distributed over $(-\pi, \pi]$. The transmitted waveform is a QPSK signal, filtered with a squared root raised cosine. The received signal is filtered with the same matched squared root raised cosine filter.

The signals are received by the array in the presence of additive alpha-stable noise. Since the alpha-stable family for $\alpha < 2$ defines processes with infinite variance, we define the *fractional effective* SNR (FESNR) measure as the ratio of the fractional signal power over the fractional noise power:

$$\mathrm{FESNR} = 10\log\left(\frac{\sum_{t=1}^{M}|s(t)|^{\alpha}}{\sum_{t=1}^{M}|n_{\alpha}(t)|^{\alpha}}\right). \tag{22}$$

According to this choice of an SNR metric, the SαS noise samples are power scaled by the corresponding characteristic exponent α before contributing to the SNR calculation. Note that, for Gaussian noise ($\alpha = 2$), expression (22) coincides with the usual SNR measure.

The proposed ROC MUSIC smoothing algorithm has to estimate the FLOS matrix in (19) from the sensor measurements. We used the approach followed in [7,17], by calculating the instantaneous value of the expectation $E[\boldsymbol{x}^{\langle p_1 \rangle} \cdot \boldsymbol{x}^{\langle p_2 \rangle^{\mathrm{H}}}]$, where the two parameters p_1, p_2 have to follow the constraint $p_1 < \alpha/2$ and $p_2 < \alpha/2$. In real life, the statistics of the additive noise data are unknown. This gives rise to the need for fast, simple, and efficient estimators of the alpha-stable parameters (especially, the characteristic exponent, α) from real data. Several such estimators compromising optimality for the sake of computational efficiency, have been proposed in the past and they are described in [18] and references therein.

Figs. 3–5 show results on the resolution capabilities of the methods for various values of the characteristic exponent α of the noise. First, by comparing Figs. 3(a),(b) to (c),(d), we see the improvement in coherent source DOA estimation achieved with spatial smoothing. Of course, because of the presence of heavy-tailed noise, the improvement is mostly apparent for the case of the introduced FLOS-based method. Indeed, comparing Figs. 3(c) and (d) we see that, for a fairly impulsive noise environment ($\alpha = 1.5$, FESNR $= -2$ dB), the SO-based MUSIC smoothing method exhibits low-resolution as it cannot clearly identify the multipath signals. On the other hand, the proposed ROC-MUSIC smoothing processor places clearly distinguished peaks to all four DOAs (cf. Fig. 3(d)).

For more closely spaced sources, which are located at $\vartheta = [15^\circ, -40^\circ, 40^\circ, -30^\circ]$ Fig. 4 demonstrates that the ROC-MUSIC Smoothing algorithm is still able to resolve the two adjacent paths from -40° and -30°, something that the MUSIC smoothing method is not able to achieve at this particular FESNR.

Even when the statistical behavior of the noise is close to Gaussian ($\alpha = 1.85$), the ROC-MUSIC smoothing method still performs better than the MUSIC Smoothing (cf. Figs. 5(a) and (b)). Finally, when operating in a Gaussian noise environment with finite second-order statistics, both methods exhibit good performance (cf. Figs. 5(c) and (d)).

The property of the introduced FLOS-based processor to operate robustly in various interference backgrounds, which was demonstrated in Figs. 3–5, was also quantified by performing Monte Carlo runs to measure the associated root mean-square error (RMSE) of the DOA estimates. For this part of the simulations, we used two signals coming from directions $[30^\circ\ -40^\circ]$ and an eight-element array with a sub-array dimension equal to 4. Table 1 reports the results of both SO- and FLOS-based methods when estimating the source at 30°, as a function of the additive noise characteristic exponent and the FESNR. Clearly, the table demonstrates that the DOA estimates associated with the FLOS-based processor have significantly smaller RMSE for all non-Gaussian noise backgrounds and all FESNR values. For the case of Gaussian noise, the performance of the two methods is comparable, with the SO-based processor having slightly better performance, as expected.

In conclusion, the proposed spatial smoothing method based on the fractional lower-order statistics is shown to be able to resolve coherent sources in various types of noise environments. The FLOS-based method exhibits increasing performance improvement over the conventional second-order based techniques for heavier noise backgrounds.

