Suboptimum adaptive polarisation cancellers for dual-polarisation radars

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Abstract: In the paper some suboptimum adaptive polarisation techniques for the cancellation of partially polarised disturbances are analysed. They are based on suitable use of the estimates of the crosscorrelation between the dual-polarisation received signals. The performances of these techniques are evaluated and compared with that of the optimum setting of polarisation on reception. It is shown that the performances of the suboptimum techniques are affected by a quite limited cancellation loss, and related to the antenna polarisation basis used on reception. Some implementation aspects of these techniques are also discussed.

1 Introduction

In conventional radar systems the received backscattered wave is converted to a scalar signal; in this way the wave polarisation is not recovered.

To acquire the entire information contents of the backscattered wave, the wave polarisation information has to be retained by a vector measurement process. This can allow for improving target detection probability in the presence of a disturbance, through dual-polarisation signal processing [1, 2].

This operation requires that the radar be capable of decomposing the received wave into two orthogonally polarised components, which independently feed two identical and coherent reception channels. In other words, for the above purpose a dual-polarisation receiver is needed.

When polarisation diversity is used not only on reception but also in transmission, the object's scattering properties are determined completely, but the system complexity is significantly increased.

Diversity polarisation techniques can be applied to adapt polarisation in transmission and/or on reception, as is needed for disturbance cancellation [1, 3]. Polarisation adaptation can be implemented by adapting, through a two-step procedure [3], the antenna polarisation so that the average received power of a partially polarised disturbance is minimised [3].

Polarisation adaptation can also provide significant cancellation of stationary and highly polarised disturbances, when it is applied on reception only [1, 3–5]. The optimum adaptive selection of antenna polarisation on reception is based on estimation of the averaged Stokes vector of the received disturbance. The optimum polarisation is thus determined in terms of the corresponding Stokes vector, which has to be transformed into the two components of the related canonical polarisation vector. These two components are the complex weights of the linear combination of the two orthogonally polarised signal components, simultaneously available on reception, providing the minimum output average disturbance power, subject to a normality constraint for the polarisation vector on reception.

In this paper some alternative, suboptimum adaptation procedures are considered. They are discussed and their performances compared with those of the optimum procedure.

The analysed suboptimum estimation procedures are based on deriving the linear combination weights directly from the crosscorrelation estimates of the two orthogonally polarised components of the received electromagnetic (EM) field. This approach recalls that applied for sidelobe cancellation of disturbances received through an antenna array [6–8], as well as the scheme proposed by Nathanson [9] for adaptive polarisation cancellation of rain clutter.

The results of this analysis show that the suboptimum procedures achieve slightly inferior disturbance cancellation with respect to the optimum procedure. On the other hand, the system complexity is reduced.

2 Fundamental concepts

Here some fundamental analytical tools are briefly recalled to describe the signal processing techniques proposed in the following Sections.

2.1 Partially polarised waves

In a right-handed cartesian xyz co-ordinate system, the EM field vector of a plane, harmonic wave propagating along the z-axis (positive sense) can be represented by a complex vector given by

$$\boldsymbol{E}(z, t) = \begin{bmatrix} E_H(z, t) \\ E_V(z, t) \end{bmatrix} = \boldsymbol{h}(t) \exp [j(\omega t - kz)] \tag{1}$$

where the labels 'H' and 'V' denote the horizontal and vertical electric field components, respectively, k is the propagation constant and $\boldsymbol{h}(t)$ is a time-varying vector,

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which can be expressed as

$$h(t) = \begin{bmatrix} h_H(t) \\ h_V(t) \end{bmatrix} \tag{2}$$

where $h_H(t)$ and $h_V(t)$ are the complex envelopes of the horizontal and vertical field components, respectively, considered as outcomes of stationary and ergodic random processes, with phase uniformly distributed and independent of moduli. In this condition, the propagating wave is called 'partially polarised'.

When the factorisation $h(t) = a(t)[h_H, h_V]^T$ holds, where the superscript 'T' means transposition of the vector, the wave is said to be completely polarised, and its time-invariant polarisation is specified by the constant vector $h = [h_H, h_V]^T$.

A partially polarised wave can be described through the average Stokes vector $\bar{g}$, whose components are defined as follows [10]:

$$\bar{g} = \begin{bmatrix} \bar{g}_0 \\ \bar{g}_1 \\ \bar{g}_2 \\ \bar{g}_3 \end{bmatrix} \triangleq \begin{bmatrix} \langle |h_H(t)|^2\rangle + \langle |h_V(t)|^2\rangle \\ 2\langle \mathrm{Im}\,\{h_H^*(t)h_V(t)\}\rangle \\ \langle |h_H(t)|^2\rangle - \langle |h_V(t)|^2\rangle \\ 2\langle \mathrm{Re}\,\{h_H^*(t)h_V(t)\}\rangle \end{bmatrix} \tag{3}$$

where the symbol '$\triangleq$' means definition of the parameters on the left side, and $\langle(.)\rangle$ denotes 'time average'. The total average power associated to the wave, given by $\bar{g}_0$, meets the following condition:

$$\bar{g}_0^2 \geqslant \bar{g}_1^2 + \bar{g}_2^2 + \bar{g}_3^2 \tag{4}$$

In expr. 4 the equality holds when the wave is completely polarised, while $\bar{g}_1 = \bar{g}_2 = \bar{g}_3 = 0$ when the wave is completely unpolarised.

In terms of the Stokes vectors, a partially polarised wave can be uniquely decomposed as [11]

$$\bar{g} = [g_{0P}, \bar{g}_1, \bar{g}_2, \bar{g}_3]^T + [\bar{g}_0 - g_{0P}, 0, 0, 0]^T \tag{5}$$

where $g_{0P} = (\bar{g}_1^2 + \bar{g}_2^2 + \bar{g}_3^2)^{1/2}$ and $(\bar{g}_0 - g_{0P})$ are the average powers pertaining to two independent waves, the former completely polarised and the latter completely unpolarised.

The ratio between the average power of the completely polarised wave and the total power is called the 'degree of polarisation' p of the wave, and is thus given by [11]

$$p \triangleq (g_{0P}/\bar{g}_0) = \frac{(\bar{g}_1^2 + \bar{g}_2^2 + \bar{g}_3^2)^{1/2}}{\bar{g}_0} \tag{6}$$

When a partially polarised wave backscattered by a target, with average Stokes vector $\bar{g}$, is received by an antenna with polarisation h_r (i.e. the antenna transmits waves with polarisation vector h_r), the average received power is minimised when the normalised antenna Stokes vector is given by [10]

$$f(h_r) = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -\bar{g}_1/g_{0P} \\ -\bar{g}_2/g_{0P} \\ -\bar{g}_3/g_{0P} \end{bmatrix} \tag{7}$$

while the orthogonal polarisation yields the maximum average power.

The minimum and maximum received powers are, respectively, given by [10]

$$\bar{P}_{min} = \tfrac{1}{2}\bar{g}_0(1 - p)$$
$$\bar{P}_{max} = \tfrac{1}{2}\bar{g}_0(1 + p) \tag{8}$$

A useful description of partially polarised waves is also given by the complex correlation coefficient μ between the voltages v_r and $v_{r\perp}$ received from two orthogonally polarised antennas. This is defined as [12]

$$\mu \triangleq \frac{\langle v_r \cdot v_{r\perp}^*\rangle}{[\langle |v_r|^2\rangle\langle |v_{r\perp}|^2\rangle]^{1/2}} \tag{9}$$

with $0 \leqslant |\mu| \leqslant 1$.

It is also convenient to define the transformation of a polarisation vector h from the polarisation basis AB to the vector h' represented in a new polarisation basis $A'B'$. The latter is obtained through a linear transformation [13]:

$$h' \triangleq \begin{bmatrix} h_{A'} \\ h_{B'} \end{bmatrix} = [T]h = [T]\begin{bmatrix} h_A \\ h_B \end{bmatrix} \tag{10}$$

where the matrix $[T]$ can be expressed as [13]

$$[T] = \begin{bmatrix} 1 & -\rho^* \\ \rho & 1 \end{bmatrix} \frac{1}{\sqrt{(1 + |\rho|^2)}} \tag{11a}$$

$$\rho \triangleq h_{A'}(h_A)/h_{B'}(h_A) \tag{11b}$$

where $h_{A'}(h_A)$ and $h_{B'}(h_A)$ are the components of the vector h_A in the $A'B'$ basis, while ρ represents the complex polarisation ratio of the polarisation A expressed in the new basis.

2.2 Polar representations of polarisation

Several polar representations of the polarisation state can be usefully applied to analysing the polarisation properties of signals [14].

In the Poincaré sphere representation [15], any polarisation state of a completely polarised wave can be represented by a point P with cartesian co-ordinates which are expressed through Stokes parameters as follows [1]:

$$P \equiv (g_2/g_0, g_3/g_0, g_1/g_0) \tag{12}$$

where the parameters involved are not time-averaged because the wave is supposed to be completely polarised.

Polarisations are thus mapped on to the surface of a sphere, called the Poincaré sphere (see Fig. 1).

In such a representation the sphere poles correspond to the circular polarisations; the equatorial circle is the locus of the linear polarisations.

The extrema of each diameter correspond to a pair of orthogonal polarisations. Symmetrical points with

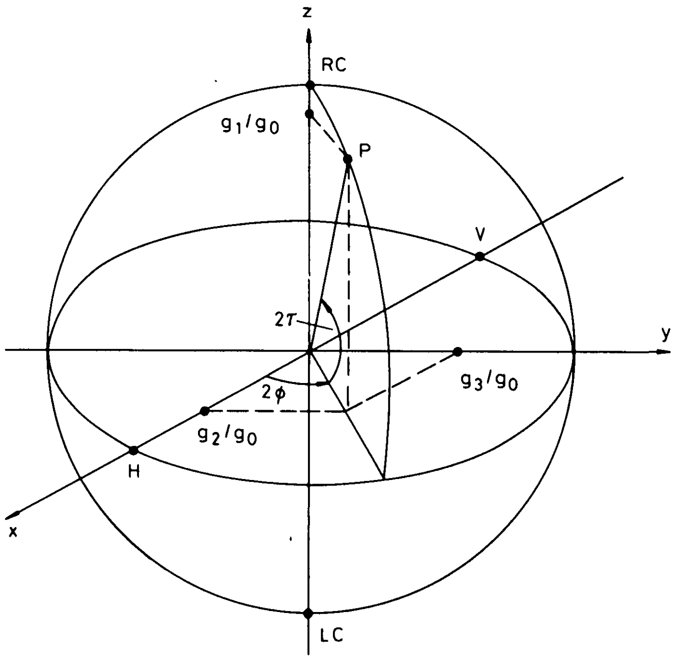

Fig. 1 *Poincaré sphere*

respect to the equatorial plane change polarisation for the rotation sense only.

