A Dynamic Channel Allocation Technique Based on Hopfield Neural Networks.

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ABSTRACT - The interest in global spectrum allocation techniques is growing with the always increasing spectrum demand for mobile communications. This paper deals with a Dynamic Channel Allocation (D.C.A.) technique based on an energy function whose minimization can be performed by a Hopfield Neural Network.
The performance of the proposed DCA technique is here derived by computer simulations. Comparisons with a classical FCA technique and a previously proposed DCA technique are given to highlight the better performance of our DCA technique.

I - INTRODUCTION.

In a cellular radio system, the territory is divided into hexagonal cells, each cell being served by a base station located at its center. Each mobile user needs a channel to communicate with the base station of the cell the user belongs to. In this paper, we refer to a channel as a generic communication resource which can be alternatively a radio channel in a specified band for a Frequency Division Multiple Access, a code for a Code Division Multiple Access or a time slot for a Time Division Multiple Access.
The same channel can be simultaneously used in different cells at a suitable distance (D). The minimum distance between cells in which the use of the same channel is possible (reuse distance) depends on the level of the co-channel interference accepted in order to allow reliable communications.
Two typical alternatives to perform the channel assignment to the cells are the Fixed Channel Allocation Technique (FCA) and the Dynamic Channel Allocation Technique (DCA) [1]. In the case of the FCA technique a set of channels is permanently assigned to each cell. The same set of channels is reused in cells at a distance D away. The basic FCA technique implies that a call attempt at a cell can only be served by an available channel belonging to the set of channels assigned to that cell. If no channel is available the call is blocked and lost. The number of channels permanently allocated to each cell is equal to M/K, where M is the number of system resources and K the *reuse factor* [1].
Differently from the FCA technique, in the DCA technique the channels are not permanently assigned to the cells. In this case, channels are assigned on a call-by-call basis in order to obtain a better performance and narrower spectrum utilization, in particular under non-uniform traffic load conditions.

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II - THE PROPOSED DCA TECHNIQUE.

The topological model we are considering is a group of hexagonal cells. We consider a number of cells along the X axis equal to those along the Y axis. The "number of cells in a side" of our radio network is a parameter. Another parameter is the "number of available channels" for the whole cellular structure.
The interference condition is taken in its simplest form. Let T be the interference order; a cell **C** interferes with the cells belonging to T rings centred in **C**.
It is not possible to use the same channel in two interfering cells. The condition cannot be violated so we refer to it as **hard condition**. Whenever the channel selected according to a suitable assignment algorithm does not satisfy this constraint, the corresponding service request (new call) is blocked. Other conditions, named **soft conditions** have also been provided. Soft conditions differ from hard ones for the fact that they can be violated at the expense of a slight decrease in the allocation performance.
The proposed technique is thought to be implemented in real cellular systems, so we limit the number of operations to be performed when a new call arrives. In particular, we do not consider any rearranging task in cells different from the one involved in the arrival as stated in [2].
Rearranging the assignment in the whole structure every time a call arrives could obviously achieve a lower blocking probability; unfortunately, the amount of time needed to perform such a task makes this impossible to be realized in a practical cellular network.
The most important soft conditions proposed are the **packing condition** and the **resonance condition**.
With the **packing condition,** assignment solutions are preferred which tend to use the minimum number of channels to satisfy the global channel demand. The impact of this condition on the assignment is to prefer channels already used in other cells, without violating any hard condition. If more choices are possible, channels used in nearest cells are taking into account. We are going to explain later how to translate this and subsequent conditions into terms of a quadratic energy function to minimize.
With the **resonance condition,** we tend to assign the same channels to cells that belong to the same reuse scheme, obtained by jumping from one cell to another with steps of length exactly equal to the reuse distance, as shown in Fig. 1.

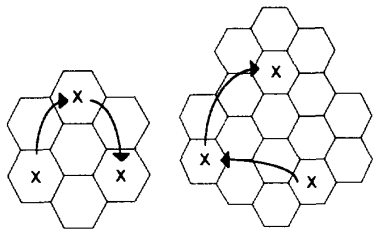

Fig. 1 - Reuse schemes.