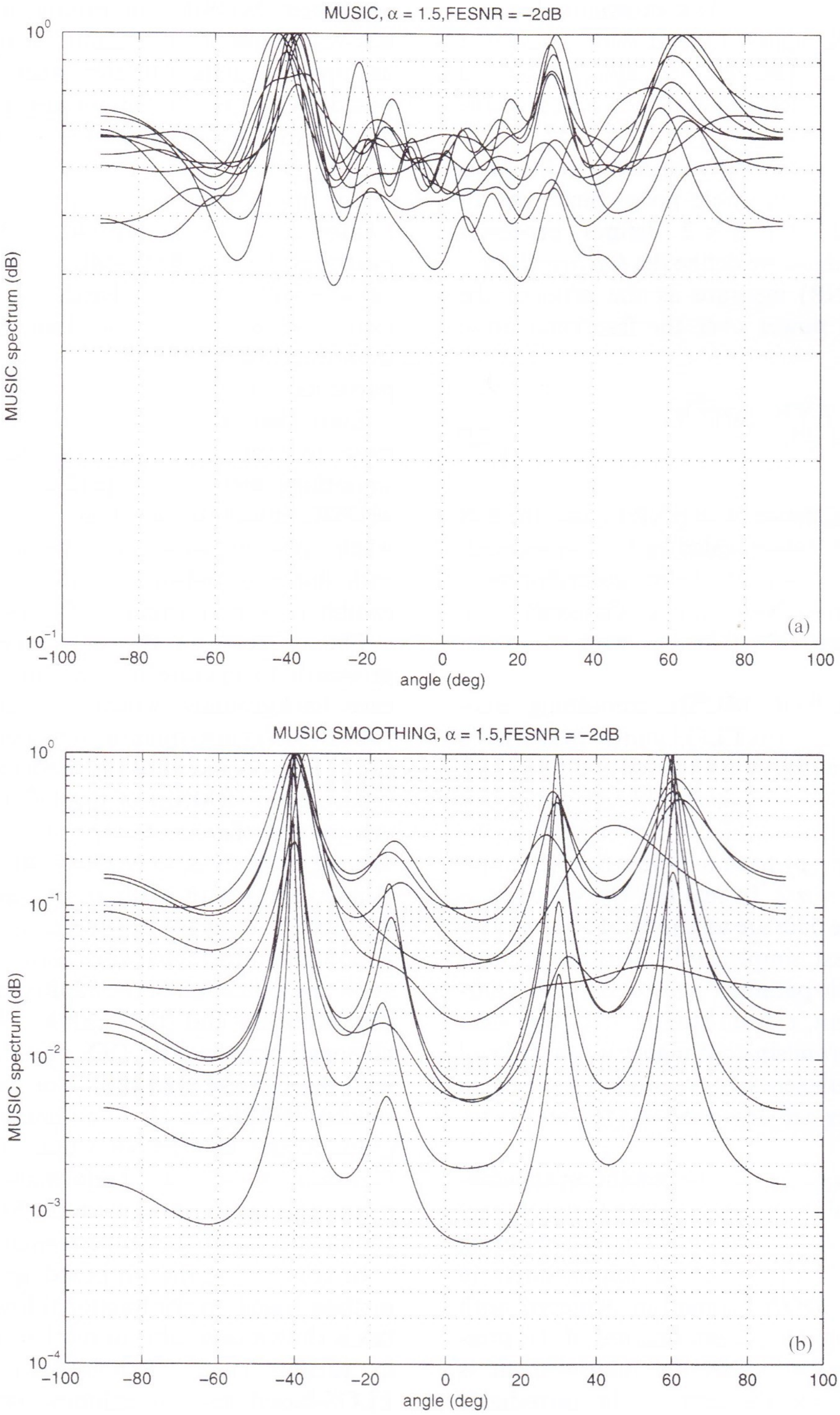


Fig. 3. (a) Spatial spectral estimates in additive alpha-stable noise with $\alpha = 1.5$ and FESNR $= -2$ dB: (a) MUSIC; (b) MUSIC smoothing; (c) ROC-MUSIC; and (d) ROC-MUSIC smoothing.