Once the average Stokes parameters are used in eqn. 12, the Poincaré sphere representation is extended to the general case of partially polarised waves [16]. A partially polarised wave is thus represented by a point inside the Poincaré sphere. In fact, from eqns. 6 and 12 it follows that the degree of polarisation ($p < 1$) of a partially polarised wave equals the distance of the representative point P from the centre of the sphere.

3 Optimum polarisation adaptation for disturbance cancellation

Radar disturbance signals, such as clutter, are partially polarised. Sometimes, as for example with rain clutter, its degree of polarisation can be near unity. In this condition optimum selection of the antenna polarisation both in transmission and on reception can provide strong selective attentuation of the disturbance (see eqn. 8*a*), while increasing the signal/disturbance power ratio [17]. The latter achievement derives from different polarisation behaviour of target and clutter.

Selection of the transmitting polarisation can be implemented by making use of antenna polarisation agility or utilising the virtual polarisation adaptation technique on reception for virtual adaptation of the transmitting polarisation [3]. The implementation of these techniques needs either a noticeable increase in the radar transmitter complexity, or a complex signal processing on reception when the VPA technique is used.

For this reason the main core of this paper deals with the polarisation adaptation on reception only, which requires the simple linear combination of the signals present on two orthogonally polarised channels. This procedure can still achieve significant cancellation when applied to partially polarised clutter signals characterised by a high degree of polarisation, such as atmospheric clutter, jamming and some kinds of ground clutter [3].

The application of the optimum procedure to adapt polarisation on reception requires the determination of the average crosscorrelation estimate $\hat{M}_{12}$, and the estimation of the average powers $\hat{P}_1$ and $\hat{P}_2$ of two orthogonally polarised signal components of the disturbance. This is typically achieved through averages performed on signal samples of the observed radar signals, within a time-space window where stationary behaviour of the disturbance is expected.

If we denote by $s_1(i)$ and $s_2(i)$ the generic samples of the two orthogonally polarised signals available on reception, the following time averages can then be computed within the said azimuth-range window of the radar coverage:

$$\begin{aligned} \hat{M}_{12} &= \frac{1}{N} \sum_{i=1}^{N} [s_1(i) s_2^*(i)] \\ \hat{P}_1 &= \frac{1}{N} \sum_{i=1}^{N} |s_1(i)|^2 \\ \hat{P}_2 &= \frac{1}{N} \sum_{i=1}^{N} |s_2(i)|^2 \end{aligned} \tag{13}$$

where N is the number of samples within the set window.

Under the following hypotheses:

(*a*) the observed disturbance has stationary polarisation behaviour within the set window;

(*b*) the signal samples are mainly contributed by the disturbance to be cancelled (superclutter or superjamming visibility condition);

(*c*) the observed disturbance presents a sufficiently high degree of polarisation when measured within the set window;

then the quantities given by eqns. 13 can usefully be employed for the selective cancellation of disturbances in the central radar resolution cell contained in the set window, to enhance the signal/disturbance power ratio.

3.1 Implementation aspects

The following computational steps are then performed for optimum adaptation of polarisation on reception [3]:

(1) Calculate the estimates of the elements of the wave average Stokes vector as follows:

$$\begin{aligned} \bar{g}_0 &= \hat{P}_1 + \hat{P}_2 \\ \bar{g}_1 &= 2\,\mathrm{Im}\,\{\hat{M}_{12}\} \\ \bar{g}_2 &= \hat{P}_1 - \hat{P}_2 \\ \bar{g}_3 &= 2\,\mathrm{Re}\,\{\hat{M}_{12}\} \end{aligned} \tag{14}$$

(2) Calculate the Stokes polarisation vector $\boldsymbol{f}(\boldsymbol{h}_r)$ of the optimum polarisation through eqn. 7, with the Stokes vector elements $\{\bar{g}_i\}$ given by eqns. 14, i.e.

$$\begin{cases} g_{0P} = \sqrt{(\bar{g}_1^2 + \bar{g}_2^2 + \bar{g}_3^2)} \\ f_0 \;\; = 1 \\ f_i \;\; = -\bar{g}_i/g_{0P} \qquad i = 1, 2, 3 \end{cases}$$

(3) Given $\boldsymbol{f}(\boldsymbol{h}_r)$, calculate the optimum canonical polarisation vector $\boldsymbol{h}_r \triangleq [h_{r1}, h_{r2}]^T$ for optimum reception. This can be obtained through the following relationships:

$$\begin{aligned} |h_{r1}| &= \sqrt{[\tfrac{1}{2}(f_0 + f_2)]} \\ |h_{r2}| &= \sqrt{[\tfrac{1}{2}(f_0 - f_2)]} \\ \arg(h_{r1}) - \arg(h_{r2}) &= \arg(h_{r1} h_{r2}^*) = \tan^{-1}(f_1/f_3) \end{aligned} \tag{15}$$

where $\arg(h_{r1})$ or $\arg(h_{r2})$ can be chosen arbitrarily: the values of h_{r1} and h_{r2} are thus specified.

(4) Adapt correspondingly the polarisation through the following linear combination of the two orothogonally polarised signals $s_1(t)$ and $s_2(t)$ available on reception:

$$s_c(t) = h_{r1} s_1(t) + h_{r2} s_2(t) \tag{16}$$

where $s_c(t)$ is the residual output signal after adaptive polarisation cancellation of the disturbance.

The above computational steps are summarised in the scheme reported in Fig. 2. In such a scheme the following definitions are adopted: (i) CHS = (change sign), and (ii) a/b = (division). A double-frame box is also used to point out the operations giving rise to a complex-valued output quantity or signal.

3.2 Performance evaluation

The performance of any polarisation-based cancellation procedure can be suitably evaluated through a disturbance cancellation ratio expressed in decibels, and defined for polarisation adaptation on reception as [1]

$$C_r \triangleq 10 \log_{10} \frac{\bar{P}}{\bar{P}_c} \tag{17}$$

where $\bar{P}_c$ is the average power of the cancelled output signal, obtained after adapting the polarisation on reception, while $\bar{P}$ denotes the average power received through the least-powered between the two orthogonally polarised channels.

The total received power $\bar{g}_0$ is independent of the receiver antenna polarisation basis; i.e.

$$\bar{g}_0 = \bar{P}_1 + \bar{P}_2 = \bar{P}_{min} + \bar{P}_{max} \tag{18}$$

In eqn. 18 $\bar{P}_1$ and $\bar{P}_2$ are the average powers received through the actual orthogonally polarised channels;† $\bar{P}_{min}$ and $\bar{P}_{max}$, defined in eqns. 8, represent the minimum and maximum average received powers on the orthogonally polarised channels, when the antenna polarisation basis is the optimum one.

Then we have

$$\begin{aligned} \bar{P}_1 &= \bar{P}_{min} + \bar{P}_\Delta \\ \bar{P}_2 &= \bar{P}_{max} - \bar{P}_\Delta \end{aligned} \tag{19}$$

where $\bar{P}_\Delta$ is a positive quantity that accounts for the mismatch of the actual antenna basis, with respect to the optimum one.

If we express the actual and optimum antenna polarisation bases through two pairs of orthonormal polarisation vectors, the basis transformation of eqn. 10 can be applied to obtain the representation in the new basis of a generic vector $\boldsymbol{h}_{opt}$, expressed in the optimum basis. in such circumstances the transformation is completely specified by an appropriate value, ρ_0, of the complex parameter ρ defined in eqns. 11.

Based on this consideration, the expression of the average power $\bar{P}_\Delta$ can easily be derived (see Appendix 8.1) as a function of ρ_0:

$$\bar{P}_\Delta = p\,\frac{\bar{g}_0}{2}(1 - \alpha) \tag{20}$$

where

$$\alpha \triangleq \left[\frac{1 - |\rho_0|^2}{1 + |\rho_0|^2}\right] \tag{21}$$

Note that the parameter α expresses the mismatch between the actual and optimum antenna polarisation bases (for $|\alpha| = 1$, the actual antenna polarisation basis coincides with the optimum one).

From eqns. 8, 19 and 20 we also obtain

$$\bar{P}_1 = \frac{\bar{g}_0}{2}(1 - \alpha p) \tag{22a}$$

$$\bar{P}_2 = \frac{\bar{g}_0}{2}(1 + \alpha p) \tag{22b}$$

$$r \triangleq \bar{P}_2/\bar{P}_1 = \frac{1 + \alpha p}{1 - \alpha p} \tag{22c}$$

where r represents the power ratio between the signals received through the dual polarisation channels.

