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Color matching of fabric blends: hybrid Kubelka-Munk + artificial neural network based method

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Abstract. Color matching of fabric blends is a key issue for the textile industry, mainly due to the rising need to create high-quality products for the fashion market. The process of mixing together differently colored fibers to match a desired color is usually performed by using some historical recipes, skillfully managed by company colorists. More often than desired, the first attempt in creating a blend is not satisfactory, thus requiring the experts to spend efforts in changing the recipe with a trial-and-error process. To confront this issue, a number of computer-based methods have been proposed in the last decades, roughly classified into theoretical and artificial neural network (ANN)–based approaches. Inspired by the above literature, the present paper provides a method for accurate estimation of spectrophotometric response of a textile blend composed of differently colored fibers made of different materials. In particular, the performance of the Kubelka-Munk (K-M) theory is enhanced by introducing an artificial intelligence approach to determine a more consistent value of the nonlinear function relationship between the blend and its components. Therefore, a hybrid K-M+ANN-based method capable of modeling the color mixing mechanism is devised to predict the reflectance values of a blend. *© 2016 SPIE and IS&T* [DOI: 10.1117/1.JEI.25.6.061402]

Keywords: computer-based color assessment; Kubelka-Munk theory; spectrophotometer; fabric blend; neural networks.
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1 Introduction

High-quality color reproduction on textiles is one of the most relevant topics to confront the textile industry. Generally speaking, it consists of finding the appropriate amount or proportion of the colorants required to obtain an exact color match. When dealing with textile fabrics or yarns, the final product to be introduced in the market often consists of a blend obtained by mixing together precolored fibers. In such cases, so-called recipe-based mixing, i.e., the process of mixing together a number of differently colored fibers in different proportions, is the preferred method for manufacturing fabrics and yarns. Usually, textile companies perform color mixing on the basis of their know-how: when a desired color (i.e., the so-called color target) has to be obtained, the colorist typically starts to search the company storehouse for correctly colored fibers to be mixed according to a given recipe. Modern companies, in effect, are provided with databases where spectrophotometric information regarding precolored fibers is stored together with suggested percentages to be used for obtaining a given color target. In other words, textile companies have historical recipes to be used for obtaining (using processes such as combing, drawing, roving, spinning, or carding[1]) a final blend.

Once a blend is created, colorists always perform an instrumental comparison with respect to the color target with the final aim of determining the color difference. The comparison is assessed under several color spaces such as CIELAB or CMC$(2:1)$[2] under a number of standard illuminants such as D65 or TL84[3] (see Fig. 1).

Unfortunately, more often than desired, the result obtained by mixing the fibers is quite different from the reference, even when companies adopt a consolidated recipe. In most circumstances, color differences between the recipe-based reflectance factors and the spectrophotometric response of the reference, in terms of CIELAB and CMC$(2:1)$ distances, may be >0.8 when measured using a spectrophotometer (see, for instance, Fig. 2).

Therefore, it is necessary to change the original recipe to match the desired color; this leads to the need to produce several samples in order to reduce the gap between the color of the final product and the desired one. This trial-and-error approach is a bottleneck for the entire fabric production process, since each trial involves time-expensive manufacturing and measuring processes.

2 Background

To speed-up the color matching process, several computer-based approaches have been proposed in literature, mainly dealing with the spectrophotometric prediction of dyed fabrics and with particular focus on the study of the color mixing model. Roughly, such methods can be divided into two categories: theoretical methods and artificial neural network (ANN)–based methods. Theoretical methods are mostly grounded on the widely known Kubelka-Munk (K-M) theory,[4,5] generally used for the analysis of diffuse reflectance spectra obtained from weakly absorbing samples.

As described in Ref. 6, K-M establishes that internal reflectance of a colorant composing a shade $\mathbf{p}(\lambda)$ depends on absorption K_λ and scattering S_λ coefficients according to the following equation:

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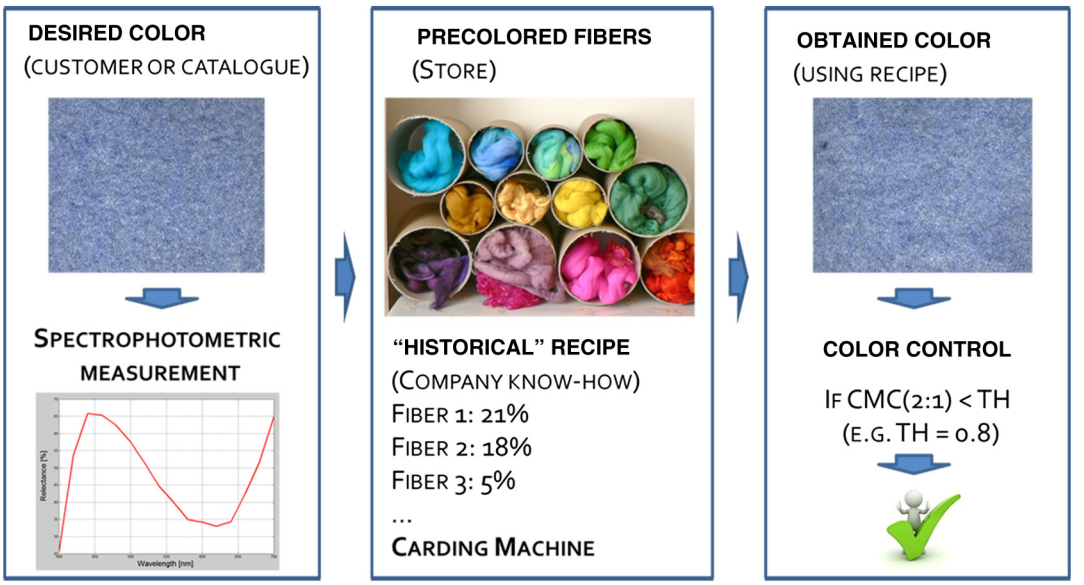

Fig. 1 Recipe-based mixing conceptual process.

