ADVANCED MAP RESTORATION TECHNIQUES WITH APPLICATION TO DIGITAL IMAGE PROCESSING.

V.Cappellini, E. Del Re, P.Francolini and G.Taiuti
Istituto di Elettronica - Università di Firenze, Italy

MAP techniques, based on the criterion of maximizing a-posteriori probability, are considered for digital image restoration. In particular MAP sectioned algorithms are examined in detail with special reference to the parameters involved in the restoration which are adapted to the local characteristics of each image section. Modifications in the above algorithms and techniques are described, regarding the efficient control of the iterative procedure in connection with the degradation level in the different image parts, taking into account that the implementation is performed by means of minicomputer systems. The efficiency of the MAP restoration techniques was also evaluated in conjunction with a pre-processing or post-processing 2-D low-pass circular symmetry digital filter. Results obtained in digital image processing tests are finally reported.

1. INTRODUCTION

Digital processing of noisy images is of increasing importance in many working areas such as biomedicine, remote sensing and image transmission to obtain higher quality enhanced or restored images.

The image formation and recording model we are considering is described by the following relation [1]

$$g(x,y) = s \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha,\beta) f(x-\alpha,y-\beta) d\alpha d\beta + n(x,y) \qquad (1)$$

where: $f(x,y)$ is the original image or object, $g(x,y)$ is the actually available recorded image, $h(x,y)$ is the impulse response of a linear system, s is the non-linear characteristic of another system (as recording device, film or electronic camera), $n(x,y)$ is the additive noise [2] [3] .

The linear system, having $h(x,y)$ as impulse response (or point-spread-function, PSF), can take into account different image linear transformations such as blurrings introduced in the picture formation process (due to optical systems). Two blurring effects of interest are linear motion and circular defocusing [4] .

The digital processing of images requires the space sampling of the image and of PSF according to the sampling theorem. By arranging the obtained samples in a lexicographical order we can obtain the following relation [5]

$$\underline{g} = s(\underline{b}) + \underline{n} \qquad (2)$$

where: $\underline{b} = Hf$ (H is the PSF array lexicographically ordered, $\underline{f}$ is the vector of the original $f(x,y)$ image), $\overline{s}$ is the non linear function above considered, $\underline{n}$ is the vector of the noise random process, $\underline{g}$ is the vector of the available recorded image.

The restoration problem we are considering can be now stated in the following general form: "estimate f, given the PSF h, the recorded image g, the response function s and some a-priori knowledge (generally of statistical type) about the nature of the noise n and about f".

In the following we examine the restoration techniques based on the criterion of maximizing a-posteriori probability (MAP).

2. MAP TECHNIQUES

The MAP criterion in image restoration maximizes the a-posteriori density probability $p(\underline{f}/\underline{g})$, which by using the bayesian law becomes

$$p(\underline{f}/\underline{g}) = \frac{p(\underline{g}/\underline{f})p(\underline{f})}{p(\underline{g})} \qquad (3)$$

Under the assumptions of Gaussian distribution for noise and image and of uncorrelation for noise samples (for this purpose see [6]), taking the logarithm of both sides of eq.(3) and looking for the aximum value, we obtain

$$H^{T}S_{b}R_{n}^{-1}\left[\underline{g} - s(H\underline{f})\right] - R_{f}^{-1}(\underline{f} - \overline{f}) = 0 \qquad (4)$$

where: S_b is the Jacobian matrix of s, $\overline{f}$ is the mean value of the vector $\underline{f}$, R_f and R_n are the

variance arrays of the image and the noise.

Eq.(4) can be solved by means of an iterative method. There are two parts: the first is pertinent to the likelihood solution, the second regards the MAP solution[6]. From any iteration we like a proper balancing between these two solutions. It is therefore natural to write the iterative equation as a weighted average of the two solutions, obtaining the following relation

$$\left[\underline{g} - s(H\underline{f})\right]^T R_n^{-1} \left[\underline{g} - s(H\underline{f})\right] = 1 \qquad (5)$$

