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LETTER

Letter to the Editor

Dynamical response of a stellar atmosphere to pressure perturbations: numerical simulations

L. Del Zanna[1,⋆], **M. Velli**[1], **and P. Londrillo**[2]

[1] Dipartimento di Astronomia e Scienze dello Spazio, Università di Firenze, Largo E. Fermi 5, I-50125 Firenze, Italy
[2] Osservatorio Astronomico, Via Zamboni 33, I-40126 Bologna, Italy

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Abstract. The time dependent reactions of an isothermal spherically symmetric stellar atmosphere to perturbations of the external (interstellar) pressure are analysed by means of computer simulations. The system is seen to evolve, through the phases of wind, breeze, accretion and back, according to an hysteresis type cycle with two catastrophe points: the value of the external pressure relative to a static atmosphere and that corresponding to the fastest (critical) breeze. This behaviour is proved to be due to the instability of the outflow breeze solutions (due to their unfavourable stratification), while subsonic accretion is stable. A crucial factor of this instability is the position of the outer boundary: if this is placed too close to the base of the atmosphere the inflow/outflow breeze stability is reversed. These simulations confirm a scenario first proposed by Velli (1994).

Key words: hydrodynamics – instabilities – (*Sun:*) solar wind – sun: atmosphere – stars: atmospheres – ISM: jets and outflows

1. Introduction: steady state solutions and breeze instability

Four decades ago Parker predicted the existence of the solar wind, that is the supersonic outflow of plasma continuously emanating from the Sun (Parker 1958). The reason for this outflow is the impossibility for the interstellar pressure, due to its extremely low value, to confine a static atmosphere with a radial temperature profile decaying less rapidly than $1/r$. A numerical simulation attempting to follow equilibrium flows set up by a given pressure difference between the coronal base and the interstellar medium, as this difference is varied, was presented by Korevaar (1989). He was able to obtain shocked wind solutions, both outflow and accretion breezes and shocked accretion flows.

Send offprint requests to: L. Del Zanna (e-mail: ldz@arcetri.astro.it)
⋆ *Present address:* International Space Science Institute, Hallerstrasse 6, CH-3012 Bern, Switzerland

Recently, however, Velli (1994) has shown that the transitions from one type of flow to another are more subtle and involve the presence of catastrophe points and instabilities of subsonic flows. In the remainder of this section a brief summary of the possible stationary flows is presented and their stability is discussed. In the next section some time dependent numerical simulations will be shown and the results confirm the Velli (1994) cyclic behaviour of the wind-accretion flows.

Consider, for simplicity, an isothermal spherically symmetric stellar atmosphere (or corona). By combining the momentum and continuity equations, the stationary solutions are derived from the Parker wind equation

$$\left(M - \frac{1}{M}\right) M' = \frac{2}{r} - \frac{g}{r^2}, \tag{1}$$

where r is nondimensionalised against the stellar radius $R_\star$ (the prime indicates a radial derivative), $M = v/c$ is the Mach number (c is the *isothermal* sound speed, for which $p = c^2\rho$) and $g = GM_\star/c^2R_\star$ is the nondimensional gravity constant ($M_\star$ is the mass of the star). If the pressure p is normalised against its base value p_0, an integral of Eq. (1) is

$$\log p + \frac{M^2}{2} - \frac{g}{r} = \frac{M_0^2}{2} - g, \tag{2}$$

where M_0 is the base Mach number. Breeze solutions have the asymptotic behaviour $|M| \sim 1/r^2$, thus the range of the outer pressure for both subsonic outflows ($0 < M < 1$) and inflows ($-1 < M < 0$) is

$$p_\infty^s < p_\infty < p_\infty^c \equiv p_\infty^s \exp(\frac{M_0^{c2}}{2}), \tag{3}$$

where $p_\infty^s = \mathrm{e}^{-g}$ is the asymptotic static pressure and p_∞^c is the *critical* pressure, corresponding to the fastest possible breeze, that reaching the sonic point $M = 1$ at $r_s = g/2$. Note that this range is usually very narrow, since for realistic values of g (≈ 10 in the solar case) $M_0^c \ll 1$.