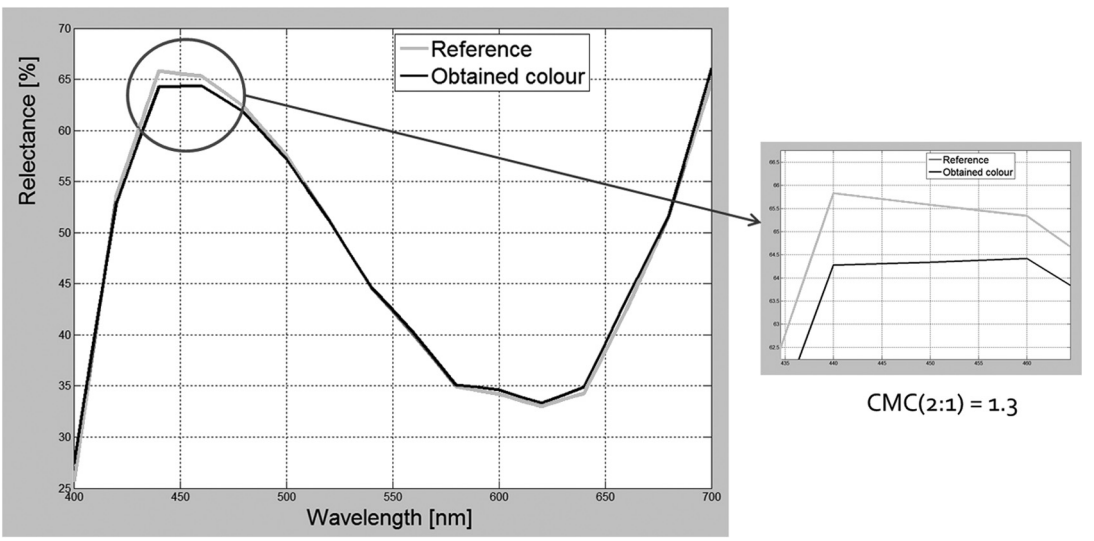

Fig. 2 Example of spectral responses, obtained using a spectrophotometer, for color target (reference) and obtained fabric.

$$\left(\frac{K}{S}\right)_{\lambda} = \frac{[1-\mathbf{p}(\lambda)]^2}{2\mathbf{p}(\lambda)}, \tag{1}$$

where λ is the wavelength in the visible range (usually 380 to 750 nm) and $(K/S)_{\lambda}$ is the ratio between the absorption and the scattering (also known as the K-S ratio) coefficients for a given wavelength. For opaque materials, where the colorants do not scatter in comparison to the substrate, the K-M equation may be written as follows:[7]

$$\left(\frac{K}{S}\right)_{\lambda,\mathrm{mix}} = \frac{k_{\lambda,t}}{s_{\lambda,t}} + \sum_{i=1}^{n} \alpha_i \left(\frac{k_{\lambda,i}}{s_{\lambda,i}}\right), \tag{2}$$

where $(K/S)_{\lambda,\mathrm{mix}}$ and $k_{\lambda,t}/s_{\lambda,t}$ are, respectively, the K-S ratio of the fabric (obtained by mixing the colored fibers) and of the fabric substrate, and α_i is the percentage of the i'th precolored fiber used for composing the blend.

Equation (2), often referred to as the two-constant K-M function, leads to problems in practical use due to the irrationality of the additivity assumption, rather than the K-M turbid medium theory itself.[8] Moreover, it is a valid choice when fiber blends are obtained by adding colorants on a substrate, but tends to fail when fabrics are obtained by mixing together precolored fibers.

To overcome these limitations, several studies related to the tristimulus-matching algorithm based on the Stearns-Noechel (S-N) model[9] (and its implementations[10]) have been proposed to predict formulas for matching a given color standard by blending predyed fibers.

The S-N model estimates the spectrophotometric response of a blend obtained by mixing differently colored fibers once an empiric constant M is known. In more detail,

S-N proposes the following empirical additive formula for the function $\mathcal{F}$:

$$\mathcal{F}(\lambda) = \frac{(1-\lambda)}{[M(\lambda-0.01)+0.01]}, \tag{3}$$

where M is an empirical constant to be determined experimentally (for instance, its value is estimated equal to 0.15 for fine wool blends and 0.109 for cotton). The value of the empirical constant has been evaluated for a range of different materials in the literature.[11,12] Nevertheless, even when M is accurately determined, the S-N model provides reliable results only for blends composed of a maximum of five to six differently colored fibers of the same material, thus limiting the approach to a lower number of cases since most companies mix together up to 15 to 20 differently colored fibers, often made of different materials such as wool, nylon, polyester, etc.[13,14]

Therefore, although the above-mentioned theoretical approaches provide excellent results for predicting the color of turbid media, they may lead to unsuitable results in forecasting the reflectance factors of blends obtained by mixing precolored fibers. As a consequence, further experiments and studies need to be carried out to reach accurate recipe predictions. With these aims in mind, recent approaches have focused on the integration of standard theories with more practical methods. In fact, since textile companies always create a first-attempt fabric using their recipe, it is possible to have as additional information the actual spectrophotometric response of at least one blend (whose reflectance values are often near the reference ones). This allows, for instance, introducing theoretical approaches using a comparison between expected and actual blend reflectance values. In effect, as demonstrated in Ref. 15, methods like K-M and even subtractive mixing may be successfully applied to the problem of blend color matching, especially using some exemplificative hypothesis, e.g., (1) the turbid mixing mechanism of fibers only slightly changes by varying the original recipe and (2) the nonlinear function relationship between the reflectance spectrum of a blend and the components is approximately constant when the recipe is varied within a limited range (e.g., 5%).

ANN-based methods have proven to provide a reliable and very practical approach for helping the colorist in color matching. This is demonstrated in Ref. 13, where a transfer function linking the color spectrum obtained by a linear combination of the spectra of each component with the measured reflectance values of a first-attempt blend is determined. The approach proved to be effective for blends composed of more than 15 components, since the average color difference between the predicted spectra and the real spectra of the carded fibers for an experimental set of blends is <0.55 for the CMC(2:1) tolerancing system. The main limitation is that the proposed approach is reliable only for variations of a recipe in the range $\pm 5\%$ and using only a single material (e.g., wool). When the operators need to create a fabric composed of a combination of different materials, the transfer function defined in Ref. 13 cannot be used even for a small variation of the recipe.

