Design of Turbo-MUD Receivers with Density Evolution in Overloaded CDMA Systems

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Abstract— In this paper an original Turbo Multiuser Detector for Overloaded Coded DS-CDMA systems is presented and studied: the performance of the proposed system is tested by Monte Carlo simulations and by means of the Density Evolution theory. The Turbo-MUD receiver is based on the use a combination of a linear MMSE detector in the first stage and an Interference Cancellation scheme in the successive ones. The inputs of both receivers are the soft information from a bank of turbo decoders. The performance of the proposed receiver can be effectively predicted in overloaded communication systems by means of density evolution technique: the use of this technique also permits to evaluate the proper number of MMSE iterations, making easer the design of the receiver.

I. Introduction

Code Division Multiple Access (CDMA) communication systems are vulnerable to Multiple Access Interference (MAI): hence, Multiuser Detection (MUD) techniques received great attention, also because of the strong emphasys on the 3G cellular systems; in the MUD receivers the intereference is considered as an additional information source and good performance are obtained by means of an increased complexity. Particularly, optimal joint decoding/detection is an excellent solution to this problem, as shown in [1]. However, this scheme results in a prohibitive computational complexity for actual implementation. Conversely, the suboptimal solution, which separates the operations of symbol detection and channel decoding, appears more attractive for practical applications.

The successful proposal of Turbo codes [2] naturally suggests the idea of an iterative (Turbo) processing techniques in the design of multiuser receivers. In the iterative multiuser detection, the extrinsic information is determined in each detection and decoding stage and used as a priori information for the next iteration. This procedure is adopted at each iteration as in Turbo codes: this detection philosophy is defined as Turbo MUD and, in the last years, many iterative receivers have been investigated: between the schemes which have been proposed, it is important to remind some Interference Cancellation schemes [3], [4], [5], [6], that are characterized by a lower complexity.

The benefits of Turbo-MUD are particularly relevant in the overloaded systems, i.e., for the systems whose number of users is greater than the spreading factor [7]. In this

paper, we propose a new iterative multiuser detector based on the combination of linear MMSE blocks with Interference Cancellation (IC) schemes and a bank of Turbo decoders. Firstly, the MMSE outputs are used to reconstruct the signal to be cancelled from the the received signals: this solution, anyway, is not optimal due to the negative effects of noise enhancements caused by MMSE filtering; hence, after some cancellation iterations, the detector begins considering the MF outputs, for the cancellation: this solution permits to retain the positive effects of the MMSE introduction in the first iterations. Though this receiver affords performance improvement for all the considered systems, it is difficult to identify the number of MMSE iterations to be performed before switching to the ordinary IC. Due to the very low error probability of the proposed receiver, definition of the optimal structure by means of Monte Carlo simulations would require very long simultations and a huge processing time.

We decide to study the proposed receiver from an analytical point of view by means of Density Evolution (DE) technique [8], [9]: though the results of DE analysis holds for the asymptotic regime, i.e., after some Turbo-MUD iterations, it is possible to compare the different MUD approaches and to draw general conclusions about the optimum number of linear MMSE iterations to be performed.

II. System Model

We consider an up-link DS-CDMA communication system with N synchronous turbo-coded users. Timing, amplitudes, carrier phases and spreading sequences of all the users are assumed to be perfectly known at the receiving end in the base station. Each user encodes blocks of information bits $u_k(i)$ with a Parallel Concatenated Convolutional Code (PCCC) and transmits the resulting codewords composed of M coded bits over a common AWGN channel with BPSK modulation. The equivalent baseband received signal can be written as

$$r(t) = \sum_{k=1}^{N} A_k \sum_{i=0}^{M-1} c_k(i)p(t - iT_b)s_k(t - iT_b) + n(t) \quad (1)$$

where:

- T_b is the bit interval;
- A_k is the kth user received amplitude;
- $c_k(i) \in \{+1, -1\}$ is the bit transmitted by kth user during the ith bit period;

- $p(t)$ is the unit-power rectangular pulse shape with duration T_b;
- $s_k(t)$ is the kth user unit-power spreading sequence;
- $n(t)$ is an Additive White Gaussian Noise (AWGN) process with double sided spectrum density $\sigma^2 = No/2$ [W/hz].

