Do negative and positive equity returns share the same volatility dynamics?

Published version:

DOI: 10.1016/j.jbankfin.2015.05.017

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This Version: December 19, 2014

Abstract

This paper investigates whether positive and negative returns share the same dynamic volatility process. The well established stylized facts on volatility persistence and asymmetric effects are re-examined in light of such dichotomy. To analyze the dynamics of down and up volatilities estimated from daily returns I use a bivariate generalization of the standard EGARCH model. As a robustness check, I also investigate various specifications of down and up realized measures estimated from high-frequency data. The empirical findings point to the existence of a marked diversity in the volatilities of positive and negative daily returns in terms of persistence and sensitivity to good and bad news. A simple forecasting exercise highlights the striking performance of the proposed approach even in correspondence of the crisis period.

Keywords: Volatility, Contemporaneous Asymmetry, GARCH, Realized Variation.

JEL classification: C1, C3, G1.

1 Introduction

The modeling and forecasting of volatility has received significant attention by the financial-economic literature due to its relevance in areas such as portfolio management and selection, risk analysis and hedging and the pricing of assets and derivatives. Since Engle's (1982) ARCH and Bollerslev's (1986) GARCH, the study of volatility has witnessed a multitude of contributions ranging from the parametric to the nonparametric and from the discrete to the continuous time modeling. Particular attention has been devoted to the modeling of the news impact curve, that is the reaction of future volatility to negative and positive return shocks. Among the first parametrizations capturing the asymmetric response of volatility to the arrival of news are the EGARCH of Nelson (1991), the GJR-GARCH of Glosten *et al.* (1993) and the GTARCH of Zakoian (1994). For a literature review of GARCH models see Andersen *et al.* (2006).

El Babsiri and Zakoian (2001) introduce the concept of contemporaneous asymmetry in conditional heteroskedasticity models and accordingly decompose the primitive innovations into negative (down) and positive (up) shocks. Treating the volatility of negative and positive returns as distinct processes, although not necessarily independent, has its economic motivation in the fact that for investors with long (short) positions risk is clearly associated with down (up) movements of the asset's price but not necessarily with up (down) movements. Failure to separate the two aspects of volatility results in biased measures and forecasts whenever the down and up components do not coincide. On the other hand, distinguishing between down and up moves allows for *"different volatility processes for down and up moves in equity markets (contemporaneous asymmetry)"* and *"asymmetric reactions of these volatilities to past negative and positive changes (dynamics asymmetry or leverage-effect)"*. El Babsiri and Zakoian (2001) model the contemporaneous asymmetries with an *ad hoc* generalization of the univariate GTARCH (already capturing dynamic asymmetries) specification. In their study of the CAC 40 stock index they find that bad news increase future down and up volatilities significantly more than good news[1]. Furthermore, they find that current down and up volatilities substantially enter with the same coefficients in both equations of future down and up volatilities.

More recently, the availability of high-frequency data has stimulated a growing literature interested in the nonparametric estimation of the latent volatility process

[1]Throughout the paper, good and bad news are defined as return realizations respectively above and below their conditional expectation.

and in the decoupling of discontinuous *jumps* from the continuous component. In this framework, Barndorff-Nielsen *et al.* (2010) spell out the theory of realized semivariances, that is the realized variance of negative and positive intradaily returns. Using intradaily down variance as explanatory variable in a study of General Electric share prices, they find that *"for non-leveraged based GARCH models, downside realized semivariance is more informative than the usual realized variance statistic"*. However, when *"a leverage term is introduced it is hard to tell the difference"*. Stronger evidence favoring the intradaily down and up dichotomy of explanatory variables is found in Chen and Ghysels (2011). Modeling realized measures of volatility as functions of intradaily returns measured over some time intervals, they achieve the down and up decomposition of the explanatory variables with the exception of the jump component. In their study of the Dow Jones cash market and S&P500 futures market, they find that *"moderately good news reduce volatility"* while *"both very good news (unusual high positive returns) and bad news (negative returns) increase volatility, with the latter having a more severe impact"*.