This conditions tends to give an optimum assignment in the presence of a uniform distribution of incoming calls among the cells. When a non-uniform traffic is present, this condition still seems to work well by arranging the assignment in an ordered way without interfering with the dynamic allocation concept.

Others soft conditions are formulated in order to:

- assign, where possible, the same channels assigned before, i.e. limiting the infracell rearranging;
- try to assign the exact number of channels requested in the cell involved in a new arrival (or the termination of a call). A violation of this condition means the impossibility of serving an incoming call which is obviously blocked.

All these conditions lead to the definition of a **quadratic energy function** discussed below. In performing our analysis the following definitions are assumed:

CE..... the total number of cells in the system.
CH ..... the number of channels available to the system.
$A_{i,j}$..... an element of the allocation table whose value is 1 if the channel j is allocated to the cell i (i=1,,CE and j=1,,CH), 0 otherwise.
i*..... the cell involved in a new arrival or a termination of a call.
Traf(i*) ..... the number of requested channels at cell i*.
$V_{i^*,j}$..... the assignment of the cell of interest; i.e. the variables of the assignment problem.
Interf(i,i*) . the function giving a value of 1 if cells i and i* are interfering according to the previous definitions, 0 otherwise.
Dist(i,i*).... this function gives the distance between cells i and i* normalized to the inter-center distance between two adjacent cells.
Res(i,i*) .... gives a value of 1 if cells i and i* belong to the same reuse scheme defined before, 0 otherwise.

We define an **energy function** associated with the modeled cellular network as follows:

$$E = \frac{A}{2} \cdot \sum_{j=1}^{CH} \sum_{\substack{i=1 \\ i \neq i^*}}^{CE} V_{i^*,j} \cdot A_{i,j} \cdot \text{Interf}(i,i^*) + \frac{B}{2} \cdot \left(\sum_{j=1}^{CH} V_{i^*,j} - \text{Traf}(i^*) \right)^2 - \frac{C}{2} \cdot \sum_{j=1}^{CH} \sum_{\substack{i=1 \\ i \neq i^*}}^{CE} V_{i^*,j} \cdot A_{i,j} \cdot \frac{(1 - \text{Interf}(i,i^*))}{\text{Dist}(i,i^*)} - \frac{D}{2} \cdot \sum_{j=1}^{CH} V_{i^*,j} \cdot A_{i^*,j} + \frac{F}{2} \cdot \sum_{j=1}^{CH} \sum_{\substack{i=1 \\ i \neq i^*}}^{CE} V_{i^*,j} \cdot A_{i,j} \cdot (1 - \text{Res}(i,i^*)) \qquad (1)$$

The first term adds a positive constant to the energy function if there are some interfering cells using the same channels; the second term is positive if the requested number of channels has not been assigned to cell i*; with the third term we fulfil the packing condition as stated before; the fourth term lowers the value of the energy function if the actual assignment is equal to the previous one and the fifth term accomplishes the resonance condition.

Constants A, B, C, D, and F are determined in order to decide which are the conditions that can be violated and in what order, as is shown later.

Every time a new call arrives (or ends) in a cell i*, the algorithm searches for a pattern of 1's and 0's that, substituted to $V_{i^*,j}$, minimizes the energy function *E*. The best pattern found represents the solution to the DCA problem only if it satisfies the channel demand condition. On the contrary, if the pattern violates it, the arriving call is blocked.

With this formulation the problem complexity grows exponentially with the number of channels; then we have proposed an approach based on Hopfield neural networks as described in the next section.

III - HOPFIELD NEURAL NETWORKS.

The combinatorial optimization problem, i.e. finding a solution that minimizes a cost function and whose variables assume only two possible values, falls in the more general class of NP-complete (non-deterministic polynomial time complete) problems [3]. A neural network approach formulated by Hopfield and Tank [4] claimed to be a method that could be realized by hardware circuits with response times much shorter than those of other algorithmic methods. Nevertheless the original formulation, suffered from a tendency to produce non-feasible solutions called "spurious states".

