Asymptotic Multiuser Efficiency for a Two-States CDMA System

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Abstract—**The use of CDMA makes third generation wireless systems interference limited rather than noise limited. The research for new methods to reduce interference and increase efficiency lead us to formulate a signaling method where fast impulsive silence states are mapped on zero-energy symbols. The theoretical formulation of the optimum receiver is reported and the asymptotic multiuser efficiency has been derived and applied to the optimum two-states receiver. Numerical comparisons have been performed to show the advantages of the proposed scheme over the traditional single state CDMA transmission. Several operating scenarios have been numerically analyzed and the results are reported in the paper.**

I. Introduction

Bandwidth represents the last challenge in wireless personal communications. In UMTS, due to the average increase of the radio link bandwidth requirements and the hostile urban radio channel for the interference limited W-CDMA access, the system capacity will meet its physical limitations even in a moderated deployment scenario.

Every technology able to increase the spectral efficiency of the radio link maintaining the compatibility with the approved standards will play a fundamental role for the economical aspects of the UMTS diffusion.

Those considerations lead to the development of the transmission scheme presented in this paper. The basic idea is the extension of the traditional informative symbol set with a *zero energy* symbol. The silence symbols are integrated with the informative ones and delivered to the radio link layer for transmission [1]. The end-to-end signaling between the applications can be avoided and the radio layer does not need to receive any explicit *transmit on/off* commands from higher layers.

The advantages of the proposed solution can be summed up in the following list:

- the reduction of the average transmit power from a CDMA terminal, obtained by employing silence symbols, reduces the interference on other users,
- the radio layer need not to be integrated with the silence state management function of the application layer,
- silence symbols allow very short traffic bursts and a great variety of fractional bit-rates without increasing the MAI level.

II. CDMA Two-States Reception

With the proposed scheme, the general base-band transmission signal is:

$$s(t) = \sum_{n=-\infty}^{n=\infty} s(t)^{(n)} \tag{1}$$

$$s(t)^{(n)} = Am^{(n)}b^{(n)}g(t-nT_s) \tag{2}$$

where
$s(t)$ is the signal produced by the user,
T_s is the symbol time,
A the transmitted amplitude for user k,
$g(t)$ is the complex valued shaping pulse, $g(t) \neq 0$ for $t \in [0, T_b)$,
$m^{(n)}$ is the **mask** symbol which assumes one of the two possible values $\{0,1\}$. It determines the state of the transmitter in the n-th time interval: *Talk* or *Silent*.
$b^{(n)}$ is the informative symbol transmitted during the n-th interval, chosen among the symbol alphabet of the chosen modulation (e.g. for a BPSK signaling $b^{(N)} \in \{-1,1\}$) . It has no significance when the transmitter is in the *Silent* state.

The received signal $r(t)$ expresses the observable part of the transmission chain. The unknown mask and symbol transmitted by the user over the transmission channel can be grouped in the two-state information symbol $q^{(n)}$ defined as:

$$q^{(n)} = m^{(n)}b^{(n)} \tag{3}$$

The **optimum detector** [2], for a given set of transmitted two-state symbols will choose the symbol $\hat{q}^{(n)}$ corresponding to the largest *posterior probability* based on the observation of $r(t)$ (MAP criterion). Formally:

$$\hat{q}^{(n)} = \arg\max_q P\left(q|\, r(t)^{(n)}\right) \tag{4}$$

We can assume that the two-states are alternating independently from the informative stream, constituted by M equally probable symbols. This leads to:

$$P\left(q_{talk}\right) = \frac{P\left(talk\right)}{M} \tag{5}$$

$$P\left(q_{silence}\right) = 1 - P\left(talk\right) \tag{6}$$

where $P\left(talk\right)$ is the absolute probability of a talk symbol. The two-state symbol q is thus possibly one of the equally probable M informative symbols or the single "silence" one. The transmission model described above needs a more complex performance characterization with respect to the traditional one. The receiver is characterized by a general *probability of error* which is specialized in:

- probability of false detection of a *silence state*, $P_{e,sil}$
- probability of symbol error conditioned to a talk state, $P_{e,symb}$.

TABLE I

BPSK+ SIGNALING

Symbol	Trasmitter state	Informative symbol
q_0	Talk	0
q_1	Talk	1
q_2	Silent	n.a.

