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DEGRADATION OF CONSTANT-ENVELOPE SIGNALS DUE TO BAND-LIMITED FILTERING

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Phase or frequency modulations with constant signal envelope are largely used in digital communication systems. When a modulated waveform is passed through a filtering operation, the envelope at the filter output has amplitude fluctuations, which reduce the performance of the communication system.
In this paper, the effects of filtering operations on phase or frequency modulations as PSK, FSK and MSK are theoretically analyzed when the low-pass filter is a four-pole Butterworth filter. The values of the interference intersymbol disturbance are presented. The actual bit error probability, due both to the interference intersymbol disturbance and channel noise, is derived. As a particular application of this analysis, the performance of a communication system for the simultaneous transmission of voice and data is also derived.

1. INTRODUCTION

Signals with constant envelope satisfy the following constraints [1]:

$$\begin{cases} u^2(t)+u^2(t-T) = c_1 & 0 \leq t \leq T \\ u^2(t)+u^2(t+T) = c_2 & -T \leq t \leq 0 \end{cases} \qquad (1)$$

where c_1 and c_2 are two constants and $u(t)$ denotes the baseband pulse for the modulation operation. In a typical communication chain with finite bandwidth, these signals have not constant envelopes.

Indeed, the transmitter and receiver filters employed to reduce the bandwidth of the communication channel give rise to envelope fluctuations [2],[3],[4]. Moreover, as the bit-duration T and the filter badnwidth B product decreases, these fluctuations increase. The distortions introduced by non-linearities in the communication chain, e.g. by high-power amplifiers, are then enhanced. The purpose of this paper is to evaluate the effect of band-limiting by an analytical approach and computer simulation filtering on constant envelope signals as PSK, FSK and MSK.

The block-diagram of the communication chain considered in this paper is shown in Fig. 1. The transmitting filter usually has a wideband and introduces negligible fluctuations on the transmitted signal; for this reason, it has been assumed as an ideal wideband system.

The receiving filter is, in many cases, a narrowband filter which introduces intersymbol interference and envelope fluctuations on the received signal. In this paper, the receiving filter is considered as a four-pole Butterworth low-pass filter.

If $f(t)$ denotes a shaping function defined as:

$$f(t) = \begin{cases} 0 & 0 \leq t \leq T \quad \text{PSK} \\ u(t)-u(t-T) & 0 \leq t \leq T \quad \text{FSK and MSK} \end{cases} \qquad (2)$$

the transmitted signal $s(t)$ can be written as:

$$s_k(t) = A\cos(\omega_c t + d_k \pi h f(t) t + \phi_k) \qquad (3)$$

where A denotes the signal amplitude, $f_c=\omega_c/2\pi$ the frequency of the carrier, and d_k is the k-th information symbol, which is assumed as a binary independent equiprobable random variable with values ± 1.

The modulation index h is:

$$h = \begin{cases} 0 & \text{PSK} \\ 1/T & \text{FSK} \\ 1/2\,T & \text{MSK} \end{cases} \qquad (4)$$

while the phase-term ϕ_k is defined in the following way if a PSK signal is considered:

$$\phi_k = \begin{cases} 0 & \text{if } d_k = 1 \\ \pi & \text{if } d_k = -1 \end{cases} \qquad (5)$$

ϕ_k is set equal to zero for a FSK signal, while in MSK it is defined by the following recursive relationship:

$$\begin{cases} \phi_{k+1} = \phi_k + (d_k - d_{k+1})\pi h k \\ \phi_o = 0 \end{cases} \tag{6}$$

For the previously described digital modulated signals, the equivalent baseband pulse has been considered and the output signals when using a four-pole Butterworth low-pass filter have been derived by an analytical approach.

The four-pole Butterworth impulse response is given by [3]:

$$h(t) = 2\pi f_T \{c_1 \exp(a2\pi f_T t)\cos(-b2\pi f_T + \theta) + c_2 \exp(b2\pi f_T t)\cos(-a2\pi f_T t + \phi)\} \quad \text{for } t \geq 0 \tag{7}$$

where:

$$\begin{cases} c_1 = 1 & c_2 = 2.4142 \\ a = \cos(5\pi/8) & b = \cos(7\pi/8) \\ \theta = 2.74889 & \phi = -1.1781 \end{cases} \tag{8}$$

and f_T represents the -3dB cutoff frequency of the filter.