To solve this issue, in Ref. 14, an ANN-based approach is proposed and validated. It consists of training a neural network on the basis of the information obtained by the spectra of each colored raw material composing the blend. Once trained, a system of ANNs is able to provide a predicted spectrum whose colorimetric distance with the actual blend response is, in terms of CMC(2:1), lower than 0.45 for a given set of blends.

Theoretical approaches introduce assumptions for the mixing mechanism that lead to lower performance in color matching with respect to ANN-based methods; nevertheless, these last ones require huge training sets for ANN training. As a consequence, it could be useful to devise a method able to provide reliable color matching without the need of large training datasets by combining the (valid) prediction obtained using theoretical models together with the advantages of ANN-based approaches.

Inspired by the above literature, the present paper provides a method for accurate estimation of the spectrophotometric response of a textile blend composed of differently colored fibers, made of different materials. In particular, the performance of the K-M method proposed in Ref. 15 is enhanced by introducing an artificial intelligence approach to determine a more consistent value of the nonlinear function relationship between the K-S ratio of the blend and the K-S ratio of its components. As a consequence, a hybrid K-M+ANN-based method capable of modeling the color mixing mechanism is devised to predict the reflectance values of a blend. Since the hybrid method requires a dataset to be trained, and this may be a drawback when confronted with theoretical approaches, a more practical formulation is eventually proposed to replace the ANN software in everyday common use by colorists.

3 Statement of the Problem

Let $\mathbf{p}_i(\lambda)$ $(i = 1,2,\ldots,n)$ be the spectral reflectance factors of the i'th component of a fabric (n being the total number of components). Now, let $\mathbf{R_F}(\lambda)$ be the spectral reflectance factors of the fabric obtained by mixing the components $\mathbf{p}_i(\lambda)$ according to a given recipe $\mathbf{A} = [\alpha_1, \alpha_2, \ldots, \alpha_n]$ with $\sum_{i=1}^{n} \alpha_i = 1$.

The general relationship between $\mathbf{R_F}(\lambda)$ and the vectors $\mathbf{p}_i(\lambda)$ may be stated by the following formula:

$$\mathbf{R_F}(\lambda) = \mathcal{F}[\mathbf{A}, \mathbf{p}_i(\lambda)], \tag{4}$$

where λ indicates the wavelength, varying in the range 400 to 700 nm. The size of vectors $\mathbf{p}_i(\lambda)$ and $\mathbf{R}(\lambda, \alpha_i)$ is 1×31.

According to Eq. (4) (and graphically explained in Fig. 3), the color matching problem may be expressed as the problem of finding the transfer function $\mathcal{F}$ between the spectral reflectance factors of a fabric $\mathbf{R_F}(\lambda)$ and the input data α_i and $\mathbf{p}_i(\lambda)$.

As mentioned in Sec. 1, the determination of the transfer function is not straightforward, especially for blends composed of more than five to six components (i.e., for $n > 6$) when theoretical approaches such as K-M and S-N have been demonstrated to provide inaccurate results.

For this reason, to evaluate the transfer function $\mathcal{F}$, it is necessary to derive supporting information from the spectral response of a first-attempt blend created using a given recipe. This first-attempt recipe, as explained below, allows introducing a number of simplifications to reduce the complexity of Eq. (4).

Summing up, the remainder of the paper confronts the following question:

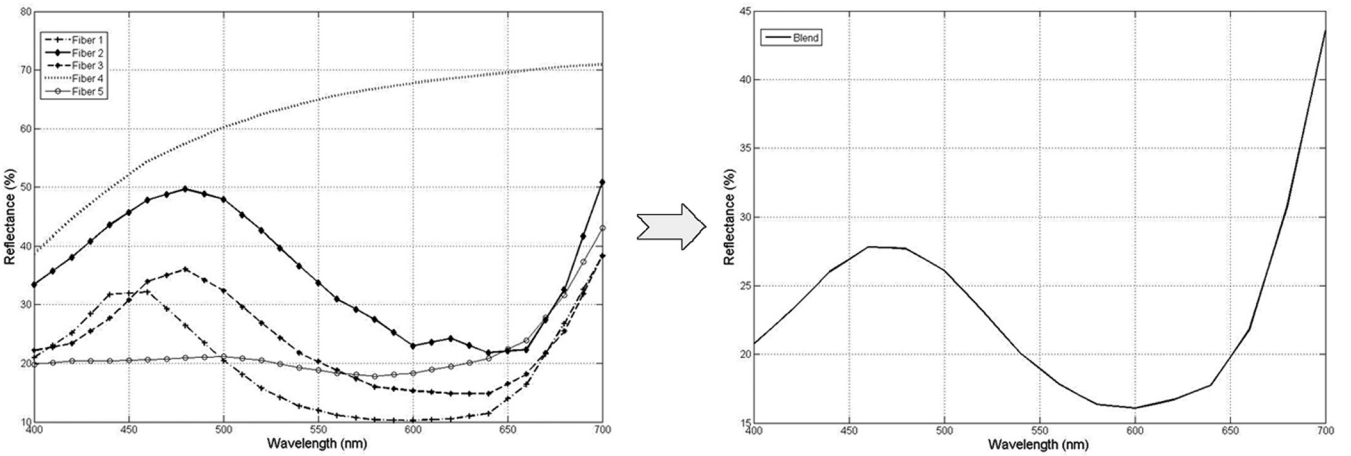

Fig. 3 Color matching consists of determining the transfer function $\mathcal{F}$ between the fiber spectra and the blend reflectance factors.

How is it possible to determine a transfer function between the reflectance factors of a blend obtained by varying a given recipe and fiber spectral responses once (1) the spectrum of each component is known, (2) the reference and first-attempt recipe spectra are measured, and (3) the original recipe is provided?