Patton and Sheppard (2011) extend the use of intradaily down and up semivariances as predictors of future realized measures by introducing *signed jump variation*, defined as the difference between positive and negative realized semivariances. From a panel regression (same coefficients) of 105 individual stocks and the S&P500 index they measure the average effects of the explanatory variables on standard measures of volatility. They conclude that intradaily down volatility *"is much more important for future volatility"* than intradaily up volatility. Furthermore, based on their definition of negative and positive jumps, they find that the former *"lead to significantly higher future volatility"* while the latter *"lead to significantly lower volatility"*.

Building on the work of El Babsiri and Zakoian (2001), this paper studies the volatility dynamics of negative and positive returns. In contrast to standard modeling, this class of volatility processes allows for time periods characterized by large (small) up and small (large) down movements. Here the UD-EGARCH, a generalization of Nelson's (1991) EGARCH, is proposed. The main differences with respect to the GTARCH generalization of El Babsiri and Zakoian (2001) are: the possibility for the realizations to have a negative impact on the volatilities without compromising their positivity and the complete separation of the model's memory parameters from the loadings of the realizations.

Stylized facts on volatility persistence and asymmetric effects are re-examined in light of the down and up dichotomy for nine major world indices. Particular attention is paid to the analysis of the memory of down and up processes. Notably, the relevance of this aspect is due to the fact that, if undetected, different levels of persistence will give investors either a false sense of calmness or a false sense of activity, resulting, among others, in costly under- and over-estimated risk exposures. Memory of the down and up processes is elicited in terms of half-lives of the shocks.

The empirical findings point to the existence of marked diversities in down and up volatilities in terms of persistence and response to good and bad news. The robustness of the In-Sample (henceforth IS) findings is evaluated by assessing their stability Out-Of-Sample (henceforth OOS). Additional robustness checks are conducted in the specific case of the S&P500 index using high-frequency observations. All results highlight significant gains in the IS estimations and OOS predictions from the separate treatment of the two aspects of volatility. Sizable gains have been identified in the measurement and prediction of volatilities for all indices for the time periods considered. Specifically, the reduction of OOS mean-squared-errors ranges between 6% to 70% and 11% to 45% (depending on the benchmark measure) and averages at more than 25%.

The paper is organized as follows. Section 2 presents the low-frequency volatility specification. The data set of daily returns and the corresponding empirical findings are described in Sections 3 and 4, respectively. Stability and predictions are discussed in Section 5. Section 6 reviews realized estimators, jump detection statistic, HAR regressions and presents high-frequency down and up volatility modeling based on unobserved and intradaily components. Sections 7 and 8 describe the high-frequency returns and present the relative empirical findings. Section 9 concludes.

2 Low-Frequency Volatility Specification

A stochastic process y_t may be described in terms of its location and scale:

$$\begin{aligned} y_t &= \mu_t + \epsilon_t \\ \epsilon_t &= h_t^{1/2} \cdot z_t \end{aligned} \tag{1}$$

where μ_t is a function describing the evolution of the mean conditional on the information set $\mathcal{I}_{t-1}$[2], h_t is the conditional variance of the process y_t and z_t is a zero-mean,

[2]The information set is defined as usual: $\mathcal{I}_t$, $t \in \mathcal{Z}^+$ is an increasing filtration of σ-fields ($\mathcal{I}_{t-1} \subset \mathcal{I}_t, \forall t$) such that $\mathcal{I}_t$ summarizes the information provided by the observation of variables of interest up to time t. For purely dynamic specifications such as conditional volatility models it is enough to define the information set generated by past realizations of the returns y_t: $\mathcal{I}_t = \{y_1, \ldots, y_t\}$.

unit-variance and serially uncorrelated innovation.