The filter response to a baseband impulse p(t) is:

$$v(t) = p(t) * h(t) \tag{9}$$

where * denotes the convolution operation. The analytical expression for p(t) is:

$$p(t) = \begin{cases} u(t)-u(t-T) & 0 \leq t < T & \text{PSK} \\ \sin\left(\frac{2\pi t}{T}\right) & 0 \leq t \leq T & \text{FSK} \\ \sin\left(\frac{\pi t}{T}\right) & 0 \leq t \leq T & \text{MSK} \end{cases} \tag{10}$$

The result of the convolution operation (9) can be written as:

$$v(t) = \begin{cases} 0 & t < 0 \\ k(t)-k(o) & 0 \leq t \leq T \\ k(t)-k(t-T) & t > T \end{cases} \tag{11}$$

where k(t) depends on the particular p(t). In the case of a PSK signal, we have:

$$\begin{aligned} k(t) &= c_1 \exp(a2\pi f_T t)\cos(-b2\pi f_T t + \theta - 5\pi/8) + c_2 \exp(b2\pi f_T t)\cos(-a2\pi f_T t + \phi - 7\pi/8) \\ k(o) &= c_1 \cos(\theta - 5\pi/8) + c_2 \cos(\phi - 7\pi/8) \end{aligned} \tag{12}$$

The analytical expression for k(t) in the case of FSK and MSK signals is given in [3].

In Fig. 2 a typical filtered MSK waveform by considering a four-pole Butterworth low-pass filter with a $f_T T$ product equal to 2.5 is shown. Fig. 3 shows the normalized response of the same filter to a MSK baseband pulse with $f_T T$ as parameter. It is easy to recognize that the shape of the normalized signal response improves as $f_T T$ grows.

2. INFLUENCE OF BAND-LIMITED FILTERING ON THE ERROR PROBABILITY

When the receiver bandwidth is limited, errors at the output of the digital detector increase because of the intersymbol interference. In the following, a suitable expression is derived analytically for the variance of the intersymbol interference and filtered noise, which is used to compute the error probability of the communication system.

The signal at the digital detector input is corrupted by a random disturbance having a variance:

$$\sigma_T^2 = \sigma_n^2 + \sigma_I^2 \tag{13}$$

where σ_n^2 is due to the filtered noise and σ_I^2 to the intersymbol interference produced by the band-limited filtering [2].

The noise variance at the filter output is given by:

$$\begin{aligned} \sigma_n^2 &= N_o/2 \int_o^\infty h^2(t)\, dt = \\ &= -2\pi f_T \left\{ \frac{c_1^2 + c_2^2}{4(a+b)} + \frac{c_1^2}{4}\left[a\cos(2\theta) - b\sin(2\theta)\right] + \frac{c_2^2}{4}\left[b\cos(2\phi) - a\sin(2\phi)\right] + \frac{c_1 c_2}{2(a+b)} \cdot \right. \\ &\quad \cdot \left[\cos(\theta+\phi) - \sin(\theta+\phi)\right] + \frac{c_1 c_2}{(a+b)^2 + (a-b)^2} \cdot \\ &\quad \left. \cdot \left[(a+b)\cos(\theta-\phi) - (b-a)\sin(\theta-\phi)\right] \right\} \end{aligned} \tag{14}$$

where $N_o/2$ is the double-side noise power density function. It is easy to note that σ_n^2 increases as the bandwidth of the receiving filter increases, as expected.

In the following, the analytical method used to derive σ_I^2 in a closed form is explained. As shown in Fig. 1, the input to the receving filter is:

$$r(t) = \Sigma_i\, d_i\, p(t-iT) + n(t) \tag{15}$$

where p(t) is given by (10) and n(t) is the white Gaussian noise with a power density function equal to $N_0/2$.

The output of the receiving filter is:

$$y(t) = \Sigma_i\, d_i\, v(t-iT) + n_o(t) \qquad (16)$$

where v(t) is given by (11) and $n_o(t)$ is defined as $n_o(t) = n(t) * h(t)$.

By sampling at the instant $t = i\,T+t_o, t_o$ being chosen over [O, T], the input to the data-demodulator is:

$$m_i = d_i\, v(t_o) + \sum_{\ell \neq o} d_{i-\ell}\, v(\ell T+t_o) + n_i \qquad (17)$$

where n_i is due to the presence of the channel noise, and the second term to the intersymbol interference introduced by the band-limited filtering. These two disturbances are considered as two statistical independent random variables in order to evaluate separately their variances. σ_I^2 is derived in the following when only the intersymbol interference occurs, and σ_n^2 has been previously evaluated.