In addition to the breeze solutions, a supersonic shocked wind is allowed for every value of the asymptotic pressure p_∞. For $p_\infty > p^c_\infty$ the only possible steady solution is a supersonic accretion inflow. To summarise, the situation is the following:

1. $p_\infty < p^s_\infty$. Only a supersonic outflow with a shock beyond r_c is allowed. The position of the shock r_s moves outwards if p_∞ is decreased.
2. $p_\infty = p^s_\infty$. The static solution with $M(r) = 0$ everywhere and a supersonic shocked solution are both allowed.
3. $p^s_\infty < p_\infty < p^c_\infty$. Three different classes of solutions are possible: an outflow breeze, an accretion breeze and again a supersonic shocked wind.
4. $p_\infty = p^c_\infty$. The shock position coincides with the critical radius $r_c = g/2$ and the shocked solutions, both outflow and inflow, collapse to the corresponding critical breezes.
5. $p_\infty > p^c_\infty$. Only the accretion shocked solution is found, with r_s moving from r_c towards the coronal base as p_∞ increases.

Despite the fact that both subsonic and supersonic outflows are allowed in the range Eq. (3), the breeze solution is unstable. Through a linear analysis, Velli (1994) showed that outflow breezes are unstable to sound waves which leave the pressure at the boundary unperturbed (standing waves). This is due to the unfavourable stratification produced by breeze solutions, resulting in $p_\infty > p^s_\infty$: given a static atmosphere, an increase in p_∞ is clearly expected to produce an inflow, not an outflow. For the same reason, inflow breeze solutions are stable.

Another crucial factor of the stability analysis is the position of the outer boundary r_b, since outflow breezes are not locally unstable everywhere. This may be understood by noticing that Eq. (2) implies:

$$p(r) = p^s(r)\, \exp\left[\frac{M_0^2 - M^2(r)}{2}\right]. \qquad (4)$$

Near the coronal base, where $M > M_0$, the pressure of a breeze solution is *lower* than the corresponding static pressure, thus the gradient of the stratification is favourable in that range. This is true out to a radius r_f where the final pressure for the breeze and static solutions coincide, that is where the Mach number drops back to its base value. By eliminating p in Eq. (2) through the continuity equation, an implicit relation for r_f is found:

$$2 \log r_f + g/r_f - g = 0. \qquad (5)$$

This is a very important point, especially for numerical simulations of stellar winds, which necessarily require a limited radial range as numerical box. For realistic values of g, r_f may be very large and if the external boundary is placed at $r_b < r_f$ outflow breezes will be stable, while inflow breezes will be unstable.

2. Time dependent simulations

The only way to follow the nonlinear time dependent evolution of stellar winds and related flows, even in the simple isothermal case, is by means of computer simulations. The code used here employs a high order shock capturing scheme, that is the WENO (*Weighted Essentially Non Oscillatory*) method proposed by Jiang & Shu (1996) with Lax-Friedrichs flux splitting, which provides an accuracy of $(\Delta x)^5$ in smooth regions (the resolution used here in all runs is 100 radial points). For the time integration, a third order TVD Runge-Kutta time stepping, developed by Shu & Osher (1988), is employed. These combined methods are known to be convergent under appropriate CFL numbers.

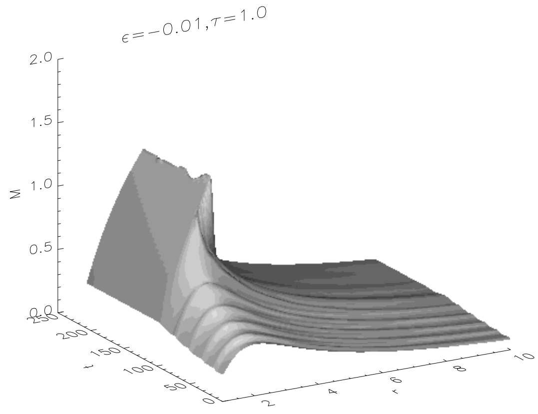

Fig. 1. Instability of a breeze solution. The outer pressure is lowered from the initial value p = 0.02782 to p = 0.02755, corresponding to $\epsilon = -0.01$. Both the initial and the final pressure are in the range allowing for steady subsonic solution, since for $g = 4$ and $r_b = 10$ the static and critical pressures at $r = r_b$ are, respectively, p^s = 0.02732 and p^c = 0.02897.