The variation proposed by Shigeo Abe [5] uses a particular formulation of Hopfield networks that eliminates all the spurious states from the solution space. This type of neural networks has been used in our simulations.
Giving an energy function :

$$E = \frac{1}{2}x'Tx + b'x \qquad (2)$$

where T is a symmetric nxn matrix; x is an n-elements vector representing our variable and b is a constant n-elements vector of inputs. The problem is equal to finding x that minimizes E for every single component of x with values 0 or 1.
We extend the range of x_i - i-th component of x - to [0,1].
Considering x_i as the output of the i-th neuron of the network the internal state u_i is introduced for which:

$$x_i = f(u_i) \quad ;i = 1\ldots n \qquad (3)$$

where $f(u_i)$ is a monotone non linear function; for istance Hopfield suggests a hyperbolic tangent function $f(u_i)=1/2(1+\tanh(u_i))$.
If we consider the quantity:

$$\widetilde{E} = E - \sum_i \int_{0.5}^{x_i} f^{-1}(X)dX$$

the temporal evolution of such an energy function is:

$$\frac{d\widetilde{E}}{dt} = \sum_{i,j} T_{i,j}x_j \frac{dx_i}{dt} + \sum_i b_i \frac{dx_i}{dt} - \sum_i u_i \frac{dx_i}{dt}$$
$$= \sum_i \frac{dx_i}{dt}\left(\sum_j T_{i,j}x_j + b_i - u_i\right)$$

thus building a system for which:

$$\frac{du_i}{dt} = -\sum_j T_{i,j}x_j - b_i - u_i \qquad (4)$$

we have:

$$\frac{d\widetilde{E}}{dt} = -\sum_i \frac{dx_i}{dt}\frac{du_i}{dt} = -\sum_i \left(\frac{du_i}{dt}\right)^2 \frac{dx_i}{du_i} \le 0$$

for the monotonicity of x=f(u). So $\widetilde{E}$ is a Lyapunov function for (4).

$$\frac{d\widetilde{E}}{dt} \le 0 \qquad (5)$$

confirming that the set of equations (4) leads E towards lower values; so, for an arbitrary initial internal state u_0, a local minimum solution is achieved.

Under the hypothesis of using a piecewise-linear function as $f(u)$, system (4) becomes a linear system operating on a closed hypercube :

$$\frac{dx}{dt} = -Tx - b \qquad 0 \le x_i \le 1 \quad i = 1\ldots n \quad (6)$$

Its equilibrium points are all in the vertex of the hypercube and we can make all the feasible solutions be the only stable points [5].
It is not difficult to show that energy function E in (1) can be expressed as a quadratic form of variables $V_{i^*,j}$ and if we call x the vector whose components are $V_{i^*,j}$ with $j=1..CH$ for a fixed i^*, expression (2) represents (1) with :

$$T_{j,j'} = B \qquad (7)$$

$$b_j = \frac{A}{2}\cdot\sum_i A_{i,j}\cdot \text{Interf}(i,i^*) - B\cdot\text{Traf}(i^*) - \frac{C}{2}\cdot\sum_i A_{i,j}\frac{\left(1-\delta_{i,i^*}\right)}{\text{Dist}(i,i^*)}\left(1-\text{Interf}(i,i^*)\right) - \frac{D}{2}\cdot A_{i^*,j} + \frac{F}{2}\cdot\sum_i A_{i,j}\cdot\left(1-\delta_{i,i^*}\right)\cdot\left(1-\text{Res}(i,i^*)\right) \qquad (8)$$

where δ_{ij} is the Kronecker delta.
In order to make system (6) stable for every possible T, we make the following substitutions without changing the values of E at the vertexes of the hypercubic domain.

$$T_{j,j'} \to 0 \quad \forall j \qquad (9)$$

$$b_j \to b_j + \frac{T_{j,j'}^{(old)}}{2} \qquad (10)$$

For the system obtained it has been proven in [6] that no stable points exist within the variability domain. On the border of the hypercube we can find vertexes and non vertexes points. The former are characterized by the fact that their component have only a value of 0 or 1. The latter have at least one component in the interval (0,1). Among vertexes, we can see feasible solutions which are compatible with the reuse and the channel demand constraints, and unfeasible ones.
The next step is to make all the feasible solutions be stable points of system (6).
Be c a vertex; c(j) an adjacent vertex obtained by substituting the j-th component of c with its complement to 1.
If c represents a feasible solution and E(c) is its energy value, in order to make c a strongly stable point we have to find values for constants A,B,C,D and F for which E(c(j))>E(c) for every j.