4 Hybrid Kubelka-Munk Artificial Neural Network–Based Method

The hybrid K-M+ANN method consists of the following three steps:

1. Definition of the equivalent fabric substrate using the simplified model described in Ref. 15.
2. Training an ANN to infer recipe variations to the substrate K-S ratio; such a ratio is then used to predict the spectral response of the fabric blend. The combination of steps 1 and 2 constitutes the hybrid K-M+ANN method.
3. Deriving (from the K-M+ANN method) a practical chart that can approximately replace the ANN software in everyday common use.

4.1 *Definition of the Equivalent Fabric Substrate*

Equation (2) states a correlation between the K-S ratio of a blend and the K-S ratio of singular components to be mixed together plus the substrate $k_{\lambda,t}/s_{\lambda,t}$. Unfortunately, the definition of substrate for fabric blends is weak, since they are obtained by mixing fibers and not by dipping a neutral monochrome fabric in a dye bath. In other words, Eq. (2) loses its physical meaning for fabric blends. Fortunately, the model proposed in Ref. 15 demonstrates that it is still possible to use Eq. (2) under the hypothesis that the turbid mixing mechanism of fibers only slightly changes by varying the original recipe (i.e., by changing the values α_i). In detail, the model proposed in the mentioned prior work takes into account that colorists always create a first-attempt blend using their historical recipe; as a consequence, the actual reflectance factors of the blend are known, together with the recipe and the spectra of each component. This additional information is used to evaluate the K-S ratio of an equivalent fabric substrate $\psi_s^*(\lambda)$ as follows:

$$\psi_s^*(\lambda) = \left(\frac{K}{S}\right)_{\lambda,\text{mix}} - \left[\alpha_1\left(\frac{k_{\lambda,1}}{s_{\lambda,1}}\right) + \ldots + \alpha_n\left(\frac{k_{\lambda,n}}{s_{\lambda,n}}\right)\right] = \psi_F(\lambda) - \psi_C(\lambda), \quad (5)$$

where $\psi_F(\lambda)$ is the K-S ratio of the fabric and $\psi_C(\lambda)$ is the weighted average of fiber K-S ratios.

In Fig. 4, the terms $\psi_s^*(\lambda)$ and $\psi_C(\lambda)$ for a given blend (i.e., sample 1 of Tables 1 and 2) are plotted in the range of 400 to 700 nm. As further explained in Sec. 4.2, Eq. (5) will be used to derive the K-M+ANN hybrid method.

4.2 *Artificial Neural Network Training and Prediction of the Blend Spectral Response*

The hypothesis that the turbid mixing mechanism of fibers changes only slightly by varying the original recipe (mathematically speaking, this means that ψ_s^* is assumed to be constant when the recipe is varied) is the main drawback of the previous work.[15] In fact, the equivalency between the actual fabric and the hypothetical fabric obtained using a dye-dipping process has proved valid only for small changes in the recipe (e.g., $\pm 5\%$ of the maximum variation for each component).

To strengthen the accuracy of the color prediction, the present paper aims to derive a more general rule for

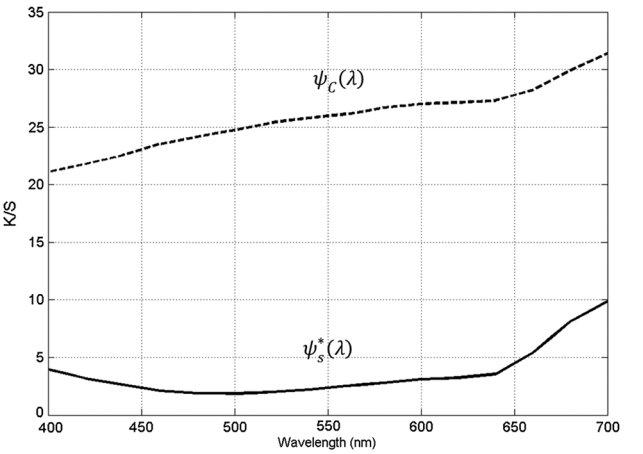

Fig. 4 Terms $\psi_s^*(\lambda)$ and $\psi_C(\lambda)$ for a given blend.

Table 1 Original and modified recipe for two samples.

Fabric	Number of raw materials	Original recipe (%)	Material	Modified recipe (%)
Sample 1	10	6.0	Wool	6.0
		3.0	Wool	7.0
		3.5	Wool	2.0
		12.0	Wool	10.0
		14.0	Wool	13.0
		11.0	Wool	11.0
		10.0	Wool	10.5
		8.0	Polyester	10.5
		22.5	Polyester	20.0
		10.0	Polyester	10.0
Sample 2	11	7.5	Wool	6.0
		4.0	Wool	3.0
		3.5	Wool	5.5
		46.5	Wool	47.0
		6.0	Wool	6.0
		4.2	Wool	4.2
		4.2	Wool	4.2
		1.1	Polyester	1.6
		13.0	Polyester	13.0
		9.0	Polyester	9.0
		1.0	Polyester	0.5

determining the function ψ_s^* valid also for large variations of recipes.

With the aim of considering the recipe variation as a variable for the problem, the definition of the equivalent fabric substrate is changed by considering it as a function not only of wavelength but also of A (original recipe) and of the vector $\hat{\mathbf{A}} = [\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_n]$ describing the new recipe

$$\psi_s^* = \mathfrak{f}[\lambda, \mathbf{A}, \mathbf{A}^*]. \tag{6}$$

Accordingly, the problem is reduced to the evaluation of the function $\mathfrak{f}$ linking the recipe variation, for a given wavelength, to the substrate K-S ratio.

To determine such a transfer function, a possible method could be to devise an ANN-based approach using a number of modified recipes as the training set and the equivalent fabric substrate obtained for the blends physically manufactured using such recipes as the target set.