In the standard decomposition of equation (1), the primitive shocks z_t are scaled by the process h_t regardless of their sign: no distinction between good and bad contemporaneous news. However, negative and positive innovations need not to be necessarily subject to the same amplification dynamics. Specifically, the return process y_t may exhibit large (small) down movements and small (large) up movements over a certain period of time. To allow for two distinct scale factors, redefine ϵ_t by:

$$\epsilon_t = \begin{cases} h_{U,t}^{1/2} \cdot z_t & \text{if } z_t > 0 \\ h_{D,t}^{1/2} \cdot z_t & \text{otherwise} \end{cases} \tag{2}$$

with:

$$\mathbb{E}[z_t^2 | z_t < 0] = 1 \quad \text{and} \quad \mathbb{E}[z_t^2 | z_t > 0] = 1 \tag{3}$$

$h_{D,t}^{1/2}$ and $h_{U,t}^{1/2}$ are the volatilities amplifying and compressing negative and positive innovations, respectively. z_t is a zero-mean and serially uncorrelated innovation satisfying the conditions in (3). These are the down and up counterparts of the standard identification condition for which primitive shocks have unit variance. It is straightforward to see that the equations in (3) imply $\mathbb{E}[z_t^2] = 1$. It must be noted that with distinct $h_{D,t}$ and $h_{U,t}$ processes additional assumptions[3] are needed to guarantee a zero conditional expectation of the shock ϵ_t. In other words, the clear-cut distinction between mean and variance of equation (1) becomes fuzzy once the constraint $h_D = h_U$ is relaxed. Further study of this salient connection between the first two conditional moments is beyond the scope of this paper which focuses on the analysis and the description of departures from the assumption of equal down and up volatilities[4].

[3]For the conditional expectation of ϵ_t:

$$\mathbb{E}[\epsilon_t | \mathcal{I}_{t-1}] = h_{U,t}^{1/2} \mathbb{E}[z_t | z_t > 0] \cdot \mathbb{P}(z_t > 0) + h_{D,t}^{1/2} \mathbb{E}[z_t | z_t \leq 0] \cdot \mathbb{P}(z_t \leq 0)$$

to be zero it is sufficient to specify the probability of observing an up movement $\mathbb{P}(z_t > 0)$ that offsets the movements in the down and up volatilities. For the unconditional expectation of ϵ_t:

$$\mathbb{E}[\epsilon_t] = h_U^{1/2} \mathbb{E}[z | z > 0] \cdot \mathbb{P}(z > 0) + h_D^{1/2} \mathbb{E}[z | z \leq 0] \cdot \mathbb{P}(z \leq 0)$$

to be zero it is sufficient to specify either $\mathbb{P}(z > 0)$ or the pair $h_U^{1/2}$, $h_D^{1/2}$ so that $\mathbb{E}[\epsilon_t] = 0$.

[4]Given that at this stage the implications of the dichotomy in (2) for the conditional mean of the process are not fully investigated, it is not possible to specify a candidate all round data generating process. Throughout the paper, the term *model* is used loosely instead of *filter* but with the understanding that it is not meant to indicate a data generating process. In addition, notice that it is not uncommon in the literature to separately model first and second conditional moments from which it follows that the difference between *model* and *filter* becomes a matter of semantics.

The methodological approach followed in the paper consists in comparing the volatility characteristics identified by a reasonable standard univariate model to those identified by its natural nesting extension which accommodates for contemporaneous asymmetry. The univariate EGARCH is recognized to provide a good description of volatility and as a baseline model leads to extensions with desirable characteristics. In particular, the conditional variances are positive by construction regardless of the sign of the model's coefficients[5] and there is complete separation between the memory parameters and the loadings of the realizations. The specific down and up volatility EGARCH extension, henceforth UD-EGARCH, is defined by:

$$\begin{bmatrix} \ln h_{U,t} \\ \ln h_{D,t} \end{bmatrix} = \omega + B \begin{bmatrix} \ln h_{U,t-1} \\ \ln h_{D,t-1} \end{bmatrix} + A \begin{bmatrix} d_{t-1} \cdot |\epsilon_{t-1}| \cdot h_{U,t-1}^{-1/2} \\ (1-d_{t-1}) \cdot |\epsilon_{t-1}| \cdot h_{D,t-1}^{-1/2} \end{bmatrix} \tag{4}$$

where d_t is a dummy variable equal to unity if $z_t \geq 0$ and zero otherwise. As in the EGARCH, past realizations enter future volatility in absolute value, however, here they are standardized by the appropriate down or up volatility. Innovations may affect down and up volatilities differently (distinct rows of A) and for each of the two they may have different effects depending on the sign of the innovation itself (distinct columns of A). Analogously, the relation between current and future volatilities may differ for future down and up volatility (distinct rows of B) and for each of the two the relation may be different with respect to current down and up volatilities (distinct columns of B). This allows for different degrees of persistence[6] in h_U and h_D which, in turn, imply different degrees of predictability for two components of volatility[7]. Desirability of these characteristics is validated in Sections 4 and 5 by the IS and OOS performance of the UD-EGARCH and the emerging stylized facts on persistence and sensitivities.

The parameters of the UD-EGARCH are estimated by Gaussian Quasi-Maximum-Likelihood (QML) associating to each return the appropriate variance. Conditional on the information set $\mathcal{I}_{t-1}$, the model generates down and up volatility predictions in unison. However, only the prediction associated with the sign of the realization

[5]For example, just like in the UD-GTARCH specification of El Babsiri and Zakoian (2001) in an UD-GARCH only the corresponding off-diagonal elements of the B-matrix in equation (4) may take negative values and the matrix as a whole is subject to positivity constraints. See Nakatani and Teräsvirta (2008).

[6]Uniform ergodicity conditions for the UD-EGARCH are derived in Appendix A.1.

[7]The model reduces to a standard EGARCH with equal down and up variances if $\omega_1 = \omega_2$, $\beta_{11} + \beta_{12} = \beta_{21} + \beta_{22}$, $\alpha_{11} = \alpha_{21}$ and $\alpha_{12} = \alpha_{22}$. Where ω_i is the i-th element of the vector ω in equation (4) and β_{ij} and α_{ij} the ij-th elements of the matrices B and A, respectively.

enters the estimation procedure. Dropping the constant term $-T/2\ln(2\pi)$ from the Gaussian likelihood yields the concentrated log-likelihood:

$$L_c = -\frac{1}{2}\sum_{t=1}^{T} d_t \left[\ln h_{U,t} + \epsilon_t^2 \cdot h_{U,t}^{-1}\right] + (1-d_t)\left[\ln h_{D,t} + \epsilon_t^2 \cdot h_{D,t}^{-1}\right] \tag{5}$$

Down (up) variances $h_{D,t}$ ($h_{U,t}$) are associated only to negative (positive) ϵ_t observations by means of the dummy variable d_t. Notice that if $h_{U,t} = h_{D,t}\ \forall t$, then L_c reduces to the usual concentrated Gaussian log-likelihood. In Appendix A.2 it is shown that the QML estimator satisfies the sample counterparts of the identification conditions in (3).

3 Low-Frequency Data

I study the down and up volatility dynamics of daily returns for nine major world indices: S&P500, Dow Jones Industrial Average (DJIA), NASDAQ, Nikkei, EuroSTOXX, DAX, FTSE, CAC and Swiss Market Index (SMI). The S&P500 and DJIA series, starting in January 1980, were obtained from WRDS. The S&P500 series ends in December 2010 whereas the DJIA ends in March 2007. All other series, obtained from Bloomberg, have the same end-of-sample in October 2011 but different beginning-of-sample periods. In particular, the series of NASDAQ, DAX and Nikkei start in January 1980, FTSE in January 1984, CAC in July 1987, EuroSTOXX in January 1987 and SMI in July 1988.