The numerical box starts at $r = 1$ (the coronal base, where the pressure is kept constant to its initial value $p = 1$) and ends at $r = r_b$. At this outer boundary the pressure is perturbed in time, at each run, according to the function

$$r = r_b: \;\; p(t) = p(0) + \epsilon\, p^s f(t/\tau); \;\; f(x) = x^2/(1+x^2), \qquad (6)$$

so that for $t \gg \tau$ the external pressure has increased by a factor ϵ, in units of the corresponding static value $p^s = \exp[-g(1 - 1/r_b)]$.

As a preliminary example, consider the effect of a small perturbation of a steady state subsonic outflow. According to the discussion of the previous section, for $r_b > r_f$, the breeze is unstable and must evolve towards either a shocked wind or to a subsonic accretion breeze, depending on the nature of the perturbation. Here the outer pressure is decreased slightly, so that a close breeze solution is mathematically accessible, and the results are shown in Fig. 1.

Here the time evolution of the velocity radial profile is shown as a shaded surface plot, with the time increasing towards the left along the y-axis. Here the value g = 4.0 is assumed, which leads to a critical radius located at $r_c = g/2$ = 2.0 and to a value for the critical outer radius $r_f \approx 5.39$. In order to demonstrate the instability of outflow breezes, the value of r_b is taken to be $r_b = 10 > r_f$. The outer pressure is decreased by using Eq. (6) with $\epsilon = -0.01$ and $\tau = 1$. A variation as small as 1% in the outer pressure is enough to destabilise the breeze solution,

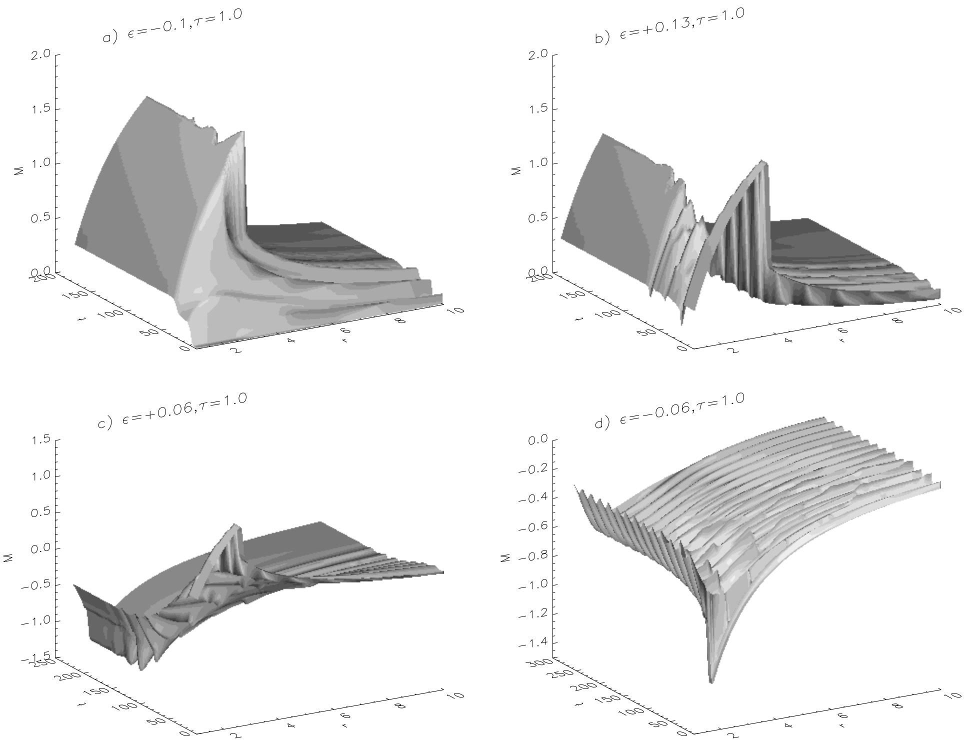

Fig. 2a–d. The hysteresis type cycle for the time evolution of a stellar atmosphere under the effect of a perturbation of the pressure at the external boundary. The values of the parameters are the same as in Fig. 1. **a** Creation of a shocked wind: the pressure is decreased from its static value, $p^s = 0.02732$, to a final value $p = 0.02459$, corresponding to $\epsilon = -0.1$. **b** The pressure is increased to a value inside the critical range ($\epsilon = 0.13 \Rightarrow p = 0.02815$): the steady final state is again a supersonic shocked wind. **c** The pressure is further increased out to a value larger than p^c_∞ ($\epsilon = 0.06 \Rightarrow p = 0.02978$): the flow reverses its direction collapsing to an accretion supersonic inflow. **d** The outer pressure is brought back to the value in the critical range ($\epsilon = -0.03$), but this time the final solution is a subsonic accretion breeze. Note that, in this last case, the oscillations at the final iteration are not damped yet.