As is widely recognized,[16] ANNs are computational systems that simulate the microstructure of a biological nervous system. ANNs can be trained to perform a particular function, either from information from outside the network or by the neurons themselves in response to the input.[17] A properly trained ANN is capable of generalizing the information on the basis of the parameters acquired during the training phase; therefore, it requires a dataset of modified recipes to be used. Therefore, the application of this method is recommended only when textile companies involved in the experimentation have a consistent database. In the present work, more than 1000 differently colored blends were available, thanks to the collaboration of an important company, New Mill S.P.A., working in Prato (Italy). Among them, a set of 120 blends with both first-attempt $\mathbf{A}_j(\lambda)$ and second-attempt $\hat{\mathbf{A}}_j(\lambda)$ recipes (with $j = 1 \ldots 120$) was selected.

To prepare the 240 fabrics composed according to the $\mathbf{A}_j(\lambda)$ and $\hat{\mathbf{A}}_j(\lambda)$ recipes, a laboratory carding machine is used. Equipped with a single licker-in, used to open the fiber stock prior to sending them to a main cylinder, this small-scale machine allows us to create a felt textile (with size approximately equal to 150×120 mm) with a density in the range of 0.150 to 0.250 g/cm^3 depending on the fiber fineness. Such a felt textile, whose thickness is in the range of 1.5 to 2 mm, is actually used for color control instead of woven fabrics. The reason behind this choice is that, usually, in textile practice, the color control is performed preferably by comparing felts (more often than desired, the customer provides the company only with small remnants). This is particularly true when dealing with yarn color control; in effect, in this case, it is rather difficult to measure the reflectance factors of yarns due to their small diameter (when compared to the spectrophotometer sensor area), thus forcing textile companies to measure felts instead of yarns. Obtained density values and fabric thickness assure a correct acquisition using the spectrophotometer (i.e., to avoid the acquisition of empty spaces between fibers and minimize the effect of transmitted light through the sample).

Felt textiles are conditioned at a temperature of 23°C for 20 min and subsequently processed by using an acquisition system consisting of a Hunterlab Ultrascan VIS reflectance spectrophotometer. The spectrophotometer provides the value of light reflectance in the wavelength range of 400 to 700 nm, with a step of 10 nm. The resulting spectrum is obtained using a scattered light measurement in specular component excluded (SCE) mode. The scan was made with a neutral white background using an 8-deg angle between the light source (D65 illuminant) and the sample. A zero calibration was used to compensate for the effects of stray light due to the changing flare characteristics of the optical system. Such a calibration is performed by removing the spectrophotometer protective cap from the aperture and aiming the aperture into the air so that no objects are within 1 m and no light source is aimed at. The white calibration of the spectrophotometer, used to set the maximum reflectance to 100%, was performed at the beginning of data acquisition using a Hunterlab-standard white cap whose reflectance is known to be equal to 1. Once acquired under controlled conditions (i.e., temperature 23 ± 1°C and SCE mode on the spectrophotometer), the actual cap spectral response is used to normalize the fabric acquisitions, i.e., to obtain a new measured

Table 2 Aggregate results obtained by comparing proposed methods with the ones described in Refs. 12, 13, and 15.

							CMC(2:1) distance from reference (actual fabric with modified recipe)			
Sample	Number of components	Mass density (g/cm^3)	Thickness (mm)	ε	K-M+ANN method	Practical chart	K-M–based approach[15]	Subtractive mixing–based approach[15]	Theoretical approach[12]	ANN-based approach[13]
1	10	0.18	1.5	2.5	0.7011	0.7095	0.7121	0.577	0.7753	0.6944
2	11	0.21	2.0	1.5	0.3714	0.3787	0.3821	0.2804	0.7266	0.2801
3	10	0.22	2.0	1.5	0.4532	0.4656	0.4797	0.4496	0.7117	0.3881
4	8	0.25	2.0	5.6	0.1113	0.1222	0.1283	0.1285	0.4243	0.1302
5	8	0.18	1.5	6.1	0.5764	0.5993	0.6003	0.6022	0.6412	0.6121
6	9	0.15	2.0	3.2	0.2123	0.2221	0.2106	0.2036	0.5511	0.1058
7	12	0.15	1.5	2.5	0.4445	0.4887	0.4914	0.4423	0.5766	0.4521
8	9	0.20	2.0	1.5	0.4287	0.4672	0.4727	0.4295	0.5444	0.4354
9	15	0.25	2.0	1.5	0.6329	0.6543	0.6632	0.6229	0.7622	0.6121
10	16	0.20	2.0	1	0.6632	0.7083	0.7102	0.6811	0.8215	0.6311
11	10	0.20	1.5	2.2	0.5255	0.5493	0.5423	0.5263	0.612	0.3342
12	12	0.20	2.0	2.3	0.3287	0.3577	0.3621	0.3323	0.5312	0.2508
13	11	0.18	2.0	2.6	0.6421	0.6421	0.6521	0.7211	0.7989	0.5559
14	18	0.20	2.0	1.5	0.9752	0.9812	0.9912	1.0121	1.1231	0.9982
15	9	0.20	2.0	6	0.3488	0.3454	0.3782	0.333	0.5212	0.3432
16	12	0.22	2.0	4.7	0.6033	0.6063	0.6169	0.6319	0.6752	0.6091
17	14	0.25	2.0	5.5	0.4532	0.4438	0.4994	0.4651	0.6982	0.2245
18	9	0.22	2.0	3	0.4992	0.5009	0.5109	0.4232	0.7129	0.3943
19	8	0.22	2.0	3	0.2989	0.3198	0.3298	0.3422	0.6633	0.3703
20	10	0.25	2.0	5.5	0.7982	0.8232	0.8827	0.8726	1.021	0.5315
21	10	0.15	1.5	6	0.5211	0.5346	0.5548	0.5291	0.9782	0.544
22	12	0.15	1.5	3.5	0.3279	0.3982	0.4002	0.3832	0.4231	0.2885
23	12	0.15	1.5	2.2	0.4387	0.4793	0.4993	0.4293	0.6752	0.4157
24	14	0.18	2.0	1.2	0.4065	0.4994	0.5024	0.5102	0.8893	0.3971
25	9	0.20	2.0	2	0.5432	0.6135	0.6235	0.5562	1.032	0.4815
26	8	0.22	2.0	2.5	0.5129	0.5438	0.5538	0.5103	0.9372	0.5079
27	9	0.25	2.0	2.4	0.6652	0.7062	0.7162	0.6239	1.132	0.527
28	10	0.25	2.0	4.5	0.3358	0.4247	0.4391	0.3722	0.7392	0.3259
29	11	0.25	2.0	6	0.5222	0.4571	0.4792	0.4825	0.7832	0.4955

Table 2 (*Continued*).