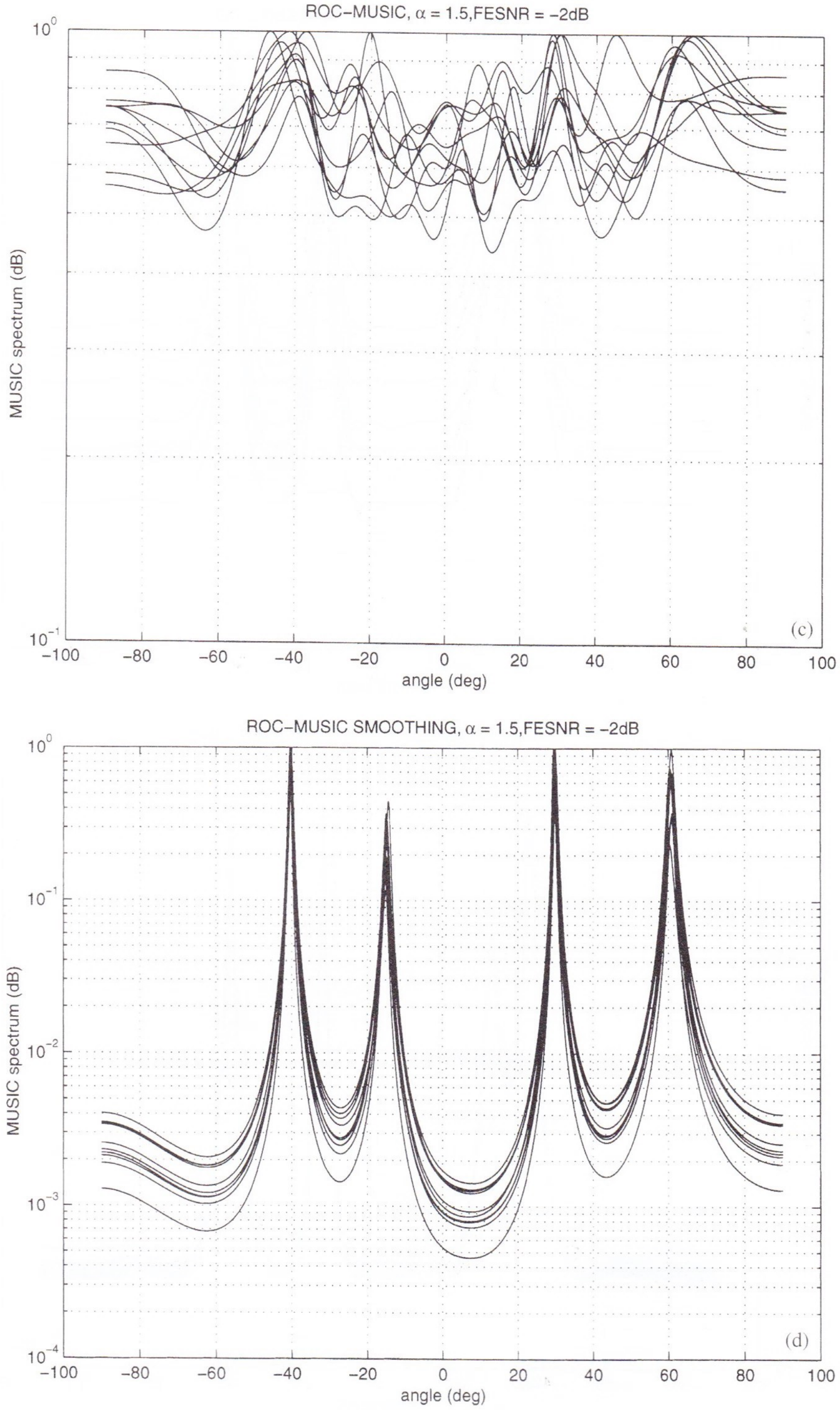
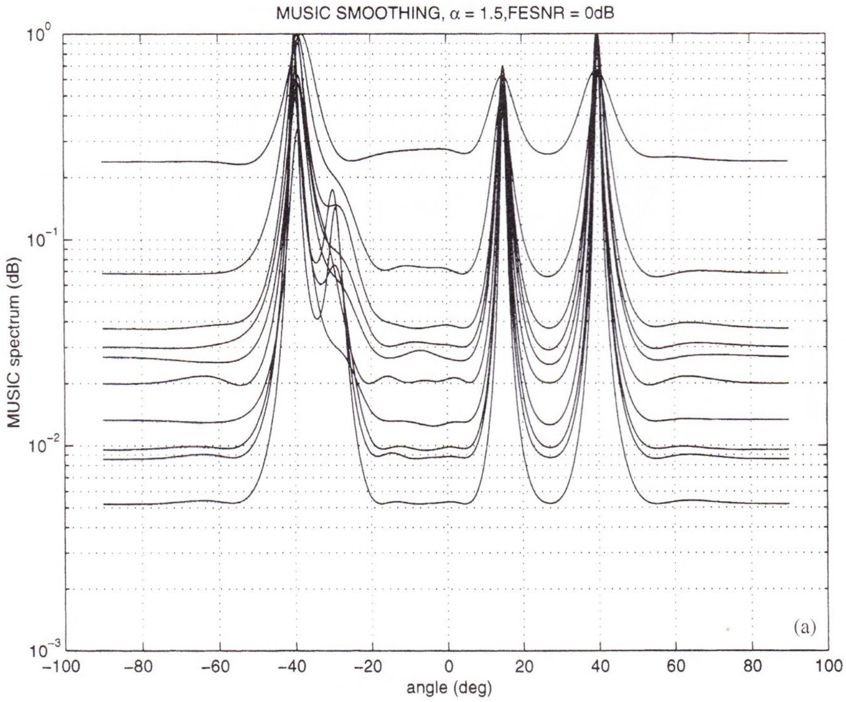