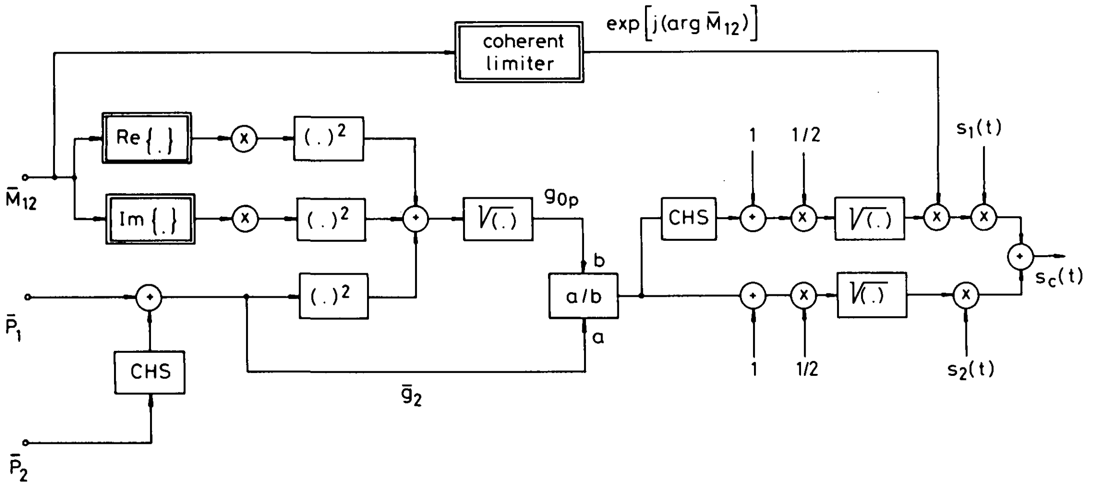

Fig. 2 *Computations involved in optimum adaptation procedure*

Based on eqns. 22, the condition $\bar{P}_1 < \bar{P}_2$ implies $0 < \alpha \leqslant 1$, while $\bar{P}_2 \leqslant \bar{P}_1$ implies $-1 \leqslant \alpha \leqslant 0$.

When the optimum adaptation procedure is applied on reception, the output power is given by

$$\bar{P}_{min} = \frac{\bar{g}_0}{2}(1 - p) \tag{23}$$

Therefore, taking into account eqns. 17 and 22*a*, the corresponding cancellation ratio thus gained on reception is expressed by

$$C_{r\,max} = 10\log_{10}\frac{\bar{P}}{\bar{P}_{min}} = 10\log_{10}\left[\frac{1 - |\alpha| p}{1 - p}\right] \tag{24}$$

In Figs. 3 and 4 the behaviour of r and $C_{r\,max}$ (expressed in decibels) are reported, respectively, as a function of $\alpha > 0$, for set values of p. The behaviour of r and $C_{r\,max}$ depicted in these Figures allows a complete evaluation of the performance that can be attained when optimally adapting polarisation on reception.

To this end, we note that r expresses the intrinsic disturbance cancellation obtained by merely selecting the least-powered signal received through the two orthogonally polarised channels. The parameter $C_{r\,max}$ instead

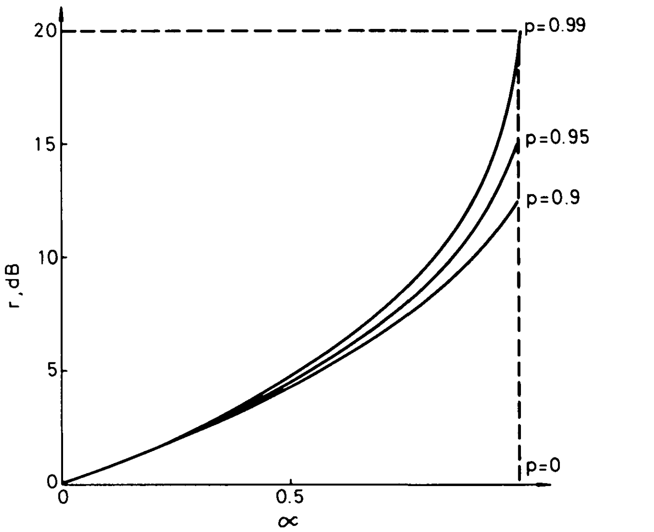

Fig. 3 *Behaviour of r (dB) as a function of α ∈ [0, 1], for set values of p*

† Note that $\bar{P}$ is equal to $\bar{P}_1$ or $\bar{P}_2$.

represents the additional cancellation improvement which can be gained through the linear combination of the received signals which synthesises the optimum polarisation on reception.

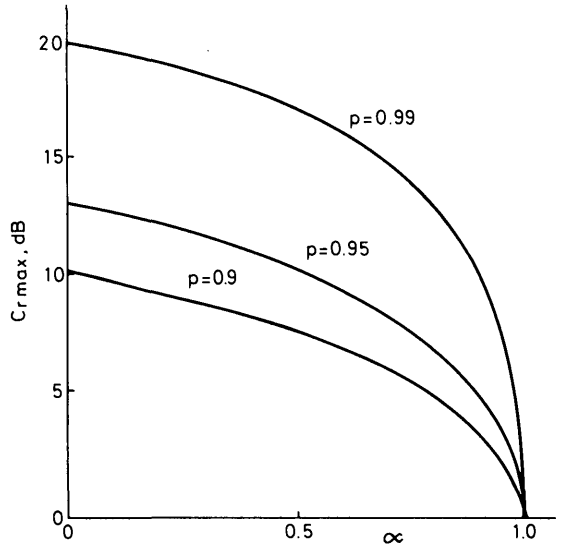

Fig. 4 *Behaviour of $C_{r_{max}}$ reported as a function of α, for different values of p*

Such an improvement increases as the polarisation basis mismatch increases ($\alpha \to 0$), and at a greater extent for higher values of p.

We also note that, when a sufficiently high value of r is observed on reception, the direct selection of the least-powered signal can be a practical solution, at least when upper-bounded values of p are expected, as in the case of the presence of background unpolarised noise.

4 Suboptimum polarisation adaptation for disturbance cancellation

In Section 3 the optimum procedure for polarisation adaptation on reception has been described. In this Section some alternative suboptimum procedures, based on the estimation of the signal crosscorrelation, are analysed. The results of the following analysis show that this achievement is accompanied by a limited loss of cancellation ratio with respect to the optimum procedure, when the hypotheses made for the application of the latter technique still hold. The system complexity of the suboptimum procedures is also examined.

4.1 First suboptimum adaptation procedure

On reception, the optimum cancellation procedure consists of linearly combining the dual polarisation signals $s_1(t)$ and $s_2(t)$, under a normality condition for the combination weights, so as to reject the signal contributed by the polarised wave component. Once the crosscorrelation parameter $\bar{M}_{12}$, defined as

$$\bar{M}_{12} \triangleq \langle s_1(t), s_2^*(t) \rangle \tag{25}$$

as well as the average powers $\bar{P}_1$ and $\bar{P}_2$ are known *a priori* or estimated, suboptimum but significant cancellation ratios can still be attained through the following alternative linear combinations [1, 5]:

$$s_{c1}(t) = s_1(t) - w_1 s_2(t) \tag{26a}$$

with

$$w_1 = \frac{\bar{M}_{12}}{\bar{P}_2} = (\bar{P}_1/\bar{P}_2)^{1/2} \mu \tag{26b}$$

or

$$s_{c2}(t) = s_2(t) - w_2 s_1(t) \tag{27a}$$

with

$$w_2 = \frac{\bar{M}_{12}^*}{\bar{P}_1} = (\bar{P}_2/\bar{P}_1)^{1/2} \mu^* \tag{27b}$$

It can easily be verified that linear processing, through eqns. 26 or 27, cancels the component of the unweighted signal ($s_1(t)$ and $s_2(t)$, respectively) which is correlated with the other signal.

This is also the basic approach followed in well known techniques applied for adaptive single-sidelobe cancellation of jamming signals received through two (main and auxiliary) antennas [6]. The weights w_1 and w_2 can also be suitably evaluated by means of a closed-loop crosscorrelation estimator, and this type of solution has also been proposed for adaptive polarisation cancellation of rain clutter [9] and jamming.

Whatever be the type of crosscorrelation estimator used to meet the adaptivity requirements, these cancellers ideally operate according to eqns. 26. Our objective is now to evaluate the ideal performance of such an operation and to compare it with the performance of the optimum procedure described in Section 3.

With reference to the suboptimum procedures implemented through eqns. 26 and 27, the following attenuation factors (expressed in decibels) are defined:

$$\begin{aligned} C_1 &\triangleq 10 \log_{10} (\bar{P}_1/\bar{P}_{c1}) \\ C_2 &\triangleq 10 \log_{10} (\bar{P}_2/\bar{P}_{c2}) \end{aligned} \tag{28}$$

where $\bar{P}_{c1}$ and $\bar{P}_{c2}$ are the corresponding average powers of the output signals $s_{c1}(t)$ and $s_{c2}(t)$.

Without loss of generality, in the following analysis we will assume that $\bar{P}_1 \leqslant \bar{P}_2$ (i.e. $0 \leqslant \alpha \leqslant 1$).

Through direct computation we obtain [1]

$$C_1 = C_2 = C_\mu = -10 \log_{10} (1 - |\mu|^2) \tag{29}$$

Note that once eqns. 26 or 27 are applied, $s_1(t)$ or $s_2(t)$ are attenuated equally, and the attenuation increases as $|\mu|$ approaches unity.

In actual fact, significant disturbance attenuation can be achieved because the complex crosscorrelation factor μ, defined in expr. 9, is strictly related to the degree of polarisation p of the received wave.

It can easily be shown (see Appendix 8.2) that the following relationship holds:

$$|\mu|^2 = p^2 \left[\frac{1 - \alpha^2}{1 - \alpha^2 p^2} \right] \tag{30}$$

which is defined when α and p are not simultaneously unity. According to eqn. 30, $|\mu|$ is plotted in Fig. 5 as a function of p for different set values of $|\alpha|$. Note that for $|\alpha| = 1$ (optimum polarisation basis) it is $|\mu| = 0$; consequently $s_{c1}(t) = s_1(t)$ and $s_{c2}(t) = s_2(t)$.

Therefore, once $s_1(t)$ is chosen (i.e. the least-powered signal), the same performance as that of the optimum cancellation procedure is achieved if $|\alpha| = 1$. This is not true for $|\alpha| \neq 1$, but, as will be shown later, $s_{c1}(t)$ provides better cancellation than $s_{c2}(t)$ in any case. This latter result is subject to the posed condition that $\bar{P}_1 \leqslant \bar{P}_2$.