Sample	Number of components	Mass density (g/cm^3)	Thickness (mm)	ε	K-M+ANN method	Practical chart	CMC(2:1) distance from reference (actual fabric with modified recipe)			
							K-M–based approach[15]	Subtractive mixing–based approach[15]	Theoretical approach[12]	ANN-based approach[13]
30	20	0.20	2.0	1.5	0.8766	0.9792	0.9992	0.9892	1.2132	0.8878
31	18	0.22	2.0	1	0.9565	1.0424	1.0924	1.028	1.3238	0.9232
32	20	0.18	1.5	4.5	0.8985	1.0232	1.0821	0.9321	1.4272	0.8872
33	9	0.20	2.0	6.4	0.4199	0.4019	0.4234	0.3992	0.8728	0.4193
34	10	0.20	2.0	7.2	0.6098	0.5452	0.5892	0.6092	0.8253	0.6131
35	10	0.20	2.0	5.4	0.3434	0.3872	0.4023	0.3452	0.5624	0.353
36	8	0.25	2.0	4.3	0.5099	0.5002	0.505	0.4491	0.8725	0.4558
37	8	0.18	1.5	7	0.3657	0.4872	0.5251	0.4039	0.7556	0.304
38	10	0.18	2.0	6	0.5324	0.7542	0.7623	0.6732	0.8843	0.5255
39	12	0.18	1.5	3	0.2998	0.3132	0.3182	0.2998	0.5421	0.3171
40	12	0.22	2.0	5	0.4309	0.4342	0.4489	0.4392	0.7825	0.4232
		Mean value			**0.5146**	**0.5478**	**0.5633**	**0.526**	**0.7886**	**0.4761**
		Median value			**0.5046**	**0.4998**	**0.508**	**0.4738**	**0.7589**	**0.4438**
		Max value			**0.9752**	**1.0424**	**1.0924**	**1.028**	**1.4272**	**0.9982**
		Min value			**0.1113**	**0.1222**	**0.1283**	**0.1285**	**0.4231**	**0.1058**
		Coefficient of variation			**0.0384**	**0.0441**	**0.0472**	**0.0454**	**0.0541**	**0.0409**

reflectance varying in the range 0 to 1. The data were stored in a PC in the form of a 31-element vector representing the reflectance values versus the wavelength for an examined sample. To ensure adequate robustness and representativeness of results, the largest (25.4 mm) measuring area of the spectrophotometer was selected. The final results of this experimental setup are the spectral responses of the 120 samples obtained using the standard recipe, the 120 spectral responses for the modified recipe, and the spectra of each raw material composing the blends. For each recipe, moreover, the substrate functions ψ_j^* and $\hat{\psi}_j^*$ are evaluated using Eq. (5).

In order to teach the ANN to provide a prediction of the transfer function ƒ, the training input data are represented by the following matrix $\mathbf{M}$:

$$\mathbf{M} = [\mathbf{A}_1(\lambda_1), \ldots, \mathbf{A}_1(\lambda_{31}), \ldots, \mathbf{A_{120}}(\lambda_1), \ldots, \mathbf{A_{120}}(\lambda_{31}), \hat{\mathbf{A}}_1(\lambda_1), \ldots, \hat{\mathbf{A}}_1(\lambda_{31}), \ldots, \hat{\mathbf{A}}_{120}(\lambda_1), \ldots, \hat{\mathbf{A}}_{120}(\lambda_{31})] \quad (7)$$

It is important to remark that the size of vectors $\mathbf{A}_j$ and $\hat{\mathbf{A}}_j$ may differ from blend to blend (some of them may, for instance, consist of five differently colored fibers mixed together, while others may be composed of 20 fibers). Accordingly, to process the data into the ANN, each vector size is considered equal to $1 \times n$, where n is the maximum size among the given recipe vectors. In the present work, n is set equal to 20. Obviously, recipes composed of $k < n$ differently colored fibers (e.g., 10) will have values in positions from $k+1$ to n equal to zero.

Accordingly, the final size of the input set $\mathbf{M}$ is 20×7440. The target set $\mathbf{T}$ (size 1×7440) consists of the actual values ψ_j^* and $\hat{\psi}_j^*$ evaluated for all wavelengths using, respectively, the recipes $\mathbf{A}_j$ and $\hat{\mathbf{A}}_j$.

$$\mathbf{T} = [\psi_1^*, \ldots \psi_{120}^*, \hat{\psi}_1^*, \ldots \hat{\psi}_{120}^*]. \quad (8)$$

The ANN, developed by using the Artificial Neural Network Toolbox® working in the MATLAB® environment, has the following characteristics:

- Three layers: input, hidden, and output.
- A hidden layer made of log-sigmoid neurons followed by an output layer of linear neurons.
- Twenty input, h hidden, and 1 output units.