Further the "sectioned" form of the MAP criterion, in which the parameters involved in the restoration are adapted to the local characteristics of each image section, is to be considered of particular interest especially when the implementation is carried out by means of mini-computers (line followed in our work). The resulting sectioned iterative procedure is defined by the following relation

$$\underline{f}_{k+1}^{(m)} = \underline{f}_k^{(m)} + \Omega_{kQ}\left\{ \frac{\sigma_f^2}{\sigma_n^2} H_Q^T S_{bQ}\left[\underline{g}^{(m)} - \right.\right.$$
$$\left.\left. - s(H_Q \underline{f}_k^{(m)})\right] - \left[\underline{f}_k^{(m)} - \overline{\underline{f}}^{(m)}\right]\right\} \qquad (6)$$

where: $\Omega = \left[(R_n)/(R_n + R_f)\right] S_b^{-2}$, σ_f^2 is the variance of the image $\underline{f}$, σ_n^2 is the variance of the noise $\underline{n}$, Q is the section size, P is the size of the PSF matrix, m = 1,2,...,T, T is the number of sections.

The corresponding stopping criterion results

$$e_m = \left[\underline{g}^{(m)} - s(H_Q \underline{f}_k^{(m)})\right]^T R_n^{-1}\left[\underline{g}^{(m)} - s(H_Q \underline{f}_k^{(m)})\right] \qquad (7)$$

The convolutions required in eq.(6) can be performed by using the overlap-save method. The computation of e_m in eq.(7) is realized taking into account only the not-overlap points in a section [7]. An estimate of the variance of the noise in a section i can be obtained by the relation

$$\sigma_{ni}^2 = \sigma^2 \overline{d}_i^{2\gamma} \qquad (8)$$

where: $\overline{d}_i$ is the mean density of the section i of $\underline{g}$, γ is a constant dependent by the model of the noise, σ^2 is the mean estimate of the noise varance.

It is interesting to observe that, when the function s is approximated in a linear form, the MAP method of restoration becomes a linear filter and more standard techniques such as Wiener

filtering can be used [8].

3. MAP MODIFICATIONS AND IMPROVEMENTS

To implement the above described MAP technique in the sectioned form the overlap-save method can be applied, which implies that each section overlaps its adjacent ones by 2(P - 1) rows and columns, where P is the dimension of the PSF.

The first modification we decided with respect to the more standard approach regards the computation of the convolutions in eq.(6) directly in the space domain instead of using the FFT transformation. Some advantages are in this way resulting, when the implementation is performed by means of mini-computers: the constraint that the dimension of the section must be a power of two is removed, the computation time is lower for small sections, it is not necessary to take in memory the matrix H that has dimensions comparable with those of the considered sections.

A second modification regards the form of the PSF matrix, which we selected with a circular symmetry, due to the fact that the blurring occurred in the formation and recording of the considered images (as remote sensing) is in general estimated to have no preferential direction. This fact implies that in eq.(6) and eq.(7) $H = H^T$.

Another interesting point regards the selection of the section size Q: from one side a value of Q very small is attractive (as 0.1N, with N the image size, resulting in section areas 0.01 of the total one) to estimate in a more accurate way the noise variance, from the other one a small value of Q is implying a great number of sections (100 in the above case) with a high resulting processing time. Therefore a compromise between these two constraints must be followed.

Further, to minimize the number of the multiplications required by every convolution, the fact that H is a sparse matrix was utilized.

Another modification we made regards the stopping criterion (eq.(7)). The following solution was defined: to stop the iteration when the stopping parameter ARR becomes less than the theoretic value plus 10% of the section size or when the diminution of ARR (from the previous iteration) becomes less than 0.5. The first bound has the purpose of avoiding an excessive removing of ARR from the theoretic value, the second one has the task to stop the iteration when a new iteration doesn't produce an appreciable improvement.

4. APPLICATIONS TO DIGITAL IMAGE RESTORATION

At first the defined MAP techniques were tested on artificial images, by using known values for s, H and $\underline{n}$ (Gaussian type).

To evaluate the improvement obtained by the application of MAP techniques as above the mean-square-error (MSE) criterion was used. More precisely, by denoting with MSE_b the mean-square-error

between the original image and the blurred-noisy one and with MSE_r the mean-square-error between the original image and the restored one, the improvement due to the restoration is evaluated by means of the following relation

$$\eta_r = \frac{MSE_r - MSE_b}{MSE_b} 100 \qquad (9)$$

measured in percentage.

