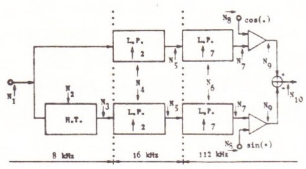
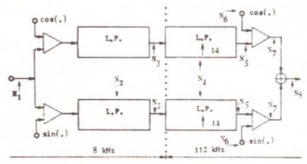
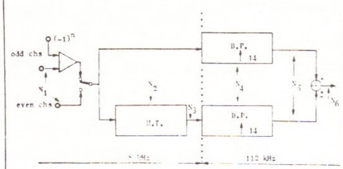
METHOD	USED FILTERS f_L = LOWER CUTOFF FREQUENCY f_U = UPPER CUTOFF FREQUENCY kHz	MULT./s PER CHANNEL	ADD./s PER CHANNEL	S/N (dB)	MULTIPLICATION BIT PRECISION					BIT STORAGE
					15	16	17	18	19	
THE HARTLEY METHOD 	- H.T. (77 coeff.) $f_L = 0$ $f_U = 4$ - 1st L.P. (55 coeff.) $f_L = 0$ $f_U = 4$ - 2nd L.P. (41 coeff.) $f_L = 0$ $f_U = 6.95$	1960×10^3	1960×10^3	71.15			1584	376		972
THE WEAVER METHOD 	- 1st L.P. (81 coeff.) $f_L = 0$ $f_U = 2$ - 2nd L.P. (83 coeff.) $f_L = 0$ $f_U = 3.435$	1904×10^3	1872×10^3	72.77			1344	336	224	1176
THE ANALYTIC SIGNAL METHOD 	- H.T. (69 coeff.) $f_L = 0$ $f_U = 4$ - B.P. (83 coeff.) $f_L = 2.35$ $f_U = 9.3$ (1st channel)	1480×10^3	1496×10^3	69.76	1344	136				1414

Table 1 - Performance comparison of the different methods.

sidered. The first ones multiply the input samples by sequences of the form ...1, 0,-1,0,1,0,-1,0...and the second ones simply imply an alternate sign inversion on the input samples. Of course both these operations do not necessitate an actual multiplying operation on the input samples.

The performance of the different methods was afterwards compared by taking into account the practical finite wordlength system implementation. In the block diagram of Table 1 the quantization bit numbers N_k of the various quantities are indicated and their actual value, determined as will be explained in the following, is reported in Table 2.

Three error sources derive from the finite length of the digital registers:

i) quantization of the input signal;
ii) quantization of the filter coefficients;
iii) rounding of the multiplication operations.

In case of the TDM to FDM conversion the input signal is already an 8-bit A-law coded PCM signal. This coding corresponds to a 13-bit linear quantization. Hence considering a 13-bit linear quantization of the input signal no additional error is produced by the conversion system. This is indicated by the quantization bit number N_1 of the input signal.

The minimum wordlengths of the filter coefficients were determined in order to guarantee that they still satisfy the frequency specification stated above. They are indicated by the N_k entering the filter blocks in Table 1.

As far as the third source of error is concerned it must be observed that a FIR digital filter implemented by P multiplications each rounded to M bits produces an output error with a r.m.s. value $P.2^{-2M}/3$. Another contribution to the output error comes from the input signal quantization error. However this contribution can be considered negligible if the input quantization error r.m.s. value is sufficiently small and the filter has a sufficiently narrow bandwidth. In these hypotheses (practically verified in the performed simulation) the output error is due only to the multiplication rounding and it corresponds to an output signals quantized to N bits given by

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{10}
Hartley Method	13	12	15	16	15	14	15	15	18	14
Weaver Method	13	14	15	16	15	15	19	14		
Analytic Signal Method	13	14	13	14	13	13				

Table 2 - Bit precision of the numbers N_k of Table 1.

$$(14)\quad 2^{-2N}/3 = P2^{-2M}/3$$

Because in the simulation it is much more convenient to quantize to N bits the filter output signals, the multiplication bit precision can therefore be calculated as

$$(15)\quad M = N + \left\lceil (\log_2 P)/2 \right\rceil$$

where $\lceil x \rceil$ indicates the minimum integer greater than or equal to x.