which steepens into a steady supersonic shocked solution, even if the final value of the pressure at r_b still allows for a breeze stationary solution. The typical time scale of the instability is, for this choice of the parameters, of the order of $100\, R_\star/c$.

In the next series of runs the typical hysteresis type cycle is presented, demonstrating that the actual state chosen by the flow depends on its history, and that the transitions from inflow to outflow and back are necessarily catastrophic in nature. Let the initial situation be static (situation no. 2 in the scheme of the previous section). First, a supersonic shocked wind is created by lowering the external pressure (situation no. 1), and then the position of the shock is moved inwards by increasing the value of p_∞ in such a way that its final value is in the range between the static and critical values (situation no. 3). The corresponding time evolution may be followed in Figs. 2a–da and 2a–db, respectively. Note that, although subsonic outflows are present at certain stages, the final stationary solution is again a shocked wind. In the third run the outer pressure is increased even further, beyond the critical value, so that the only possible solution is a supersonic accretion inflow (situation no. 5). The catastrophic nature of the hysteresis cycle is especially apparent here, since a shocked outflow collapses directly to a shocked inflow, without passing through steady subsonic solutions (see Fig. 2a–dc). Finally, in Fig. 2a–dd, the evolution to an accretion breeze type solution is shown. This has been obtained by lowering the external pressure back to the value already reached at the end of the second run.

The situation is summarised in Fig. 3, where the four stationary final states of the cycle are shown all together. Note that, if the pressure was further decreased below the static value, the accretion breeze would collapse back to a supersonic shocked wind solution (second catastrophe point).

Finally, it is interesting to verify that when the outer boundary is placed too close to the coronal base, that is when $r_b < r_f$, breeze outflows are actually stable steady state solutions. For example, consider an initial static atmosphere and allow the pressure at $r = r_b$ to decrease of a factor $\epsilon = -0.05$. This time, as it is shown in Fig. 4, the system slowly evolves towards a steady state breeze solution.

3. Discussion

We have shown via computer simulations how flows in stellar atmospheres are established by the pressure difference between the atmospheric base and the interstellar medium. Our simulations confirm in detail the scenario proposed by Velli (1994) in which the transition from outflows to inflows and *vice versa* is necessarily of a catastrophic nature, in the mathematical sense that a small variation in a control parameter, i.e. the pressure difference from atmospheric base to infinity, may cause a finite amplitude transition to a completely different flow configuration. As a consequence, there exists a range of values of the control parameters for which the equilibrium flow which is established depends not only on the value of the parameter but also on the previous history of the flow. In other words, for a cyclical behaviour of the control parameter, the flows that occur in the atmosphere follow a hysteresis cycle and the symmetry with respect to the sign of radial velocity, present in the equilibrium stationary state flow equations, is broken.

For simplicity our numerical calculations have been carried out considering isothermal flows: subsonic flows are in this case