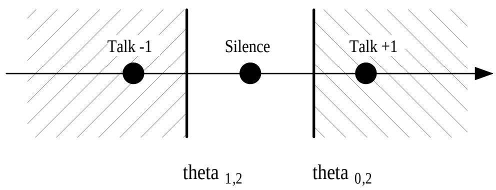

Fig. 1. BPSK+ Decision Regions

In the special case of a BPSK+ (the "plus" symbol indicates the presence of a "silent" state) operating on a AWGN channel, the optimum receiver is defined by the following thresholds:

$$\theta_{0,2} \doteq \frac{\sigma^2}{\sqrt{\varepsilon}} \ln \frac{P(q_2)}{P(q_0)} + \frac{\sqrt{\varepsilon}}{2} \quad (7)$$

$$\theta_{1,2} \doteq \frac{\sigma^2}{\sqrt{\varepsilon}} \ln \frac{P(q_1)}{P(q_2)} - \frac{\sqrt{\varepsilon}}{2} \quad (8)$$

Where the symbols are labeled as in table I, and ε is the symbol energy.

The decision regions for the described receiver, with r being the observable metric, are described by:

$$\begin{cases} r < \theta_{1,2} & \text{the symbol } q_1 \text{ is selected} \\ \theta_{1,2} \leq r < \theta_{0,2} & \text{the symbol } q_2 \text{ is selected} \\ \theta_{0,2} \leq r & \text{the symbol } q_0 \text{ is selected} \end{cases} \quad (9)$$

The decision regions are represented in Fig. 1.

III. Asymptotic Multiuser Efficiency

The well known asymptotic multiuser efficiency [3] is a measure of the influence that interfering users have on the Bit Error Rate (BER) of the user of interest. The asymptotic efficiency is defined as the limit, in the high signal-to-noise-ratio (SNR) region, of the ratio between the energy $e_k(\sigma)$ that the desired user would require to achieve the same BER of a single-user Gaussian channel and the actual energy E_k of the user:

$$\eta_k = \lim_{\sigma \to 0} \frac{e_k(\sigma)}{E_k} \quad (10)$$

and represents the performance loss when the dominating impairment is the existence of interfering users rather than the additive channel noise. The parameter η_k lies between 0 and 1, where a value of 1 indicates that the user of interest is not affected by other users presence. The kth user asymptotic efficiency can also be written as

$$\eta_k = \sup\{0 \leq r \leq 1 : \lim_{\sigma \to 0} \frac{P_k(\sigma)}{Q(\frac{\sqrt{rE_k}}{\sigma})} < +\infty\} \quad (11)$$

where $P_k(\sigma)$ is the probability of error associated to the selected detector. It is straightforward to find the kth user asymptotic efficiency achieved by a general linear transformation L [4]:

$$\eta_k(L) = max^2\{0, \frac{1}{\sqrt{E_k}} \frac{\sqrt{(LR)_{kk}} - \sum_{j \neq k} \sqrt{|(LR)_{kj}|}}{\sqrt{(LRL^T)_{kk}}}\} \quad (12)$$

where R is the cross-correlation matrix whose generic element is $R_{ij} = \int_{T_b} s_i(t)s_j(t)dt$ and $s_i(t)$ indicates the spreading waveform of the ith user. The term $(LRL^T)_{kk}\sigma^2$ is the noise variance after the received signal is passed through the linear filter.

In the two state case, the asymptotic multiuser efficiency is defined as:

$$\eta_k^{twostate} = \sup\left\{0 \leq r \leq 1 : \lim_{\sigma \to 0} \frac{P_k(\sigma)}{Q\left(\frac{\sqrt{rE_k} - \theta'}{\sigma}\right)} < +\infty\right\} \quad (13)$$

where

$$\theta' = \theta'(r) = \frac{\sigma^2}{\sqrt{r}\sqrt{E_k}} \ln\left(\frac{P(q_0)}{P(q_2)}\right) + \frac{\sqrt{r}\sqrt{E_k}}{2}$$

represents the amplitude modification due to the two-state decision region. Hence, $Q\left(\frac{\sqrt{rE_k} - \theta'}{\sigma}\right)$ is the probability of error of a two-state single-user receiver according to the asymptotic efficiency definition.

