Bandpass Signal Filtering and Reconstruction through Minimum-Sampling-Rate Digital Processing

E. Del Re
Istituto di Elettronica, Facoltà di Ingegneria
Università di Firenze, Firenze

Abstract. First a method is presented to recover a bandpass signal through the minimum number of its samples. Then it is shown how this method can be applied to perform a digital bandpass filtering of a continuous wide-spectrum signal and subsequent reconstruction of the filtered signal using the minimum theoretical number of samples. Some different implementations of this technique are shown.
At last experimental results are reported referring to the application of the above technique to the bandpass filtering of some biomedical signals (EEG).

1. Introduction

Some methods are known [2] for recovering bandpass signals from their samples. In general they use a sampling frequency higher than the minimum theoretical sampling frequency or make use in the final D/A conversion of analog bandpass filters with a frequency response depending on both the signal bandwidth and center frequency.

This paper describes a particular method for recovering a continuous bandpass signal by the minimum theoretical number of samples [1] [2] (1), employing in the final D/A conversion only lowpass filters with a frequency response depending on the signal bandwidth.

At last a simple digital implementation of the described method, simulated by the computer and applied to the processing of some biomedical signals, is shown.

2. Reconstruction of a bandpass signal by the minimum number of samples using lowpass D/A converters

Let $s(t)$ be a bandpass signal with a spectrum $S(f)$, shown in fig. 1, extending over a bandwidth $f_2 - f_1 = B$ centered on the frequency f_0. It is known [2] that the permissible values of the sampling frequency f'_s which guarantee no spectrum overlapping are given by (2)

$$(1) \qquad \frac{2 f_2}{m+1} \leqslant f'_s \leqslant \frac{2 f_1}{m}, \qquad m = 0, 1, 2, \ldots$$

and that, called M the integer part of the ratio f_1/B and $k = f_1/B - M$ its fractional part, the minimum frequency suitable for the sampling of $s(t)$ is given by [2]

$$(2) \qquad f'_{s_{min}} = 2B\left(1 + \frac{k}{M+1}\right)$$

The analog signal $s(t)$ may be recovered from its samples by means of an analog bandpass filter with bandwidth B centered on f_0.

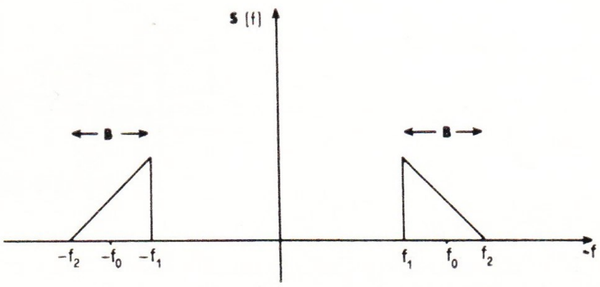

Fig. 1. - Spectrum of $s(t)$.

By the particular technique here presented it is however possible to recover the bandpass signal $s(t)$ through a suitable sampling carried out at the minimum theoretical [1] [2] rate B (3), avoiding at the same time any analog bandpass filter, as, for instance, it would be required by a direct implementation of the theoretical interpolation formula.

As is known, a bandpass signal, such as $s(t)$, may be represented in the form

$$(3) \qquad s(t) = a(t)\cos 2\pi f_0 t - b(t)\sin 2\pi f_0 t$$

where $a(t)$ and $b(t)$ are its in-phase and quadrature components defined as

$$(4) \qquad \begin{aligned} a(t) &= s(t)\cos 2\pi f_0 t + \hat{s}(t)\sin 2\pi f_0 t \\ b(t) &= \hat{s}(t)\cos 2\pi f_0 t - s(t)\sin 2\pi f_0 t \end{aligned}$$

$\hat{s}(t)$ being the Hilbert transform of $s(t)$. Since their

(1) We refer to the minimum number of samples according to the sampling theory for bandpass signals; clearly for specific signals, if special data compression techniques are used, a lower number can be eventually obtained.

(2) For $m = 0$ the right bound in (1) should be read as ∞.