A proper performance analysis cannot merely be based on the evaluation of the attenuation factors defined by eqns. 28. In fact, we have to take into account that while the linear combinations expressed by eqns. 26 and 27 perform a polarisation adaptation, they do not generally meet the normality requirement for the corresponding polarisation vectors. In particular, the polarisation settings corresponding to the linear combinations of eqns. 26 and 27 can, respectively, be expressed by the following complex polarisation ratios:

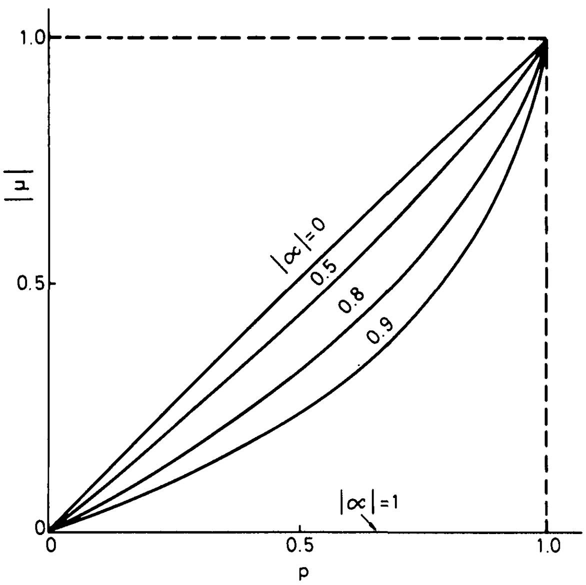

Fig. 5 *Absolute value of complex crosscorrelation factor, as a function of degree of polarisation, for different values of polarisation basis mismatch parameter* $|\alpha|$

$$\rho_1 = -w_1 = -\frac{\bar{M}_{12}}{\bar{P}_2} = -(\bar{P}_1/\bar{P}_2)^{1/2}\mu$$

$$\rho_2 = -\frac{1}{w_2} = -\frac{\bar{P}_1}{\bar{M}_{12}^*} = -(\bar{P}_1/\bar{P}_2)^{1/2}\frac{1}{\mu^*} \tag{31}$$

Exprs. 31 indicate that the polarisation settings associated with the application of eqns. 26 and 27 are generally different. These polarisations are also generally different from the optimum one: this aspect will be discussed in more detail later.

The evaluation of the cancellation ratio defined by eqn. 17 requires us to refer to the following canonical polarisation vectors, which have unitary modulus:

$$\boldsymbol{h}_{r1} \triangleq \frac{1}{(1+|w_1|^2)^{1/2}}\begin{bmatrix} 1 \\ -w_1 \end{bmatrix}$$

$$\boldsymbol{h}_{r2} \triangleq \frac{1}{(1+|w_2|^2)^{1/2}}\begin{bmatrix} -w_2 \\ 1 \end{bmatrix} \tag{32}$$

to be associated with the linear combinations of eqns. 26 and 27, respectively. In other words, the disturbance cancellation ratio has to be evaluated with respect to the following normalised output signals:

$$s_{c1n}(t) = \frac{1}{\sqrt{(1+|w_1|^2)}}\, s_{c1}(t) \tag{33a}$$

$$s_{c2n}(t) = \frac{1}{\sqrt{(1+|w_2|^2)}}\, s_{c2}(t) \tag{33b}$$

The cancellation ratio, defined according to eqn. 17, when eqns. 26 or 27 are applied, is thus, respectively, given by [1]

$$C_{r1} = C_1 + 10\log_{10}(1+|w_1|^2)$$
$$C_{r2} = C_2 + 10\log_{10}(1+|w_2|^2) - 10\log_{10}(\bar{P}_2/\bar{P}_1) \tag{34}$$

Through direct computation of eqns. 34 we then obtain (see Appendix 8.3)

$$C_{r1} = C_{r\,max} - C_\delta(p, \alpha)$$
$$C_{r2} = C_{r\,max} - C_\delta(p, -\alpha) \tag{35}$$

with $0 \leqslant \alpha \leqslant 1$, where $C_{r\,max}$ is the cancellation ratio obtained with the optimum cancellation procedure (see eqn. 24), and C_δ is given by

$$C_\delta(p, \alpha) \triangleq 10\log_{10}\left[\frac{(1+p)(1+\alpha p)}{p^2 + 2\alpha p + 1}\right] \tag{36}$$

which is defined for $-1 \leqslant \alpha \leqslant 1$. The function $C_\delta(p, \alpha)$ is always non-negative, and expresses the cancellation loss with respect to the optimum polarisation adaptation procedure. The behaviour of $C_\delta(p, \alpha)$ is shown in Fig. 6 as a function of α for different set values of p. The following properties can be demonstrated, which can also be inferred from Fig. 6:

(i) The cancellation loss C_δ decreases as α increases.

(ii) The cancellation loss is zero for $\alpha = 1$, whatever p is.

(iii) The cancellation loss is zero for $p = 0$ and $p = 1$, whatever α is.

(iv) For $\alpha > 0$ the cancellation loss takes on much lower values than for $\alpha < 0$. Since the former and latter cases apply to eqns. 26 and 27, respectively, the cancellation procedure represented by eqns. 26 has possibly to be selected.

From property (iv) it derives that the best performance is achieved when the least-powered orthogonally polarised channel is recognised, and the correspondent linear combination expressed by eqns. 26 is correspondingly and continuously applied. When the power ratio between the two orthogonally polarised channels changes with time and an adaptive operation is thus required, the above assumption is met, which is based on exchanging one input signal for the other, when passing from the case $\bar{P}_1 < \bar{P}_2$ to $\bar{P}_2 < \bar{P}_1$ or vice versa, while keeping the same processing scheme, in accordance with eqns. 26. This approach was proposed for adaptive polarisation cancellation of jamming [4, 18]. When using this switching procedure, the cancellation loss is given by $C_\delta(p, |\alpha|)$, and it

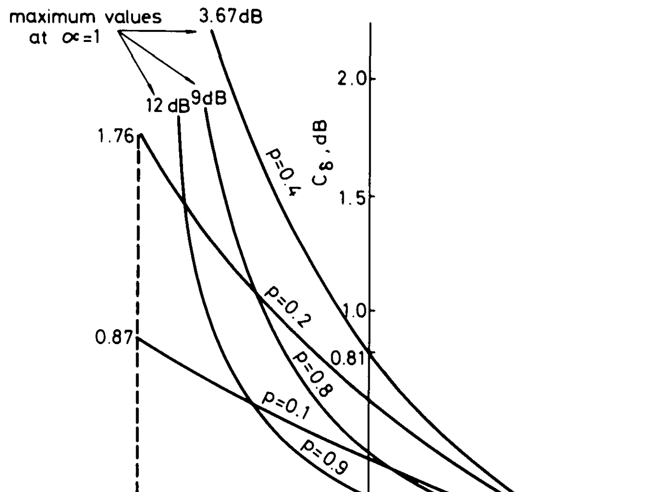

Fig. 6 *Cancellation loss* C_δ*, as a function of polarisation basis mismatch parameter* α*, for different values of polarisation degree p*

For negative values of α the curves approach different limit values of C_δ, which are indicated at top left of Figure takes on its maximum value. $C_{\delta 0}$ for $\alpha = 0$ and $p = p_0 = \sqrt{(2)} - 1$ (see Fig. 6). Such a value is given by

$$C_{\delta 0} = 10 \log_{10} \frac{\sqrt{(2)}}{4 - 2\sqrt{(2)}} = 0.81 \text{ dB} \tag{37}$$

Since the values of p providing large values of cancellation ratio approach unity, then the corresponding values of C_δ are still lower than $C_{\delta 0}$.

More explicitly, we can observe that for $p > 0.75$, corresponding to $C_{r\,max} > 6$ dB, the cancellation loss takes on a maximum of 0.5 dB at $\alpha = 0$; on average, it ranges from 0.2 to 0.3 dB, if α is uniformly distributed.

The above results show that disturbance cancellation by means of the proposed polarisation adaptation is quite effective.

We now evaluate the difference between the polarisation set on reception realised through the suboptimum procedure or the optimum one. If we represent these polarisations on the Poincaré sphere by the points P' and P, respectively, a proper measure of their difference is provided by their angular distance, $\delta > 0$, evaluated along the great circle joining P and P', regardless of the direction (see Fig. 7). In other words, δ is the angular distance, on the Poincaré sphere, between the suboptimum polarisation and the optimum one, providing the minimum received power.

The value of δ can be interpreted as the polarisation angle error introduced by the suboptimum procedure in the estimation of the reception polarisation. The value of δ is related to the cancellation loss associated with the suboptimum procedure used. This relationship is now exploited to evaluate δ.

Note that for this analysis δ is meaningful only when $p \neq 0$. Once both the wave decomposition expressed by eqn. 5 and the relationship between the received power and the geometric representation of polarisation on the Poincaré sphere [16] are taken into account, the following expression for the output power contributed by the polarised wave component, after polarisation setting, is easily obtained:

$$P_{cp} \triangleq g_{0p} \frac{1 - \cos \delta}{2} = g_{0p} \sin^2 (\delta/2) \tag{38}$$

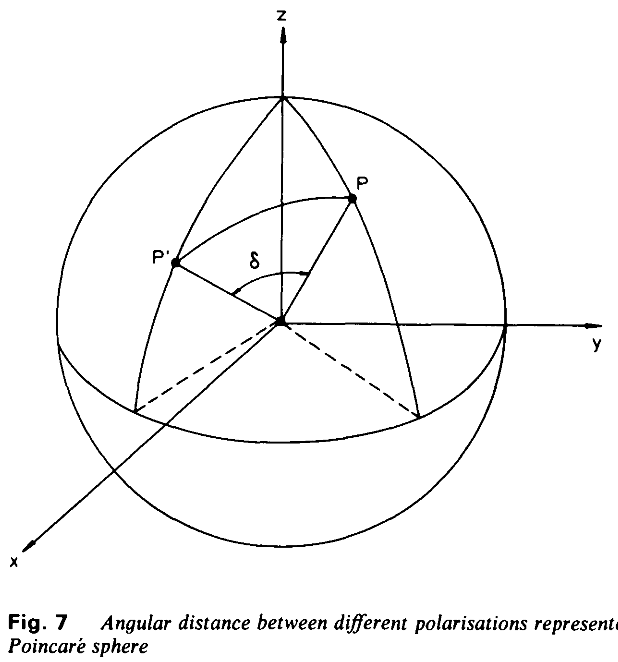

Fig. 7 *Angular distance between different polarisations represented on Poincaré sphere*

The total average output power $\bar{P}_c$ can be expressed as

$$\bar{P}_c = P_{cp} + \bar{P}_{cu} \tag{39}$$

where $\bar{P}_{cu}$ is the average power contributed by the unpolarised wave component, which, independently of the polarisation setting on reception, is given by

$$\bar{P}_{cu} = \tfrac{1}{2}(\bar{g}_0 - g_{0p}) \tag{40}$$

Once the optimum polarisation is set on reception, we obtain the minimum value of $\bar{P}_c$ (see eqn. 23).