The two-state probability of error when $N_z = z < K$ users are in the silent state, is:

$$\begin{aligned} &P_k^{twostate}(P_{talk}, N_z) \\ &\quad = Prob\{(LR)_{kk} > \theta | b_k = -1, P_{talk}, N_z\} \\ &= \sum_{\mathbf{b} \in (-1,0,1)^K, b_k = -1} Prob\{(LR)_{kk} > \theta | \mathbf{b}, P_{talk}, N_z\} \\ &\qquad Prob\{\mathbf{b} | b_k = -1, P_{talk}, N_z\} \end{aligned} \quad (14)$$

where

$$\theta = \theta_{0,2} = \frac{\sigma^2}{\sqrt{E_k}} \ln\left(\frac{P(q_0)}{P(q_2)}\right) + \frac{\sqrt{E_k}}{2}$$

. The second term of (14) is the probability to have $N_z = z$ users in the silent state when K users are transmitting their information:

$$Prob\{\mathbf{b} | b_k = -1, P_{talk}, N_z\} = \binom{K-1}{z} P_{talk}^{K-1-z} (1 - P_{talk})^z \quad (15)$$

The first term of (14) is the probability of error of a two-state receiver when a certain transmission pattern **b** is sent:

$$Prob\{(LR)_{kk} > \theta|\mathbf{b}, P_{talk}, N_z\} = Q\left(\frac{\sqrt{(LR)_{kk}} - \sum_{j\neq k}\sqrt{|(LR)_{kj}|} - \theta}{\sqrt{(LRL^T)_{kk}}\sigma}\right) \quad (16)$$

Thus, the asymptotic multiuser efficiency conditioned on $N_z = z$ can be written as:

$$\eta_k^{twostate}(P_{talk}, N_z) = \sup\left\{0 \le r \le 1 : \lim_{\sigma\to 0}\frac{Q\left(\frac{\sqrt{(LR)_{kk}} - \sum_{j\neq k}\sqrt{|(LR)_{kj}|} - \theta}{\sqrt{(LRL^T)_{kk}}\sigma}\right)}{Q\left(\frac{\sqrt{r}\sqrt{E_k} - \theta'}{\sigma}\right)} < \infty\right\} \quad (17)$$

Noting that $\theta \to \frac{\sqrt{E_k}}{2}$ and $\theta' \to \frac{\sqrt{r}\sqrt{E_k}}{2}$ as $\sigma \to 0$, equation (17) becomes

$$\eta_k^{twostate}(P_{talk}, N_z) = max^2\left\{0, \frac{1}{\sqrt{E_k}} \cdot \frac{2\sqrt{(LR)_{kk}} - 2\sum_{j\neq k}\sqrt{|(LR)_{kj}|} - \sqrt{E_k}}{\sqrt{(LRL^T)_{kk}}}\right\} \quad (18)$$

The mean value of the asymptotic multiuser efficiency for a two-state linear receiver can be written as,

$$\bar{\eta}_k^{twostate} = \sum_{N_z}\eta_k^{twostate}(P_{talk}, N_z) \cdot Prob\{N_z = z\} = \sum_{N_z=z}\binom{K-1}{z}\eta_k^{twostate}(P_{talk}, N_z) \cdot P_{talk}^{K-1-z}(1-P_{talk})^z \quad (19)$$

This value has to be compared to that in eq. (12) in order to highlight the advantage of the proposed two-state receiver.

The conventional single-user detector consists on a filter matched to the desired user spreading waveform. In this case, the asymptotic multiuser efficiency can be simply recovered by substituting $L = I$ in eq. (12) and (19). So,

$$\eta_k^{onestate}(conv) = max^2\left\{0, \frac{\sqrt{E_k} - \sum_{j\neq k}\sqrt{|R_{kj}|}}{\sqrt{E_k}}\right\} \quad (20)$$

is the asymptotic multiuser efficiency of the one state conventional detector, while

$$\bar{\eta}_k^{twostate}(conv) = \sum_{N_z}\eta_k^{twostate}(P_{talk}, N_z) \cdot Prob\{N_z = z\} = \sum_{N_z=z}\binom{K-1}{z}P_{talk}^{K-1-z}(1-P_{talk})^z \cdot max^2\left\{0, \frac{1}{\sqrt{E_k}} \cdot \frac{2\sqrt{E_k} - 2\sum_{j\neq k}\sqrt{|R_{kj}|} - \sqrt{E_k}}{\sqrt{(LRL^T)_{kk}}}\right\} \quad (21)$$

is the asymptotic multiuser efficiency of the two state conventional detector.

IV. Numerical Results

The dependence of the asymptotic efficiency from the operating point of the proposed receiver has been analyzed numerically; the same operating condition have been then applied to the conventional single state receiver and the resulting performances compared to those obtained by the proposed scheme.