(3) Of course, as it is theoretically required, two sets of suitable samples are necessary at the sampling rate B.

Estratto da « ALTA FREQUENZA », N. 9 vol. XLVII - 1978 - da pag. 395 E — 675 a pag. 678 — 398 E

spectra extend over the frequency range $(-B/2, B/2)$, the two lowpass signals $a(t)$ and $b(t)$ are completely determined by their samples at the Nyquist rate B. According to the two relations (4), the samples of $a(t)$ and $b(t)$ at the sampling points $1/B$ apart can be easily determined by the knowledge of the samples of both $s(t)$ and $\hat{s}(t)$ taken at the same sampling frequency B, and they are given by

$$(5)\quad \begin{aligned} a\left(\frac{n}{B}\right) &= s\left(\frac{n}{B}\right)\cos\left(2\pi f_0 \frac{n}{B}\right) + \\ &\quad + \hat{s}\left(\frac{n}{B}\right)\sin\left(2\pi f_0 \frac{n}{B}\right) \\ b\left(\frac{n}{B}\right) &= \hat{s}\left(\frac{n}{B}\right)\cos\left(2\pi f_0 \frac{n}{B}\right) - \\ &\quad - s\left(\frac{n}{B}\right)\sin\left(2\pi f_0 \frac{n}{B}\right) \end{aligned}$$

By means of a lowpass filter and a usual D/A converter, it is then possible to recover the two continuous lowpass signals $a(t)$ and $b(t)$ from their samples (5). Finally the in-phase and quadrature components permit to determine the continuous signal $s(t)$ through the relation (3).

both signals results to be 11.33 Hz. In this case by the proposed technique a factor of $11.33/6 \simeq 2$ for the gain in the frequency rate reduction is achieved.

Further the described method does not require the use of an analog bandpass filter with more stringent specifications as the center frequency f_0 increases. The reconstruction of the two lowpass signals $a(t)$ and $b(t)$ requires a lowpass filter whose characteristics depend only on the bandwidth B and not on the center frequency f_0.

From the preceeding results it is clear that this method is particularly favourable in digital transmissions (especially in time-division multiplexed PCM systems), as it reduces the transmission rate of the signal samples to the minimum value and requires simple and economical systems to recover the signal at the receiver.

3. Analog and digital implementations

The determination of the continuous Hilbert transform $\hat{s}(t)$ or of its samples $\hat{s}(n/B)$, n integer, may be carried out by analog or digital techniques.

The first technique makes use of an analog $-90°$ phase shifter (fig. 2). The two analog signals $s(t)$ and $\hat{s}(t)$ are then sampled at the frequency B and processed to give — and to transmit or store — the re-

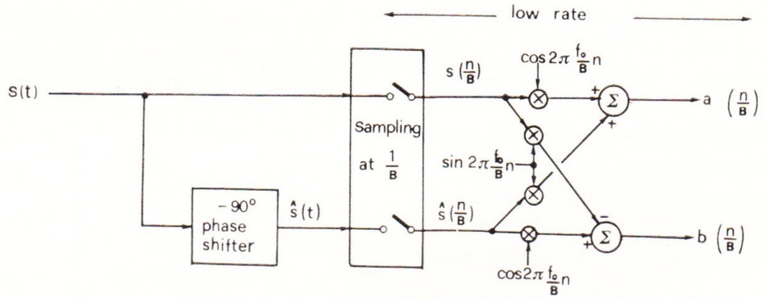

Fig. 2. - Analog implementation.