In the performed tests two types of PSF function were used: one corresponding to a space shift, the other of Gaussian form. The first one is very common in blurring effects, the second can be considered a good approximation of real blurred images (as in remote sensing, biomedicine).

The noise variance δ_n^2 was chosen on the basis of signal-to-noise (S/N) ratios, by computing the signal variance δ_f^2, that is

$$\delta_n^2 = \frac{\delta_f^2}{S/N} \qquad (10)$$

Moreover with the term S/N we mean the ratio between the signal variance and the variance of white Gaussian zero-mean noise.

Table 1 reports results obtained on a test image (letter C) with a PSF of size P=3, in the case of shifting or Gaussian form and $\gamma = 1$.

Table 1 - Percentual improvement for MSE on a test image (64x64)

S/N	shifting PSF	Gaussian PSF
10	25.77	19.79
20	39.25	30.13
40	52.59	37.64
60	62.25	43.46
80	68.44	46.18

In the application of the above restoration technique we examined also the utility of using a pre-processing digital filter. In particular a 2-D low-pass digital filter of FIR type designed by means of the window technique (Cappellini window [9]) was applied before the restoration: the filter has a circular symmetry in the frequency domain and a cut-off frequency such that to not eliminate the space frequencies of interest. The results obtained in a test image processing as above for a Gaussian PSF are reported in Table 2, in comparison with those obtained by using the 2-D digital filter alone or the restoration technique only.

As the Table 2 shows clearly the combination of the pre-processing 2-D digital filter and MAP restoration gives very good efficiency (high improvement for MSE) especially at S/N of low value (where the MAP alone as also shown by Ta-

Table 2 - Percentual improvement for MSE on a test image (64x64)

S/N	2-D filter	restorat.	2-D filter with restor.
10	15.35	19.79	34.10
20	10.45	30.13	36.94
80	6.28	46.18	47.30

ble 1 is not giving good improvements. Analogous results were obtained also by applying after the MAP restoration a 2-D low-pass digital filter, due to the fact that the filter is reducing the high frequency components and fluctuations introduced by the restoration process.

The above MAP techniques, also with a pre-processing or post-processing 2-D low-pass digital filter were applied to real images (remote sensing maps or biomedical images as scintigraphies of 64x64 size) with good enhancement and restoration results.

In conclusion the described techniques using MAP criterion, implemented by using minicomputers (a PDP 11-34 or HP 2100 A were in particular employed), can be considered very useful alternatives to other techniques such as inverse frequency filtering (Wiener type or other ones) or Kalman filtering for the restoration of blurred noisy images, the combination of 2-D low-pass filtering and MAP techniques appearing a good processing strategy in the case of low S/N values.

REFERENCES

[1] H.C. Andrews and B.R. Hunt, Digital Image Restoration, (Prentice Hall, Englewood Cliffs N.J., 1977).
[2] B.R. Hunt, The application of constrained-least-squares estimation to image restoration by digital computer, IEEE Trans. on Computers C-22 (1973) 805-812.
[3] F.M. Tomnovec and R.L. Mather, Experimental gamma-ray collimator sensitivity patterns, J. Appl. Phys. 28 (1957) 1208-1211.
[4] C.E.K. Mees, The Theory of the Photographic Process, (Mac Millan, New York, 1954).
[5] V. Cappellini, E. Del Re, P. Francolini and G. Taiuti, Some MAP Restoration Techniques with Application to Image Processing, Report Istituto di Elettronica, Firenze, R15-79 (1979).
[6] H.J. Trussell, Improved Methods of Maximum a Posteriori Image Restoration, Ph.D. Thesis, Univ. New Mexico, Albuquerque (1976).
[7] B.R. Hunt and H.J. Trussell, Sectioned methods for image restoration, IEEE Trans. Acoust. Speech Sign. Proces. ASSP-26 (1978).
[8] H.J. Trussell, Notes on linear image restoration by maximizing the a posteriori probability, IEEE Trans. Acoust. Speech Sign. Proces. ASSP-26 (1978).
[9] V. Cappellini, A.G. Constantinides and P. Emiliani, Digital Filters and Their Applications, (Academic Press, London-New York, 1978).