By recalling that $E\{d_i^2\} = 1$, we have:

$$\sigma_I^2 = \sum_{\ell \neq o} v^2(\ell T+t_o) \qquad (18)$$

The analytical expression of σ_I^2 for PSK, FSK and MSK signals is too complex and is reported in [3]. Nevertheless, a suitable approximation which holds for not too small $f_T T$ values can be obtained in all cases. In the following, the σ_I^2 approximate expression for a PSK signal is reported as an example:

$$\sigma_I^2 \simeq \frac{c_1^2\, \ell^{4a\pi f_T T}}{2} \{\ell^{4a\pi f_T T} \cdot [1+\cos(-8b\pi f_T T+2\varepsilon_1)] + [1+\cos(-4b\pi f_T T+2\varepsilon_1)]\} + \frac{c_2^2\, \ell^{4b\pi f_T T}}{2} \{\ell^{4b\pi f_T T} \cdot [1+\cos(-8a\pi f_T T+2\varepsilon_2)] + [1+\cos(-4a\pi f_T T+2\varepsilon_2)]\} \qquad (19)$$

where:

$$\varepsilon_1 = \theta - \frac{5\pi}{8} \quad ; \quad \varepsilon_2 = \phi - \frac{7\pi}{8} \qquad (20)$$

and t_o is chosen equal to T.

The error probability of a modulated band-limited waveform [5] is given by:

$$P_e = Q\left(\sqrt{\frac{v2(t_o)}{\sigma_t^2}}\right) \qquad (21)$$

Q(x) being the Q-function defined as:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-(z^2/2)}\, dz \qquad (22)$$

3. RESULTS

In this section, the error probability evaluated by (21) for PSK, FSK and MSK signals is presented when both the intersymbol interference and channel noise are considered. In Fig. 4 the error probability P_e is reported versus the signal-to-noise ratio (S/N) for PSK and MSK modulations with $f_T T$ as parameter. The theoretical error probability for a PSK signal with infinite bandwidth is reported (dotted line) for comparison. It can be noted that the better behaviour of MSK with respect to PSK and FSK depends on the more compact spectrum of the MSK modulation.

The previous results were used to evaluate the performance of the communication system shown in Fig. 5. In this system a single carrier is employed for simultaneous transmission of voice and data [6],[7]. The voice modulates in amplitude and the data signal modulates the phase of the same carrier.

The band-pass filters were taken to be Butterworth filters with the following characteristics:

1) The transmitter filter was a fourth-order filter, with a -3dB bandwidth of 7.5 kHz (single side).
2) The receiver filter was an eight-order filter, with a -3dB bandwidth of 5 kHz (single side).

In order to characterize the effect of band-limited filtering for the envelope fluctuations, an ideal transmission channel was considered. The mean power of the disturbance on the filtered signal is denoted by N_d and its behaviour versus the f_T T product is reported in Fig. 6. In order to simulate the voice signal which modulates the amplitude of the carrier, a tone at 937.5 kHz was chosen.

In Fig. 7 N_d is reported versus the $f_T T$ product when the modulating signal is a real voice signal; in this case the modulation index is supposed constant and equal to 0.9. Both these figures have shown a good behaviour when a MSK signal is used.

In order to evaluate the effect of the amplitude variation due to AM-modulation and band-limited filtering, the bit error probability P_e has been evaluated when the transmission channel is assumed to introduce an additive white Gaussian noise with a power density function equal to $N_0/2$. Fig. 8 shows P_e versus the signal-to-noise ratio S/N_0; a $f_T T$ product equal to 4.17 is considered in the case of curve a, and to 16.67 for curve b. The P_e for a binary PSK system is also reported for comparison. These results have shown that the loss due to the presence of the AM-modulation and finite bandwidth is not too high. In particular, it is acceptable if a modulation technique with limited spectrum shape such as MSK is employed.

4. CONCLUSIONS

In this paper, an analytical method has been proposed in order to derive in closed form the expression of the disturbance variance due to noise and intersymbol interference. Results were obtained in the case of a four-pole Butterworth low-pass receiving filter and PSK, FSK or MSK as transmitted signals.

As a particular application of our results, a communication system has been considered in which the envelope fluctuations of the received signal due to band-limited filtering have a great influence on its performance [7].

In all the examined cases, the influence of band-limited filtering is not too high in terms of envelope fluctuations and P_e degradation, if a suitable modulation technique, such as MSK, is used.