To reduce the possibility of results substantially driven by *outliers*, such as extreme down and up price swings, all time series have been Winsorized at the 0.1-th and 99.9-th percentile. Table 1 reports the number of Winsorized observations together with the corresponding percentiles for each of the nine indices. To conduct stability checks of the proposed methodology, observations have been partitioned into IS and OOS for each of the nine indices. Specifically, the last 1260 daily observations (roughly 5 years) of each series have been set aside for the OOS analysis. Thus, the OOS periods of all indices, with the exception of DJIA, include the financial crisis period. Table 2 reports the *per mille* occurrence of extreme observations IS and OOS. Returns are categorized as extreme if they are smaller than the 0.1-th Winsorization percentile or greater than the 99.9-th Winsorization percentile. For all indices, the OOS periods are characterized by a greater number of extreme price swings than the IS periods. In particular, the OOS periods are characterized by relatively more frequent large positive returns: on average 50% more than large negative returns. Consequently, the OOS periods constitute a significant challenge for any IS-estimated model. This provides an interesting environment to evaluate stability and performance of the UD-EGARCH specification with respect to the nested EGARCH.

4 Low-Frequency Findings

Tables 4-6 report QML parameter estimates for the standard EGARCH, unconstrained- and constrained-UD-EGARCH specifications. For all series of returns considered, UD-EGARCH is BIC/SIC preferred to standard EGARCH. The constrained UD-EGARCH specification is the result of BIC/SIC model selection performed on the B-matrix, capturing the persistence in the down and up volatilities, and the A-matrix, capturing the impact of negative and positive realizations[8]. The reason to perform constrained estimations, beyond statistical model selection, is to obtain more precise estimates of the set of parameters governing the interdependence between the two types of volatilities and their dynamics. For all nine indices the data supports (Wald tests and BIC/SIC) exclusion and equality constraints on the B- and A-matrix.

Tables 7-9 report QML parameter estimates for the standard-TGARCH, unconstrained- and constrained-UD-TGARCH specifications. For all series of returns considered, UD-TGARCH is BIC/SIC preferred to standard TGARCH. However, the UD-EGARCH specification is BIC/SIC preferred to the UD-TGARCH of El Babsiri and Zakoian (2001) for all indices considered with the exception of SMI.

Throughout the rest of the paper, β^D and α^D (β^U and α^U) indicate the coefficients of lagged down (up) volatility and the realizations of bad (good) news, respectively. Whether the $\beta^{U/D}$ and $\alpha^{U/D}$ coefficients refer to left-hand-side down or up volatilities will be made clear from the context.

4.1 Realizations

The coefficients α^D, capturing the impact of bad news on the two volatilities, are positive for all indices. Furthermore, from the Wald tests, they are statistically equal in h_U and h_D for all indices but NASDAQ and Nikkei. For the SMI, the difference is statistically significant at 10% but not at 5%. These findings are paralleled by the information criterion with the only exception of the SMI for which distinct α^D coefficients in the two volatilities are BIC/SIC preferred. Overall, this suggests that,

[8]Table 3 lists the constrained B and A-matrix specifications that have been estimated.

just like in standard volatility models, negative shocks equally increase the one-step-ahead down and up conditional variances.