We define the cancellation loss factor as

$$c_\delta \triangleq \log_{10}^{-1} \left[\frac{C_\delta}{10}\right] \tag{41}$$

Once eqns. 6, 38 and 40 are taken into account, we obtain

$$c_\delta \triangleq \frac{\bar{P}_c}{\bar{P}_{min}} = \frac{P_{cp}}{\bar{P}_{min}} + \frac{\bar{P}_{cu}}{\bar{P}_{min}} = 1 + \left(\frac{2p}{1 - p}\right) \sin^2 (\delta/2) \tag{42}$$

From the above relations, for $p \neq 0$ and $p \neq 1$, we obtain

$$\delta = 2 \sin^{-1} \left\{\left[\frac{1 - p}{2p} (c_\delta - 1)\right]^{1/2}\right\} \tag{43}$$

Taking into account eqns. 36 and 41, we then obtain

$$\delta = \delta(p, \alpha) = \begin{cases} 2 \sin^{-1} \left\{\left[\dfrac{(1 - \alpha)(1 - p)^2}{2(p^2 + 2\alpha p + 1)}\right]^{1/2}\right\} & p \neq 0, p \neq 1 \\ 0 \quad p = 1 \end{cases} \tag{44}$$

which expresses the polarisation angle error introduced by the suboptimum polarisation procedure described previously. In eqns. 44 the same limitations apply for the range of α, as indicated in applying the expression of $C_\delta(p, \alpha)$. The angular polarisation distance δ is plotted in Fig. 8 as a function of α for different set values of p.

The following properties can be demonstrated which can also be inferred from Fig. 8:

(*a*) δ is a decreasing function of p for any $\alpha \neq 1$.

(*b*) δ is a decreasing function of α for any $p \neq 1$.

(*c*) δ can take on large values for low values of p, corresponding to quite limited cancellation losses.

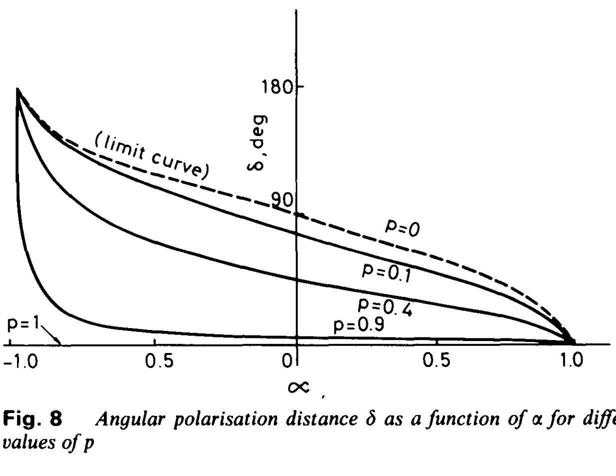

Fig. 8 *Angular polarisation distance δ as a function of α for different values of p*

(*d*) Once the least-powered channel is selected and eqns. 26 are correspondingly applied, the value of δ meets the following relation, whatever the value of $\alpha > 0$ and p:

$$\delta < 90^\circ \tag{45}$$

while the maximum value of δ, δ_{max}, attained for $p_0 = (\sqrt{(2)} - 1)$ and $\alpha = 0$, is given by

$$\delta_{max} = 2 \sin^{-1}\left(\frac{\sqrt{(2)} - 1}{\sqrt{(6) - 2\sqrt{(2)}}}\right) \simeq 45^\circ \tag{46}$$

The above properties show that even when the least-powered channel is selected for applying this suboptimum procedure, a significant polarisation error bias in δ can occur, but only when the degree of polarisation is low.

4.2 Second suboptimum adaptive procedure

An alternative procedure can be used, which reduces the cancellation loss for values of $|\alpha|$ approaching zero (i.e. large antenna polarisation basis mismatch). The following new (but alternative) linear combinations are considered:

$$s'_{c1}(t) = s_1(t) - w'_1 s_2(t) \tag{47a}$$

$$s'_{c2}(t) = s_2(t) - w'_2 s_1(t) \tag{47b}$$

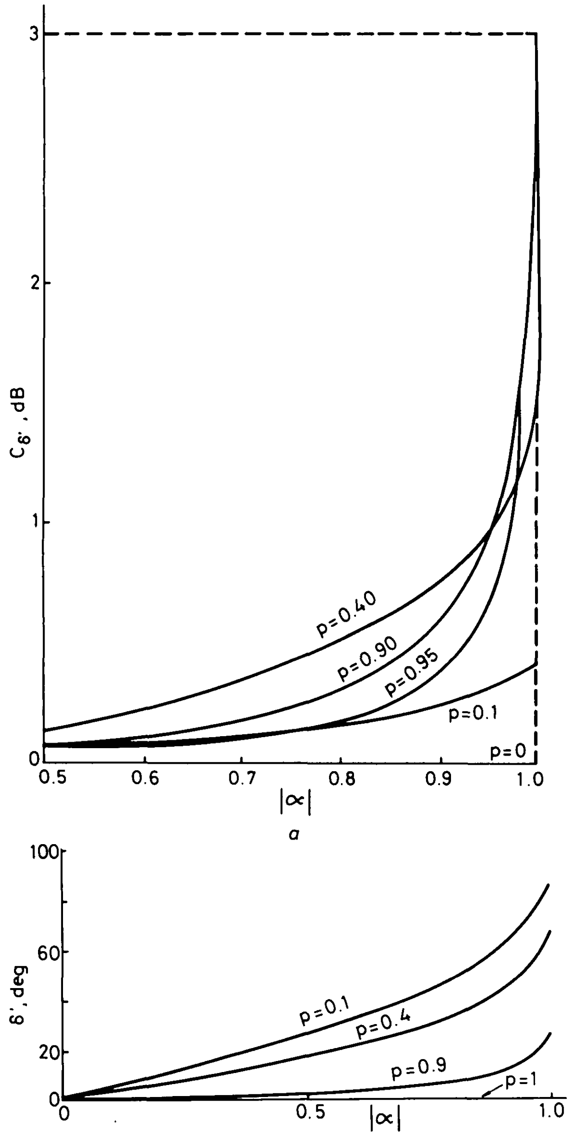

Fig. 9 *Cancellation loss $C_{\delta'}$, and angular polarisation distance δ' as functions of $|\alpha|$, for different values of p*
a Cancellation loss $C_{\delta'}$ *b* Angular polarisation distance δ'

where $-w'_1$ and $-w'_2$ are chosen so that the corresponding complex polarisation ratios are both equal to ρ', given by the complex geometric average of ρ_1 and ρ_2 (see eqns. 31); i.e.

$$-w'_1 = -(1/w'_2) = \rho' \triangleq \sqrt{(\rho_1 \rho_2)} \tag{48a}$$

Note that from eqns. 31 we obtain

$$\begin{aligned} \rho' &= -\sqrt{(\bar{P}_1/\bar{P}_2)}\,\frac{\mu}{|\mu|} \\ &= -\sqrt{(\bar{P}_1/\bar{P}_2)} \exp\,[j \arg(\mu)] \\ &= -\sqrt{(\bar{P}_1/\bar{P}_2)} \exp\,[j \arg(\bar{M}_{12})] \\ &= -\sqrt{(\bar{P}_1/\bar{P}_2)} \exp\,[j \arg(w_1)] \\ &= -\sqrt{(\bar{P}_1/\bar{P}_2)} \exp\,[-j \arg(w_2)] \end{aligned} \tag{48b}$$

The corresponding cancellation ratio can be expressed as

$$C'_r = C_{r\,max} - C_{\delta'}(p, \alpha) \tag{49a}$$

where $C_{\delta'}$ is the resulting cancellation loss. Based on eqn. 48*a*, both the exprs. 47*a* and 47*b* set the same polarisation on reception. Therefore the same value of C'_r is achieved, when applying either eqns. 47*a* or 47*b*.

Through simple computations we achieve the following explicit expression for $C_{\delta'}(p, \alpha)$ [see Appendix 8.4]:

$$C_{\delta'}(p, \alpha) = \begin{cases} 10 \log_{10}\left\{\dfrac{1 - \alpha^2 p^2}{1 - p}\left[1 - p\left(\dfrac{1 - \alpha^2}{1 - \alpha^2 p^2}\right)^{1/2}\right]\right\} & p \neq 0, p \neq 1 \\ 0 \quad p = 1 \end{cases} \tag{49b}$$

with $-1 \leqslant \alpha \leqslant 1$. Eqn. 49*b* applies to both procedures defined by eqns. 47*a* and 47*b*.

Once eqns. 42 and 43 are applied again to this case, an expression for the corresponding polarisation angle error δ' is then obtained:

$$\delta' \triangleq \delta'(p, \alpha) = \begin{cases} 2 \sin^{-1}\left\{\dfrac{1 - \alpha^2 p^2}{2p}\left[1 - p\left(\dfrac{1 - \alpha^2}{1 - \alpha^2 p^2}\right)^{1/2}\right] + \dfrac{p - 1}{2p}\right\}^{1/2} & p \neq 0, p \neq 1 \\ 0 \quad p = 1 \end{cases} \tag{50}$$

In Figs. 9*a* and *b*, respectively, $C_{\delta'}$ and δ' are plotted as functions of $|\alpha|$ for different set values of p.

