics. Whereas other test functions often have a particular set of characteristics, this function generation method is parameterized with respect to several function features, such that the influence of separate characteristics or combinations of characteristics can be investigated in a systematic way.

The presented method enables the systematic design/construction of test problems of varying structures with parameterized variance contribution distributions, higher order interactions and heterogeneous modality in a general way. Furthermore multiple problem instances with the same problem specifications can be generated, which facilitates the statistical assessment of MHA performance on different instances of a problem type. Besides as stand-alone test functions, the fields or functions can be added to existing test functions to enrich their complexity and increase the level of difficulty. The aim of this chapter is to demonstrate/introduce the concept to construct structured functions that are based on the superposition of random fields in order to apply the resulting fields as test functions in the field of evolutionary global optimization. In section 5.2.1 a simple and practical algorithm implementation of a discrete basic random field creator that can generate fields with higher order interactions is described. In section 5.2.2 a “smoothing” method to obtain continuous fields is presented, together with several composition techniques to create structured fields by combining different types of basic random. In section 5.3 the optimization performance of several example functions generated with the presented method is investigated, followed by a discussion, outlook, and conclusions.

objective function are all fully parameterized, and the general idea of the method can be scaled in order to create test functions for very large scale problems.

5.2. Description of the Random Field Composition method

In this section a concept for test function generation based on the composition of random fields is presented. This section is divided in three parts:

1. The description of a basic multidimensional discrete random field generator capable to produce parameterized fields with higher order interactions
2. The description of a "smoothing" method to obtain continuous and differentiable fields, by means of weighting functions.
3. A description of several composition techniques to create structured fields by combining different types of basic random fields

5.2.1. A basic Multidimensional Discrete Random Field (MDRF) generator

Random fields are of interest in various branches of Mathematics, Physics and Engineering. A random field is a stochastic process taking values in a Euclidean space [Adl09]. Elementary discrete random fields can be interpreted as a list of "random" numbers with the indices mapped onto an n-dimensional space. The general idea presented in this communication is to compose a number of Discrete Random Fields (DRF) of different spatial resolutions and dimensionality, in order to construct fields with particular structures. Such fields can serve as test functions of highly variable difficulty in terms of spatial nonlinearity, variance contribution distributions, and higher order interactions.

To model computationally affordable fields which can possess higher order interactions we describe a MDRF generator. This generator function (referred to as operator $\mathbf{H}$) takes a multidimensional vector $\boldsymbol{x}$ of floating point values from the unit hypercube domain as an input, and maps it to a value from a given finite set $\boldsymbol{S}$ with an arbitrary discrete probability distribution and a computational type (e.g. binary integer or float) of choice:

$$y = \mathbf{H}(x_1, x_2, x_d, \dots x_n) \text{ where } x_d \in [0,1] \text{ for d}=1,2,\dots,n \text{ and } y \in \boldsymbol{S} \tag{5.1}$$

To explain the concept of the implementation, a related discretized version of this idea can be defined as:

$$y = \mathbf{A}(j_1, j_2, j_d, \dots j_n) \text{ where } j_d \in \mathbb{N} \text{ and } j_d \leq r_d \text{ and } y \in \boldsymbol{T} \tag{5.2}$$

Here operator $\mathbf{A}$ can be interpreted as high-dimensional random "array" A with indices j of which each index j_d is bounded by the maximum array size r_d for dimension d. In expression 5.2, $\boldsymbol{T}$ is a finite set of successive integers pointing to the distinct elements of the set $\boldsymbol{S}$.

The concept chosen for the MDRF generator algorithm is to compute and reproduce the pseudo-random values of the high-dimensional arrays "on the fly" instead of storing a potentially huge passive map in the computer memory[14]. Another alternative interpretation

[14] Such a passive map would be very memory intensive since the required memory scales with the number of elements $m = \prod_{d=1}^{n} r_d$ or $m = r^n$ for a uniform resolution r and field dimension n. which already becomes problematic at modest resolutions and problem dimensions. A discrete field array of resolution $r = 10$ and dimension $n = 12$ would already require 8 TB (terabyte) of memory when each element takes 8 bit of storage.

would be to consider operator $\mathbf{A}$ as a pseudo-random number generator with a high-dimensional vector as its generating seed.

The ideas in equations 5.1 and 5.2 can be combined to establish a parameterized MDRF operator: $y = \mathbf{H}(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{\varphi}, \boldsymbol{S})$ where the operator on input variables $\boldsymbol{x}$ is parameterized with respect to the discretization resolution $\boldsymbol{r}$, discretization offset $\boldsymbol{\varphi}$ and codomain set $\boldsymbol{S}$. The concept of the algorithm can be explained by the following steps:

1. Addition of an optional offset or shift φ_d to design variable vector x_d: $\hat{x}_d = x_d + \varphi_d$
2. Discretize the resulting floating point input variables $\hat{x}_d$ to integers with respect to the resolution r_d or corresponding array size : $j_d = \mathrm{ROUND}(\hat{x}_d, r_d)$, where j_d represents an index vector.
3. Map the resulting index vector $\boldsymbol{j}$ to an integer index i of $\boldsymbol{T}$ by a Pseudo-Random Mapping (PRM) : $i = \mathrm{PRM}(\boldsymbol{j})$
4. Return the element $y \in \boldsymbol{S}$ to which the resulting integer index i pointed: $y = S_i$

The pseudo-randomness and higher order interactions of the resulting discrete field are introduced by the PRM (step 3). The PRM can be achieved by using a Pseudo-Random Number Generator (PRNG), with a multivariate random seed mechanism. Depending on the program implementation such an approach would easily enable the use of discrete random fields with a total array size m of 10^{1000} or more ($m = r^n$ where n is the dimensionality of the problem). In the following sections the parameterized implementation of the MDRF generator is denoted by $\mathbf{H}(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{\varphi}, \boldsymbol{S})$ or primitives thereof (when parameters not of interest in a particular context, they are omitted for better readability).

5.2.2. Obtaining continuous and differentiable random fields

The algorithm presented in the previous section can generate multidimensional discrete random fields, with specific probability density distributions. Figure 25 shows on the left a two-dimensional example of such a discrete field, with an array size or spatial resolution of 5 intervals per dimension in the domain.

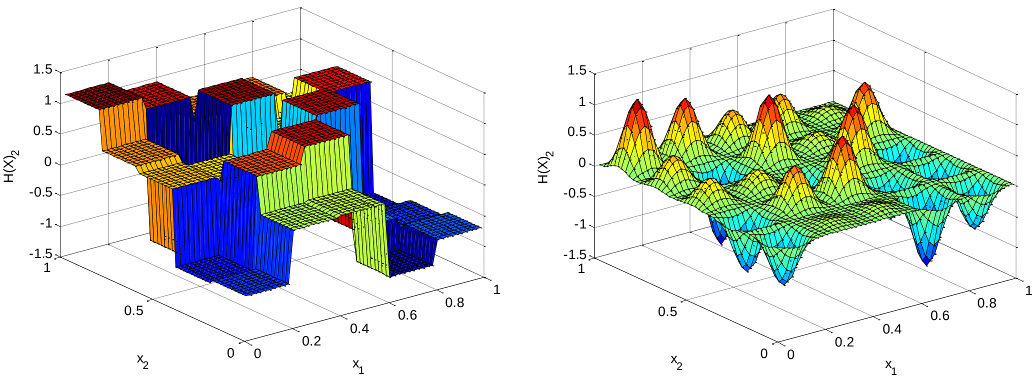


Figure 25 Left: Examples of a discrete random field in 2 dimensions. Right: the corresponding continuous field after smoothing

Using a multiplicative weighting function as expressed in equation 3 these discrete random fields can be transformed to "smooth" differentiable continuous random fields.

$$\mathbf{C}(\boldsymbol{x},\boldsymbol{r},\boldsymbol{\varphi},p) = \left(\prod_{d=1}^{n}\left(\left(1 - e^{cos(2\pi(r_d x_d+\varphi_d))-1}\right) * \left(\frac{1}{1-e^{-2}}\right)\right)\right)^{p} \qquad (5.3)$$

Where $\boldsymbol{\varphi}$ denotes the vector of the (phase) shift of the discrete field and the $\mathbf{C}(\,)$ operator such that also fields with non-zero values along the domain boundaries can be constructed ($\varphi_d \in [0,1]$). With parameter p the shape of the function can be adjusted. This weighting function has the properties, that at each location where the discrete random field is not differentiable in one or more directions, the value and corresponding derivative of the weighting function are equal zero for all values[15] of $p > 0$. Figure 25 (right) shows a "smoothened" version of the discrete field using this method. The multiplicative composition of operators $\mathbf{H}()$ and $\mathbf{C}()$ is abbreviated as $\widetilde{\mathbf{H}}()$**.** Although the application of the $\mathbf{C}(\,)$ operator or weighting function, is technically not smoothing, we will refer to it as smoothing to avoid misunderstandings with the weighting factors introduced later. This "smoothing" operator works to generate continuous fields from the discrete fields in arbitrary dimensions, however it should be noted that in this context as an effect of high dimensionality the integral over the product of the smoothing or weighting function can become vanishingly small with a rate that depends on the choice of exponent p. This effect is similar to the decreasing relative

[15] Although the above statement is true for all $p > 0$ the author recommends to use as a rule of thumb $p \geq 1/d$, since for very small values of p the smoothness vanishes.

volume of the n-dimensional hyper-sphere with respect to the volume of the unit hypercube for high dimensions. The smoothing operator also affects the probability density distribution of the resulting field w.r.t. the original discrete field distribution. For high-dimensional spaces these effects can be controlled by choosing appropriate values for exponent p. Alternatively other smoothing approaches could be considered.

5.2.3. Random Field Composition (RFC) based test functions

The application of the “bare bones” discrete random fields generated by the algorithm in the previously described sections, as optimization test functions is of little practical interest because of the primitive problem structure. The message of this section is however that compositions of such fields of different and heterogeneous resolutions, dimensions and codomain distributions can provide test functions with interesting problem structures.

Continuous fields with different spatial resolution can be created, and compositions can be made by for example multiplication or by weighted addition such as for example:

$$\widetilde{\mathbf{H}}^{\boldsymbol{comp}}(\boldsymbol{x}) = \sum_{\boldsymbol{k=1}}^{\boldsymbol{m}} w_k * \widetilde{\mathbf{H}}_{\boldsymbol{k}}(\boldsymbol{x}, \boldsymbol{r}_k, \boldsymbol{\varphi}_k) \tag{5.4}$$

where $\boldsymbol{r}_k$ and $\boldsymbol{\varphi}_k$ (both in bold) denote the vectors with the array size and shifts for each dimension of composition fields k. A graphical example of such a weighted field summation in 2d is displayed in Figure 26.

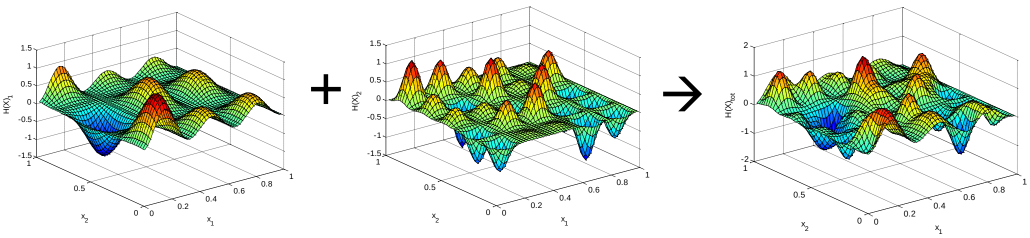


Figure 26 Graphic example of the addition of 2 smoothened discrete fields of different resolutions and the resulting composition

For clearness of visualization, only two fields of low resolution were added, but additions of many fields, with higher and distinct resolution are possible. Besides composition of fields over a fixed set of dimensions or design variables such as in the previous example, also fields over different variable subsets can be composed in order to generate fields targeting variance contributions by specific interaction terms.

According to the Sobol-Hoeffding decomposition [Hfd48, Sbl90], it is possible to decompose a vector valued function $f(\boldsymbol{x})$, into summands of increasing dimensions.

$$f(\boldsymbol{x}) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{1\le i<j\le n} f_{i,j}(x_i, x_j) + \cdots + f_{i,j,\ldots,n}(x_1, x_2, \ldots, x_n) \tag{5.5}$$

In expression 5.5 multi-index notation is used[16] [EOM15]. The different terms of the summation refer to the interaction terms of all possible combinations of variable subsets. When the summands are orthogonal this decomposition is unique. In the field of sensitivity analysis, this idea is commonly used to decompose the variance contribution of the function output variance w.r.t. the individual summands, to identify the variance contribution of variables and interactions of variable subsets. Such results are commonly expressed by means of quantitative importance measures named Sobol indices D, or sensitivity indices, that can be defined as in eq. 5.6 [Sbl90]:

$$D_{i,j,\dots,n} = \mathrm{Var}\left(f_{i,j,\dots,n}(x_1, x_2, \dots, x_n)\right) / \mathrm{Var}\left(f(\boldsymbol{x})\right) \text{ with } 1 \le i < j < \dots \le n \tag{5.6}$$

This expresses the variance contribution of a subset of variables as: the ratio of the variance of the corresponding term[17] from the Sobol-Hoeffding decomposition to the variance of the total function. This concept can be used to quantify (additive) function (output) separability w.r.t. its (input) design variables.

In this context, we apply these ideas in order to construct functions with predefined variance contribution distributions of the first and higher order interaction terms by specifying the weights in equations 5.4 and 5.7 accordingly. Besides composition of fields over a fixed set of dimensions or design variables (such as the example in Figure 26), also fields over different variable subsets can be composed in order to generate fields targeting variance contributions by specific interaction terms. Equation 5.7 shows how a random field can be composed of weighted sums of random fields over variable subsets.

$$\widetilde{\mathbf{H}}_{tot}^{comp}(\boldsymbol{x}) = w_0 + \sum_{i=1}^{n} w_i * \widetilde{\mathbf{H}}_{i}^{comp}(x_i) + \sum_{1 \le i < j \le n}^{i..n} w_{i,j} * \widetilde{\mathbf{H}}_{i,j}^{comp}(x_i, x_j) + \dots + w_{i,j,\dots,n} * \widetilde{\mathbf{H}}_{i,j,\dots,n}^{comp}(x_1, x_2, \dots, x_n) \tag{5.7}$$

Particular variance contribution distributions over a selection of subsets can be achieved by applying the weights according to one's needs. Each of the subset fields $\widetilde{\mathbf{H}}_{i,j,\dots,n}^{comp}$ can themselves also be composed of a summation of fields over the corresponding variable subset (see equation 5.4). A notable point is however, the uniqueness and orthogonally of the summands. In general, the random vectors or fields generated for variables or variable subsets are not necessarily orthogonal to each other. For subfields of high resolution, or high dimensionality the lower-order interaction effects will average out and will become approximately orthogonal w.r.t. lower-order effects of other fields in the composition. For lower resolutions and dimensionality such separable effects cannot be neglected and have to be accounted for by for example an *a posteriori* sensitivity analysis on the final composed function, or a covariance/correlation coefficient analysis between the composition summands.