In the receiver a bank of matched filter is used for despreading. Without loss of generality, we can assume that the first bit-interval is observed. As a result, the output of the kth matched filter is given by

$$y_k = \frac{1}{T_b} \int_0^{T_b} r(t)s_k(t)dt = A_k c_k + \sum_{\substack{j=1 \\ j \neq k}}^{N} A_j c_j \rho_{jk} + n_k \quad (2)$$

where ρ_{jk} is the normalized crosscorrelation coefficient between the jth and kth users and n_k is the noise Gaussian sample of the kth user with distribution $N(0,\sigma^2)$. The second term in eq.(2) represents the MAI, that has to be cancelled.

III. A NEW IC ITERATIVE RECEIVER

An iterative cancellator consists of an IC receiver followed by N single-user turbo decoders: each constituent block iteratively provides soft informations to the others.

In the very first multiuser detection iteration, the a priori information of the coded bits is not available, i.e. $L_{ap}(c_k(i)) = 0$, k=1,2,...,N, i=0,1,...M-1. The IC stage delivers interference-cancelled soft outputs $\tilde{y}_k(i)$ to the input of the turbo decoders. After a fixed number of turbo decoder iterations, the extrinsic information of coded bits at the output of turbo decoders are fed back to the input of the IC detector as the a priori information for the next receiver iteration.

The considered turbo codes are composed of two Recursive Systematic Convolutional (RSC) codes connected by an interleaver while a MAP based algorithm is used for iterative decoding [10]. Since the IC receiver requires soft information about reliability of both the systematic and the parity bits, the decoding algorithm is properly modified to produce also the extrinsic information about the latter [11]. At each new iteration, the iterative structure permits the multiuser receiver to have a more reliable a priori information and the decoders to operate on the soft inputs, in which a greater amount of interference has been cancelled.

A. The Conventional Iterative PIC Receiver

In the conventional iterative Parallel Interference Cancellation (PIC) receiver [12], at each IC stage, MAI is removed simultaneously from each user. Therefore, at the mth receiver iteration, the PIC soft output, i.e., the turbo decoders input, can be expressed as

$$\tilde{y}_k^{(m)} = y_k - \sum_{\substack{j=1 \\ j \neq k}}^{N} A_j \rho_{kj} \hat{c}_j^{(m)}$$

$$= A_k c_k + \sum_{\substack{j=1 \\ j \neq k}}^{N} A_j \rho_{kj} \left(c_j - \hat{c}_j^{(m)} \right) + n_k \quad (3)$$

where $\hat{c}_j^{(m)}$ is the estimate of bit c_j at iteration m. Note that the second summation represents the residual MAI after cancellation.

The data estimates $\hat{c}_k^{(m)}$ have been chosen in [12] as the expectation of the coded bits, that is

$$\hat{c}_k^{(m)} = \text{E}\left\{ c_k^{(m)} \right\} = \tanh \left[\frac{1}{2} \text{L}_{ap}^{(m)}(c_k) \right] . \quad (4)$$

The term $\text{L}_{ap}^{(m)}(c_k)$ is the a-priori *Log-Likelihood Ratio* of the bit c_k at the mth iteration, defined as

$$\text{L}_{ap}^{(m)}(c_k) \triangleq \log \frac{P^{(m)}(c_k = +1)}{P^{(m)}(c_k = -1)} . \quad (5)$$

In the first receiver iteration no a priori information is available from the decoder output: hence, for the initializing condition, it is assumed $\text{L}_{ap}^{(0)}(c_k)$=0, k=1,2,...N. Instead, in the successive iterations the extrinsic information coming from the decoders can be used, leading to $\text{L}_{ap}^{(m)}(c_k)$=$\text{L}_{ex}^{(m-1)}(c_k)$.

B. The Proposed Iterative MMMSE+PIC Receiver

The coding gain due to turbo codes becomes larger as the number of iterations increases. However, the performance improvement obtained by the turbo codes is remarkable in the first iterations, and more and more negligible in the successive ones. This remark suggests to concentrate the significant part of interference-cancellation in the first iterations: for this reason many IC based iterative receivers with a first linear stage have been proposed [5], [6], [13]. Nevertheless, a linear MUD has the drawback of an extremely high computational complexity.