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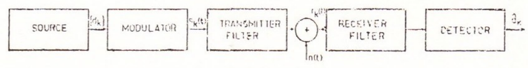

Fig. 1

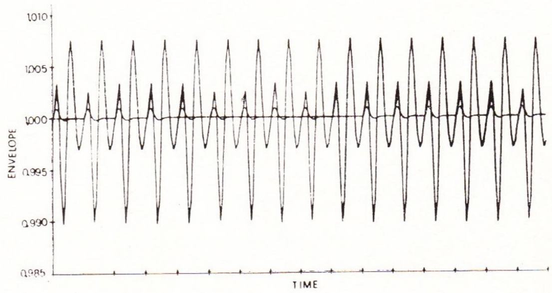

Fig. 2

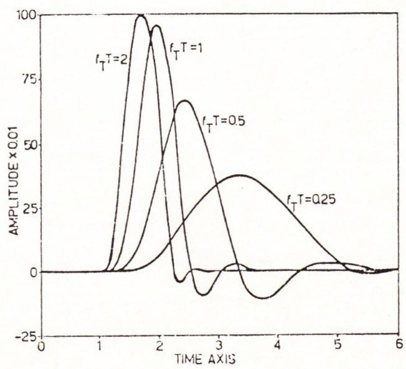

Fig. 3

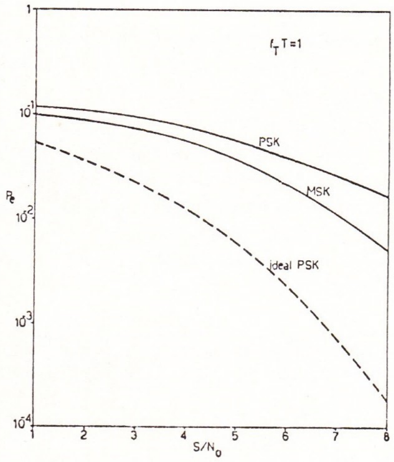

Fig. 4

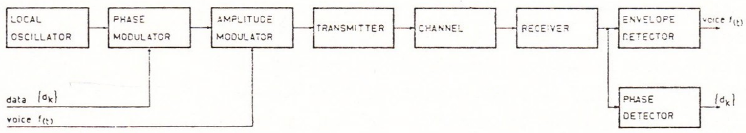

Fig. 5

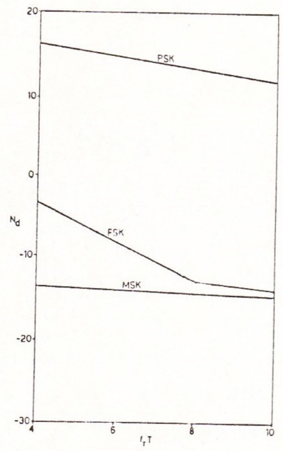

Fig. 6

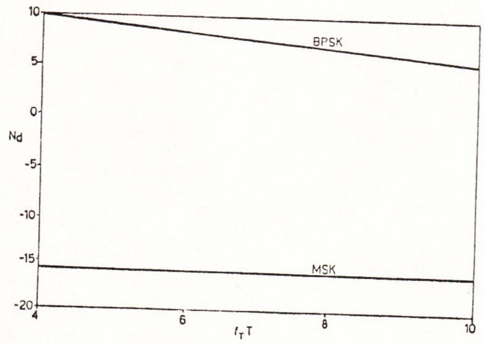

Fig. 7

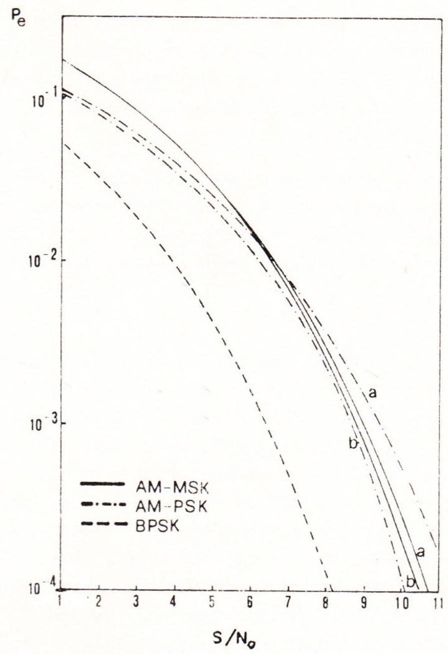

Fig. 8