Table 1 shows the filter output signals quantized to the indicated N_k bits. The N_k values were determined for each conversion method through simulation. In particular, considering only one channel active at a time, an error signal was defined as

$$(16)\quad e(nT/L) = y(nT/L) - y'(nT/L)$$

where y(nT/L) and y'(nT/L) are the system outputs, without and with multiplication rounding respectively. For both outputs the input 'active' signal $x_i(nT)$ was quantized to $N_1 = 13$ bits and the filter coefficients were quantized to their appropriate number of bits (see Tables 1 and 2).

The values of the output signal quantization bits N_k were determined requiring that the power of the error (16) in the signal range of the active channel (corresponding to the baseband frequency interval 200-3400 Hz) should give a signal power to error power ratio S/N of the order of 70 dB for each channel.

For each method Table 2 summarizes the values N_k for all the quantized quantities indicated in Table 1.

The values of the signal to error power ratios S/N obtained with the quantization bits of Table 2 are reported in the fifth column of Table 1. They refer to the first FDM channel (4 to 8 kHz).

From the number N_k of Table 2 of the quantization bits of the filter output signals it is possible to derive, according to (15), the corresponding multiplication bits. The sixth column of Table 1 reports the number of multiplications per channel that must be performed at the indicated bit precision.

The last column of Table 1 indicates the amount of bit memory per channel required to store the quantized filter coefficients and the quantized sinusoidal samples for the product modulators.

The system performances, summarized in Table 1, show that the advantages of the proposed analytic signal method of TDM-FDM transmultiplexing are:

i) a simpler system implementation structure,
ii) a smaller number of the required operations,
iii) a smaller length of the digital words;

whereas the only disadvantage is an increase of the required digital memory. However today this is not a really limiting factor.

Finally for the three methods respectively Figs. 10, 11 and 12 show as an example the spectral plots of the conversion system output signals: the spectra Y(f) and Y'(f) without and with multiplication rounding respectively and the error signal spectrum E(f) = Y(f) - Y'(f). The example refers to the case of only the first FDM channel (4-8 kHz) being active.

5 - CONCLUSIONS

The comparison of the performance of the different non-FFT methods of TDM-FDM transmultiplexing has shown the advantage of the new proposed method based on the properties of the sampled analytic signal, which leads to a simpler system structure without any product modulator and achieves a definite saving in the computational complexity.