In conclusion the technique outlined above recovers exactly the original signal $s(t)$ from the sequences of samples $s(n/B)$ and $\hat{s}(n/B)$, n integer, — or equivalently $a(n/B)$ and $b(n/B)$ — taken at the minimum sampling rate B. From the point of view of processing complexity and information transmission, the equivalent sampling rate is therefore $2B$ samples/sec, less than or equal to the minimum sampling rate given by (2). However in case of several bandpass signals, having the same bandwidth and different center frequencies, the gain of sampling rate reduction is greater than that predicted by the comparison with expressione (2). In fact it may happen that the minimum sampling frequencies, given for each signal by (2), are not equal. In this case the minimum sampling frequency for all the signals is given by the minimum value of the permissible frequency sets, determined for each signal by the relation (1). In some cases this minimum frequency is much higher than the individual minimum sampling frequencies given by (2). For example, if we consider two bandpass signals extending from 8 to 11 Hz and from 14 to 17 Hz respectively, the minimum sampling frequency for

quired values of the samples of the in-phase and quadrature components $a(t)$ and $b(t)$, as shown in fig. 2. It must be pointed out that in an actual (either analog or digital) system implementation only the samples $s(n/B)$ and $\hat{s}(n/B)$ may be stored or transmitted. The processing for the evaluation of the samples $a(n/B)$ and $b(n/B)$ may in fact be performed at the receiver, simplifying the transmitter structure [4].

In digital implementations we are often in presence of the samples $s(n/f_s)$ obtained by an oversampling of the signal $s(t)$ at a sampling frequency f_s. In these cases, provided that f_s is a multiple of the signal bandwith B, it is necessary to retain only one sample every f_s/B available samples and to determine the Hilbert transform samples by means of a digital Hilbert filter at the sampling instants relevant to the retained samples.

As is known [3] [4], FIR digital Hilbert filters can be easily designed, having an optimum frequency response and a relatively small impulse response length

[4] The $-90°$ phase shifter unavoidably also introduces delay. This must exactly be compensated for in the second transmission path.

(total number of coefficients of the order of 20). An FIR digital filter is highly suitable [5] [6] for such an application, because it allows to compute the output sample values only at the required sampling instants for the transmission or storage. Fig. 3 shows the digital realization of the proposed technique.

Of course also a bandpass filtering of a (wide-spectrum) signal $x(t)$ can be carried out at the minimum sampling rate. Two solutions can be adopted for the evaluation of the samples of the bandpass component $s(t)$ and of its Hilbert transform $\hat{s}(t)$ at the required sampling frequency B from the samples of the wide-spectrum signal taken at a higher frequency f_s. In the first solution the samples $s(n/f_s)$ can be produced at the output of a digital bandpass filter at the same sampling rate f_s of the input wide-spectrum signal and then the processing for the evaluation of $s(n/B)$ and $\hat{s}(n/B)$ is as that shown in fig. 3. In the second solution, shown in fig. 4, an FIR bandpass digital filter and an FIR bandpass Hilbert transformer are used to produce the samples $s(n/B)$ and $\hat{s}(n/B)$ at the required sampling frequency directly from the samples of the wide-spectrum signal $x(t)$ sampled at f_s. This solution avoids the intermediate production of the bandpass signal samples $s(n/f_s)$ at the frequency f_s. Therefore a more efficient system implementation is achieved, even if the FIR bandpass Hilbert transformer requires generally a greater number of coefficients than the wideband Hilbert transformer shown in fig. 3.

Finally we can observe that in some applications the samples of $a(t)$ and $b(t)$ may be required at a higher rate than the frequency B, as, for example, for an easier reconstruction of the continuous signals $a(t)$ and $b(t)$ by a D/A converter or for a subsequent digital signal processing. By using FIR interpolation digital filters, practically any desired interpolated sampling frequency can be easily obtained at the receiver, maintaining at a minimum value the transmission rate or the memory size required for an exact reconstruction of a bandpass signal.

4. Experimental results

The described method has been applied to wide-spectrum signals. More in particular an electroencephalogram (EEG), originally sampled at 96 Hz, was processed to obtain the 6 to 14 Hz component (alpha rythm). An FIR linear phase bandpass digital filter and an FIR linear phase bandpass Hilbert transformer (see fig. 4) have been employed to extract the in-phase and quadrature components directly from the wide-spectrum signal. The impulse response length of the two digital filters has been of 63 samples.