A test function $Z(\boldsymbol{x})$ based on the presented concept of parameterized MDRF composition can then be described by the general expression:

---

[16] Explanation to the multi-index notation: The expression $\sum_{1 \le i < j \le n} f_{i,j}(x_i, x_j)$ indicates a sum over all function decomposition terms with two variables for which $1 \le i < j \le n$. This applies similarly to all pairs of higher order interactions $f_{i,j,\dots,n}$.

[17] The variance for the terms expression 5.6, w.r.t. the corresponding sub domain in the unit hypercube can be expressed as: $\mathrm{Var}\left(f_{i,j,\dots,n}(x_1, x_2, \dots, x_n)\right) = \int f^2{}_{i,j,\dots,n}(x_1, x_2, \dots, x_n)\ \mathrm{d}x_i \dots \mathrm{d}x_n$.

$$Z(\boldsymbol{x}) = \widetilde{\mathbf{H}}^{\boldsymbol{comp}}_{\overline{\boldsymbol{w}},\overline{\boldsymbol{r}},\overline{\boldsymbol{\varphi}},\overline{\boldsymbol{S}}}(\boldsymbol{x}) = \widetilde{\mathbf{H}}(\boldsymbol{x},\overline{\boldsymbol{w}},\overline{\boldsymbol{r}},\overline{\boldsymbol{\varphi}},\overline{\boldsymbol{S}}) \tag{5.8}$$

where now the composition parameters $\overline{\boldsymbol{w}},\overline{\boldsymbol{r}},\overline{\boldsymbol{\varphi}},\overline{\boldsymbol{S}}$ indicate arrays/structures containing all the parameter vectors of the composed fields.

The 2d “landscapes” from the previous visualization examples, are not really any more spectacular than landscapes of existing test functions. The novelty of the method lies in the parameterization of the function structure (with respect to variance contribution distributions, function modality and higher order interactions) combined with the straightforward scalability to create high-dimensional test problems.

5.3. Examples and case studies in meta-heuristic optimization algorithm performance analysis

For a few example problems, the isolated effects of some function features on the optimization performance of a genetic algorithm are demonstrated. The optimization algorithm used is a simple genetic algorithm from the publicly available Genetic Algorithm Toolbox for MATLAB developed by Chipperfield et al. [Chi95]. For the investigations a population size of 1000, combined with default settings were used. The optimization performance is measured in the number of function evaluations N that is required to find a solution within ε of the best known solution($\varepsilon = 10^{-3}$).

Variance contribution distributions

The first example shows the influence of the different distributions of the first order sensitivity indices or variance contribution, on the genetic algorithm performance. For a given instance of the first order term related fields, the weights w_k can be optimized such that a particular distribution for the sensitivity indices for the first order terms D_i can be obtained. The target first order sensitivity distributions $\widehat{D}_i$ of the small 10 dimensional example problem are chosen according to:

$$\widehat{D}_i = \frac{(i/n)^k}{\sum_{i=1}^{n}\left(\frac{i}{n}\right)^k} \text{ and } k \geq 0 \tag{5.9}$$

Such that different types of distributions (uniform for k=0, linear for k=1, and skewed for k>1) can be obtained (see also Figure 27). Although here only demonstrated for first order sensitivity index distributions, also the distribution of higher order effects are expected to be function features that influence the optimization performance on a particular problem. The example in Figure 28 shows that for increasing values of k, and decreasing effective dimension, the problem gets significantly easier to solve for the selected optimization algorithm.

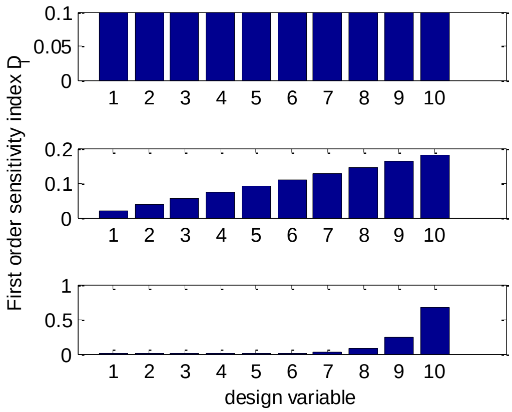


Figure 27 Variance contribution distribution examples for k values: 0 (top); 1 (center); 10 (bottom)

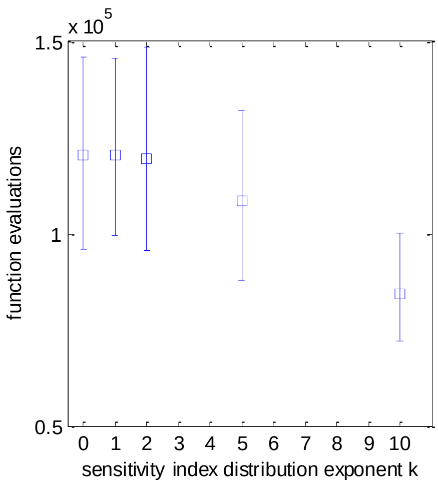


Figure 28 The average number of required GA function evaluations and the 20% and 80% percentiles, on random fields with an increasing distribution exponent *k* (MDRF settings: *n*=10, *r*=20, *q*= 1)

Variable-interaction order

The second example shows the effect of increasing design variable interactions on the level of difficulty of the problem expressed in the number of function evaluations required to converge. In equation (5.5) the different term types represent different interaction orders. Each interaction order q adds $\binom{q}{n}$ interaction terms/or discrete random fields of interaction order q (that each have r^q degrees of freedom). For each interaction term the corresponding weights $(w_{i,j,k,l,m})$ are chosen such that the variance contribution of the corresponding weighted field is: $\frac{1}{Q\binom{q}{n}}$ where Q is the maximum order of interaction of the problem. For the small example problem of dimension 5 with interaction orders up to 5, Figure 29 shows as expected that for increasing interaction order both the mean level of difficulty as well as the variance increase.

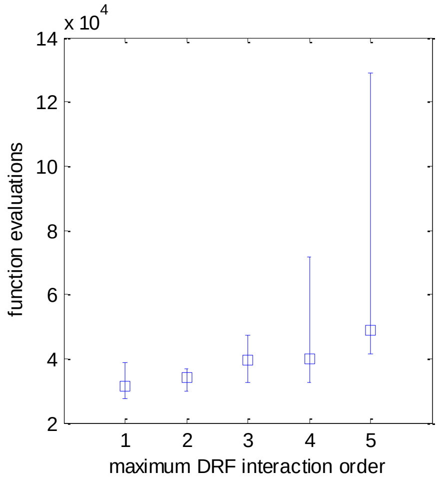


Figure 29 The average number of required GA function evaluations, on fields with an increasing interaction order *q* (MDRF settings: *n*=5 and *r*=5)

Multimodality and MDRF resolution

The third example shows how for smoothened random field containing only first-order interactions the optimization algorithm efficiency scales with respect to the chosen base resolution of the discrete random field. In this 5 dimensional example, the resolution is homogeneous and the DRF values are taken from a uniform distribution. Thus the "resolution" directly affects the multimodality of the resulting test function. The modality is further dependent on the choice for the targeted distribution ***S*** of the discrete field. Figure 30 shows the required number of function evaluations that the genetic algorithm requires to reach convergence, for various resolutions.

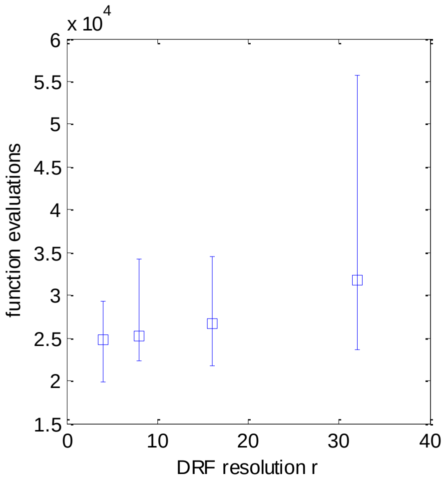


Figure 30 The required number of GA function evaluations on random fields with increasing resolution *r* (MDRF settings: *n*=5, max. q=1)

Dimensional scalability

A common algorithm performance discrimination criterion is the scaling behavior of optimization algorithms on test function instances with increasing dimensionality. In the presented approach parameterization of the dimensionality of the problem is straightforward. A fourth example shows the number or function evaluations required by the genetic algorithm until the convergence criterion is met. Figure 31 shows super-polynomial scaling.

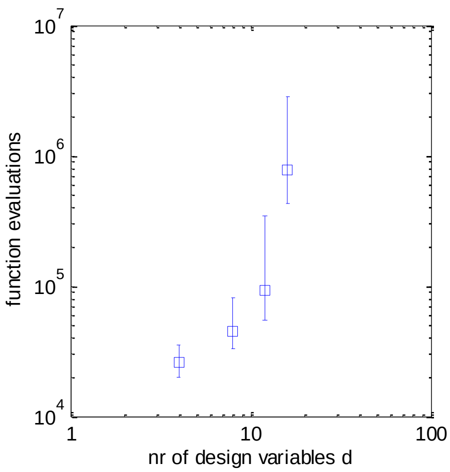


Figure 31 The required number of GA function evaluations on random fields with increasing dimension *n* and interaction order *q* (MDRF settings: *r*=10, interaction order q~*n*)

In this example each test problem was based on a single MDRF, such that the interaction order increases proportionally to the number of design variables. The example shows that quite easily difficult problems can be created that require many function evaluations to solve.

It is however also possible to limit the interaction order of the problems so that the scaling will be less strong.

Deceptive functions

Also test functions containing the established function features such “deception” can be constructed. The general idea behind deceptive functions is a global trend or “larger” function basin that distracts from the smaller basin of the true global optimum. This effect can be achieved in a statistical sense by the composition of smoothened lower resolution fields, combined with high resolution fields of which the probability density function is such that at most points in space the “amplitude” of these fields is negligible, except at a single or few places where the magnitude of the amplitude dominates the amplitudes of the lower resolution smoothened fields (see also Figure 32).

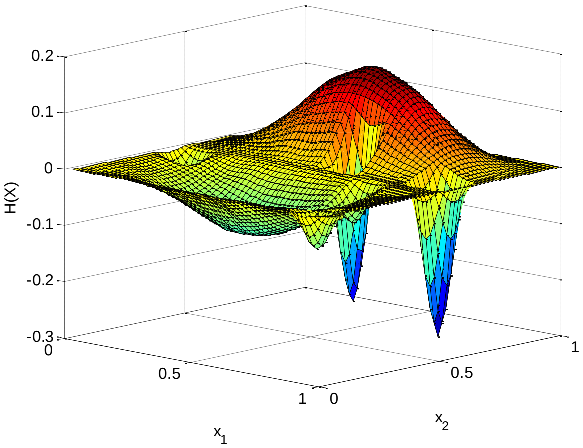


Figure 32 Example of a RFC-based deceptive function

These few examples briefly demonstrate the potential of the concept of RFC-based test problems to systematically construct optimization problems with several different properties that could influence the performance of optimization algorithms. In most examples isolated properties are assessed, but the presented function generation concept covers a vast function space containing many combinations of different “landscape” properties.

5.4. Discussion and outlook

When analyzing the performance of optimization algorithms on different problems, one is interested in which algorithms or algorithm operators can exploit certain properties of the problem structure. As was presented in chapter 4, several characteristic properties of the simulation responses could be identified for the car body design optimization application. But, how do variations of such properties influence the optimization performance of a particular algorithm? What are the isolated effects and how do particular combinations of problem characteristics influence the optimization performance?

In the application example of chapter 4, particular sensitivity (or variance contribution) distributions for the simulation responses were observed (see figure 17). Example 1 in section 5.3 shows by means of a simple case study how the influence of different distributions on the optimization performance of a genetic algorithm can be investigated using the random field composition method presented. The response characterization results from chapter 4, quantified importance of first and second order variable interactions for the different simulation responses. The second example in section 5.3 shows by means of a simple case study how the RFC method can be applied to investigate the influence of variations in variable interaction order of an objective function on the optimization performance of an genetic algorithm. The third and fourth examples in section 5.3 provided case studies in which the application of the RFC to investigate the influence of nonlinearity, and problem dimensionality was demonstrated.

The novelty and additional value of the method presented in this chapter, over the test functions in the literature (see section 5.1), is the tunable level of difficulty regarding modality, variance contribution and interaction order of the design variables of the resulting test functions. A variety of test problems can be generated, which ranges from simple smooth uni-modal problems, up to apparent difficult structureless (pseudo-random) hyper dimensional problems. Because the functions are parameterized w.r.t. the function features, the effect of the function features can be investigated by means of parameter studies. Furthermore various problem instances of similar problems can be generated to investigate the effect of the function properties in a statistical sense, rather than merely on a single anecdotal test function.

A drawback of the presented method is that for compositions of these discrete random field based test functions the global optimum is not necessarily known (depending on the composition type). Probability based estimations can however been made, or brute force evaluation could be used to explore the details of the generated search space. Although a priori knowledge of the global optimum is desired for test functions, such knowledge is often also not available for real-world problems, and thus this disadvantage could therefore also be seen as a feature of realism.

Another property of the test function generating concept presented here, is that according to one's wishes it is possible to generate test function with a huge amount of descriptive parameters. Although this property is typically classified as unwanted, because it’s potential complexity in use. One could also argue using the concept of Kolmogorov complexity [Kol65] that simple test functions with few parameters are intrinsically very unlikely to represent the difficulties that can arise in highly specified complex computational models, and are therefore strongly limited in their scope. In this context, the author regards the possibility of creating highly parameterized test problems as a necessary feature to specify problems of highly structured complexity.

The presented concept enables the construction of complex problem types with structures that have been rarely investigated. As also expressed in [Bar11] the development of more challenging test functions could lead to the development of more robust and effective optimization algorithms. The author hopes that this communication will motivate others to use and extend the concepts presented, in endeavors to analyze and develop useful met heuristic algorithms.

Although the description of the method, only covers a few pages, an endless variety of test problems with different properties can be created using the concept presented. The large function space that the method covers, naturally leads to the need for the specification of subproblem classes (parameter ranges) and to define standardized test function instances for algorithm benchmarking. Such a classification could also contribute to compare this method with the many available “anecdotal” test functions, and it could place those functions in a more general function feature context. Other topics for further research based on the presented method, involve the construction of constrained and multi-objective test functions using a similar systematic approach. Besides investigations on the performance of different optimization algorithms, also the relation of function characteristics and optimization algorithm operators can be investigated, in order to analyze the mechanisms of metaheuristic algorithms in a more intrusive manner. The flexible parameterization, of the dimensionality, and higher order interactions between groups of design variables is also of interest for the generation of test problems for large scale global optimization.