The following properties can be demonstrated which can also be inferred from Fig. 9:

(*a*) $C_{\delta'}$ and δ' are symmetrical functions of α for any p.

(*b*) $C_{\delta'}$ and δ' are increasing functions of $|\alpha|$ for any p ($p \neq 0, p \neq 1$).

(*c*) $C_{\delta'}$ is within the bound of 3 dB (this is the limit maximum value asymptotically attained at $|\alpha| = 1$ for $p \to 1$).

(*d*) δ' is an increasing function of p for any α.

(*e*) $C_{\delta'}$ and δ' are zero for $\alpha = 0$ and any p.

In contrast to the suboptimum procedure above, we note that the performance of this alternative suboptimum procedure is near-optimum when the antenna polarisation basis mismatch is near-maximum ($|\alpha| = 0$).

If reliable information on the range of the antenna polarisation basis mismatch is available *a priori*, then the more convenient suboptimum cancellation procedure can be selected accordingly. On the other hand, when such a mismatch can vary widely according to the unforeseeable polarisation behaviour of the disturbance, the first suboptimum cancellation procedure provides more bounded values of cancellation loss and polarisation angle error.

However, a proper choice of the suboptimum procedure is also to be based on the implementation aspects, which we now discuss.

4.3 Implementation aspects

Some aspects are now considered with reference to the implementation of several adaptive polarisation schemes for disturbance cancellation on reception, based on the previously described suboptimum procedures.

4.3.1 Open-loop cancellers: An open-loop implementation of the proposed suboptimum adaptation procedures is obtained once the estimates, $\bar{M}_{12}$, $\bar{P}_1$ and $\bar{P}_2$, of the crosscorrelation parameters are known *a priori*, or estimated through the avaiable time averages, determined according to eqns. 13.

The following steps then apply to the first described suboptimum procedure:

(i) Compute $r = \bar{P}_2/\bar{P}_1$.
(ii) Exchange the inputs if $r < 1$.
(iii) pply eqns. 26.

The resulting computation schemes are reported in Figs. 10*a* and 10*b*; they refer to the case where the condition $\bar{P}_1 < \bar{P}_2$ is known *a priori* and the case where no information on the relation between $\bar{P}_1$ and $\bar{P}_2$ is *a priori* available, respectively.

As can be inferred from such schemes, the involved computations are less complex than those of the optimum adaptation scheme reported in Fig. 2. This is partially connected with the absence of the normalisation of the cancelled output signal, which would require the application of eqns. 33, so as to obtain

$$s_{cn}(t) = \frac{s_c(t)}{c} \tag{51}$$

with

$$c = (1 + |w_1|^2) \tag{52a}$$

or

$$c = (1 + |w_2|^2) \tag{52b}$$

However, owing to the following considerations, the application of eqn. 51 is not strictly needed:

(*a*) The ratio between the target signal power and the disturbance signal power is not sensitive to such a normalisation.

(*b*) Taking into account both the relations between $\bar{P}_1$ and $\bar{P}_2$ and eqns. 26*b* and 27*b*, the factor c can vary within a quite limited range ($1 \leqslant c \leqslant \sqrt{2}$).

(*c*) In the presence of unpolarised background noise only, $\alpha = 1$, then $s_c(t)$ coincides with one of the input signals (this is 'transparent' operation of the canceller).

(*d*) In the presence of a partially polarised disturbance (i.e. clutter, jamming), some sort of constant false alarm ratio technique is to be adopted, when further processing the cancelled signal, which can automatically account for the dynamic range of the processed signal (this technique is usually applied in the target detection process).

In the second adaptation procedure described in Section 4.2, the following computational steps are required instead:

(i) Compute $r = \bar{P}_2/\bar{P}_1$.
(ii) Exchange the inputs if $r < 1$ (optional operation).
(iii) Compute $w_1' = (1/r)^{1/2} \exp [j \arg (\bar{M}_{12})]$.
(iv) Apply eqn. 47*a*.

In the case that option (ii) is omitted, the resulting computation scheme is that reported in Fig. 11*a*. Such a scheme, which refers to the application of eqn. 47*a*, shows that the necessary computations are simpler than those of the optimum adaptation procedure, but are slightly more complex than those of the preceding suboptimum schemes.

As far as the output signal normalisation is concerned, the considerations made for the preceding procedure still apply: some additional considerations need to be devoted

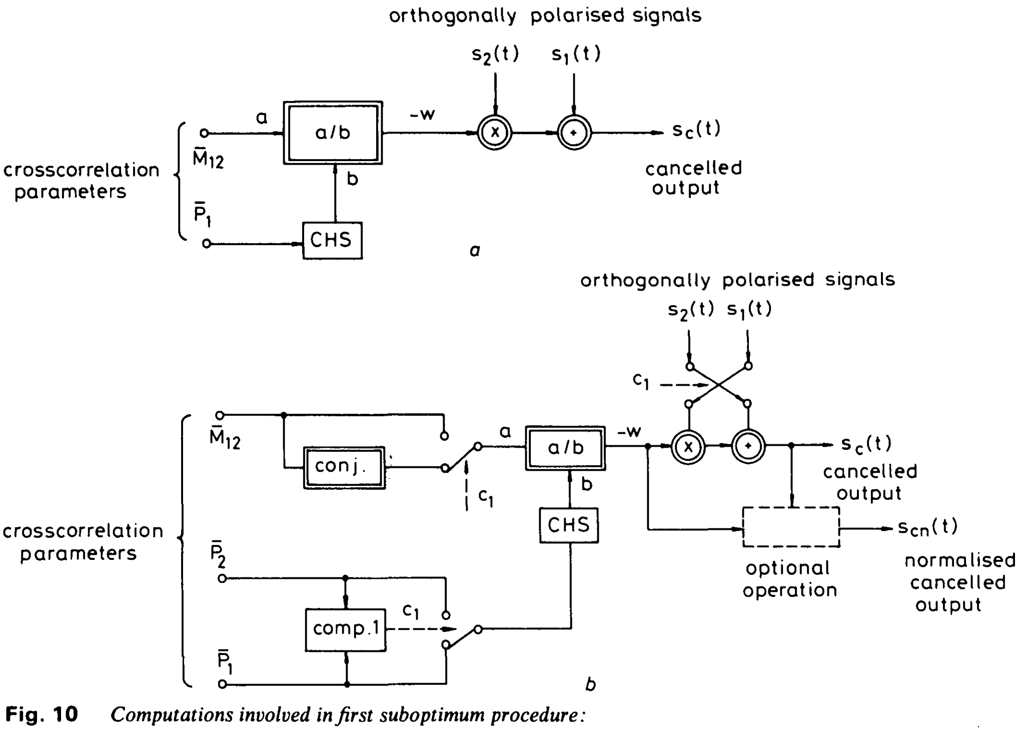

Fig. 10 *Computations involved in first suboptimum procedure:*
a $\bar{P}_1 < \bar{P}_2$ *b* Relation between $\bar{P}_1$ and $\bar{P}_2$ not known *a priori*

to point (*b*). above. We note indeed that the scheme of Fig. 11*a* can work with any limitation for the actual suboptimum procedure is operated, according to the scheme of Fig. 11*a*.

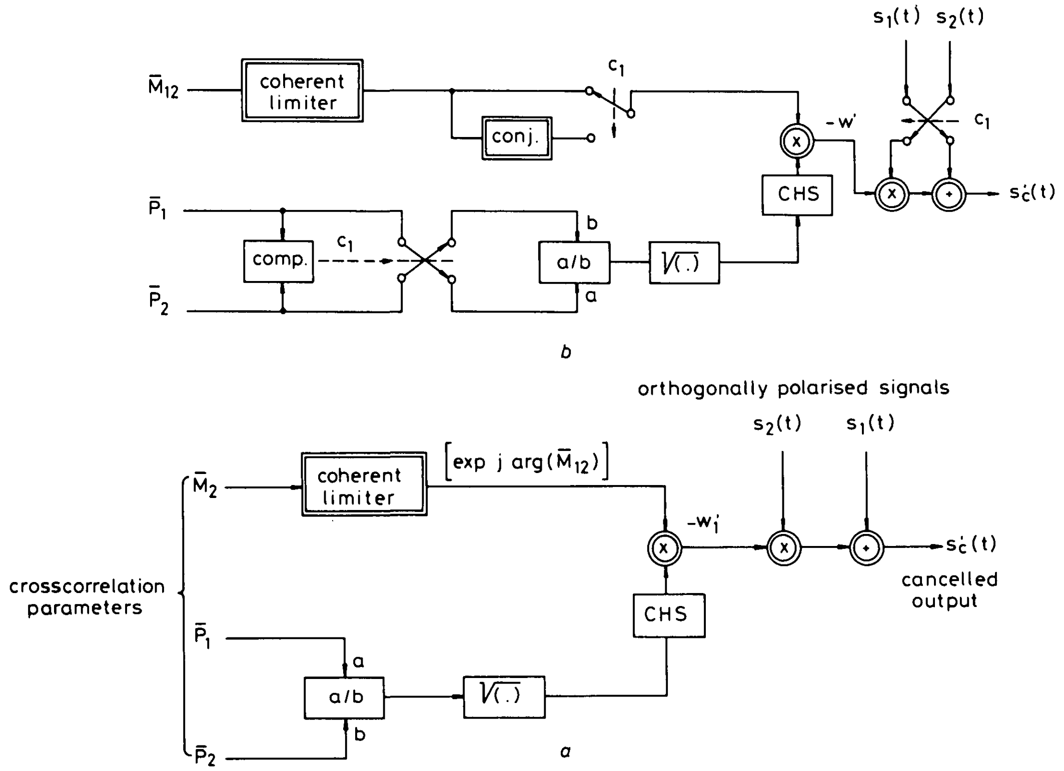

Fig. 11 *Computations involved in second suboptimum procedure*
a Canonical scheme
b Scheme limiting dynamic range of cancelled output

value of the power ratio r; this intrinsically implies a more extended dynamic range of the normalisation factor c and, consequently, of the non-normalised output signal $s_c(t)$. However, under suitable operating conditions of such a canceller, a significant polarisation basis mismatch is expected (low values of $|\alpha|$), which limits the dynamic range of r (see eqn. 22*c*), and consequently of both $|w_1'|$ (see eqns. 31) and c (see eqns. 52, where w' has to be substituted for w_1 or w_2). If this problem causes some concern, the alternative scheme of Fig. 11*b*, which applies option (ii), can be adopted to limit the dynamic range of the cancelled output signal. In fact, such a scheme achieves the same cancellation as that of the scheme of Fig. 11*a*, but the normalisation factor would now range as in the case of Fig. 10 ($1 \leqslant c \leqslant \sqrt{2}$).