We decide whether a vertex represents a feasible solution on the basis of five rules:

Rule:	Constant:	Description:
R1	A	no interference
R2	B	channel demand satisfied
R3	C	reuse packing preferred
R4	D	minimum variation from previous assignment preferred
R5	F	resonance condition preferred

We start from a generic solution c; then we change one (or more) of its components from 0 to 1 or vice versa. Due to the priorities of our rules, we are interested in four types of transitions :

- transitions that violate rule R1, but lower energy value for rules R2,R3,R4, and R5;
- transitions that increase energy for R2, but lower energy for R3,R4, and R5 leaving the term associated with R1 unchanged;
- transitions that increase energy for R5, but lower energy for R3,R4 leaving the terms associated with R1 and R2 unchanged;
- transitions that increase energy for R3, but lower energy for R4 leaving the terms associated with R1, R2, and R5 unchanged.

Since system (6) tends to lower energy during its evolution we have to find some relations among the constants of (1) so that each of the above named transitions coincides with an energy increment. These relations make all our feasible solutions be the only stable points in the system.
The relations are:

$$\begin{cases} C > D\left(\frac{1}{d_1} - \frac{1}{d_2}\right)^{-1} \\ E > \frac{C}{d_{reuse}}\left(\left\lceil \frac{n_{cells}}{k_{reuse}} \right\rceil - 1\right) + D \\ B > E\left(\left\lceil \frac{n_{cells}}{k_{reuse}} \right\rceil\right) \\ B > \frac{C}{d_{reuse}}\left(\left\lceil \frac{n_{cells}}{k_{reuse}} \right\rceil - 1\right) + D \\ A > B(2n_{channels} + 1) + \frac{C}{d_{reuse}}\left(\left\lceil \frac{n_{cells}}{k_{reuse}} \right\rceil - 1\right) + D \end{cases} \qquad (11)$$

where d_{reuse} and k_{reuse} are the distance and reuse factors related to the reuse pattern of the cellular network.
In order to simulate the behaviour of our neural network, we have built a software model of the network using the Euler's method to solve the set of differential equations of (6).
After one time step Δt we have:

$$x'_i = x_i - \Delta t\left\{T_i \cdot \bar{x} + b_i\right\}$$

$$New \quad x_i = \begin{cases} x'_i & \text{for } 0 < x'_i < 1 \\ 1 & \text{for } x'_i \geq 1 \\ 0 & \text{for } x'_i \leq 0 \end{cases}$$

where

$$\Delta t = \frac{R}{Max_i\left\{\left(\frac{1}{2}T \cdot \bar{1} + b\right)_i\right\}}$$

$$\bar{1} = [1,1,1,1,,,1]'$$

For the initial value of x we chose $x_i(0) = 0.5 + r_i$; where r_i is a uniformly distributed pseudo-random value in the range [-α/2, α/2].
In our simulation, we chose a value of R=0.3 and $\alpha = 10^{-9}$.

IV - SIMULATIONS AND PERFORMANCE EVALUATION.