5.5. Summary and conclusions

Because theoretical optimization performance analysis is difficult for non-convex problems, and since problems based on models of real-world systems are often computationally expensive, in the literature several artificial performance test problems and test function generators have been proposed for empirical comparative assessment and analysis of meta-heuristic optimization algorithms. These test problems however often lack the complex function structures and forthcoming difficulties that can appear in real-world problems.

In the chapter 4 an approach was presented to construct test problems that have similar characteristics as simulation response based optimization problems for vehicle structures. The response characterization criteria encountered during the development of that approach, inspired the development of a more general method to formulate optimization test problems with parameterized function features, targeting the wider framework of global optimization. Whereas the focus of the RSP approach (see chapter 4) lies on the construction of surrogate problems for a particular problem type, the method presented in this chapter enables the generation of parameterized test problems in a more general sense. This enables systematic investigation of the influence of selected function features on the performance of optimization algorithms.

In this chapter, a method is presented to systematically construct test problems with varying types of difficulty, based on the composition of parameterized random fields (Q6). An algorithm is described that can be used to generate and high-dimensional pseudo-random discrete fields of heterogeneous resolution. The resulting discrete fields can be combined with suitable weighting functions to obtain continuous and differentiable fields. By parameterized composition of these random fields, various interesting test functions can be generated. The presented method provides means to construct test functions with a variety of problem structures, with respect to modality, variance contribution distribution, and variable interactions. The concepts and the potential of the method was demonstrated by means of a few examples, in which the influence of different function parameters on the performance of a simple genetic algorithm was investigated.

The developed method is of relevance to the field of optimization because the methodology can be used as a tool for systematic optimization algorithm performance analysis, and for the development and identification of efficient global optimization algorithms for particular problem types. The method enables the construction of parameterized global optimization test functions with features combinations that were previously unavailable in the optimization literature. Furthermore the concept has potential to be extended and applied in multi-objective and large-scale optimization problems, and is a step towards the generation of optimization test problems with structure and level of difficulty relevant for industrial optimization problems.

6. Overall summary, discussion, and conclusions

"As our circle of knowledge expands, so does the circumference of darkness *surrounding it."*

-Albert Einstein

This dissertation presented numerical investigations and new methodologies related to the computational efficiency of multidisciplinary design optimization of car body structures. The focus has been on the performance analysis of optimization algorithms for simulation-based car body design optimization problems involving mass, vibration, and crash criteria. In addition, some of the developed ideas have been generalized to be of value for the field of global optimization in general. By answering the targeted research questions, this work set an explorative step in the direction of problem orientated optimization performance analysis, which is in the view of the author necessary to achieve improved optimization efficiency, in a systematic way, rather than by opportunism. Although, within the limits of the presented investigations and methodologies, significant improvements in optimization performance could be achieved for car body design problems by means of proper algorithm selection and parameter tuning, the path towards more efficient optimization methods for complex optimization problems involving computationally expensive simulations in general still requires further exploration.

The motivation for the presented work is the need in the automotive industry to improve the efficiency and methodology for complex computationally expensive optimization problems. The automotive industry is stimulated in various ways to design and construct fuel efficient, lightweight and safe vehicles, with profitable margins under tight time constraints. To achieve effective vehicle design solutions, efficient computer-aided engineering, simulation and optimization methods are of paramount importance. For industrial optimization problems that involve highly nonlinear, non-convex design criteria, such as (but not limited to) car body optimization, the selection and development of efficient optimization algorithms are still challenging open research topics of practical relevance.

In chapter 2 a survey on the literature regarding optimization performance analysis for crashworthiness optimization and related topics was presented, in which several open issues in the field have been identified. In the different studies in the literature on optimization of car body structures, many different optimization algorithms were used. There is no clear consensus on which algorithms are the most effective, and significant comparative studies are rare. Although there are many works in the literature that demonstrate the application of optimization algorithms to car body optimization problems, there are very few review papers or reference books targeting automotive crashworthiness optimization problems in a broader sense.

Comparative optimization performance assessments are not only rare for vehicle crashworthiness problems, but also for problems that involve computationally expensive simulations in general. As was stated in [Wan13] about simulation-based optimization: there are many different optimization algorithms developed, but there are not enough comparisons. In a more general scope performance comparisons between optimization algorithms are done quite commonly, using computationally inexpensive standard test problems. Many of these test problems have however been criticized for their lack of representativity for complex problems of industrial relevance [Lia05, Bar11, Die12]. Another point of criticism stated in [Sha10] was that not enough attention was given to the analysis of the structure of the underlying optimization problems.

In this thesis the primary aim was to address these open challenges for the particular application of optimization of car body structures. While the secondary aim was to make contributions that could also be extended to be of relevance for a wider range of problems.

6.1. Conclusions: the research questions revised

Based on the literature analysis (chapter 2), six research questions were formulated in the introduction (chapter 1), which were answered based on the presented work. In the following paragraphs, the questions and corresponding conclusions are recapitulated.

1. **Are the relative optimization algorithm performances on a particular vehicle design problem correlated with the relative performance on a similar vehicle design problem involving another vehicle model?**

   In chapter 3 the performance of 8 different optimization algorithms was assessed for 6 different optimization formulations, using meta-model based comparisons on two distinct vehicle models. The results indicated that the optimization performance distributions of corresponding optimization formulations over different vehicle models were significantly correlated with on average CC>0.93 and p<0.002. Additional validation results using a direct simulation approach on a third independent vehicle model confirmed the results

   Although by means of examples no general validity among all similar problems can be claimed. The presented results support the assumption that experiences based on an optimization problem of one vehicle, could be relevant for similar optimization problems of other vehicles. Although this has been silently assumed in previous works, the particular issue has not been explicitly addressed or investigated in the available literature before. The investigation and results were relevant since correlation significance among problem instances is a required justification for comparative assessments and benchmark studies on these problem types.

2. **How representative are meta-model optimization benchmarks for vehicle design problems compared to full direct simulation-based optimization performance benchmarks?**

The statistical correlation between meta-model based optimization algorithm performance, and direct simulation-based algorithm performance was high and significant for those optimization formulations that did not include crashworthiness related responses ($CC>0.84$ $p<0.0082$). The significance of the correlation was however marginal ($p>0.05$) for the problem formulations that included nonlinear crash responses.

These results are of relevance because several of the few available comparative studies in the literature that dealt with vehicle crash-optimization were based on meta-models using only up to 40 construction points, whereas these results obtained with 1000 construction points indicated that these meta-models were still not representative with sufficient significance. This implies that for car body optimization problems involving crashworthiness simulation responses, meta-model based comparisons should not implicitly be assumed to be representative for the algorithm performance on the corresponding simulation-based optimization problem.

3. **Are the differences in performance between meta-heuristic algorithms on various problem formulations of typical car body design optimization problems involving crashworthiness responses, of practical relevance?**

The comparative assessment studies in chapter 3 and 4 indicated significant performance differences among the optimization efficiency for the investigated problem formulations. For the investigated case studies the difference in optimization algorithm efficiency in terms of relative objective improvement at a given function evaluation budget ranged between 8% and 98% in terms of relative objective improvement. Moreover the results clearly showed that the best-performing optimization algorithms were dependent on the problem formulation, and no algorithm could be selected that was top performing among all problem formulations. The performance difference between average algorithm performance and the best-performing algorithm was larger than the average performance increase when doubling the function evaluation budget from 250 to 500.

The results showed that the differences in optimization performance are of practical significance. This emphasizes the importance of comparative testing to identify suitable optimization algorithms for particular problem types and function formulations, because proper optimization algorithm selection enables significant improvements in optimization efficiency, thus improved design solutions and/or savings in computational cost and time can be achieved.

4. What are the characteristics of the simulation responses of the selected design criteria w.r.t. changes in the design variables? (Are there any typical response characteristics over similar problems involving different vehicle models?)

An analysis of the simulation responses was provided in chapter 4. By means of global sensitivity analysis, parameter studies, harmonic analysis, and covariance or correlation analysis, several response characteristics have been identified and quantified, based on investigations on two different vehicle models. The simulation responses investigated were: BIP mass, (free-free) 1st natural torsion eigenfrequency, peak acceleration[18], and A-B pillar deformation[18]. The design variables were scaling factors on the thickness of BIP components. Briefly summarized the results of the simulation response characterization indicated that:

- For all simulation responses, the variance contribution distribution of first-order effects was unbalanced. Which implies that a small subset of design variables have a large influence in terms of variance contribution while many design variables only have a small influence.
- The influence of second and higher order variable interactions had characteristic distributions for each of the investigated simulation responses. In detail mass, and eigenfrequency responses had nearly no interaction terms, whereas the crashworthiness responses were significantly influenced by variable interactions in terms of total variance contribution.
- Each of the simulation responses had characteristic behavior w.r.t. design variable changes in terms of nonlinearity. Mass responses varied linearly; eigenfrequency responses varied mildly nonlinear; and both crashworthiness responses had highly nonlinear behavior.
- Using simulation-based function evaluations based on pseudo-random sampling the normalized covariance between the different simulation responses was estimated. Significant similarities between the normalized covariance distributions for the simulation responses for two vehicle models were identified.
- Similar results were obtained for the normalized covariance between the first-order sensitivity index distributions among the different simulation responses. Although such correlations are not identical for the different vehicle models, clear similarities could be observed and quantified.

Although by the use of other or additional analysis techniques possibly also other response characteristics could be identified, the presented response analysis and resulting set of response characteristics are unprecedented in the literature. The

[18] Both peak acceleration at the vehicle tunnel, and A-B pillar deformation responses for the load case of a 56km/h frontal crash against a rigid wall.

characterization approach and results are of relevance to practitioners in the application field of car body design, as well as to those interested in optimization problem and performance analysis in general, since the methods used characterization are not problem specific, and could also be used for other problem types.

5. **How to formulate computationally affordable test problems which are representative for simulation-based car body design optimization problems and their response characteristics?**

A new approach to construct representative surrogate test problems based on response characteristics was proposed in chapter 4 and evaluated by means of several case studies. In the approach basis functions and parameter constraints for each of the targeted simulation responses are selected based on the response characterization results. These basis functions and parameter constraints are combined with additional constraints that enforce the other response characteristics, such as the variance contribution distributions and response correlations, to describe a constraint satisfaction problem. By the solution of the constraint satisfaction problem, problem instances can be obtained which are similar w.r.t. the selected response characteristics. The results of the case studies indicated that the use of the approach for comparative algorithm performance assessments led to performance distributions which were significantly correlated ($CC \geq 0.910$, $p \leq 0.0012$) with the performance of the validation study on an independent vehicle model.

The function characterization requires a considerable function evaluation investment (and thus computational effort), however once the function characterization is established the resulting test functions are computationally affordable. For industrially relevant problems where many instances of similar optimization problems have to be solved, such an approach could however pay off, since significant increases in optimization efficiency can be achieved.

The presented approach provides insight in the optimization problem characteristics and not merely the optimization algorithm performance. Therefore the presented approach is a contribution in the direction of the research targets set in [Sha10] aiming towards a more problem oriented application of optimization methods.

6. **How to construct optimization test functions with relevant problem features, in a way that enables systematic performance analysis w.r.t. particular response characteristics?**

The previous point (5) addressed the construction of test functions based on particular real-world problem types. A further key issue is the systematic analysis of optimization performance based on problems with particular response characteristics. A new method has been proposed (in chapter 5) which enables the construction of a broad range of different test problems with parameterized characteristics such that their influence can be studied systematically. The method is based on the superposition or composition of different random fields. An algorithm is described to generate discrete random fields of varying nonlinearity, spatial resolution and dimensionality. By the use of weighting functions, continuous and differentiable random fields can be obtained. These continuous random fields can be combined and weighted to compose test functions of various complexity. The provided framework allows the parameterized control over function features such as nonlinearity, variance contribution distributions, variable interactions, and dimensionality. These function features can also be combined. The results in chapter 4 indicated that these function features can be characteristic for some problem types. In chapter 5 a few example investigations are presented to demonstrate the effect that some of these individual features have on optimization performance.

The presented method contributes to a more systematic analysis of optimization algorithm performance on problems with various types of complexity. The scope of the method is not limited to automotive or engineering applications, and is relevant for the field of global optimization in general.

6.2. Main contributions, discussion, and outlook

The objective of this work was to make scientific contributions targeting one or more challenges related to multidisciplinary vehicle design involving weight, crash and vibration criteria. The main contributions of this work were:

1. **A meta-model based comparative assessment on the performance of optimization algorithms for car body design problems, involving lightweight, vibrational comfort and crashworthiness criteria.**

   Compared to past comparative studies on car body optimization problems [Dud08, Gu13, Kia15] the presented meta-model based comparison, extended the previously available work by several aspects: the use of multiple vehicle models for similar problem types; the number of compared optimization algorithms; the number of construction points used for the meta-models was an order of magnitude larger than previous investigations; the algorithm comparison results based on meta-models were compared with results using direct simulation-based results. Due to these features of the study, the comparative assessment addressed several open issues in this application field (see research questions 1-3, in the previous section).

   The results showed that the performance differences between the different algorithms are large. Choosing the most suitable algorithm from those investigated would lead to larger objective improvements, then doubling the number of optimization iterations of an average algorithm. The results also demonstrated a significant correlation between the optimization performance over similar problems formulations on different vehicle models. Which indicates that comparative assessments and benchmark studies, can prove of value for algorithm selection on similar problems. The results however also indicated that in comparative optimization algorithm assessments, the performance based on meta-model responses is not necessarily representative for the direct simulation based performance.

   Although the results of the comparative assessment study, extended the available literature in various ways, the value of such comparative assessments is only limited to similar optimization problems. Furthermore, comparative assessment results by themselves do not provide insight in the problem characteristics and efficiency of the mechanisms and operators of which the optimization algorithms are composed. However, in the absence of better comparisons and analysis, the presented study provides useful results for practitioners in industry dealing with these problem types.

   Outlook

   The presented comparative study could be extended in various ways. The representativity of meta-model based performance comparisons with additional or other design criteria, load cases, and design variable types, could be investigated and compared with simulation-based comparisons on the corresponding problems. Also, a larger variety of optimization algorithms could be tested. The value of such comparative studies could be increased if the used meta-models, construction point datasets and algorithms were shared and made publicly available. Although meta-model based comparative assessments of optimization algorithms can be useful, the performance results should be compared and validated with direct simulation based results, since representativeness of the results cannot be taken for granted. In the author's perspective, the application and extension of more intrusive and systematic performance analysis techniques such as the RSP approach seems beneficial over the use of meta-models, because it provides additional insights into the problem structure.

2. **The development of a novel Representative Surrogate Problem (RSP) approach to construct test problems for comparative assessments based on simulation responses related to car body design problems.**

Comparative optimization algorithm assessments in which the simulation responses are estimated using meta-models are limited in their validity, depending on the response types involved. A new surrogate problem modeling approach for such comparisons was presented, that is also suitable for problems with high nonlinearities, such as the case with crashworthiness responses. A detailed description of the approach was given in chapter 4.