Two examples of the results obtained through the described method are shown in figs. 5 and 6. The top signal is the original wide-spectrum EEG. The third and fourth lines represent the decimated samples of the in-phase and quadrature components respectively, corresponding to a sampling frequency of 8 Hz. The bottom signal is the 6 to 14 Hz bandpass signal reconstructed from the in-phase and quadrature samples. Only for comparison purposes, the second line shows the 6 to 14 Hz EEG component obtained by a direct bandpass filtering of the top signal.

For clearness and for requirements of subsequent signal processing the sampling interval in the representation of figs. 5 and 6 was chosen to correspond to a sampling frequency of 288 Hz. The original EEG

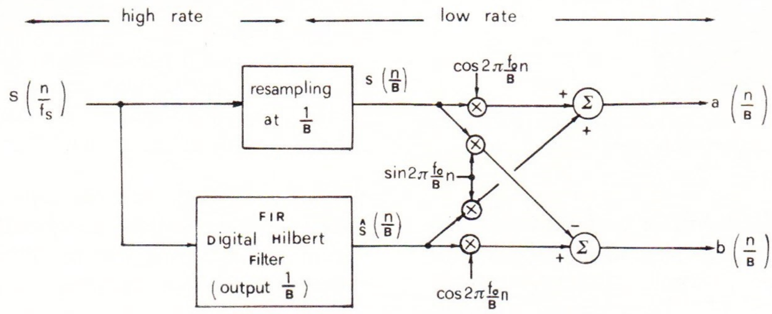

Fig. 3. - Digital implementation.

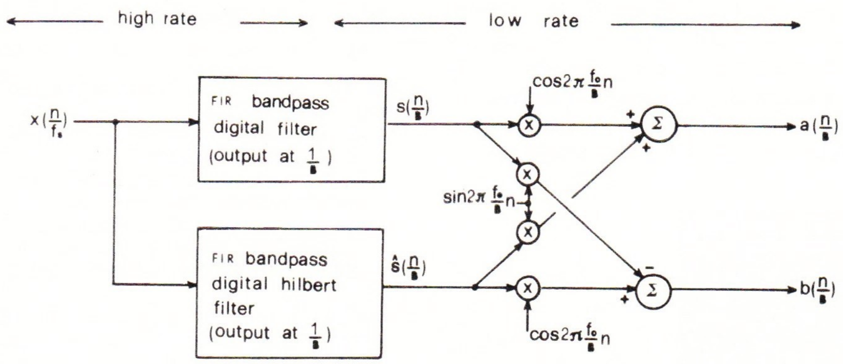

Fig. 4. - Direct evaluation of $s(n/B)$ and $\hat{s}(n/B)$ from a wide-spectrum signal.

signal of the first line and the directly bandpass filtered signal of the second line were interpolated to the desired sampling frequency by means of a digital interpolating filter with an interpolating factor of 3. The bandpass signal of the last line was reconstructed by first interpolating the in-phase samples $a(n/B)$ and quadrature samples $b(n/B)$ to the

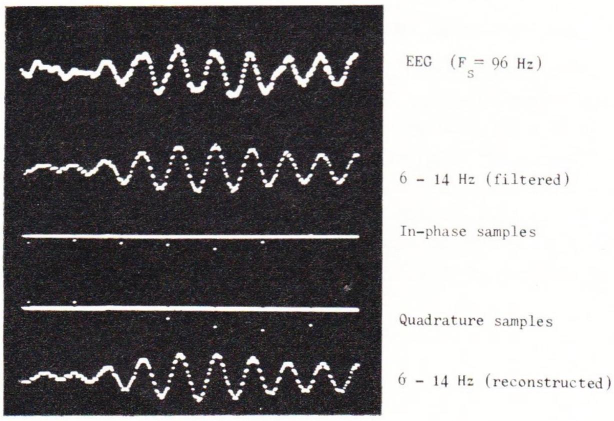

Fig. 5. - Example of bandpass signal reconstruction by the minimum number of samples.

288 Hz frequency by a digital filter with an interpolating factor of 36 and then applying the synthesis relation defined by (3) to the interpolated samples of $a(t)$ and $b(t)$. In both cases FIR linear phase window-designed interpolating digital filters were employed, using the high efficiency Cappellini window [7].