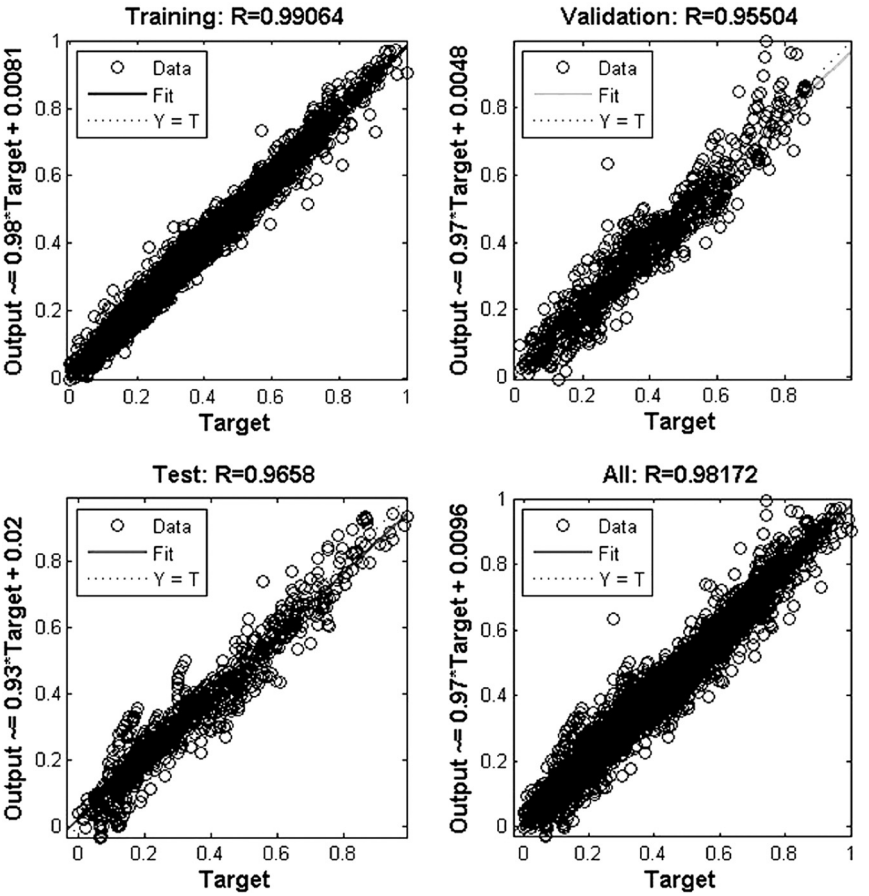

Fig. 5 ANN performance in terms of linear regression.

It is possible to select the best network within a set of candidate configurations. For this purpose, the number of hidden units was varied from 9 to 30 with a step of 3 units, monitoring the performance of response using the training subset. As is known, during the training, the weights and biases of the network are iteratively adjusted to minimize the network error function. The network error used in this work is the mean square error corresponding to the training set elements. This error is monitored during the training process and will normally decrease during the initial phase of the training. However, when the network becomes excessively specialized in reproducing the training data, the early stopping error will typically begin to rise. When the early stopping error increases for a specified number of iterations, the training is stopped, and the weights and biases at the minimum early stopping error are returned. The selected network is characterized by $h = 15$ units. The training was carried out using a training rule based on the gradient descent backpropagation algorithm with an adaptive learning rate.[18] Optimal training was achieved in 64 epochs. In Fig. 5, the ANN performance is depicted in terms of linear regression between datasets (split into the typical subsets: training, validation, and testing).

Once trained, the network is able to correlate the training set elements to the target ones. In other words, the ANN is able to receive (as input) any vector of 20 elements (e.g., a modified recipe) and to give as output the prediction of the corresponding $\psi_s^*(\lambda)$ value.

Finally, the K-S ratio for any given variation of the recipe $\psi_{F\text{new}}^*(\lambda)$ can be evaluated as follows:

$$\psi_{F\text{new}}^*(\lambda) = \psi_s^*(\lambda) + \bar{\psi}_C(\lambda), \tag{9}$$

where $\bar{\psi}_C(\lambda)$ is the linear combination of fibers' K-S ratio using the modified recipe.

Solving Eq. (1) for $\mathbf{R_F}(\lambda)$ allows the estimation of the blend reflectance factors using the hybrid K-M+ANN method.

$$\psi_{F\text{new}}^*(\lambda) = \frac{[1 - \mathbf{R_F}(\lambda)]^2}{2\mathbf{R_F}(\lambda)}. \tag{10}$$

4.3 *Deriving a Practical Chart to Be Used by Colorists*

As already stated, the above-described procedure, i.e., the hybrid K-M+ANN method, is based on a training procedure requiring experimental data whose retrieval is time-consuming. As a consequence, even when the prediction is extremely improved with respect to similar approaches, the method still has the drawback of requiring training sessions and ANN simulation. Therefore, it is useful to derive a more practical (still approximate) method to determine the $\psi_s^*(\lambda)$ value. As a consequence, a practical chart to be used by practitioners without the need of using ANN is proposed. The chart is built using the information coming from ANN, but, once built, it can be used with new inputs without

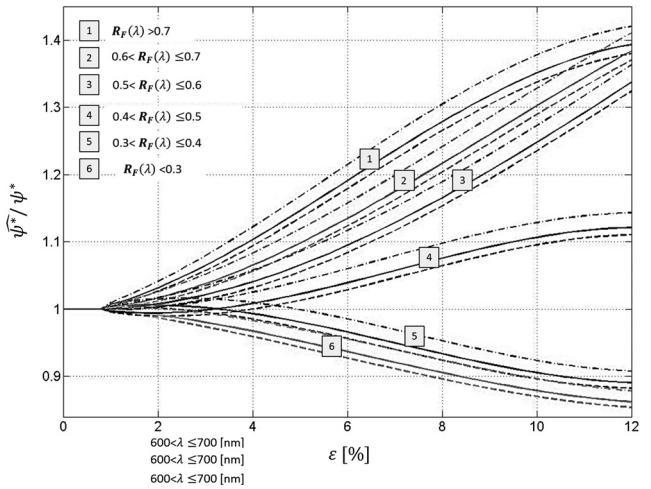

Fig. 6 Practical chart obtained by simulating the hybrid K-M+ANN method: it can be considered a simplified model of the turbid mixing mechanism (simulated by using ANN).

simulating the ANN (i.e., it can also be used in cases where the ANN has not been devised at all).