Briefly summarized the approach is composed of 3 phases:

a. Response characterization based on function evaluations on the simulation models;
b. Construction of a representative surrogate response system by formulating and solving a constraint satisfaction problem, with selected basis functions, parameter bounds constraints that enforce selected response characteristics based on the response characterization;
c. Combining surrogate response system with an optimization formulation of interest to construct an optimization problem;
d. Corroboration of the representativeness of the obtained optimization problem.

The presented RSP approach is partly related to surrogate data generation techniques for time series such as [Pri94], but strongly adapted for the application to multivariate and multidisciplinary optimization problems. This approach to generate optimization test functions derived from real-world problems by means of function characterization and the incorporation of selected function characteristics by means of solving a constraint satisfaction problem is new in the field of structural optimization.
For the simulation response analysis: global sensitivity analysis methods and parameter studies have been applied to analyze the characteristics of the simulation responses w.r.t. changes in the design variables for several vehicle models. Although the analysis of optimization problem structures in order to improve optimization efficiency has been suggested in the literature [Sha10], such investigations are still very scarce. The simulation response analysis is to the knowledge of the author the first study of its kind for the application of car body design problems.

A proof of concept was presented, in which the optimization efficiency of 8 different optimization algorithms was compared using the RSP approach, (calibrated using 2 vehicle models) and a comparison using full vehicle simulation-based results on an independent corroboration vehicle model. The presented results indicated that for this problem statistically significant predictions on optimization performance, could be made w.r.t a similar optimization problem on the independent vehicle model. Using these prediction results to select a suitable optimization algorithm led to efficiency increases between 16% and 32% w.r.t. the average performance over the investigated algorithms.

Quantitative representativeness of the RSP was also demonstrated for a meta-optimization of the vehicle model optimization. In this case study the parameters of the optimization algorithm (Differential Evolution) were optimized using the RSP problem, and the optimized parameters were applied to perform optimization runs of the corroboration vehicle model. The results indicated that additional optimization efficiency improvements in the order of 4% can be achieved by using the optimized meta-parameters, based on the application of the RSP approach.

The proposed RSP approach provides a new perspective on optimization performance assessment for car body design problems, and the results indicate potential for applications in other fields of (design) optimization. The presented RSP approach, the investigated application case studies, and their validations, showed encouraging results. Therefore further investigations on other applications and problem types are of interest, in order to evolve the approach to a more general method suitable for other applications. The concept of the presented approach and ideas do not contain problem specific methods, such that further investigations on other problem types are straight forward.

Outlook

The idea of the RSP approach can be extended and applied for the use of meta-simulation of optimization processes for complex industrially relevant problems dealing with computationally expensive simulations. The solution of such problems is often restricted by time and computational resources (hardware infrastructure, software licenses). For such problems, there are often many possibilities to apply parallel computing: parallelization of the simulation solution process, parallelization of the different simulation responses, or parallelization in the optimization process using population-based approaches. For complex simulation-based optimization problems, the efficiency of the solution approach is not only problem dependent but also resource dependent, using RSP-based optimization test problems, tailored strategies for a given problem and simulation environment could be established. In the presented work the focus was on multidisciplinary optimization problems with a single objective and various constraints, for some problems also multi-objective optimization problems can be of interest. Therefore an extension of the presented approach for such problems would be of interest. Also of interest are investigations on the RSP approach targeting robust design optimization (RDO) problems, which are also highly relevant for car body design [Asp12, Hun13, And15]. Investigations on MOP and RDO problems however usually require even more computational effort than the presented single objective multidisciplinary problems addressed in this work, and are therefore an important challenge for future work.

3. **The development of a new method based on random field composition to construct global optimization test functions with a wide variety of function characteristics.**

   Most optimization test functions that have been proposed in the literature deal with a particular combination of function characteristics. Few attempts have been made to construct parameterized functions that could be used to investigate optimization performance in a systematic way.
   A novel method proposed in chapter 5, uses concepts from random field theory to construct basic discrete random fields with probability distributions of choice. By means of the described random field composition method, fields with specified features can be constructed, such as particular variance contribution distributions, higher order interactions, anisotropic multimodality. The method provides means to construct test functions with high complexity and features of realism representative for real-world problems, which in its turn enables a systematic investigation of optimization algorithm performance w.r.t. variations in problem function features. Such systematic investigations with relate problem features to optimization algorithm performance, are important for the development of efficient optimization strategies.

   Outlook
   The description of the proposed RFC-based method for the generation of global optimization based test functions only requires a few pages, the amount of different test functions and problem types that can be generated are however endless. In the scope of this thesis, only a few different possibilities to study the effect of different problem characteristics have been investigated for a single optimization algorithm.
   With the provided methodology it is straightforward to extend this work with studies that include the combination of different optimization problem features and different optimization algorithms. Such studies are relevant because the influence of different function features and their combinations, on the performance of various optimization algorithms can be investigated in a systematic way. This in its turn could provide to be a valuable tool, for the development of efficient optimization algorithms for problem types with particular characteristics. Another point of interest for future research, is to apply the presented concept to large scale global optimization problems, and to extend it to multi-objective optimization problems.

6.3. Final remarks, and overall outlook

This thesis addressed multidisciplinary car body design optimization problems, which regard crashworthiness, vibration comfort, and lightweight design criteria. In a full industrial vehicle design process, even more criteria are of practical relevance. Besides additional criteria from the selected fields, also design criteria such as: structural durability, production cost and the environmental impact during the product life cycle are of relevance to the car body design process. Studies on highly interdisciplinary optimization problems, which take many or all of the relevant car body design criteria in to consideration, are presently still rare in the literature. Future studies could investigate the potential of new optimization strategies for problems with such high complexity. However, it is often easier to create complex problems, than it is to solve them. To find efficient optimization approaches, it is necessary to investigate and analyze the underlying structure and characteristics of such problems. This thesis provides new approaches to compare optimization strategies, using test functions with particular problem characteristics. To experiment with more complex optimization problems, and their solution techniques, representative surrogate problems that mimic the real problem characteristics can be of practical value, for the selection and development of optimization strategies. The case studies presented in this thesis indicated that significant performance gains can be achieved, for industrially relevant problems, by the selection and tuning of optimization algorithms, using appropriate test and benchmark problems. For a more intrusive and systematic optimization algorithm analysis, besides testing optimization algorithm performance on problems with fixed characteristics, also investigations on problems with parameterized characteristics are of importance. The presented method to construct random field composition based test functions, can be a valuable aid to evaluate how optimization algorithm performance depends on various optimization problem characteristics and sources of complexity.

Specific outlooks to each of the contributions were already stated in the previous section. As a general recommendation for future research the author would like to emphasize the potential benefits of inter- and trans-disciplinary perspectives to better exploit the data and methods that are involved with computationally expensive simulation based optimization problems. In the scope of this work, numerical simulation, meta-modeling, sensitivity analysis, variable screening, and optimization methods were applied in an engineering context. Merging competences and techniques from subfields as computational mechanics, machine learning, statistics, and mathematical optimization could result in further novel techniques and tools valuable for applications in automotive and industrial engineering.

During the activities for this thesis, the author came across various novel strategies related to the optimization and analysis for industrial complex large scale optimization problems that seem promising but are not commonly applied yet. Examples are various distributed optimization architectures (see for example [Mar13] for a review) and co-evolutionary optimization strategies (e.g. [Cao15]). Although these strategies have not been addressed in the scope of this thesis (because the involved computation cost for validation studies would have exceed the available resources of the author), the ongoing further development of these strategies, could however strongly benefit from the presented approaches to construct realistic and challenging optimization test problems. As a first and straightforward step, comparative studies of such optimization methods on test functions based on the in this thesis described approaches could be performed in a similar way as presented in this thesis. A next objective could be: to gain further insight in the relation between optimization performance and problem characteristics in order to develop, guidelines or methodologies that enable the construction of efficient optimization algorithms and architectures for particular problem types, based on a priori known problem characteristics. A further step would be the development of highly adaptive meta-optimization-algorithms that during the optimization process on a new unknown problem, attempt to classify the problem type, and change the applied optimization strategy accordingly, while also regarding the available computation and simulation resources. To achieve these and other goals, there is still much research to be done in the field of optimization of complex problems, and in the meantime the complexity of industrial problems will only increase even further. Challenging test problems with realistic problem features (such as contributed in this thesis) are therefore indispensible to achieve these goals, and further improvements.

This thesis targeted to contribute to the topic of: simulation based mathematical design optimization of car body design structures. An important open issue in this area, is the efficiency and selection of the optimization algorithms that are used for these type of problems. Because such problems involve computationally expensive simulations, relevant comparative assessments are cumbersome, and rare in the literature. Although the complexity, computational cost and depth of the open problems in this field span a challenge that is too large to tackle in the scope of a single thesis, the investigations, approach and methods presented in this work, provide previously unavailable insights, new ideas and methods that are aimed to set a step towards more efficient multidisciplinary car body design optimization. Furthermore several concepts are presented in a general way, in order to also be of benefit to further optimization research for other applications that deal with complex and computationally expensive problems.

Acknowledgements

In a world with so many talented people in difficult situations, I consider the opportunity to do research work as an undeserved privilege, resulting from many fortunate and unfortunate circumstances. Although time and unforeseen occurrences can befall us all rather randomly, it feels unfair to those less fortunate to "thank" these circumstances, but it would be ignorant not to appreciate the circumstances favorable and supportive to life and happiness. Furthermore, much of the fortunate circumstances that helped me, were the direct or indirect result of the perseverance, patience and love of many fellow humans to which I owe gratitude. At this occasion I would like to take the opportunity to mention a few of them.

Besides a number of restless nights at home, most of the present thesis work was carried out in the Department of Industrial Engineering of the University of Florence. Foremost, I would like to express gratitude to my supervisors for the trust, support and opportunity they give me to do this work: Prof. Marco Pierini for his patient guidance, and the open-minded, encouraging while firm discussions. Prof. Baldanzini for the direct, critical but kind and constructive discussions on research ideas and preliminary manuscript texts. Regarding the manuscripts I have prepared in the scope of this work, (the thesis and various publications) I also owe much gratitude to the reviewers, for their effort, perseverance and feedback.

Also, I would like to thank all my colleagues of the MOVING group, for the nice atmosphere in in via Rossi 1. In specific I would like to thank Ilaria and Pavlina for their assistance and tips on administrative issues during the first months in Italy; Francesco, Giorgio and Federico for their kind companionship during the doctorate studies. I will particularly cherish good memories to Lorenzo "Berzipedia", for sharing his omnivalent quick accessible knowledge on nearly any subject, and the hilarious jokes during lunch.

I would like to thank all my teachers for their effort, but especially for their patience deciphering my handwriting. In particular I would like to thank "Juf" Monique van Lammeren, "Juf" de Graaf-Bierbrouwer, "meester" Willem Gabes, Erik Wezenbeek, Wilke van Muiswinkel, Dr. Anna Dall'Acqua, Dr Paul Heuler, Prof. Raymond Freymann and Prof. Gerhard Müller for their enthusiasm and their style of teaching, which helped and inspired me tremendously during my studies. In addition to my regular studies, also the video lectures by Prof. Gilbert Strang, Prof. Arthur Mattuck and Prof. Stephen Boyd were a great help. Also I would like to thank Dr. Andreas Mackensen, Dr. Helmut Gese, Gernot Oberhofer, Martin Oehm, Harald Fleischer, Matthias Metschkoll, Günter Jäger, and Dr. Helge Oppermann for their mentorism and support in various ways after my studies.

During the research for the thesis I have spent too little time with my friends. I would like to thank my friends for their patience, and I am happy that our friendship survived in a sort of "hibernation" mode during the last years. Kazuhito, Mahmoud, Tim, Jorge: the few moments together and your words of support meant a lot to me and helped me through when difficult times occurred.

In Germany there was a custom that also the wife of man who successfully finished their doctorate was called with the title of their husband ("Frau Doktor"). And although it is fortunate that mentioning titles in daily is out of fashion, I like the idea of how that custom expressed the solidarity with the hardships that the partner of a doctorate candidate has to endure. I thank my wife Siyana sincerely for all her patience and love, and all the work she did to support me. Also I want to thank Kristina for her cheerfulness and her help with writing and checking the formulas in my dissemination. Furthermore I would like to thank my grandparents and my aunt for their kindness and financial support during my studies.

Above all I would like to express my gratitude to my parents Adrie en Joke for their abundance of love, guidance and support in any possibly imaginable way. Any finite statement would fall short to describe what I owe you.

Bibliography

“there is no end to the crafting of many books, and too much study wearies the body.”

Solomon 970-930 B.C.

[Abb14] Abbasi M., Ghafari-Nazari A., Reddy S., & Fard M. (2014). A new approach for optimizing automotive crashworthiness: concurrent usage of ANFIS and Taguchi method. Structural and Multidisciplinary Optimization, 49(3), 485-499.

[Ack87] Ackley D. (1987). A connectionist machine for genetic hillclimbing. Springer Science & Business Media.

[Add07] Addis B., Locatelli M. (2007). A new class of test functions for global optimization. J. Glob. Optim. 38, 2007479–501

[Adl09] Adler R. J., and Taylor J. E. (2009). Random fields and geometry. Springer Science & Business Media.

[Agt10] Agte J., De Weck O., Sobieszczanski-Sobieski J., Arendsen P., Morris A., and Spieck M. (2011) "MDO: assessment and direction for advancement—an opinion of one international group." Structural and Multidisciplinary Optimization 40.1-6 (2010): 17-33.

[Ahr10] Ahrari A., Ahrari R.: On the utility of randomly generated functions for performance evaluation of evolutionary algorithms. Optim. Lett. 4, 531–541 (2010).

[Ali10] Alimoradi A., Foley C. M., and Pezeshk S. (2010) Benchmark Problems in Structural Design and Performance Optimization: Past, Present, and Future—Part I. 19th ASCE Conf. Proc., State of the Art and Future Challenges in Structure. ASCE Publications. 2010.: pp. 455-466. doi: 10.1061/41131(370)40

[All82] Allemang R. J., and Brown D. L. (1982, November). A correlation coefficient for modal vector analysis. In Proceedings of the 1st international modal analysis conference (Vol. 1, pp. 110-116). (IMAC), Orlando.

[And08] Andrei N.: An unconstrained optimization test functions collection. Adv Model Optim. 10, 147–161 (2008).

[And53] Anderson, R. L. (1953). Recent advances in finding best operating conditions. Journal of the American Statistical Association, 48(264), 789-798.

[And15] Andricevic N., Duddeck F., and Hiermaier S. (2015). A novel approach for the assessment of robustness of vehicle structures under crash. International Journal of Crashworthiness, 1-15.

[Arf13] Ariffin R. N. R., and Zahari R. K. Towards a Sustainable Urban Transport System in the Klang Valley, Malaysia: The Key Challenges.Procedia-Social and Behavioral Sciences 85 (2013): 638-645.