Fig. 3. (continued).



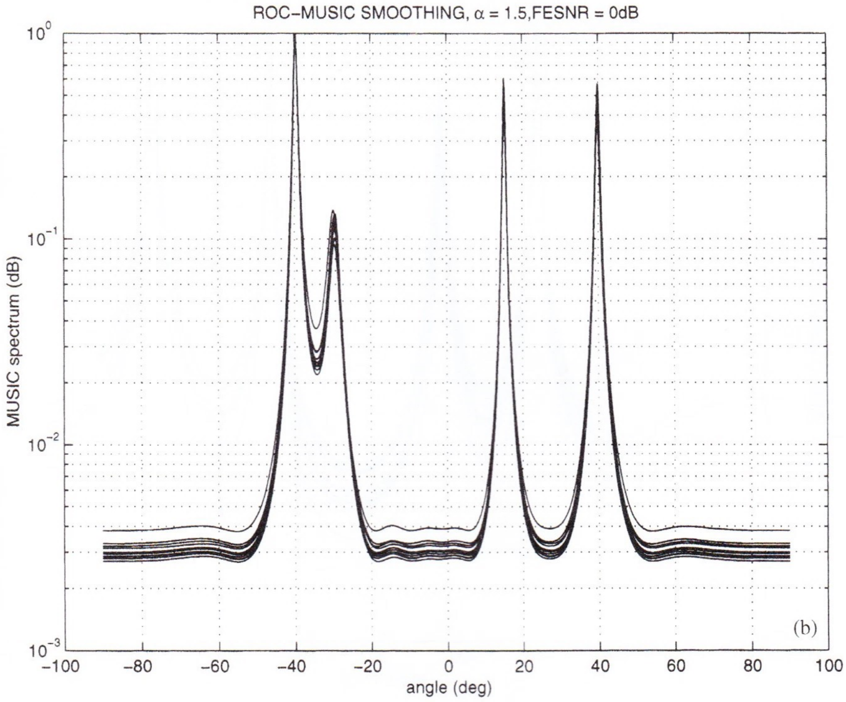


Fig. 4. Spatial spectral estimates in additive alpha-stable noise with $\alpha = 1.5$ and FESNR = 0 dB. (a) MUSIC smoothing; (b) ROC-MUSIC smoothing.