In this paper we present an iterative PIC receiver with a linear MMSE as the first stage; as it is known [7], the linear MMSE receiver is well suited for overloded systems, where it is not possible to allocate orthogonal spreading waveforms to all the users, whereas a conventional iterative PIC receiver can not afford satisfactory performance.

Nevertheless, if the MMSE filter outputs are used as inputs for the Parallel Interference Cancellation block, we can observe that BER performance tends to increase after some iterations: we can deduce that this phenomenon is caused by the noise enhancement due to the linear MMSE introduction. This impairment can be avoided by switching to use the Matched Filters outputs as the inputs to the PIC blocks after a proper number of MMSE iterations: we can summarize that, for the first iterations, MAI effects are prevalent over noise whereas after some iterations MAI can be assumed to be cancelled and noise enhancements is the main impairments in the system. The structure of this receiver is shown in Fig. 1.

One crucial design element of the prposed receiver is the optimum number of MMSE iterations to be performed: we have defined this parameter as *mmse-iter*: basically, if a too low parameter was choosen, MAI wouldn't be suppressed completely; conversely, a too high value of *mmse-iter* would cause sensible noise enhancement. In the following, we will report the results of the *mmse-iter* optimization.
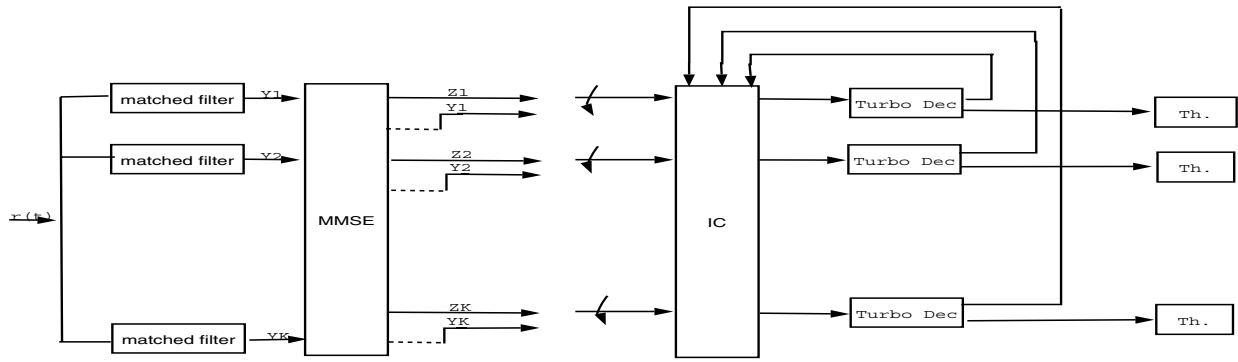
Fig. 1. The block diagram of the proposed iterative MUD.

C. System Description

After the mth signal PIC iteration the kth signal can be expressed as:

$$z_k^{(m)} = A_k c_k + \sum_{j \neq k} A_j T_{k,j} (c_j - \hat{c}_j) + \tilde{n}_k \qquad (6)$$

where

- $\hat{c}_k$ is the soft estimate of the bit of the kth user based on the $L_{ap}^{(m)}(c_k)$
- n_k is a zero-mean white Gaussian noise process whose covariance matrix is equal to:

$$E\left[n_{MMSE} n_{MMSE}^T\right] = E\left[EMnn^T M^T E\right] = \\ EME\left[nn^T\right] M^T E = \sigma^2 EMRM^T E \qquad (7)$$

where $E = diag\left((MR)_{1,1}^{-1}, \ldots, (MR)_{K,K}^{-1}\right)$ is the normalization matrix.