[Arg60] Argyris J. H., and Kelsey S. (1960). Energy theorems and structural analysis (Vol. 960). London: Butterworths.

[Aro90] Arora J. S. (1990). Computational design optimization: a review and future directions. Structural Safety, 7(2), 131-148.

[Asp12] Aspenberg D., Jergeus J., and Nilsson L. (2013). Robust optimization of front members in a full frontal car impact. Engineering Optimization, 45(3), 245-264.

[Back96] Bäck T. (1996). Evolutionary algorithms in theory and practice: evolution strategies, evolutionary programming, genetic algorithms. Oxford University Press.

[Bal05] Balasundaram B., Butenko S.: Constructing test functions for global optimization using continuous formulations of graph problems. Optim. Methods Softw. 20, 439–452 (2005).

[Bal60] Ball W. W. R. (1960). A short account of the history of mathematics. Courier Corporation. http://www.gutenberg.org/ebooks/31246

[Bao09] Bao L., and Zeng J. C. (2009, August). Comparison and analysis of the selection mechanism in the artificial bee colony algorithm. Ninth International Conference on Hybrid Intelligent Systems, 2009. HIS'09. (Vol. 1, pp. 411-416). IEEE.

[Bar11] Barrera J., Coello C.A.C.: Test Function Generators for Assessing the Performance of PSO Algorithms in Multimodal Optimization. In: Handbook of Swarm Intelligence. pp. 89–117. Springer (2011).

[Bar54] Barricelli N.A. Esempi numerici di processi di evoluzione. Methodos6.21-22 (1954): 45-68.

[Bar57] Barricelli N.A. Symbiogenetic evolution processes realized by artificial methods. Methodos 9.35-36 (1957): 143-182.

[Bar93] Barthelemy J. F., and Hafkta R. T. (1993). Approximation concepts for optimum structural design—a review. Structural and Multidisciplinary Optimization, 5(3), 129-144.

[Barr95] Barr R., Golden B., Kelly J., Rescende M. and W. Stewart, (1995) Designing and Reporting on Computational Experiments with Heuristic Methods, Journal of Heuristics, *1*(1), 9-32.

[Bck11] Backlund P. B., Shahan, D. W., and Seepersad, C. C. (2012). A comparative study of the scalability of alternative metamodelling techniques. Engineering Optimization, 44(7), 767-786.

[Bel54] Bellman R. The Theory of Dynamic Programming. Bull. Am. Math. Soc., 60, 503-516, 1954.

[Bel76] Belytschko T. (1976). A survey of numerical methods and computer programs for dynamic structural analysis. Nuclear Engineering and Design, 37(1), 23-34.

[Bel78] Belytschko T., and Mullen R. (1978). Stability of explicit-implicit mesh partitions in time integration. International Journal for Numerical Methods in Engineering. 12(10), 1575-1586.

[Ben14] Bengler K., Dietmayer K., Farber B., Maurer M., Stiller C., and Winner H. (2014). Three decades of driver assistance systems: Review and future perspectives. Intelligent Transportation Systems Magazine, IEEE, 6(4), 6-22.

[Ben77] Bennett J. A., Lin K. H., and Nelson M. F. (1977). The application of optimization techniques to problems of automotive crashworthiness (No. 770608). SAE Technical Paper.

[Bend88] Bendsøe M. P., and Kikuchi N. (1988). Generating optimal topologies in structural design using a homogenization method. Computer methods in applied mechanics and engineering, 71(2), 197-224.

[Bey02] Beyer H. G., and Schwefel H. P. (2002). Evolution strategies–A comprehensive introduction. Natural computing, 1(1), 3-52.

[Big76] Biggs Norman, Lloyd E. K., and Wilson R.J. Graph Theory, 1736-1936. Oxford University Press, 1976.

[Bir06] Birge B., Particle Swarm Optimization Toolbox, MATLAB file exchange (2006) http://www.mathworks.com/matlabcentral/fileexchange/7506-particle-swarm-optimization-toolbox

[Bis07] Biswas A., Dasgupta S., Das S., and Abraham A. (2007). Synergy of PSO and bacterial foraging optimization—a comparative study on numerical benchmarks. In Innovations in Hybrid Intelligent Systems (pp. 255-263). Springer Berlin Heidelberg.

[Bli15] Blincoe L. J., Miller T. R., Zaloshnja E., and Lawrence B. A. (2015, May). The economic and societal impact of motor vehicle crashes, 2010. (Revised) (Report No. DOT HS 812 013). Washington, DC: National Highway Traffic Safety Administration

[Blu01] Blumhardt R (2001). FEM-crash simulation and optimization. International Journal of Vehicle Design 26.4: 331-347.

[Bor04] Borenstein Y., Poli R.: Fitness distributions and GA hardness. In: Parallel Problem Solving from Nature-PPSN VIII. pp. 11–20. Springer (2004).

[Box13] Box E. Mortality statistics and road traffic accidents in the UK. RAC Foundation Report (2013).

[Box51] Box G. E., and Wilson K. B. (1951). On the experimental attainment of optimum conditions. Journal of the Royal Statistical Society. Series B (Methodological),13(1), 1-45.

[Box66] Box G. E. (1966). Use and abuse of regression. Technometrics, 8(4), 625-629.

[Box87] Box G. E. and Draper (1987), Empirical model-building and response surfaces, Wiley, (quote at p. 424)

[Boy09] Boyd S., L. Vandenberghe. Convex optimization. Cambridge university press, 2009. https://web.stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf

[Brb77] Brebbia C. A., and Dominguez J. (1977). Boundary element methods for potential problems. Applied Mathematical Modelling, 1(7), 372-378.

[Bre06] Brest J., Greiner S., Boškovic B., Mernik M., and Zumer V. (2006). Self-adapting control parameters in differential evolution: a comparative study on numerical benchmark problems. Evolutionary Computation, IEEE Transactions on, 10(6), 646-657.

[Bre62] Bremermann H. J. Optimization through evolution and recombination. Self-organizing systems 93 (1962): 106.

[Brn95] Braun R. D., and Krooy I. M. (1995). Development and Application of the Collaborative Optimization Architecture in a Multidisciplinary Design Environment. Nasa Technical report 20040111288

[Bro11] Brownlee J. Clever Algorithms: Nature-Inspired Programming Recipes, First Edition, Lulu Enterprises, January 2011. ISBN: 978-1-4467-8506-5

[Bro88] Broomhead D., and Lowe D. (1988). Multivariable functional interpolation and adaptive networks. Complex Systems, 2, 321-355.

[Brt87] Burnett D. S. (1987). Finite element analysis: from concepts to applications. Prentice Hall.

[Buh00] Buhmann M. D. (2000). Radial basis functions. Acta Numerica 2000, 9, 1-38.

[Buh13] Buehren M., Differential evolution, MATLAB file exchange (2013) http://www.mathworks.com/matlabcentral/fileexchange/18593-differential-evolution

[Cao15] Cao, Z., Wang, L., Shi, Y., Hei, X., Rong, X., Jiang, Q., & Li, H. (2015, May). An effective cooperative coevolution framework integrating global and local search for large scale optimization problems. In *Congress on Evolutionary Computation (CEC), 2015 IEEE* (pp. 1986-1993).

[Cau1847] Cauchy B. A. L. (1847). Exercices d'analyse et de physique mathématique(Vol. 3). Bachelier.

[Cce15] Center for climate and energy solutions Transportation overview 2015 http://www.c2es.org/energy/use/transportation (accessed 06.10.2015)

[Cel73] Cella A., and Soosar K. (1973). Discrete variables in structural optimization.in Gallagher RH, Zienkiewicz O. C. Optimal Structure Design—Theory and Application. New York: John Wiley and Sons.

[Cer85] Cerný V., Thermodynamical approach to the traveling salesman problem: An efficient simulation algorithm. Journal of optimization theory and applications 45.1 (1985): 41-51.

[Cgh60] Clough R. W., The finite element method in plane stress analysis, in Proceedings of the American Society of Civil Engineers 2nd Conference on Electronic Computation, September, Pittsburgh, PA, 1960.

[Ch10] Cheah Lynette W. Cars on a diet: the material and energy impacts of passenger vehicle weight reduction in the US. Diss. Massachusetts Institute of Technology, 2010.

[Chi95] Chipperfield A.J., Fleming P.J.: The MATLAB genetic algorithm toolbox. IEEE Colloquium on applied control techniques using MATLAB,. pp. 10/1–10/4. IET (1995).

[Cmp07] Campolongo, F., Cariboni, J., & Saltelli, A. An effective screening design for sensitivity analysis of large models. Environmental Modelling & Software 22.10 (2007): 1509-1518.

[Coh95] Cohen, P.: Empirical Methods for Artificial Intelligence. The MIT Press (1995)

[Cor03] Corne D. W., and Knowles J. D.. No free lunch and free leftovers theorems for multiobjective optimisation problems. Evolutionary Multi-Criterion Optimization. Springer Berlin Heidelberg, 2003.

[Cra02] Craig K. J., Stander N., Dooge D., and Varadappa S. (2002). MDO of automotive vehicle for crashworthiness and NVH using response surface methods. AIAA paper, 5607.

[Cra02b] Craig K. J., Nielen S., Dooge D. A., and Varadappa S. (2002, May). Multidisciplinary design optimization of automotive crashworthiness and NVH using LS-Opt. In 7th Int. LS-DYNA Users Conf., Michigan, US.

[Cra03] Craig K. J., Stander N., and Balasubramanyam S. (2003). Worst-case design in head impact crashworthiness optimization. International journal for numerical methods in engineering, 57(6), 795-817.

[Crm94] Cramer E. J., Dennis Jr, J. E., Frank P. D., Lewis R. M., and Shubin G. R. (1994). Problem formulation for multidisciplinary optimization. SIAM Journal on Optimization, 4(4), 754-776.

[Crn02] Crandall J. R., Bhalla K. S., and Madeley N. J. (2002). Designing road vehicles for pedestrian protection. BMJ: British Medical Journal, 324(7346), 1145.

[Csf96] Crisfield M.A, (1996) Non-Linear Finite Element Analysis of Solids and Structures (Vol 1) Wiley

[Cul96] Culberson J. C. (1998). On the futility of blind search: An algorithmic view of “no free lunch”. Evolutionary Computation, 6(2), 109-127.

[Dal05] Daly C.: Higher order models using entropy, Markov random fields and sequential simulation. In: Geostatistics Banff 2004. pp. 215–224. Springer (2005).

[Dan55] Dantzig G. B., Orden A., and Wolfe P. (1955). The generalized simplex method for minimizing a linear form under linear inequality restraints. Pacific Journal of Mathematics, 5(2), 183-195.

[Daw65] Dawe D. J. (1965). A finite element approach to plate vibration problems. Journal of Mechanical Engineering Science, 7(1), 28-32.

[Deb00] Deb, K., Agrawal, S., Pratap, A., and Meyarivan, T. A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. Lecture Notes in Computer Science 1917 (2000): 849-858.

[Die12] Dieterich J.M., Hartke B.: Empirical Review of Standard Benchmark Functions Using Evolutionary Global Optimization. Appl. Math. 03, 1552–1564 (2012).

[Dnc34] Duncan W. J., and Collar A. R. (1934). LXXIV. A method for the solution of oscillation problems by matrices. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 17(115), 865-909.

[Dok89] Dokainish M. A., and Subbaraj K. (1989). A survey of direct time-integration methods in computational structural dynamics—I. Explicit methods. Computers and Structures, 32(6), 1371-1386.

[Don09] Donders S., Takahashi Y., Hadjit R., Van Langenhove T., Brughmans M., Van Genechten B., and W. Desmet, A reduced beam and joint concept modeling approach to optimize global vehicle body dynamics. Finite Elements in Analysis and Design, 45(6), (2009): 439-455.

[Dro02] Droste S., Jansen T., and Wegener I. (2002). Optimization with randomized search heuristics—the (A) NFL theorem, realistic scenarios, and difficult functions. Theoretical Computer Science, 287(1), 131-144.

[Dro99] Droste S., Jansen T., Wegener I.: Perhaps Not a Free Lunch But At Least a Free Appetizer. In: GECCO, July 13-17, vol. 1, pp. 833–839. Morgan Kaufmann, San Francisco (1999)

[DuB04] Du Bois P., Chou C. C., Fileta B. B., Khalil T. B., King,A. I., Mahmood H. F., and Belwafa J. E. (2004). Vehicle crashworthiness and occupant protection. American Iron and Steel Institute, Michigan , http://www.autosteel.org/~/media/Files/Autosteel/Research/Safety/safety_book.pdf

[Dud03] Duddeck F., Heiserer D., and Lescheticky J. (2003, June). Stochastic methods for optimization of crash and NVH problems. In Bathe KJ (ed) 2nd MIT Conference on Computational Fluid and Solid Mechanics (pp. 2265-2268).

[Dud08] Duddeck F. (2008). Multidisciplinary optimization of car bodies. Structural and Multidisciplinary Optimization, 35(4), 375-389.

[Dud13] Duddeck F., and Zimmer H. (2013, January). Modular Car Body Design and Optimization by an Implicit Parameterization Technique via SFE CONCEPT. In Proceedings of the FISITA 2012 World Automotive Congress (pp. 413-424). Springer Berlin Heidelberg.

[Dud15] Duddeck F.; Wehrle E.J. (2015): Recent Advances on Surrogate Modeling for Robustness Assessment of Structures with respect to Crashworthiness Requirements. 10th European LS-DYNA Conference, DYNAmore GmbH, Würzburg, Germany, 15-17 June 2015

[Dun96] Duncan Alan E., Frank C. Su, and Walter L. Wolf. Understanding NVH basics. International Body Engineering Conference Proceedings, IBEC. 1996.

[ECP15] European commission Press release (2015) "How safe are your roads? Commission road safety statistics show small improvement for 2014" IP/15/4656

[EdL07] Editorial, On the road: accidents that should not happen. The Lancet 369.9570 (2007): 1319.

[Eli15] Eliasson J, and Proost S. Is sustainable transport policy sustainable? Transport Policy 37 (2015): 92-100.

[Eng96] English T. M. (1996). Evaluation of Evolutionary and Genetic Optimizers: No Free Lunch. In Evolutionary Programming (pp. 163-169).

[EOM15] Encyclopedia of Mathematics. Multi-index notation. URL: http://www.encyclopediaofmath.org/index.php?title=Multi-index_notation&oldid=30979 accessed on December 2015

[Etm96] Etman L. F. P., Adriaens J. M. T. A., Van Slagmaat M. T. P., and Schoofs, A. J. G. (1996). Crash worthiness design optimization using multipoint sequential linear programming. Structural Optimization, 12(4), 222-228.

[ETSC2011] European Transport Safety Council, Jost, G., Allsop, R., Steriu, M., & Popolizio, M. 2010 road safety target outcome: 100,000 fewer deaths since 2001: 5th road safety PIN report. 2011.