Some consideration has to be devoted to step (ii) of both procedures (exchange of the input signals when $r < 1$).

As shown by the expression of w_1, w_2, w_1' and w_2' (see eqns. 31 and 48), in the case of strong polarisation basis mismatch ($|\alpha| \simeq 0$), this transient operation can also determine an abrupt change of both the amplitude and the absolute phase of the output signal. Under typical operating conditions such a transient operation is occasional, and it is caused by slow variations of the polarisation state of the disturbance. Under such particular conditions the transition typically occurs with a value of r near unity: according to eqns. 26*b*, 27*b* and 48*b* only the absolute phase of the output signal can then change significantly due to the phase conjugation of the weight w or w' after that transition.

The above transient behaviour has to be accounted for, when trying to mitigate its effect, especially when further coherent processing of the cancelled output is requested. This problem does not occur when the second suboptimum procedure is operated, according to the scheme of Fig. 11*a*.

4.3.2 Closed-loop cancellers: As proposed by Nathanson [9] for rain clutter cancellation, a closed-loop implementation of the first suboptimum procedure is given by the scheme in Fig. 12*a*, called an adaptive polarisation canceller (APC): this corresponds to the closed-loop adaptation scheme of Fig. 10*a*.

In contrast to the latter scheme of Fig. 10*a*, the closed-loop scheme embodies the crosscorrelation parameter estimation process. This is achieved by recursively processing the dual-polarisation signals along the sweep time, based on the locally stationary behaviour of the disturbance in the range domain. This behaviour is indeed typically shown by rain clutter and barrage jamming. When a significant antenna polarisation mismatch can

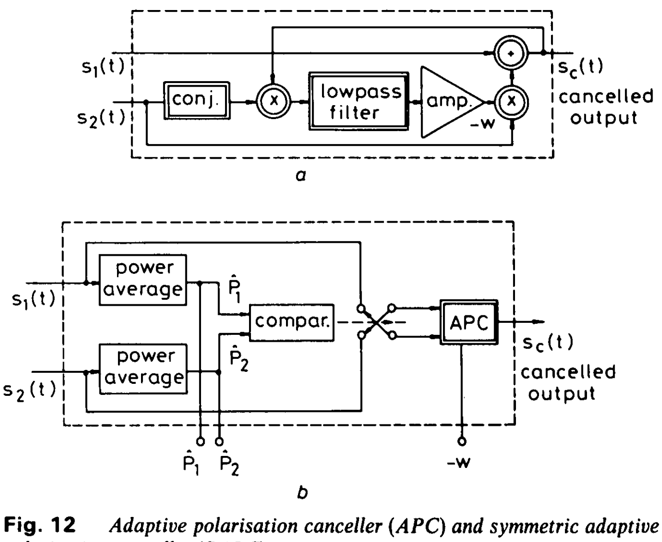

Fig. 12 *Adaptive polarisation canceller (APC) and symmetric adaptive polarisation canceller (SAPC)*
a APC *b* SAPC

occur (for example with jamming), the adaptation scheme of Fig. 12*b* is preferred [19], which is called the symmetric adaptive polarisation canceller (SAPC): that is the closed-loop implementation of the scheme of Fig. 10*b*.

Note that, together with the cancelled output, the open-loop cancellers inherently provide, through w and w', the estimated complex ratio of the polarisation providing the maximum cancellation: this recursive estimate can also be a useful first step to be exploited in devising more complex adaptive polarisation procedures, such as those proposed by Poelman and Guy [20]. In particular, such an estimate can be exploited for the application of the second suboptimum procedure, according to the scheme of Figs. 13*a* and *b*, derived from that of Figs. 11*a* and *b*, respectively, based on the relationships of eqns. 48.

On account of the possibility of significant antenna polarisation mismatch, assumed in both schemes of Figs. 13*a* and *b*, the SAPC is used: this ensures more stable operation of the closed-loop estimator, and consequently, more limited errors of the estimate of the polarisation ratio ρ' associated with w' [6].

When comparing the schemes of Fig. 12 with those of Fig. 13, we note that the former are less complex than the latter. Furthermore, based on the results of performance analysis of the first and second suboptimum adaptation procedures, evaluated in terms of cancellation loss, the former schemes achieve a quite limited loss, even in the case of a significant antenna polarisation mismatch (i.e. α around zero).

Therefore these considerations, based on the steady-state operation of the adaptive cancellers, suggest the use of the adaptation schemes of Fig. 12, which operate according to the first suboptimum procedure.

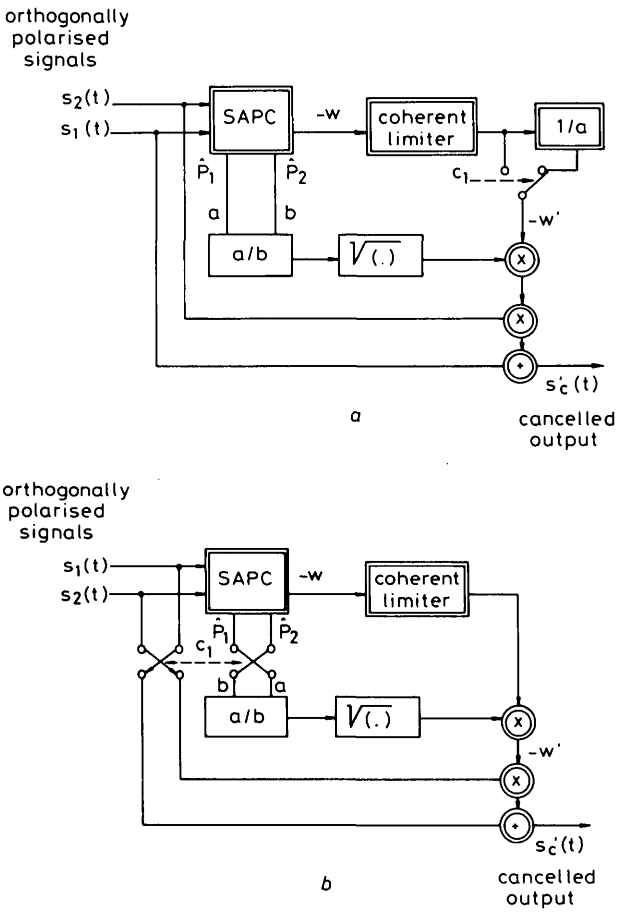

Fig. 13 *Closed-loop implementation of second suboptimum procedure*
a Canceller obtained by scheme of Fig. 11*a*
b Canceller obtained by scheme of Fig. 11*b*

However, some additional considerations are to be accounted for in the case that time nonstationary polarisation behaviour of the disturbance is expected with time, accompanied by a high antenna polarisation mismatch (i.e. α around zero). Under this condition the transient behaviour of the adaptive canceller should be considered. Some insight is obtained when considering the results of the polarisation angle error (see Figs. 8 and 9*b*). Note that such an error can take on large values only for low values of the degree of polarisation.

Generally this error is not of concern, since under a low degree of polarisation the disturbance cannot be adequately cancelled, even by the optimum polarisation adaptation procedure. However, while the polarisation error bias is not of concern to the stationary, fairly polarised disturbance, it can perhaps degrade the transient behaviour of closed-loop adaptive polarisation cancellers devised to track the polarisation of a limited nonstationary disturbance. In particular, the above polarisation error bias can result in an enhanced cancellation loss, when the degree of polarisation of the disturbance changes with time (range) at a rate that is comparable with the adaptation time of the crosscorrelation estimate performed by the APC or the SAPC. This can sometimes occur both for fast variation of the intrinsic polarisation behaviour of the disturbance as well as for the variation of the disturbance/background power ratio, which affects the degree of polarisation of the observed signals. In these cases reducing the polarisation angle error could then be advisable, even in the case of a low degree of polarisation. These considerations would suggest the use of the schemes of Fig. 13.

5 Comparisons and conclusions

The analysis carried out in this paper shows that the proposed suboptimum procedures of adapting polarisation on reception for the cancellation of a disturbance introduce a quite limited cancellation loss with respect to the optimum adaptation procedure, while presenting several advantages for actual implementation.

The proposed suboptimum adaptation procedures perform a linear combination of the orthogonal polarisation signals available on reception, based on the estimated crosscorrelation parameters of such signals, while removing the normality constraint for the polarisation vector correspondingly synthesised on reception.

The performance analysis carried out shows that the antenna polarisation basis mismatch plays a dominant role in the cancellation loss with respect to the optimum procedure, as well as in the proper choice between the alternative suboptimum schemes.

In the case that a limited polarisation basis mismatch is expected, application of the first suboptimum procedure is more convenient, in terms of both cancellation loss and system complexity (the schemes of Fig. 10). For example, this condition applies to the case of rain clutter cancellation, when the antenna polarisation basis is circular.

In the case that a strong antenna polarisation basis mismatch is expected, application of the second suboptimum procedure (the schemes of Fig. 11) generally provides minimum cancellation loss.

The antenna polarisation basis mismatch, represented by the parameter α, can be quite variable, due to the nonstationary polarisation behaviour of the disturbance specifically being handled. This condition usually applies to the case of ground clutter and barrage jamming. Under such a condition, application of either the first or second suboptimum procedure can be considered (the schemes in Figs. 10 and 11). A proper choice between the different schemes can be made when accounting for the following features:

(*a*) Application of the first suboptimum procedure strictly requires a switching operation to exchange the input signal when the average power of one set signal overcomes that of the other signal.