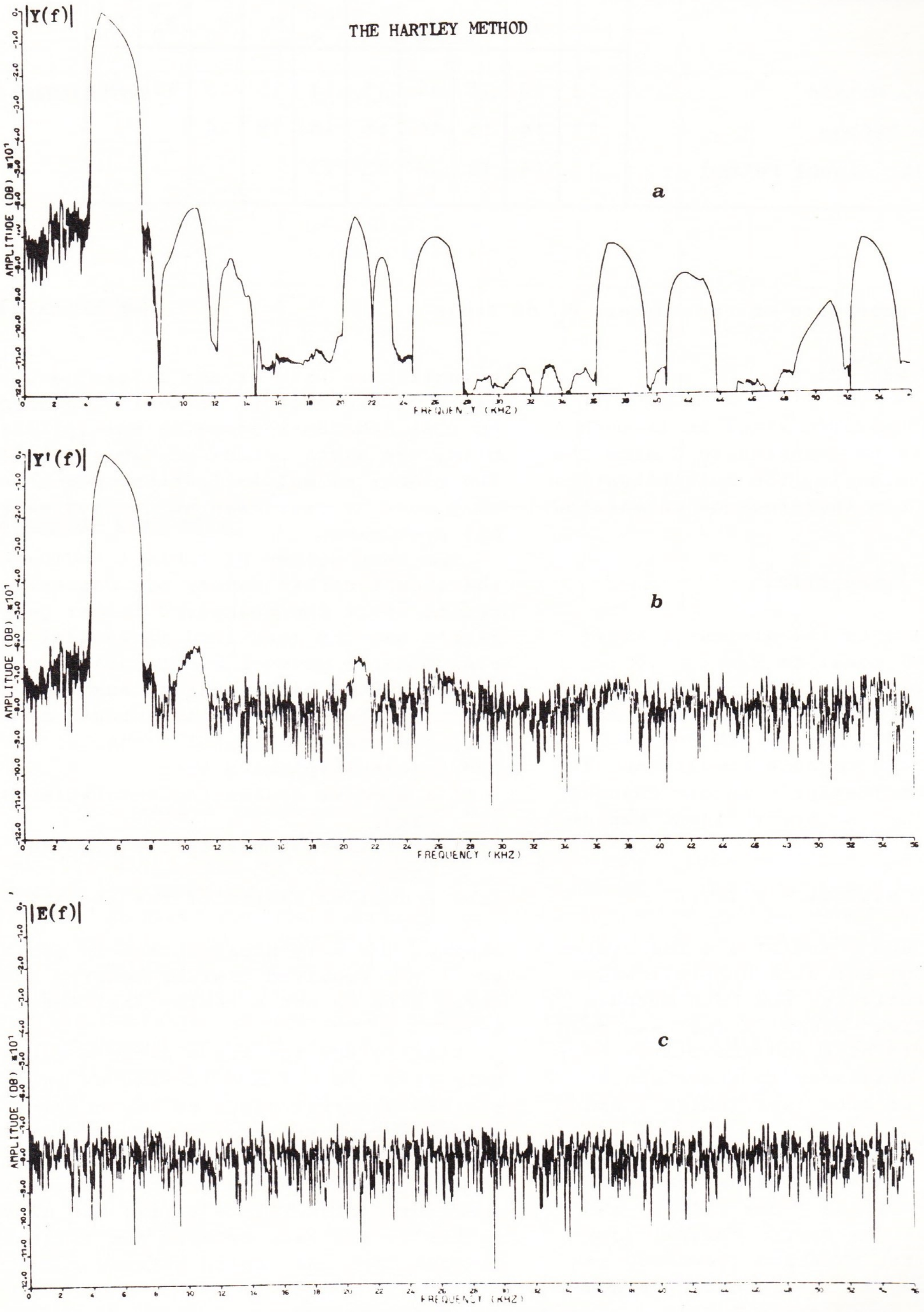


Fig. 10 - Performance of the conversion system with only the first FDM channel active: a) output without multiplication rounding; b) output with multiplication rounding; c) error signal.