[ETSC2015] European Transport Safety Council, Position Paper: "Revision of the General Safety Regulation 2009/661"March 2015 http://etsc.eu/wp-content/uploads/2015_06_gsr_review_pp.pdf

[Eul1744] Euler L. (1952). 1744 Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes sive solutio problematis isoperimetrici latissimo sensu accepti. Bousquet, Lausannae e Genevae, E65A. OO Ser. I, 24.

[EUP14] Regulation (EU) No 540/2014 of the European Parliament and of the council of 16 April 2014 on the sound level of motor vehicles and of replacement silencing systems, and amending Directive 2007/46/EC and repealing Directive 70/157/EEC

[Eur15a] Eurostat Final energy consumption by sector 2.4.1-r2150-2015-09-28 (PROD) code: tsdpc320 retrieved 10.11.2015 on: http://ec.europa.eu/eurostat/tgm/table.do?tab=table&init=1&plugin=1&pcode=tsdpc320&language=en

[Eur15b] Eurostat (2015) Energy consumption of transport, by mode 2.4.1-r2150-2015-09-28 code tsdtr250 retrieved 10.11.2015 on: http://ec.europa.eu/eurostat/tgm/refreshTableAction.do?tab=table&plugin=1&pcode=tsdtr250&language=en

[Fen01] Fenton J., and Hodkinson R. (2001). Lightweight electric/hybrid vehicle design. Elsevier.

[Fis50] Fisher R. A. (1950). Statistical methods for research workers. Eleventh revised edition, Oliver and Boyd,

[Flo09] Floudas C. A., and Gounaris C. E. (2009). A review of recent advances in global optimization. Journal of Global Optimization, 45(1), 3-38.

[Flou99] Floudas C. A., Pardalos P.M., Adjiman C.S., Esposito W.R., Gümüs Z.H., Harding S.T., Klepeis J.L., Meyer C.A., Schweiger C.A.: Handbook of Test Problems in Local and Global Optimization. Springer US, Boston, MA (1999).

[Flp01] Felippa C. A. (2001). A historical outline of matrix structural analysis: a play in three acts. Computers and Structures, 79(14), 1313-1324.

[Flt10] Fletcher R. The sequential quadratic programming method. Nonlinear Optimization. Springer Berlin Heidelberg, 2010. 165-214. (DOI: 10.1007/978-3-642-11339-0_3)

[Fng05a] Fang H., Rais-Rohani, M., Liu, Z., and Horstemeyer, M. F. (2005). A comparative study of metamodeling methods for multiobjective crashworthiness optimization. Computers and Structures, 83(25), 2121-2136.

[Fng05b] Fang H., Solanki K., and Horstemeyer M. F. (2005). Numerical simulations of multiple vehicle crashes and multidisciplinary crashworthiness optimization. International Journal of Crashworthiness, 10(2), 161-172

[Fng06] Fang H., and Horstemeyer M. F. (2006). Global response approximation with radial basis functions. Engineering Optimization, 38(04), 407-424.

[Fog62] Fogel L.J. (1962). Autonomous automata. Industrial Research, 4(2), 14-19.

[Fog66] Fogel L. J., Owens A. J., and Walsh M. J. (1966). Artificial intelligence through simulated evolution. Chapter 14 of Biophysics and Cybernetic Sciences Symposium, edited by M. Maxfield, A. Callahan, and L. J. Fogel, Spartan Book Co., Washington, D.C., pages 131-155.

[For06] Forsberg J., and Nilsson L. (2006). Evaluation of response surface methodologies used in crashworthiness optimization. International Journal of Impact Engineering, 32(5), 759-777.

[Fu02] Fu M. C. (2002). Optimization for simulation: Theory vs. practice. INFORMS Journal on Computing, 14(3), 192-215.

[Gal06] Gallagher M., Bo Yuan: A general-purpose tunable landscape generator. IEEE Trans. Evol. Comput. 10, 590–603 (2006).

[Gao07] Gao W., and Mi C. (2007). Hybrid vehicle design using global optimisation algorithms. International Journal of Electric and Hybrid Vehicles, 1(1), 57-70.

[Gel09] Gelman A. (2009) Statistical Modeling, Causal Inference, and Social Science, http://andrewgelman.com/2009/11/30/just_one_of_the/

[Gho08] Gholizadeh S., Salajegheh E., and Torkzadeh P. (2008). Structural optimization with frequency constraints by genetic algorithm using wavelet radial basis function neural network. Journal of Sound and Vibration, 312(1), 316-331.

[Gho13] Gholizadeh S., and Barzegar A. (2013). Shape optimization of structures for frequency constraints by sequential harmony search algorithm. Engineering Optimization, 45(6), 627-646.

[Gld06] Goldman T., and Roger Gorham. Sustainable urban transport: Four innovative directions. Technology in society 28.1 (2006): 261-273.

[Gld88] Goldberg DE, Holland J. H., Genetic algorithms and machine learning. Machine learning, 3(2), 95-99 (1988)

[Glk1915] Galerkin B. G. (1915). Series solution of some problems of elastic equilibrium of rods and plates. Vestnik Inzhenerov i Tekhnikov, 19(7), 897-908. (in Russian) Translated in English: Galerkin, B. G. (1968). Rods and Plates: Series in Some Questions of Elastic Equilibrium of Rods and Plates. National Technical Information Service.

[Gnd12] Gander M. J., and Wanner G. (2012). From Euler, Ritz, and Galerkin to Modern Computing. SIAM Review, 54(4), 627-666.

[Goe09] Goel T., and Stander N. (2009). Comparing three error criteria for selecting radial basis function network topology. Computer Methods in Applied Mechanics and Engineering, 198(27), 2137-2150.

[Gom11] Gomes H. M. (2011). Truss optimization with dynamic constraints using a particle swarm algorithm. Expert Systems with Applications, 38(1), 957-968.

[Gra70] Grattan-Guinness I. (1970). Joseph Fourier's anticipation of linear programming. Operational Research quarterly, 361-364.

[Gra93] Grandhi R. (1993). Structural optimization with frequency constraints-a review. AIAA journal, 31(12), 2296-2303.

[Gre97] Greene David L., and Michael Wegener. Sustainable transport. Journal of Transport Geography 5.3 (1997): 177-190.

[Gret71] Grether W. F. (1971). Vibration and human performance1. Human Factors: The Journal of the Human Factors and Ergonomics Society, 13(3), 203-216.

[Gu13] Gu X., Sun G., Li G., Mao L., and Li Q. 2013). A comparative study on multiobjective reliable and robust optimization for crashworthiness design of vehicle structure. Structural and Multidisciplinary Optimization, 48(3), 669-684.

[Gud13] Gudmundsson H., and Claus Hedegaard Sørensen. Some use—Little influence? On the roles of indicators in European sustainable transport policy. Ecological Indicators 35 (2013): 43-51.

[Haf86] Haftka R. T., and Grandhi R. V. (1986). Structural shape optimization—a survey. Computer Methods in Applied Mechanics and Engineering, 57(1), 91-106.

[Hal06] Hallquist J. O. (2006). LS-DYNA theory manual. Livermore software Technology corporation

[Hal07] Hallquist J. O. (2007). LS-DYNA keyword user’s manual. Livermore Software Technology Corporation, 970.

[Ham03] Hamza K., and Saitou K. (2003, January). Design optimization of vehicle structures for crashworthiness using equivalent mechanism approximations. In: ASME 2003 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (pp. 459-472). American Society of Mechanical Engineers.

[Ham04] Hamza K. and Saitou K., (2004) Design for crashworthiness of vehicle structures via equivalent mechanism approximations and crush mode matching. Proceedings of the IMECE04 ASME Congres 13-20 November 2004

[Ham04b] Hamza K., and Saitou K. (2004). Crashworthiness design using meta-models for aprroximating the response of structural members. In Cairo University Conference on Mechanical Design and Production.

[Hap01] Happian-Smith, J. (2001). An introduction to modern vehicle design. Elsevier.

[Har04] Harrison M. Vehicle refinement: controlling noise and vibration in road vehicles. Elsevier, 2004.

[Har71] Hardy R. L. (1971). Multiquadric equations of topography and other irregular surfaces. Journal of geophysical research, 76(8), 1905-1915.

[Haug86] Haug E., Scharnhorst T., and Du Bois P. (1986). FEM-Crash, Berechnung eines Fahrzeugfrontalaufpralls. VDI Berichte, 613, 479-505.

[Hck80] Hock W., and Schittkowski K. (1980). Test examples for nonlinear programming codes. Journal of Optimization Theory and Applications, 30(1), 127-129.

[Hes15] Hesse S. H., Lukaszewicz D. J., and Duddeck F. (2015). A method to reduce design complexity of automotive composite structures with respect to crashworthiness. Composite Structures, 129, 236-249.

[Hfd48] Hoeffding W.: A class of statistics with asymptotically normal distribution. Ann. Math. Stat. 293–325 (1948).

[Hil66] Hill W. J., and Hunter W. G. (1966). A review of response surface methodology: a literature survey. Technometrics, 8(4), 571-590.

[Him72] Himmelblau D. M. (1972). Applied nonlinear programming. McGraw-Hill Companies.

[Hir13] Hirz M., Dietrich W., Gfrerrer A., and Lang J. (2013). Integrated Computer-Aided Design in Automotive Development: Development Processes, Geometric Fundamentals, Methods of CAD, Knowledge-Based Engineering Data Management. Springer Publishing Company, Incorporated.

[Hol75] Holland John H. Adaptation in natural and artificial system: an introduction with application to biology, control and artificial intelligence. Ann Arbor, University of Michigan Press (1975).

[Hoo95] Hooker J. N. (1995). Testing heuristics: We have it all wrong. Journal of Heuristics, 1(1), 33-42.

[Hor09] Horstemeyer M. F., Ren X. C., Fang H., Acar E., and Wang P. T. (2009). A comparative study of design optimisation methodologies for side-impact crashworthiness, using injury-based versus energy-based criterion. International Journal of Crashworthiness, 14(2), 125-138.

[Hor61] Hornick R. J., Boettcher C. A., and Simons A. K. (1961). The effect of low frequency, high amplitude, whole body, longitudinal and transverse vibration upon human performance. Bostrom Research labs Milwaukee WI.

[Hou08] Hou S., Li Q., Long S., Yang X., and Li W. (2008). Multiobjective optimization of multi-cell sections for the crashworthiness design. International Journal of Impact Engineering, 35(11), 1355-1367.

[Hua02] Huang M. (2002). Vehicle crash mechanics. CRC press.

[Hun13] Hunkeler S., Duddeck F., Rayamajhi M., and Zimmer H. (2013). Shape optimisation for crashworthiness followed by a robustness analysis with respect to shape variables. Structural and Multidisciplinary Optimization, 48(2), 367-378.

[Ige01] Igel C., and Toussaint M. (2001). On classes of functions for which no free lunch results hold. arXiv preprint cs/0108011.

[Irr02] Irretier H. D. (2002). History and development of frequency domain methods in experimental modal analysis. Journal de Physique 4, 12(11), Pr11-91

[Jin11] Jin Y. (2011). Surrogate-assisted evolutionary computation: Recent advances and future challenges. Swarm and Evolutionary Computation, 1(2), 61-70.

[Joh02] Johnson D. S.(2002) A Theoreticians guide for experimental analysis of algorithms, in Proceedings of the 5th and 6th DIMACS Implementation Challenges, Vol 59. Series in Discrete Mathematics and Theoretical Computer Science

[Jon75] De Jong K.A. (1975). Analysis of the behavior of a class of genetic adaptive systems.. Ph. D. Thesis University of Michigan

[Kar39] Karush W. (1939). Minima of functions of several variables with inequalities as side conditions. Master Thesis, University of Chicago

[Ken95] Kennedy J., and Eberhart R. C. (1995). Proceedings of IEEE International Conference on Neural Networks. Perth, Australia.

[Kep1515] Kepler J. (1615) “Nova stereometria doliorum vinariorum, in primis Austriaci, figurae omnium aptissimae; et usus in eo virgae cubicae compendiosissimus & plane singularis” retrieved from e-rara.ch persistenter Link: http://dx.doi.org/10.3931/e-rara-11051 http://www.e-rara.ch/zut/content/titleinfo/3298814

[Kia15] Kiani M., and Yildiz A. R. (2015). A Comparative Study of Non-traditional Methods for Vehicle Crashworthiness and NVH Optimization. Archives of Computational Methods in Engineering, 1-12.

[Kim03] Kim H. M., Rideout D. G., Papalambros P. Y., and Stein, J. L. (2003). Analytical target cascading in automotive vehicle design. Journal of Mechanical Design, 125(3), 481-489.

[Kir83] Kirkpatrick S., Gelatt C. D., and Vecchi, M. P. (1983). Optimization by simulated annealing. Science, 220(4598), 671-680.

[Kno05] Knowles, J., and Hughes E. J. (2005, January). Multiobjective optimization on a budget of 250 evaluations. In Evolutionary Multi-Criterion Optimization (pp. 176-190). Springer Berlin Heidelberg.

[Kno09] Knowles J., Corne D., and Reynolds A. (2009, January). Noisy multiobjective optimization on a budget of 250 evaluations. In Evolutionary Multi-Criterion Optimization (pp. 36-50). Springer Berlin Heidelberg.

[Kod04] Kodiyalam S., Yang R. J., Gu L., and Tho C. H. (2004). Multidisciplinary design optimization of a vehicle system in a scalable, high performance computing environment. Structural and Multidisciplinary Optimization, 26(3-4), 256-263.

[Kof10] Koffler C., and Rohde-Brandenburger K. (2010). On the calculation of fuel savings through lightweight design in automotive life cycle assessments. The International Journal of Life Cycle Assessment, 15(1), 128-135.

[Kok02] Kokkolaras M., Fellini R., Kim H. M., Michelena N. F., and Papalambros P. Y. (2002). Extension of the target cascading formulation to the design of product families. Structural and Multidisciplinary Optimization, 24(4), 293-301.

[Kol65] Kolmogorov A.N.: Three approaches to the quantitative definition of information’. Probl. Inf. Transm. 1, 1–7 (1965).

[Kol92] Kollerstrom N. (1992). Thomas Simpson and ‘Newton's method of approximation’: an enduring myth. The British Journal for the History of Science,25(03), 347-354.

[Kri03] Krishnamurthy, T. (2003, April). Response surface approximation with augmented and compactly supported radial basis functions. In Proceeding of the 44th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference. AIAA Paper (Vol. 1748).

[Krn14] Kiranyaz S., Ince T., and Gabbouj M. (2014). Multidimensional particle swarm optimization for machine learning and pattern recognition. Springer.

[Kry72] Kreyzig E, Advanced Engineering Mathematics (1972), Wiley

[Kyo97] Kyoto Protocol, United Nations framework convention on climate change.Kyoto Protocol, Kyoto (1997).