First, the ANN is simulated in the whole wavelength range by randomly varying values $\hat{\alpha}_i$ so that

$$\max|\hat{\mathbf{A}} - \mathbf{A}| = \varepsilon, \tag{11}$$

with ε varying in the range 1 to 12 with a step of 1.

Results of simulations are then compared in order to evaluate how the equivalent fabric substrate changes when recipe changes are artificially imposed. Accordingly, it is possible to build a set of curves showing the influence of recipe changes on the substrate K-S ratio. In detail, the curves show the trend of the ratio $\hat{\psi}^*/\psi^*$ as a function of ε for different ranges of reflectance and for varying values of reflectance for the first-attempt recipe. The chart depicting the resulting curves (see Fig. 6) represents a simplified model of the turbid mixing mechanism and, as demonstrated in Sec. 5, proves to be useful for technicians, enabling them to forecast blend color without the need to use ANN. It is also worth noting that for small variations of recipes, e.g., $\max|\hat{\mathbf{A}} - \mathbf{A}| < 2$, the equivalent fabric substrate remains approximately constant, thus proving the correctness of the assumptions made in Ref. 15, at least for these cases.

5 Results

The prediction method described above has been validated, thanks to the collaboration of the staff of the textile company New Mill s.p.a., Prato (Italy), by using the (validation) set of 40 samples obtained, respectively, adopting their standard recipe and a modified version using the same procedure described in Sec. 4.2. In particular, the validation set consists of felt textiles typically processed by the company in its everyday work. To maximize the validation as much as possible, samples with different numbers of mixed fibers and with different colors have been selected.

In Table 1, the original and modified recipes of two samples among the 40 chosen for validating the procedure are listed.

The predicted spectra obtained using the proposed approaches (hybrid method and practical chart) are compared in terms of CMC(2:1) distance with the actual measurement of the real fabrics obtained using the modified recipes. Moreover, a comparison between the results obtained using the four approaches provided, respectively, by Refs. 13–15 is proposed. In Table 2, the results of the comparison for the 40 samples are shown.

A comparison between the results obtained using the proposed K-M+ANN method and the ANN-based approach described in Ref. 15 has been assessed using a paired t-test. The evaluated p value for such a test result equals 0.0013, thus implying that the difference, in terms of performance, is statistically significant. Therefore, it can be stated that the proposed method outperforms the ANN-based ones. In addition, the use of a practical chart roughly equals the K-M-based approach for small variations of recipes and allows more accurate prediction when $\varepsilon > 4.5$, thus proving its effectiveness in forecasting the final color of the mixed blend.

It is worth noting that the prediction error, i.e., the color distance, increases for blends characterized by a high number of raw materials as shown, for example, for samples 30, 31, and 32, where the CMC(2:1) distance exceeds the recommended value of 0.8. However, a prediction error <1.2 for such cases (i.e., samples with a large number of components) is considered still acceptable by many companies[19] working in the textile field. As a final remark, the results demonstrate that both methods (K-M+ANN and practical chart) average a closer prediction when compared with the theoretical approach proposed in Ref. 13.

6 Conclusions

The present paper described a hybrid K-M+ANN method for predicting the spectral response of a fabric obtained by

Table 3 Results obtained by using a second-attempt recipe as reference for evaluating the blend color using the hybrid K-M+ANN method.

		CMC(2:1) distance from reference (actual fabric with modified recipe)		CMC(2:1) distance from reference (actual fabric with second-attempt modified recipe)	
Sample	Number of components	K-M+ANN method	Practical chart	K-M–based approach (using second-attempt recipe)	Subtractive mixing–based approach (using second-attempt recipe)
30	20	0.8766	0.9792	0.3234	0.2998
31	18	0.9565	1.0424	0.3562	0.3223
32	20	0.8985	1.0232	0.2903	0.2237

mixing precolored fibers. In detail, the simplified K-M method proposed in Ref. 15 is enhanced by introducing a more intelligent evaluation of the K-S ratio of the equivalent substrate. Moreover, a practical chart is built by simulating the ANN under specific recipe variations so that a method not requiring the ANN can be stated. The approach proved to be effective in forecasting the spectral response of the blend within an average color distance in terms of CMC(2:1), <0.6.

The effectiveness of the method is mainly due to the fact that the knowledge of the first-attempt spectrum allows one to immediately validate the exemplificative assumptions for any color blend. On the other hand, the main limitation is that it is not able to estimate the spectrophotometric response of a blend when the information on the reflectance factors measured for the first-attempt fabric is not provided. Accordingly, even if the method described here overcomes the performance of the K-M and S-N approaches, it is on the understanding that literature-established methods also work well when the first-attempt spectrum is not known. Accordingly, for general-purpose color matching problems, the literature methods still have to be considered the best available option.

The authors, consequently, want to encourage other researchers working in the field of colorimetry and spectrophotometry to provide a large number of results of their experiments using the provided equations and to propose improvements to the system. By way of example, possible improvements could be linked to textile praxis in creating fabric blends. In fact, in cases where the prediction is not so accurate (e.g., samples 30, 31, and 32), it could be possible to further explore practical solutions based on the creation of a second-attempt fabric to be used as the new reference for the proposed methods. On the basis of a preliminary study, proposed here and shown in Table 3, such an approach could allow one to substantially reduce the color distance between the predicted spectrum and the actual one (i.e., the third-attempt fabric). Examples in Table 3 refer to results obtained by applying the proposed hybrid method using as equivalent fabric substrate the mean value between the first- and second-attempt recipes. This allows us to obtain a closer prediction of the term $\psi_s^*(\lambda)$, thus leading to more accurate predictions of $\mathbf{R_F}(\lambda)$.

Of course, this last approach requires the technicians to physically create a new fabric, thus increasing the overall time for assessing the final recipe. It should be noted, however, that for a high number of precolored materials to be mixed together, the traditional definition of the final recipe usually involves a high number of attempts (often 8 to 10). Diminishing the number of attempts from such a number to 3 could be an effective strategy anyway. Further analysis is, however, required prior to drafting more consistent conclusions about this possible improvement.

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