It should be noted that the asymptotic efficiency permits a significant comparison between the single and the two states receiver since it takes into account the performance degradation introduced by multiple access interference. The comparisons reported in this document, however, do not take into account the additional information available at the proposed receiver concerning the status of the transmitter. This additional information in a conventional receiver requires a signaling which has an impact on the overall performance. In this sense the results shown below are not completely fair to the proposed receiver as concerns the offered service.

In fig. 2 are reported the curves of the averaged asymptotic efficiency for both the single and two states receivers. The curves are plotted versus the P_{talk} probability, defined by the absolute probability of a non-silence symbol for each user. The ρ parameter expresses the maximum cross correlation value among the spreading signatures of the active users.

As shown, the low activity region ($P_{talk} < 0.5$) is characterized by a substantial improvement of the proposed transmission scheme over the traditional "always on" transmission. As the probability of a non-silence symbol increases, the increase of interfering power and the smaller decision regions for the non-silence information symbols introduce a degradation over the traditional reception schemes.

The dependence of the asymptotic efficiency from the increasing number of users is reported in the fig. 3.

Again, the increase of the MAI interference is mitigated by the average reduced activity of the sources as shown by the curves for low values of P_{talk}

V. Conclusions

In this paper is presented a new CDMA transmission scheme based on a three symbols constellation called "two states"

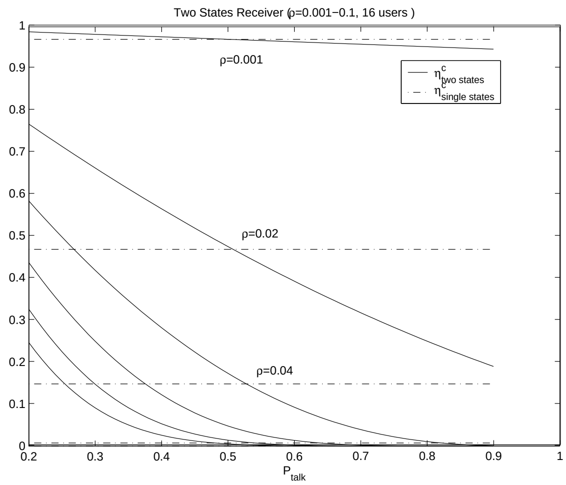

Fig. 2. P_{talk} and ρ influence on asymptotic efficiency for the conventional receiver

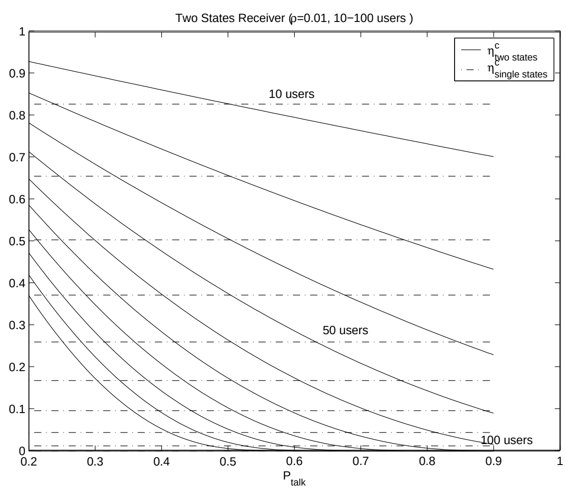

Fig. 3. P_{talk} and number of interfering users influence on asymptotic efficiency for the conventional receiver

transmission. The advantages over the traditional single state scheme are described and a performance evaluation based on the definition of asymptotic multiuser efficiency is derived. The analytical expression have also been applied to various operating scenarios in order to evaluate the benefits over traditional transmission schemes for the conventional matched filter detector. A further analysis over a larger number of multi-user receivers will be published shortly.

REFERENCES

[1] Brady P. T., "A statistical analysis of On-Off patterns in 16 conversations," *Bell Syst. Tech. J.*, vol. 47, pp. 73–91, 1968.

[2] C. W. Helstrom, *Statistical Theory of Signal Detection*, Pergamon, London, 2nd edition edition, 1968.

[3] Sergio Verdú, *Multiuser Detection*, Cambridge University Press, Cambridge, UK, 1998.

[4] Ruxandra Lupas and Sergio Verdú, "Linear multiuser detectors for synchronous code division multiple access channels," *IEEE Trans. on Inf. Theory*, vol. 35, no. 1, pp. 123–136, January 1989.