The cellular network used by our simulations is made by a 7x7 portion of a hexagonal cell layout. We have considered two rings of interfering cells and 70 channels are available to the whole system. In order to evaluate the performance of our technique, the blocking probability as a performance index was used.
In the analytical model of a telephone system, the call arrivals are assumed to follow the Poisson process with mean arrival rate λ (calls/min.). The call duration is an exponentially distributed statistical process with mean $\bar{x}$ (min.). The quantity $\bar{x} \cdot \lambda = \rho$ expresses the load offered to the cellular network.
We computed the probability of refusing an incoming call for the 9 central cells only and for a uniform traffic distribution among all the cells. Under these simulation hypotheses, we can compare our results with those obtained by Zhang and Yum [7] with the "Locally Optimized Dynamic Assignment" (LODA), "Borrowing with Channel Ordering" (BCO) and "Borrowing with Directional Channel Locking" (BDCL) allocation algorithms.
Curves for theoretical FCA (ERLANG-B) are added to the numerical results obtained from the simulations.
In Fig.2, the behaviour of our technique is shown with respect to others DCA techniques for a uniform traffic distribution.

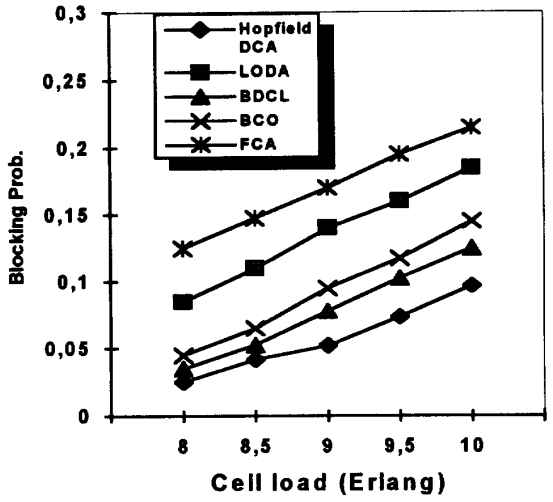

Fig. 2 - Performance with uniform traffic

For a non-uniform traffic distribution we considered the traffic pattern proposed by Zhang and Yum [7] shown in Fig.3.

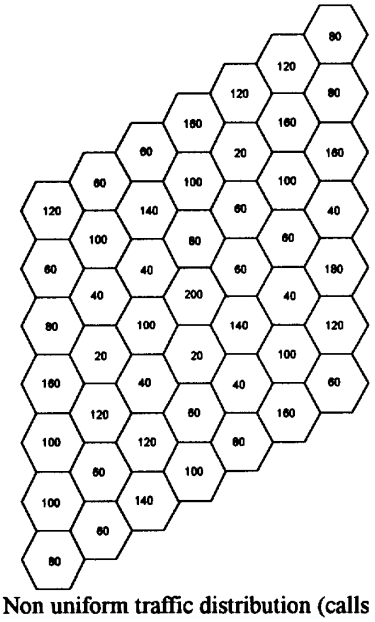

Fig. 3 - Non uniform traffic distribution (calls per hour)

The traffic load for each cell increased by a percent factor ranging from 0 to 140. The simulations results are summarized in Fig. 4.

V - CONCLUSIONS

In quadratic optimization problems, hopfield neural networks are not expected to perform much better than other methods for minima finding, since they effect a local search and the choice of a correct starting point is sometimes a hard task to be accomplished. However for a dynamic channel assignment, decision times are essential for the practical utilization of such techniques, and a neural approach like the one proposed in this paper seems to achieve a good level of performance with short processing times due to a massively-parallel computational structure.

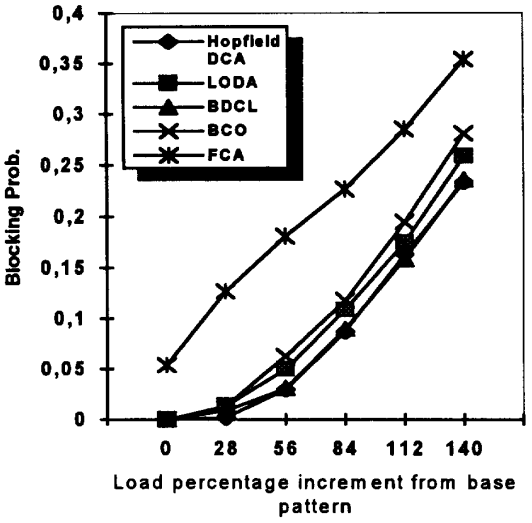

Fig. 4 - Performance with non uniform traffic load.

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