Finally, if the Interference Cancellation is performed on the MMSE outputs, the matrix T is equal to $M * R$ where $M = (ARA + \sigma^2 I)^{-1} A$, R and A are the MMSE filter matrix, the crosscorrelation and the diagonal matrix of the received amplitude; on the other hand, if the IC is performed on the matched Filter outputs we have that $T = R$. Aiming to characterizing the MAI contribution, (6) can be rewritten as:

$$z_k^{(m)} = A_k c_k + \eta_k \qquad (8)$$

where $\eta_k \sim N(\mu_k, \sigma_k^2)$. If the channel noise and the MAI are supposed to be uncorrelated we have:

$$\mu_k = \sum_{j \neq k} A_j T_{k,j} E\left[(c_j - \hat{c}_j)\right] + E\left[n_j\right] = \\ = \sum_{j \neq k} A_j T_{k,j} (E\left[c_j\right] - \hat{c}_j) = \\ = \sum_{j \neq k} A_j T_{k,j} (\bar{c}_j - \hat{c}_j) \qquad (9)$$

$$\sigma_k^2 = \sigma^2 + \sum_{j \neq k} A_j^2 T_{k,j}^2 (1 - \bar{c}_j^2) = \\ = \sigma^2 + \sigma_{MAI}^2 \qquad (10)$$

From (9) it can be observed that the closer the estimated bit is to the mean value, the more accurate the interference cancellation is. Moreover, from (10) we can deduce that also σ_{MAI}^2 decreases as $\bar{c}_j \to \pm 1$.

IV. SIMULATION RESULTS

In this section the effectiveness of the proposed receiver will be investigated by computer simulations. In order to mitigate the complexity burden due to the implementation of a non-linear decision device, the tanh() function has been approximated through an eight-values look-up table. For all simulations we use a rate R_c=1/2 turbo code, composed by two 8-state RSC codes with generator polynomials $G_0 = (13)_8$ $G_1 = (15)_8$, and the Block Interleaver addressed by the UMTS standard [14].

The performance of the proposed receiver has been analyzed in a synchronous AWGN channel in the cases of 10, 20, 24 and 28 equal-power users, assuming Pseudo-Noise short codes, processing gain G and frame lenght respectively equal to 16 and 800. The quantized Log-MAP algorithm is used for the decoding [10].

Fig. 2 shows the BER performance of the MMSE+PIC receiver for different values of *mmse-iter*: the optimum value of the parameter increases as the number of users grows; this behaviour can be explained by noting that a system with a higher number of active users is characterized by a greater MAI so that also a higher number of MMSE iteration is required to perform effective interference cancellation. On the contrary, underloaded system does not need too many MMSE iterations: particularly, performance gain due to one mmse iteration introduction is almost negligible while more iterations cause noise enhancement and performance loss.
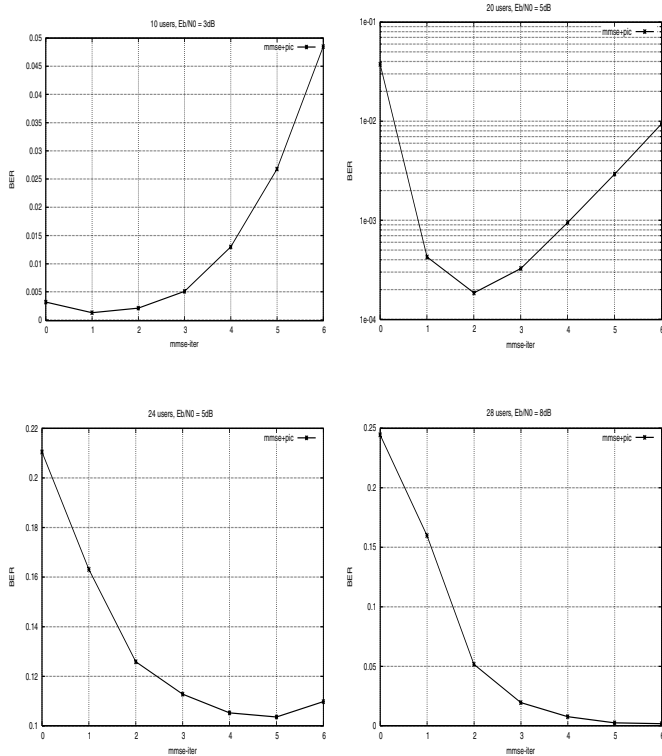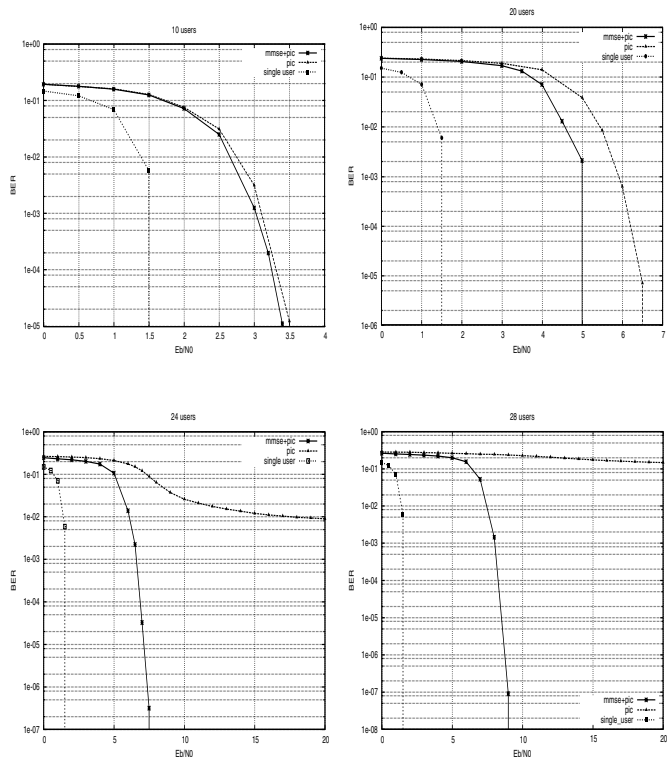
Fig. 2. Ber Performance