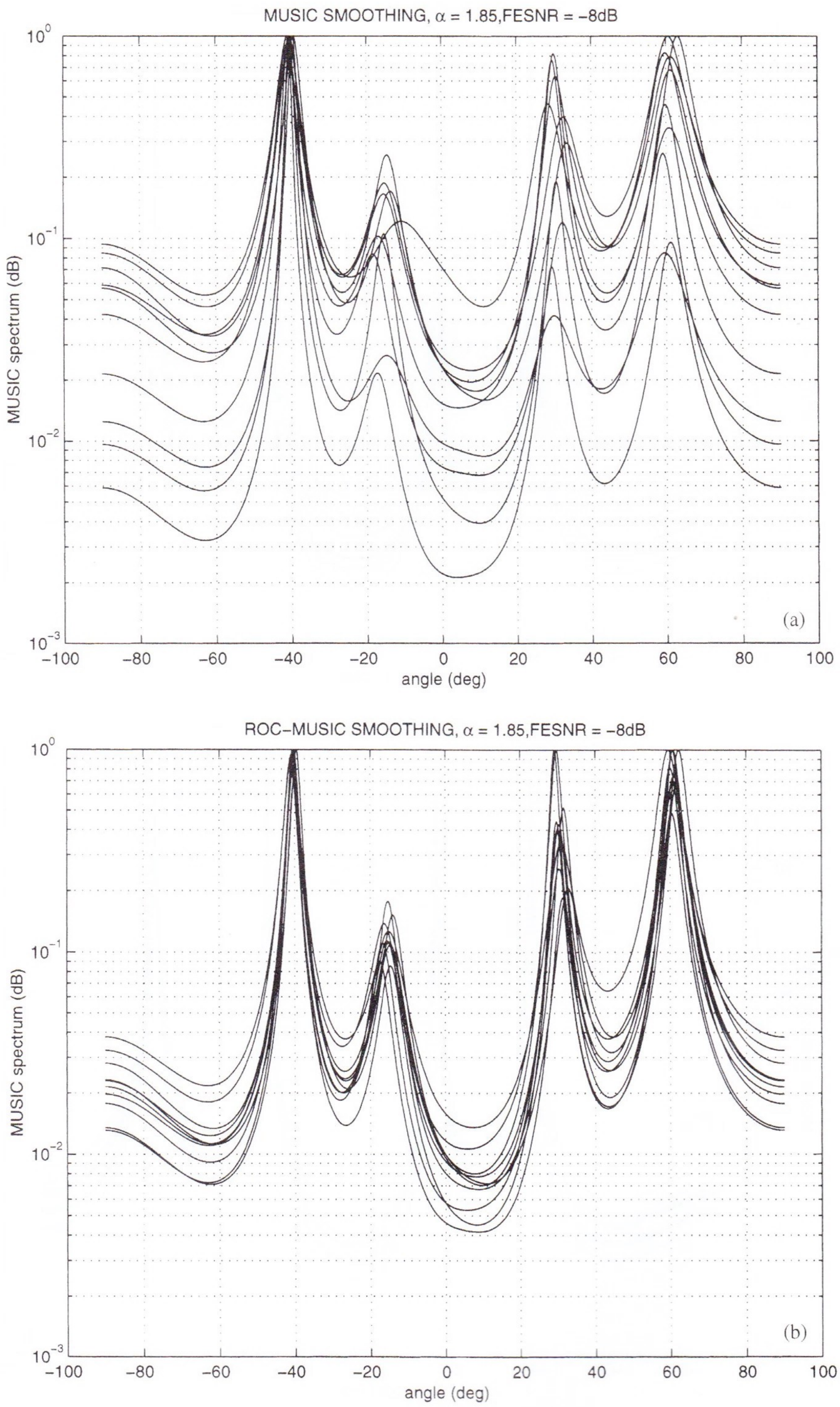
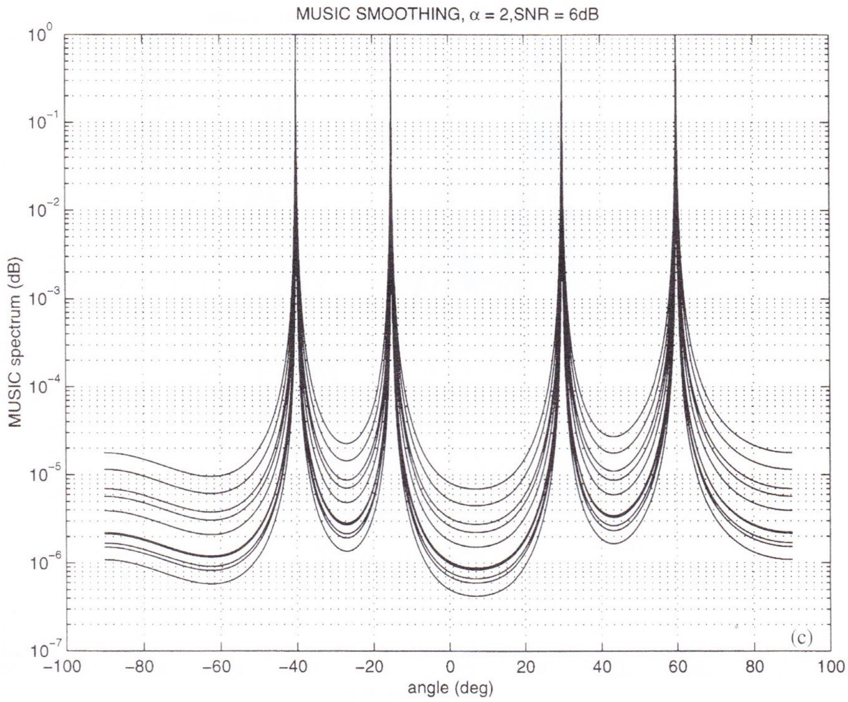