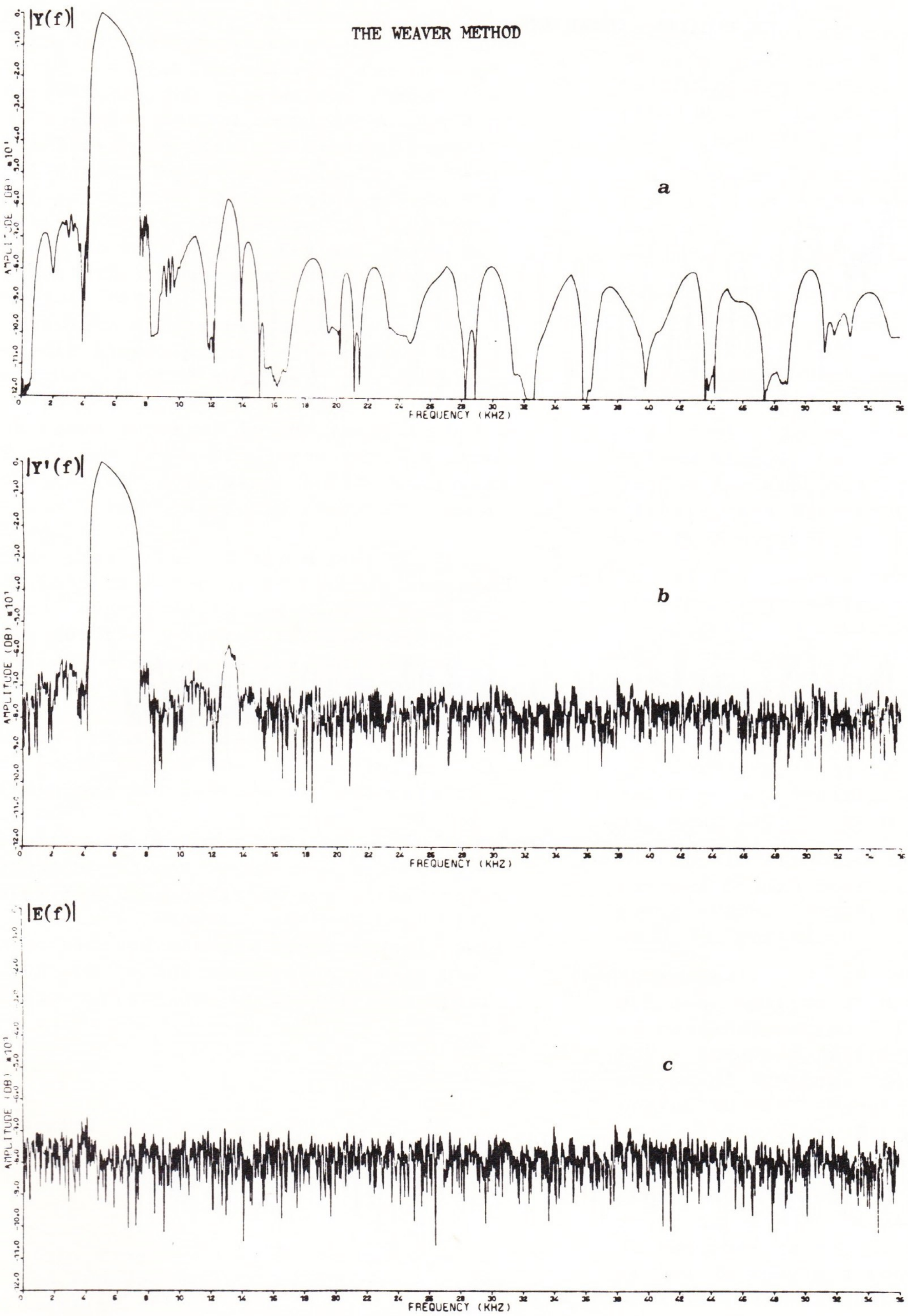


Fig. 11 - Performance of the conversion system with only the first FDM channel active: a) output without multiplication rounding; b) output with multiplication rounding; c) error signal.