(*b*) Through operation (*a*) the first suboptimum procedure attains a more bounded cancellation loss (<0.81 dB) than the second procedure.

(*c*) The superior limit of the cancellation loss with the second procedure is 3 dB, attained with a matched antenna polarisation basis. However, even with only a slight antenna polarisation basis mismatch, such a loss decreases significantly for a high degree of polarisation.

(*d*) The switching operation described in (*a*) can determine an abrupt change of both the amplitude and the absolute phase of the output signal.

(*e*) The switching described in (*a*) is not required for application of the second suboptimum procedure. However, in such an application the switching operation may be desirable to limit the dynamic range of the output signal.

(*f*) Application of the first suboptimum procedure generally results in a less complex implementation scheme.

An advantage of the proposed adaptation procedures is also given by the possibility of their direct implementation through closed-loop cancellers, which perform recursive processing of the orthogonally polarised signals along the sweep time (the scheme in Fig. 12), while intrinsically estimating the necessary crosscorrelation parameters. This type of implementation is of interest for the cancellation of rain clutter [9] and barrage jamming [5]. The resulting schemes derived from the first procedure, called APC (see Fig. 12*a*) and SAPC (Fig. 12*b*), are quite simple. They also intrinsically provide an estimate of the polarisation ratio of the polarisation providing the minimum output signal power. This estimate can also be used to device alternative schemes operating according to the second suboptimum procedure (see Figs. 13*a* and *b*).

A polarisation angle error has been defined, and explicitly evaluated for both the suboptimum procedures, which expresses the difference between the polarisation actually set and that providing the minimum output power on reception. This error can take large values only for low values of the degree of polarisation. Therefore such an error is not generally of concern. However, such an error may cause a transient performance degradation of the closed-loop adaptive polarisation cancellers, as in the case where the polarisation state of the disturbance is not stationary.

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8 Appendix

8.1 Demonstration of eqn. 20

When considering a backscattered wave with polarisation $\boldsymbol{h}_s(t)$ (defined in the target co-ordinate system), the voltages v_{1opt} and v_{2opt} and the average powers $\bar{P}_{min}$ and $\bar{P}_{max}$ received by the optimum orthogonally polarised antennas (with polarisation vectors $\boldsymbol{i}_{1opt}$ and $\boldsymbol{i}_{2opt}$) are given by

$$\begin{aligned} v_{1opt} &= \boldsymbol{h}_s \cdot \boldsymbol{i}_{1opt} \qquad & v_{2opt} &= \boldsymbol{h}_s \cdot \boldsymbol{i}_{2opt} \\ \bar{P}_{min} &= \langle v_{1opt} v_{1opt}^* \rangle \qquad & \bar{P}_{max} &= \langle v_{2opt} v_{2opt}^* \rangle \end{aligned} \tag{53}$$

where v_{1opt} and v_{2opt} are the components of the vector $\boldsymbol{h}_s$ in the optimum antenna polarisation basis. This basis gives rise to the minimum and maximum average powers received by the orthogonally polarised channels.

When operating a transformation of the optimum polarisation basis to the $\boldsymbol{i}_1 - \boldsymbol{i}_2$ basis of the actual antenna, through eqn. 10, we obtain

$$\boldsymbol{h}'_s = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [\boldsymbol{T}]^{-1}\boldsymbol{h}_s$$
$$= [1 + |\rho_0|^2]^{-1/2} \begin{bmatrix} v_{1opt} - \rho_0^* v_{2opt} \\ -\rho_0 v_{1opt} + v_{2opt} \end{bmatrix} \quad (54)$$

where ρ_0 is the proper value of the complex polarisation ratio in the optimum basis, given by eqn. 11*b*. It follows that the average powers $\bar{P}_1$ and $\bar{P}_2$, received through the actual orthogonally polarised channels, can be expressed as

$$\bar{P}_1 = \frac{\langle |v_{1opt}|^2 \rangle + |\rho_0|^2 \langle |v_{2opt}|^2 \rangle}{1 + |\rho_0|^2} = \frac{\bar{P}_{min} + |\rho_0|^2 \bar{P}_{max}}{1 + |\rho_0|^2}$$
$$\bar{P}_2 = \frac{\langle |v_{2opt}|^2 \rangle + |\rho_0|^2 \langle |v_{1opt}|^2 \rangle}{1 + |\rho_0|^2} = \frac{\bar{P}_{max} + |\rho_0|^2 \bar{P}_{min}}{1 + |\rho_0|^2} \quad (55)$$

where we have accounted for incorrelation between the signals received through the optimum orthogonally polarised channels.

From eqns. 55, once both the definition expr. 21 and eqns. 19 are taken into account, eqn. 20 is then derived.

8.2 Demonstration of eqn. 30

Here we derive the expression for $|\mu|$ as a function of both the polarisation degree p and the parameter α. From eqn. 9 we have

$$|\mu|^2 \triangleq \frac{|\langle v_r v_{r\perp}^* \rangle|^2}{[\langle |v_r|^2 \rangle \langle |v_{r\perp}|^2 \rangle]} \quad (56)$$

Moreover, from eqns. 8, 20 and 21 it follows that

$$\langle |v_r|^2 \rangle = \bar{P}_1 = \frac{\bar{g}_0}{2}(1 - \alpha p)$$
$$\langle |v_{r\perp}|^2 \rangle = \bar{P}_2 = \frac{\bar{g}_0}{2}(1 + \alpha p) \quad (57)$$

Based on the definition of the average Stokes vector, we can write

$$\langle |v_r v_{r\perp}^* \rangle|^2 = \frac{\bar{g}_1^2 + \bar{g}_3^2}{4} \quad (58)$$

which, through the definition of p given by eqn. 6, becomes

$$|\langle v_r v_{r\perp}^* \rangle|^2 = \frac{p^2 \bar{g}_0^2 - \bar{g}_2^2}{4} \quad (59)$$

Moreover, from the definition of the Stokes vector it results that

$$\bar{g}_2^2 = [\bar{P}_2 - \bar{P}_1]^2 = \alpha^2 p^2 \bar{g}_0^2 \quad (60)$$

Finally, from eqns. 56, 57, 59 and 60, eqn. 30 is derived, which is defined when α and p are not both unity.

8.3 Demonstration of eqns. 35 and 36

Eqns. 35 are directly derivable from eqns. 34 when eqn. 29 is substituted in eqns. 34, thus giving

$$C_{r1} = -10 \log_{10}(1 - |\mu|^2) + 10 \log_{10}(1 + |w_1|^2)$$
$$C_{r2} = -10 \log_{10}(1 - |\mu|^2) + 10 \log_{10}[(1 + |w_2|^2)(\bar{P}_1/\bar{P}_2)] \quad (61)$$

Then the expressions for w_1 and w_2, given by eqns. 26*b* and 27*b*, can be substituted in eqns. 61:

$$C_{r1} = 10 \log_{10}\left[(1 - |\mu|^2)^{-1}\left(1 + \frac{\bar{P}_1}{\bar{P}_2}|\mu|^2\right)\right]$$
$$C_{r2} = 10 \log_{10}\left[(1 - |\mu|^2)^{-1}\left(\frac{\bar{P}_1}{\bar{P}_2} + |\mu|^2\right)\right] \quad (62)$$

Because $|\mu|^2$ can be expressed as in eqn. 30, while $\bar{P}_1$ and $\bar{P}_2$ are given by eqns. 22*a* and *b*, respectively, it follows that

$$C_{r1} = 10 \log_{10}\left[\frac{(1 + 2\alpha p + p^2)(1 - \alpha p)}{(1 - p^2)(1 + \alpha p)}\right]$$
$$C_{r2} = 10 \log_{10}\left[\frac{(1 - 2\alpha p + p^2)}{(1 - p^2)}\right] \quad (63)$$

where α is positive for the posed hypothesis.

Then, because eqn. 24 holds, eqns. 35 are obtained, where $C_\delta(p, \alpha)$ is defined as in eqn. 36.

8.4 Demonstration of eqns. 49

The expressions for C'_{r1} and C'_{r2} derive from those of eqns. 34 defining C_{r1} and C_{r2}, and can be written as

$$C'_{r1} = 10 \log_{10}\left(\frac{\bar{P}_1}{\bar{P}'_{c1}}\right) + 10 \log_{10}(1 + |w'_1|^2)$$
$$C'_{r2} = 10 \log_{10}\left(\frac{\bar{P}_2}{\bar{P}'_{c2}}\frac{\bar{P}_1}{\bar{P}_2}\right) + 10 \log_{10}(1 + |w'_2|^2) \quad (64)$$

The expressions for $\bar{P}'_{c1}$ and $\bar{P}'_{c2}$ can be calculated as functions of $|\mu|$, $\bar{P}_1$ and $\bar{P}_2$ from eqns. 47:

$$\bar{P}'_{c1} = (1 - |\mu|)2\bar{P}_1$$
$$\bar{P}'_{c2} = (1 - |\mu|)2\bar{P}_2 \quad (65)$$

When substituting in eqns. 64 the expression obtained for $|\mu|$ (see eqn. 30), $\bar{P}_1$ and $\bar{P}_2$ (eqns. 22), and $\bar{P}_{c1}$ and $\bar{P}_{c2}$ (eqns. 65), it follows that

$$C'_r = C'_{r1} = C'_{r2} = -10 \log_{10}[2(1 - |\mu|)] + 10 \log_{10}[(\bar{P}_1 + \bar{P}_2)/\bar{P}_2]$$
$$= -10 \log_{10}\left[(1 + \alpha p)\left(1 - p\sqrt{\left[\frac{1 - \alpha^2}{1 - \alpha^2 p^2}\right]}\right)\right] \quad (66)$$

Through these equations the expression for $C_{\delta'}$ (eqn. 49*b*) is easily derived from eqn. 49*a*.