Fig. 5. (a) Music smoothing and ROC-MUSIC smoothing for $\alpha = 1.85$ and FESNR $= -8$ dB (a)–(b), and for $\alpha = 2$ and SNR $= 6$ dB (c)–(d).



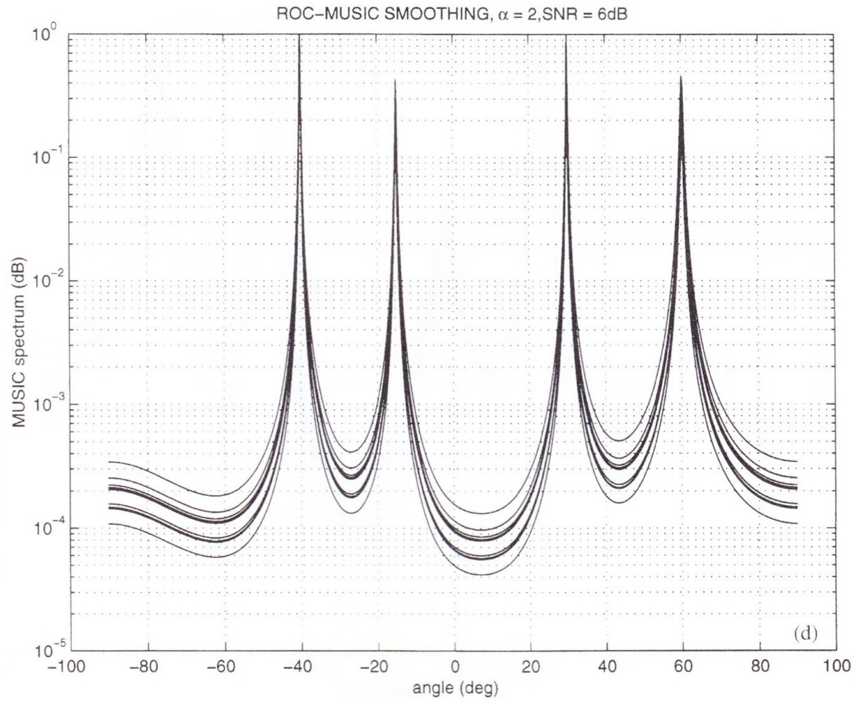


Fig. 5. (continued).

Table 1
RMSE of DOA estimation as a function of the noise characteristic exponent and the FESNR (MS: MUSIC Smoothing, R-MS: ROC-MUSIC Smoothing)

	$\alpha = 1.5$		$\alpha = 1.85$			$\alpha = 2$	
dB/	R-MS	MS	R-MS	MS	dB/	R-MS	MS
− 12	1.8	23.8	5.6	13.11	4	0.14	0.1
− 8	0.76	15.72	1.1	2.01	6	0.12	0.07
− 4	0.46	2.5	0.4	0.7	8	0.09	0.05
0	0.26	1.06	0.22	0.25	10	0.08	0.04
4	0.2	0.42	0.09	0.08	12	0.05	0.02

6. Conclusion

Conventional high-resolution eigen-decomposition techniques perform poorly in coherent receiving environments. Spatial smoothing methods proposed in the past address the signal coherence problem but fail to operate reliably in a heavy-tailed non-Gaussian noise. The method proposed in this paper is able to achieve high resolution performance when operating in both multipath and impulsive noise environments. The new algorithm is based on spatial smoothing of the FLOS matrix of an antenna array and it is shown to exhibit better resolution performance in a wide range of noise environments without considerably increasing the complexity of the system. Several limitations of the proposed method need to be addressed in the future, including unequally spaced non-linear arrays, and correlated additive noise structures.

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