Fig. 3. Ber Performance

Fig.3 shows that MMSE+PIC results useful for an over-loaded system while conventional PIC is characterized by a poor performance; on the contrary, the benefits of MMSE+PIC introduction in an underloaded system are negligible

V. DENSITY EVOLUTION APPROACH

An iterative decoder can be seen as a nonlinear dynamic feedback system: extrinsic information messages λ_i are passed from one constituent decoder to the other. The message λ_i measures the log-likelihood ratio for the ith bit based on input messages from all other bits. If a proper interleaver is used, the extrinsic information messages λ_i are indipendent and identically distribuited, with symmetric probability density function $f(\lambda)$ [8]. As shown in [9], the (empirical) probability densities $f(\lambda)_{in}$ and $f(\lambda)_{out}$ evolve with successive decoder iterations starting from narrow densities concentrated around $\lambda = 0$ to broader Gaussian-shaped densities with increasing means as the iterations continue.

If $f(\lambda)$ can be approximated by a Gaussian density function, then it statistically depends on two parameters only, namely the mean $\mu = E(\lambda)$ and the variance $\sigma^2 = Var(\lambda)$. Based on this distribution, a proper signal-to-noise ratio can be defined as $SNR = \mu^2/\sigma^2$: a high value of SNR implies that $f(\lambda)$ is easily discriminated from $f(-\lambda)$. If $f(\lambda)$ is both Gaussian and symmetric, then $\sigma^2 = 2\mu$ and $SNR = \mu/2$.

We can define the SNRs at the inputs and the outputs of each decoder at each iteration as $SNR1_{in}$, $SNR1_{out}$, $SNR2_{in}$ and $SNR2_{out}$. If we assume that E_b/N_0 at the

receiver is greater than 0, the decoder 1 will generate a value of $SNR1_{out}$ gretear than zero. More generally, for a given value of E_b/N_0, the output SNR of each decoder is a nonlinear function at its input SNR, denoted as G_1 for decoder 1 and as G_2 for decoder 2. Hence, we have

$$SNR1_{out} = G_1(SNR1_{in}, E_b/N_0) \qquad (11)$$

$$SNR2_{out} = G_2(SNR2_{in}, E_b/N_0) \qquad (12)$$

Since, in an iterative decoder we can assume that $SNR1_{out} = SNR2_{in}$, we have