[Lan50] Lanczos C. (1950). An iteration method for the solution of the eigenvalue problem of linear differential and integral operators. Journal of Research of the National Bureau of Standards Vol. 45 No. 4. 2133.

[Lev53] Levy S. Structural Analysis and Influence Coefficients for Delta Wings, Journal of the Aeronautical Sciences, Vol.20, No. 7 (1953), pp. 449-454.

[Lia05] Liang J.J., Suganthan P.N., Deb K.: Novel composition test functions for numerical global optimization. In: Swarm Intelligence Symposium, 2005. SIS 2005. Proceedings 2005 IEEE. pp. 68–75. IEEE (2005).

[Lia08] Liao X., Li Q., Yang X., Li W., and Zhang W. (2008). A two-stage multi-objective optimisation of vehicle crashworthiness under frontal impact. International Journal of Crashworthiness, 13(3), 279-288.

[Lia08B] Liao X., Li Q., Yang X., Zhang W., and Li W. (2008). Multiobjective optimization for crash safety design of vehicles using stepwise regression model. Structural and Multidisciplinary Optimization, 35(6), 561-569.

[Liang13] Liang J. J., Qu B. Y., and Suganthan P. N. (2013). Problem definitions and evaluation criteria for the CEC 2014 special session and competition on single objective real-parameter numerical optimization. Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou China and Technical Report, Nanyang Technological University, Singapore.

[Lin11] Lin S. , (2011) NGPM A NSGA-II Program in Matlab, MATLAB file exchange. http://www.mathworks.com/matlabcentral/fileexchange/31166-ngpm-a-nsga-ii-program-in-matlab-v1-4

[Liu08] Liu Y. (2008). Optimum design of straight thin-walled box section beams for crashworthiness analysis. Finite Elements in Analysis and Design, 44(3), 139-147.

[Ljung07] Ljungberg J. K., and Neely G. (2007). Stress, subjective experience and cognitive performance during exposure to noise and vibration. Journal of Environmental Psychology, 27(1), 44-54.

[Lng05] Longhitano D., Henary B., Bhalla K., Ivarsson J., and Crandall, J. (2005).Influence of vehicle body type on pedestrian injury distribution (No. 2005-01-1876). SAE Technical Paper.

[Lnz04] Lanzi L., Castelletti L. M. L., and Anghileri M. (2004). Multi-objective optimisation of composite absorber shape under crashworthiness requirements. Composite Structures, 65(3), 433-441.

[Lom01] Lomonaco C., and Gianotti E. (2001, June). 5-years status report of the advanced offset frontal crash protection. In 17th ESV conference, Amsterdam, Netherlands.

[Mah15] Mahdavi S., Shiri M.E., Rahnamayan S.: Metaheuristics in large-scale global continues optimization: A survey. Inf. Sci. 295, 407–428 (2015).

[Mal10] Mallick P. K. (Ed.). (2010). Materials, design and manufacturing for lightweight vehicles. Elsevier.

[Mar03] Marsaglia G. (2003) Xorshift rngs. Journal of Statistical Software 8.14:1-6.

[Mar04] Marler R. T., and Arora J. S. (2004). Survey of multi-objective optimization methods for engineering. Structural and Multidisciplinary Optimization, 26(6), 369-395.

[Mar11] Marzbanrad J., and Ebrahimi M. R. (2011). Multi-objective optimization of aluminum hollow tubes for vehicle crash energy absorption using a genetic algorithm and neural networks. Thin-Walled Structures, 49(12), 1605-1615.

[Mar13] Martins J. R., and Lambe A. B. (2013). Multidisciplinary design optimization: a survey of architectures. AIAA journal, 51(9), 2049-2075.

[Mea75] Mead R., and Pike D. J. (1975). A biometrics invited paper. A review of response surface methodology from a biometric viewpoint. Biometrics, 803-851.

[Mig12] Miguel, L. F. F., and Miguel, L. F. F. (2012). Shape and size optimization of truss structures considering dynamic constraints through modern metaheuristic algorithms. Expert Systems with Applications, 39(10), 9458-9467.

[Mih10] Mihaylova P., Baldanzini N., Pratellesi A., and Pierini M., On the improvement of concept modeling of joints within simplified finite element models with application to structural dynamics. Proceedings of the

International Conference on Noise and Vibration Engineering—ISMA. Vol. 2010.

[Mih12] Mihaylova P., Baldanzini N., Pratellesi A., and Pierini M., Beam Bounding Box–a novel approach for beam concept modeling and optimization handling. Finite Elements in Analysis and Design 60 (2012): 13-24.

[Mit92] Mitchell M., Forrest S., and Holland J. H. (1992, December). The royal road for genetic algorithms: Fitness landscapes and GA performance. In Proceedings of the first European Conference on Artificial Life (pp. 245-254). Cambridge: The MIT Press.

[Mor65] Moore G. E. (2006). Cramming more components onto integrated circuits, Reprinted from Electronics, volume 38, number 8, April 19, 1965, pp. 114 ff. IEEE Solid-State Circuits Newsletter, 3(20), 33-35.

[Mor99] Morris R.D.: Auxiliary variables for Markov random fields with higher order interactions. In: Energy Minimization Methods in Computer Vision and Pattern Recognition. pp. 131–142. Springer (1999).

[Mye89] Myers R. H., Khuri A. I., and Carter W. H. (1989). Response surface methodology: 1966–l988. Technometrics, 31(2), 137-157.

[Nad64] Nadaraya E. A. (1964). On estimating regression. Theory of Probability & Its Applications, 9(1), 141-142.

[Nah09] Nahvi Hassan, Mohammad Hosseini Fouladi, and Mohd Jailani Mohd Nor. Evaluation of whole-body vibration and ride comfort in a passenger car. International Journal of Acoustics and Vibration 14.3 (2009): 143-149.

[NCA12] National Crash Analysis Center, Finite Element Model Archive, http://www.ncac.gwu.edu/vml/models.html , retrieved August 2012

[NCA15] Euro N. C. A. P. (2009). Assessment protocol-AOP, v7.02 April 2015.

[Nel65] Nelder J. A., and Mead, R. (1965). A simplex method for function minimization. The Computer Journal, 7(4), 308-313.

[New1726] Newton I, Cohen, I. Bernard, and A. Koyr. Isaac Newton's Philosophiae naturalis Principia Mathematica (with variant readings), Vol. 1. (1972).

[Nil04] Nilsson L., and Redhe M. (2004). An investigation of structural optimization in crashworthiness design using a stochastic approach. In Proceeding of Eighth international LS-DYNA users conference simulation, Hyatt Regency-Dearborn Dearborn, Michigan (pp. 43-54).

[Nkp94] Nijkamp Peter. Roads toward environmentally sustainable transport. Transportation Research Part A: Policy and Practice 28.4 (1994): 261-271.

[Nwm59] Newmark N. M. (1959). A method of computation for structural dynamics. Journal of the Engineering Mechanics Division, 85(3), 67-94.

[Odn03] Oden J. T., Belytschko, T., Babuska, I., and Hughes, T. J. R. (2003). Research directions in computational mechanics. Computer Methods in Applied Mechanics and Engineering, 192(7), 913-922.

[Oed15] Online etymology dictionary retrieved at november 2015 from www.etymonline.com

[Ore94] Oreskes N., Shrader-Frechette K., and Belitz K. (1994). Verification, validation, and confirmation of numerical models in the earth sciences. Science, 263(5147), 641-646.

[Pag06] Pagerit S., Sharer P., and Rousseau A. (2006). Fuel economy sensitivity to vehicle mass for advanced vehicle powertrains. Society of Automotive Engineers, Paper, (2006-01), 0665.

[Pan05] Panneton F., L'Ecuyer P.: On the Xorshift random number generators. ACM Trans. Model. Comput. Simul. TOMACS. 15, 346–361 (2005).

[Pat80] Patankar S. (1980). Numerical heat transfer and fluid flow. CRC Press.

[Per13] Pereira N. Q., and Callaghan B. (2013). A Comparison of New Car Assessment Program NCAP Requirements and Procedures Around the World (No. 2013-36-0499). SAE Technical Paper.

[Pin13] Pintér J. D. (2013). Global optimization in action: continuous and Lipschitz optimization: algorithms, implementations and applications (Vol. 6). Springer Science & Business Media.

[Pit12] Pitzer E., and Affenzeller, M. (2012). A comprehensive survey on fitness landscape analysis. In Recent Advances in Intelligent Engineering Systems(pp. 161-191). Springer Berlin Heidelberg.

[Pli10] Plischke E (2010) An effective algorithm for computing global sensitivity indices (EASI). Reliab Eng Syst Saf 95(4):354–360

[Pon1890] Poincaré H. (1890). Sur le problème des trois corps et les équations de la dynamique. Acta Mathematica, 13(1), A3-A270.

[Pri94] Prichard D. and Theiler J. (1994). Generating surrogate data for time series with several simultaneously measured variables Physical Review Letters 73.7: 951.

[Ptr40] Petrov G. I. (1940). Application of the method of Galerkin to a problem involving the stationary flow of a viscous fluid, Prikl. Matem. Mekh. 4, 3.

[Rak15] Rakowska J., Chator A., Barthelemy B., Lee M., Morgans S., Laya J., Zinn G. Chuang C.H. and Gondipalle S. R. (2015). An Iterative Application of

Multi-Disciplinary Optimization for Vehicle Body Weight Reduction Based on 2015 Mustang Product Development. SAE International Journal of Materials and Manufacturing, 8(2015-01-0470).

[Ras64] Rastrigin L. A. (1964). Convergence of random search method in extremal control of many-parameter system. Automation and Remote Control, 24(11), 1337.

[Ras74] Rastrigin L.A. (1974) Systems of extremal control, in Theoretical Foundations of Engineering Cybernetics Series, (In Russian) Nauka, Moscow.

[Rat10] Ratto M and Pagano A (2010) Using recursive algorithms for the efficient identification of smoothing spline ANOVA models AStA Advances in Statistical Analysis 94.4: 367-388.

[Rch65] Rechenberg I. (1965). Cybernetic solution path of an experimental problem. Royal Aircraft Establishment, Farnborough p. Library translation 1122 (secondary reference, as cited in [Bey02])

[Rch71] Rechenberg I. (1971). Optimierung technischer Systeme nach Prinzipien der biologischen Evolution. 1971. Ph. D. Thesis Technical University of Berlin, Published by Frommann-Holzboog, Stutgart 1973.

[Red04] Redhe M., Giger M., and Nilsson L. (2004). An investigation of structural optimization in crashworthiness design using a stochastic approach. Structural and Multidisciplinary Optimization, 27(6), 446-459.

[Ric05] Richardson Barbara C. Sustainable transport: analysis frameworks. Journal of Transport Geography 13.1 (2005): 29-39.

[Rio12] Rios L. M., and Sahinidis N. V. Derivative-free optimization: A review of algorithms and comparison of software implementations. Journal of Global Optimization (2012): 1-47.

[Rod88] Rodgers L.J. and Nicewander W.A. (1988). Thirteen ways to look at the correlation coefficient. The American Statistician, 42(1), 59-66.

[Ros60] Rosenbrock H.H. (1969). An automatic method for finding the greatest or least value of a function. The Computer Journal 3.3: 175-184.

[Roz01] Rozvany G.I.N. (2001). Aims, scope, methods, history and unified terminology of computer-aided topology optimization in structural mechanics. Structural and Multidisciplinary Optimization, 21(2), 90-108.

[Roz09] Rozvany G.I.N. (2009). A critical review of established methods of structural topology optimization. Structural and Multidisciplinary Optimization, 37(3), 217-237.

[Rtz1908] Ritz W. (1908). Über eine neue Methode zur Lösung gewisser Variationsprobleme der mathematischen Physik. Jour. Reine Angew. Math.,-135.

[Rus08] Rusinek A., Zaera, R., Forquin P., and Klepaczko J. R. (2008). Effect of plastic deformation and boundary conditions combined with elastic wave propagation on the collapse site of a crash box. Thin-Walled Structures, 46(10), 1143-1163.

[Sac89] Sacks J. Welch W. J., Mitchell T. J., and Wynn H. P. (1989). Design and analysis of computer experiments. Statistical science, 409-423.

[Sai05] Saitou K., Izui K., Nishiwaki S., and Papalambros P. (2005). A survey of structural optimization in mechanical product development. Journal of Computing and Information Science in Engineering, 5(3), 214-226.

[Sal14a] Sala R., Baldanzini N., Pierini M. *An intuitive variance based variable screening method.* Poster session presented at the MascotNum workshop, Computer Experiments and Meta-models for uncertainty quantification; 2014 April 23-25; Zurich, Switzerland

[Sal14b] Sala R., Pierini M., Baldanzini N. *The development and application of tailored test problems for meta-simulation of multidisciplinary optimization of vehicle structures.* Presentation at the (XI) World Congress on Computational Mechanics; 2014 July 20-25; Barcelona, Spain

[Sal14c] Sala R., Pierini M., Baldanzini N.. *Optimization efficiency in multidisciplinary vehicle design including NVH criteria.* Proceedings of the Leuven Conference on Noise and Vibration Engineering (ISMA); 2014 September 15-17; Leuven, Belgium

[Sal15] Sala R., Baldanzini N., Pierini M. *On variable screening and optimization of Car Body Structures subject to multidisciplinary constraints*. Presentation at the International Conference on Engineering Vibration Ljubljana, Slovenia, 7-10 Sept. 2015.

[Sal16a] Sala R., Baldanzini N., Pierini M. *Representative Surrogate Problems as test functions for expensive simulators in multidisciplinary design optimization of vehicle structures.* In Press, Structural Multidisciplinary Optimization, Springer 2016

[Sal16b] Sala R., Baldanzini N., Pierini M. *Global optimization test problems with higher order interactions based on random field composition.* In Press, Optimization Letters, Springer 2016

[Sba03] Shcherbina O., Neumaier A., Sam-Haroud D., Vu X. H., and Nguyen, T. V. (2003). Benchmarking global optimization and constraint satisfaction codes (pp. 211-222). Springer Berlin Heidelberg.

[Sbh1851] Schellbach K. (1851). Probleme der Variationsrechnung. Journal für die Reine und Angewandte Mathematik, 41, 293-363.

[Sbi00] Sobieszczanski-Sobieski J., Agte J. S., and Sandusky R. R. (2000). Bilevel integrated system synthesis. AIAA journal, 38(1), 164-172.

[Sbi01] Sobieszczanski-Sobieski J., Kodiyalam S., and Yang R. Y. (2001). Optimization of car body under constraints of noise, vibration, and harshness (NVH), and crash. Structural and Multidisciplinary Optimization, 22(4), 295-306

[Sbi87] Sobieszczanski-Sobieski J., James B. B., and Riley, M. F. (1987). Structural sizing by generalized, multilevel optimization. AIAA Journal, 25(1), 139-145.

[Sbi95] Sobieszczanski-Sobieski J. Multidisciplinary design optimization: an emerging new engineering discipline. Springer Netherlands, 1995.