The reconstruction error (defined as the maximum value of the magnitude of the difference between the reconstructed signal and the directly bandpass filtered signal) was evaluated: it always resulted to be within the value allowed by the specifications of the frequency response of the used filters.

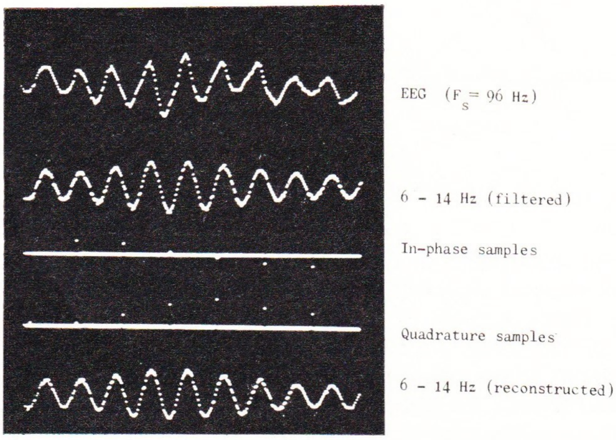

Fig. 6. - Example of bandpass signal reconstruction by the minimum number of samples.

Also a power spectral estimation for the bandpass signal was carried out by means only of the decimated samples of the in-phase and quadrature components. The average power has been correctly estimated within an approximation error always less than 1% [8] [9].

5. Conclusions

The described method permits to recover any bandpass signal through the minimum theoretical number of samples, exploiting the properties (in particular the lowpass property) of the in-phase and quadrature components of the bandpass signal.

The method is in particular attractive in the case of time-division multiplexed PCM transmission: the required transmission rate can be kept at the minimum value consistent with the bandwidth extension of the individual signals, while relatively simple systems are required at the receiver for signal reconstruction.

Finally an attractive aspect of the method regards its discrete implementation by using CCD devices: a single « chip » could be constructed performing all the required operations (as in figs. 3 and 4).

Acknowledgment

The author wishes to thank Prof. V. Cappellini for the helpful suggestions and discussions and for the revision of this paper.

Work supported by Italian CNR under contract 77.01482.07.

The paper was first received on February 28, 1978.

REFERENCES

[1] C. Atzeni, V. Cappellini: *Elementi di Teoria delle Comunicazioni.* Ed. G & G, Firenze, 1971.

[2] P. F. Panter: *Modulation, Noise and Spectral Analysis.* McGraw-Hill, N.Y., 1965.

[3] J. H. McClellan, T. W. Parks, L. R. Rabiner: *A Computer Program for Designing Optimum FIR Linear Phase Digital Filters.* « IEEE Trans. Audio and Electroac. » AU-21, n. 6, Dec. 1973.

[4] L. R. Rabiner, R. W. Schafer: *On the Behaviour of Minimax FIR Digital Hilbert Transformer.* « BSTJ » vol. 53, n. 2, Feb. 1974.

[5] R. W. Schafer, L. R. Rabiner: *A Digital Signal Processing Approach to Interpolation.* « Proc. IEEE » vol. 61, n. 6, June 1973.

[6] L. R. Rabiner, B. Gold: *Theory and Application of Digital Signal Processing.* Prentice-Hall, Englewood Cliffs, N.J., 1975.

[7] E. Borchi, V. Cappellini, P. L. Emiliani: *A New class of FIR Digital Filters Using a Weber-type Weighting Function.* « Alta Frequenza » vol. 44, n. 9, Oct. 1975.

[8] V. Cappellini, E. Del Re: *Sample Rate Reduction for Spectral Estimation performed through Digital Filtering.* « Alta Frequenza » vol. 45, n. 10, Oct. 1976.

[9] G. D. Cain, V. Cappellini, E. Del Re: *A Digital Bandpass Analysis Approach for Biomedical Signal Processing and Telemetry.* In: Proc. EUROCON '77, Venezia, May 1977.

Arti Grafiche Stefano Pinelli via Farneti 8 20129 Milano