$$SNR2_{out} = G_2(G_1(SNR1_{in}, E_b/N_0), E_b/N_0). \qquad (13)$$

VI. DENSITY EVOLUTION IN ITERATIVE MULTIUSER DETECTOR

Density Evolution approach can be applied to the proposed receiver : particularly, this technique permits to study asymptotical behaviour. Hence, the SNR_{in}-SNR_{out} relations have to be estimated both for the SISO decoder and for the SISO multiuser detector. For what concerns SISO decoder, this relation can be achieved, by assuming to have a gaussian distribution with the mean $\mu = E(\lambda)_{in} = 2SNR_{in}$ and the variance $\sigma^2 = Var(\lambda)_{in} = 4SNR_{in}$ and obtaining a SNR_{out} estimate by averaging the soft outputs $SNR1_{out} = \frac{\overline{\lambda_{out}}}{2}$.

On the other hand, SISO multiuser detector relation is dependent on the value of E_b/N_0; hence, we have to generate a proper gaussian distribution $N(c_i * 2SNR2_{in}, 4SNR2_{in})$, where $c_i \in \{\pm 1\}$ is coded bit and $SNR2_{out} = \frac{c_i*\lambda_{out}}{2}$.
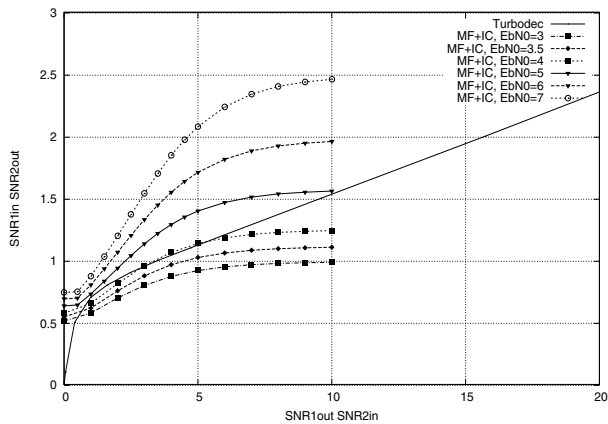
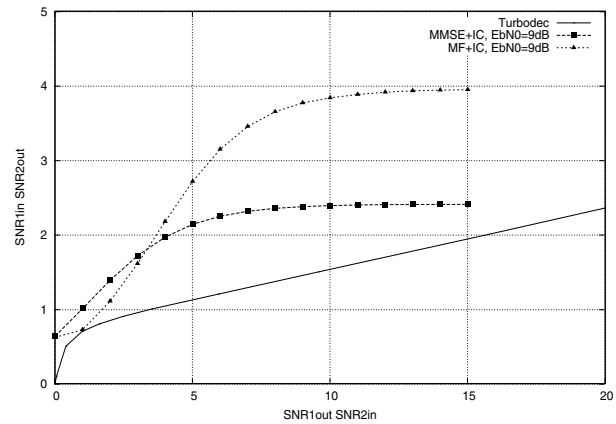Fig. 4.   DE analysis for MMSE+PIC ina 24-user system

Fig. 5.   Performance Comparison between MMSE+PIC and MF+PIC in a 32- user system with $E_b/N_0 = 9dB$

In Fig. 4 we reported the curves of the SISO decoder and of the SISO multiuser detector: only the latter depends on the value of E_b/N_0.

When the curve $TurboDec$ is touching the $MF+IC$ curve, the value E_b/N_0 corresponds to the waterfall threshold.

Finally, from Fig. 5, we can deduce some interesting characteristics of the proposed receiver. When SNR_{in} is low, DE analysis proves how useful the MMSE introduction is: particularly, the MMSE ends up boosting the convergence of the iterative detection and decoding.

Nevertheless, MF solution is characterized by a higher asymptotic value of the SNR_{out}, so confirming that switching to MF Interference Cancellation after some iterations is beneficial.

VII. CONCLUSION

In this paper an original Turbo Multiuser Detector for Overloaded Coded DS-CDMA systems has been presented: the performance of the proposed system has been tested by means of Monte Carlo simulations and DE analysis. The asymptotical behaviour of the proposed receiver in a overloaded scenario has been studied with a DE analysis which also permits to estimate the proper number of MMSE iterations.

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