[Sbi97] Sobieszczanski-Sobieski J., and Haftka R. T. (1997). Multidisciplinary aerospace design optimization: survey of recent developments. Structural optimization, 14(1), 1-23.

[Sbl01] Sobol I.M. (2001) Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates, Mathematics and Computers in Simulation 55.1-3: 271-280.

[Sbl90] Sobol I.M.: On sensitivity estimation for nonlinear mathematical models. Mat. Model. 2, 112–118 (1990). English translation in: Sensitivity Estimates for nonlinear Mathematical Models. Math. Mod. And Computational Experiment 1993, Vol 1. N1. 407-414

[Sch14] Schuh G., Korthals K., and Arnoscht J. (2014, June). Contribution of body lightweight design to the environmental impact of electric vehicles. In Advanced Materials Research (Vol. 907, pp. 329-347).

[Schm60] Schmit L. A. (1960, September). Structural design by systematic synthesis. In Proc. of the Second ASCE Conference on Electronic Computation (pp. 105-122).

[Schm81] Schmit L. A. (1981). Structural synthesis-its genesis and development. AIAA Journal, 19(10), 1249-1263.

[Schr96] Schramm U., and Pilkey W. D. (1996). Optimal design of structures under impact loading. Shock and Vibration, 3(1), 69-81.

[Schr98] Schramm U., Thomas H., and Schneider D. (1998). Structural optimization in occupant safety and crash analysis. Design Optimization, 1(4), 374-387.

[Schw12] Schwefel H.-P. (2012) “Life lessons thought by simulated evolution” in Burgin, M., and Eberbach, E. (2012). Ubiquity symposium: Evolutionary Computation and the Processes of Life: opening statement. Ubiquity, 2012 (August), 1.

[Schw65] Schwefel H.-P. Kybernetische Evolution als Strategie der experimentellen Forschung in der Strömungstechnik. Master's thesis, Technical University of Berlin (1965).

[Schw75] Schwefel H.-P. (1975). Evolutionsstrategie und numerische Optimierung. Technische Universität Berlin. (PhD. Thesis)

[Sei07] Wech L., and Seiffert U. W. (2007). Automotive Safety Handbook (Vol. 377). SAE Technical Paper.

[Sha10] Shan S., and Wang G. G. (2010). Survey of modeling and optimization strategies to solve high-dimensional design problems with computationally-expensive black-box functions. Structural and Multidisciplinary Optimization, 41(2), 219-241.

[Shi13] Shiau Tzay-An. Evaluating sustainable transport strategies for the counties of Taiwan based on their degree of urbanization. Transport Policy 30 (2013): 101-108.

[Sim08] Simpson T. W., Toropov V., Balabanov V., and Viana F. A. (2008, September). Design and analysis of computer experiments in multidisciplinary design optimization: a review of how far we have come or not. In 12th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (Vol. 5, pp. 10-12).

[Slt06] Saltelli A., Ratto M., Tarantola S., and Campolongo F. Sensitivity analysis practices: Strategies for model-based inference. Reliability Engineering & System Safety 91.10 (2006): 1109-1125.

[Slt10] Saltelli A., Annoni P., Azzini I., Campolongo F., Ratto M. and Tarantola S. (2010) Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index. Computer Physics Communications 181.2: 259-270.

[Srb96] Schreiber T., Schmitz A. (1996) Improved surrogate data for nonlinearity tests. Phys Rev Lett 77(4):635

[Sri94] Srinivas N., Deb K. (1994) Multiobjective optimization using nondominated sorting in genetic algorithms. Evolutionary Computation 2.3 221-248.

[Sta03] Stander N., Roux W. J., Giger M., Redhe M., Fedorova N., and Haarhoff, J. (2003, May). Crashworthiness optimization in LS-OPT: Case studies in metamodeling and random search techniques. In proceedings of the 4th European LS-DYNA Conference, Ulm, Germany.

[Sta04] Stander N., Roux W., Giger M., Redhe M., Fedorova N., and Haarhoff J. (2004, August). A comparison of metamodeling techniques for crashworthiness optimization. In Proceedings of the 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, Albany, NY (Vol. 30).

[Sti81] Stigler, S. M. (1981). Gauss and the invention of least squares. The Annals of Statistics, 465-474.

[Sti02] Stillwell J. (2002). Mathematics and its History. The Australian Mathem. Soc, 168.

[Stk12] Schittkowski K. (2012). More test examples for nonlinear programming codes(Vol. 282). Springer Science & Business Media.

[Sto97] Storn R., and Price K. (1997). Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. Journal of Global Optimization, 11(4), 341-359.

[Str68] Strømholm P. (1968). Fermat's methods of maxima and minima and of tangents. A reconstruction. Archive for History of Exact Sciences, 5(1), 47-69.

[Str73] Strang G., and Fix G. J. (1973). An analysis of the finite element method (Vol. 212). Englewood Cliffs, NJ: Prentice-Hall.

[Stre03] Streeter M. J. (2003, January). Two broad classes of functions for which a no free lunch result does not hold. In Genetic and Evolutionary Computation—GECCO 2003 (pp. 1418-1430). Springer, Berlin, Heidelberg.

[Sun10] Sun G., Li G., Hou S., Zhou S., Li W., and Li Q. (2010). Crashworthiness design for functionally graded foam-filled thin-walled structures. Materials Science and Engineering: A, 527(7), 1911-1919.

[Sun11] Li G., Zhou S., Li H., Hou S., and Li Q. (2011). Crashworthiness design of vehicle by using multiobjective robust optimization. Structural and Multidisciplinary Optimization, 44(1), 99-110.

[Swz92] Schweizerhof K., Nilsson L., and Hallquist J. O. (1992). Crashworthiness analysis in the automotive industry. International Journal of Computer Applications in Technology, 5(2-4), 134-156.

[Tang13] Li X., Tang K., Omidvar M. N., Yang Z., Qin K., and China H. (2013). Benchmark functions for the CEC 2013 special session and competition on large-scale global optimization. gene, 7(33), 8.

[Ted06] Tedford N. P., and Martins J. R. R. A. (2006, September). On the common structure of MDO problems: a comparison of architectures. In Proceedings of the 11th AIAA/ISSMO multidisciplinary analysis and optimization conference, Portsmouth, VA, AIAA (Vol. 7080, p. 2006).

[Tha95] Thanedar P. B., and Vanderplaats, G. N. (1995). Survey of discrete variable optimization for structural design. Journal of Structural Engineering, 121(2), 301-306.

[Tho1883] Thomson W. (1883) "The practical applications of Electricity" Transcript of a lecture delivered at the institution of Civil engineers https://archive.org/stream/popularlecturesa01kelvuoft#page/72/mode/2up

[Tho95] Thomke S. H. (1998). Simulation, learning and R&D performance: Evidence from automotive development. Research Policy, 27(1), 55-74.

[Tje98] Tjelmeland H., Besag J.: Markov random fields with higher-order interactions. Scandinavian Journal of Statistics. 25, 415–433 (1998).

[Tng09] Tang W. J., and Wu Q. H. Biologically inspired optimization: a review. Transactions of the Institute of Measurement and Control 31.6 (2009): 495-515.

[Top83] Topping B. H. V. (1983). Shape optimization of skeletal structures: a review. Journal of Structural Engineering, 109(8), 1933-1951.

[TRB97] Transportation Research Board, 1997. Toward a sustainable future, addressing the long-term effects of motor vehicle transportation on climate and ecology. Special Report 251. National Academy Press, Washington, DC http://www.trb.org/Publications/Blurbs/153301.aspx

[Tur56] Turner M. J., Clough, R. W., Martin, H. C., and Topp, L. J. Stiffness and Deflection Analysis of Complex Structures. Journal of the Aeronautical Sciences, Vol. 23, No. 9 (1956), pp. 805-823.

[Tur67] Turner M. J. Design of minimum mass structures with specified natural frequencies. AIAA Journal, Vol. 5, No. 3 (1967), pp. 406-412. doi: 10.2514/3.3994

[USE15] U. S. Environmental Protection Agency Inventory of US Greenhouse Gas Emissions and Sinks 1990-2013. (2015) Washington.

[Ven78] Venkayya V. B. (1978). Structural optimization: a review and some recommendations. International Journal for Numerical Methods in Engineering, 13(2), 203-228.

[Var12] Varis T. and Tuovinen T. (2012) Open Benchmark database for multidisciplinary optimization problems, Proceedings of the International Conference on Modeling and Applied Simulation, September 2012 , Vienna, Austria

[Ven99] Venkataraman S., and Haftka R. T. (1999, September). Optimization of composite panels-a review. In Proceedings of the American Society for Composites (pp. 479-488).

[Ver10] Veres S. M., Molnar L., Lincoln N. K., and Morice, C. P. (2011). Autonomous vehicle control systems—a review of decision making. Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, 225(2), 155-195.

[Ves04] Vesterstrøm J., and Thomsen R. (2004, June). A comparative study of differential evolution, particle swarm optimization, and evolutionary algorithms on numerical benchmark problems. In Evolutionary Computation, 2004. CEC2004. Congress on (V. 2, pp. 1980-1987). IEEE.

[Via14] Viana F. A., Simpson T. W., Balabanov V., and Toropov V. (2014). Metamodeling in multidisciplinary design optimization: how far have we really come?. AIAA Journal, 52(4), 670-690.

[Vnm03] Venema V (2003) IAAFT implementation inMATLAB retrieved January 2014 from http://www.meteo.uni-bonn.de/victor

[Wan13] Wang LF, Shi LY (2013). Simulation optimization: a review on theory and applications. Acta Automatica Sinica, 39(11), 1957-1968.

[WBC04] World Business Council for Sustainable Development. Mobility 2030: Meeting the Challenges to Sustainability. World Business Council for Sustainable Development, 2004.

[WCE87] World commission on environment and development. Our common future. New York: Oxford University Press; 1987

[Web14] Weber J. (2014). Automotive development processes. Springer.

[Weh15] Wehrle E. J., Han Y. H., and Duddeck F. (2015). Topology optimization of transient nonlinear structures—A comparative assessment of methods. 10th European LS-DYNA Conference, DYNAmore GmbH, 2015Würzburg, Germany, 15-17 June 2015

[WHO04] World health Organization, World report on road traffic injury prevention. 2004.

[WHO12] World Health Organization, Burden of disease from environmental noise: Quantification of healthy years life lost in Europe. World Health Organization, 2012. ISBN: 978 92 890 0229 5

[Wng07] Wang G. G., and Shan S. (2007). Review of metamodeling techniques in support of engineering design optimization. Journal of Mechanical Design,129(4), 370-380.

[Wol95] Wolpert D.H. and Macready W.G. (1995) No Free Lunch Theorems for Search, Santa Fe Institute Technical Report SFI-TR-05-010,SantaFe Institute, SantaFe, NM.

[Wol97] Wolpert D.H. and Macready W.G. (1997) No Free Lunch Theorems for Optimization, IEEE Transactions on Evolutionary Computation 1(1):67–82.

[Yan09] Yang X. S. (2009). Firefly algorithms for multimodal optimization. In Stochastic algorithms: foundations and applications (pp. 169-178). Springer Berlin Heidelberg.

[Yan10a] Yang X. S., Nature-inspired metaheuristic algorithms. Luniver press, 2010. http://web.info.uvt.ro/~dzaharie/cne2013/proiecte/tehnici/FireflyAlgorithm/Yang_nature_book_part.pdf

[Yan10b] Yang X. S., Engineering optimization: an introduction with metaheuristic applications. John Wiley & Sons, 2010.

[Yan13] Yang G., Wang Y., Zeng Y., Gao GF., Liang X., Zhou M., Wan X., Yu S., Jiang Y., Naghavi M., Vos T., Wang H., Lopez A.D., Murray CJ. Rapid health transition in China, 1990–2010: findings from the Global Burden of Disease Study 2010. The lancet 381.9882 (2013): 1987-2015.

[Yao99] Yao X., Liu Y., and Lin G. (1999). Evolutionary programming made faster. IEEE Transactions on Evolutionary Computation. 3(2), 82-102.

[Yil12] Yildiz A. R., and Solanki K. N. (2012). Multi-objective optimization of vehicle crashworthiness using a new particle swarm based approach. The International Journal of Advanced Manufacturing Technology, 59(1-4), 367-376

[Yng01] Yang R. J., Gu L., Tho, C. H., and Sobieszczanski-Sobieski, J. (2001). Multidisciplinary design optimization of a full vehicle with high performance computing. AIAA paper, 1273, 2001.

[Yng02] Yang R. J., Gu L., Tho, C. H., Choi, K. K., and Youn, B. D. (2002, April). Reliability-based multidisciplinary design optimization of a full vehicle system. In 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization.

[Yng04] Yang R. J., and Gu L. (2004). Experience with approximate reliability-based optimization methods. Structural and Multidisciplinary Optimization, 26(1-2), 152-159.

[Yng05] Yang R. J., Wang N., Tho, C. H., Bobineau, J. P., and Wang, B. P. (2005). Metamodeling development for vehicle frontal impact simulation. Journal of Mechanical Design, 127(5), 1014-1020.

[Yng94] Yang RJ, Tseng L; Nagy L; Cheng J (1994) Feasibility study of crash optimization. In: Gilmore BJ, Hoetzel DA, Dutta D, Eschenauer HA (eds.) Advances in design automation, DE-Vol 69-2 pp, 549-556. ASME

[Ypm95] Ypma T.B.J. (1995). Historical development of the Newton-Raphson method. SIAM review, 37(4), 531-551.

[Yu08] Yu F., Li D. F., and Crolla D. A. (2008, September). Integrated Vehicle Dynamics Control—state-of-the art review. In Vehicle Power and Propulsion Conference, 2008. VPPC'08. IEEE (pp. 1-6).

[Zan10] Zang, H., Zhang, S., & Hapeshi, K. (2010). A review of nature-inspired algorithms. *Journal of Bionic Engineering*, *7*, S232-S237.

[Zar08] Zarei H., Kröger M., and Albertsen H. (2008). An experimental and numerical crashworthiness investigation of thermoplastic composite crash boxes. Composite structures, 85(3), 245-257.

[Zit00] Zitzler E, Deb K and Thiele L (2000) Comparison of multiobjective evolutionary algorithms: Empirical results. Evolutionary comp. 8.2: 173-195.

[Zit98] Zitzler E., Thiele L. Multiobjective optimization using evolutionary algorithms—a comparative case study. Parallel Problem Solving from Nature. Springer Berlin Heidelberg, 1998.

[Zuo11] Zuo W., Xu, T., Zhang, H., and Xu, T. (2011). Fast structural optimization with frequency constraints by genetic algorithm using adaptive eigenvalue reanalysis methods. Struct. and Multidisc. Optimization, 43(6), 799-810.

[Zwz67] Zienkiewicz O. C., and Cheung Y.K. (1967), The Finite Element Method in Structural and Solid Mechanics, McGraw Hill, London