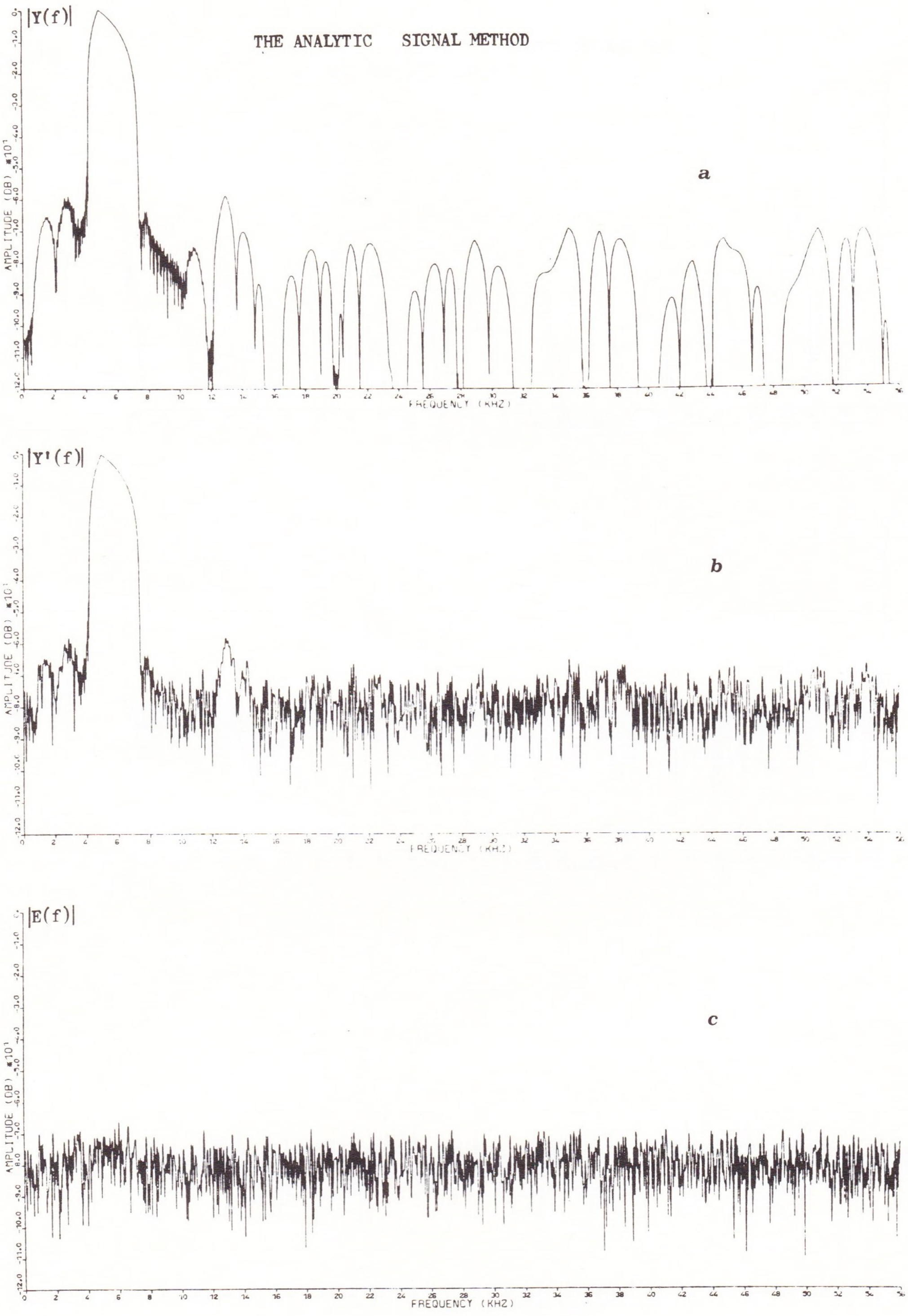


Fig. 12 - Performance of the conversion system with only the first FDM channel active: a) output without multiplication rounding; b) output with multiplication rounding; c) error signal.

Some remarks have to be made about the performed system simulation for the comparison of the different methods. As already said, a particular suboptimal filter design technique was chosen because of its specific property of high flexibility in changing the required filter characteristics. This guarantees for a correct relative system comparison, but does not achieve the absolute optimum system performance. Indeed some results of Table 1 are worse than their corresponding ones reported in the literature [1] , obtained through optimum filter design. However the comparison suggests that the use of optimized filter designs should achieve an absolute better performance for the analytic signal method with respect to the other non-FFT methods.

Also some results of Table 1 are worse than those reported in the literature for FFT methods [2] . Of course in this case too a correct comparison would require an optimized version of the analytic signal method.

For these reasons the filter optimization both in terms of design and implementation techniques is the current research topic for the proposed transmultiplexer solution to arrive at the absolute minimum computational complexity for a correct comparison with other reported FFT and non-FFT approaches. To this end it is also worth noting that a main difference between non-FFT methods and FFT ones is the complete hardware separation of all the channels for the first solution, a feature which can be advantageous in the system fault recognition and elimination.

Finally it must be recalled that all the considered methods apply equally well to the FDM to TDM conversion simply by the appropriate use of network transposition rules.

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