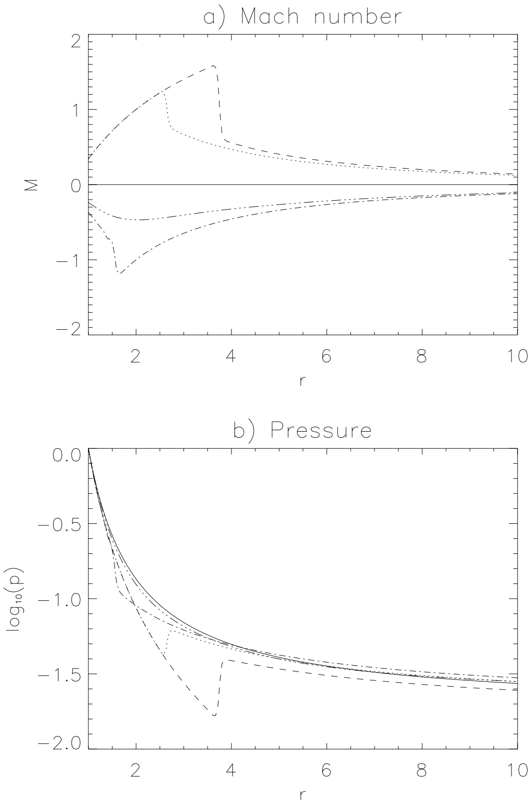

Fig. 3. Steady state solutions resulting from the four runs of the cycle shown in Fig. 2a–d. The solid line refers to the initial, static case. The dashed, dotted, dot-dashed and double dot-dashed lines refer to runs a, b, c and d, respectively.

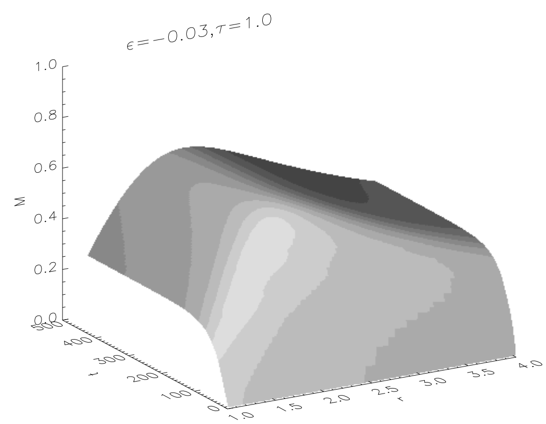

Fig. 4. Stability of a breeze solution, for $r_b = 4 < r_f$. The outer pressure of a static atmosphere is lowered from its initial value $p = 0.04979$ to $p = 0.04829$, corresponding to $\epsilon = -0.03$. The system slowly evolves towards a steady state breeze solution.

limited to an exponentially small range of values of the interstellar pressure, and only subsonic accretion flows are stable (in the absence of external influences, such as a companion, close to the star). Supersonic shocked flows, both of accretion and wind type, are stable. The physical reason for the abrupt transition from one to the other is the discontinuous nature of communication via sound waves, which is absent when the flows are supersonic, and is open when the flow is subsonic. In other words, as long as there is a supersonic shocked outflow, the lower part of the atmosphere is protected from knowing the state of the medium downwind of the shock; once the shock is pushed to the critical point, commmunication sets in, but by that time it is too late, so to speak: the pressure at the atmospheric base is too small and a collapse to accretion occurs, as is well seen in the numerical simulation.

The scenario described above also holds for polytropic flows, and may be of relevance in the discussion of more realistic treatments of atmospheric heating and corona formation, as discussed e.g. by Souffrin (1982), Hearn et al. (1983), Korevaar (1989), and references therein. Of particular interest to our discussion are the simulations by Korevaar (1989), who found a smooth transition from supersonic shocked winds to breezes and then a *rapid onset* of supersonic shocked accretion: his result may be understood indeed in terms of the dimensions of the numerical box, since the external boundary conditions were imposed well below the marginal stability radius.

We defer simulations with a more realistic energy equation, necessary to capture the additional effects of coronal relaxation oscillations, to a subsequent paper. To summarise, we might say that this paper completes, by describing in detail the outflow/inflow transition and its relation with the parameters of the stellar atmosphere and interstellar medium, the study of spherically symmetric isothermal flows initiated by Bondi (1952) and McCrea (1954) for accretion and by Parker (1958) for the solar wind.

References

Bondi H., 1952, MNRAS 112, 195
Hearn A.G., Kuin N.P.M., Martens P.C.H., 1983, A&A 125, 69
Jiang G-S., Shu C-W., 1996, J. Comput. Phys. 126, 202
Korevaar P., 1989, A&A 226, 209
McCrea W.H., 1956, ApJ 124, 461
Parker E.N., 1958, ApJ 128, 664
Roberts P.H., Soward A.M., 1972, MNRAS 328, 185
Shu C-W., Osher S., 1988, J. Comput. Phys. 77, 439
Souffrin P., 1982, A&A 109, 205
Velli M., 1994, ApJ 432, L55

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