From the Wald tests, the estimated α^U coefficients are statistically different in the down and up variances for all indices. For CAC the p-value of the Wald test is 0.112 in the unconstrained specification but 0.032 in the constrained specification selected by BIC/SIC. Specifically, the α^U coefficients in the up volatilities are positive, statistically significant and relatively smaller in magnitude than α^D in h_U for all indices. In terms of economic significance, it may be ventured that up variances are primarily driven by negative shocks. All point estimates of α^U coefficients in the down volatilities are negative. They are statistically significant for all indices except FTSE, CAC and SMI. However, even for these indices, specifications which impose $\alpha^U = 0$ in h_D are not BIC/SIC preferred. Thus, the empirical evidence suggests that the impact of good news is twofold: reduce down volatility and mildly increase up volatility. Standard models like the EGARCH, on the other hand, would suggest the analyst and enforce in their predictions that good news increase future conditional variances indiscriminately. Such effect of good news cannot be picked up by the UD-TGARCH either. In fact, the α^U in the down volatility of the UD-TGARCH hit the positivity constraint for all indices: Tables 7-9.

4.2 Persistence

In the unconstrained specifications, up variances exhibit large autoregressive coefficients β^U with values between 0.912 and 1.173. The β^D coefficients instead are considerably smaller in magnitude and take negative (in correspondence of estimated $\beta^U > 1$) and positive values. Nevertheless, for all indices, the β^D coefficients in h_U are not statistically significant. Specifications with $\beta^D = 0$ in the up variances are also BIC/SIC preferred. With this constraint in place all autoregressive β^U coefficients, while remaining large, shrink below the unit root threshold and range from 0.969 to 0.989. Thus, the empirical evidence does not support any relation or causality between $h_{D,t-1}$ and $h_{U,t}$. Instead, it outlines up volatility trajectories that do not depend on down volatility. Standard models that do not differentiate between the two volatilities implicitly impose a strong positive relation between such quantities which is then reflected in the predictions.

Down variances exhibit autoregressive coefficients that are considerably smaller than those estimated by standard specifications. β^U and β^D in the down volatilities are positive and statistically significant for all indices except DAX (only β^U significant) and CAC (only β^D) significant. The BIC/SIC identifies the specifications with $\beta^U = \beta^D$ in h_D as preferred to $\beta^U \neq \beta^D$ for all indices. The magnitude of the equal β coefficients in h_D is less than half of β^U in h_U ranging from 0.410 to 0.484. Therefore, contrary to up, down volatilities have trajectories that do exhibit interdependence: relatively higher (lower) down variances will decrease (increase) to re-align with up variances. The reaction times of these re-alignments will be substantially shorter than those of up variances. The same is true when compared to the reaction times predicted by standard models. Although the same triangular structure of the B-matrix is also BIC/SIC preferred in the UD-TGARCH it does not have the same implications. In fact, in the TGARCH family of models persistence/memory does not depend solely on B but also on the A-matrix. Since for none of the studied indices the estimated A-matrix of the UD-TGARCH is triangular, it is clear how this specification fails to pick up the distinction between up volatilities with own trajectories and down volatilities exhibiting interdependence.

Overall, a clear pattern in the structure of the B-matrix emerges from the IS empirical analysis: triangular with equal elements for the down volatility process. However, interpreting the persistence of the two processes from the estimated B-matrix directly is not immediate. The lower triangular structure of the B-matrix suggested by the data allows to identify β^U in h_U as the largest eigenvalue and $\beta^U = \beta^D$ in h_D as the smallest eigenvalue. Hence, persistence of h_U is determined by the largest eigenvalue while persistence of h_D is determined by a time-varying combination of the two eigenvalues (not to be confused with their average). Specifically, down variances have quite short own memory but, in force of their loading on lag-one up variance, they inherit its persistence.

An intuitive analysis of persistence in the world of dichotomized variances is reported in Table 10 in terms of half-lives of negative and positive shocks to the volatility processes. In the UD-EGARCH, the shocks' half-lives depend on the current state of the system (current levels of h_U and h_D) and the magnitude of the shocks themselves. I present results for half-lives computed from a system initially in equilibrium (h_U and h_D equal to their equilibrium levels) shocked by innovations of one standard deviation. For all indices, the half-lives of negative and positive shocks to up volatility are equal. Furthermore, the signature pattern of geometric decays is evident from the constancy of the half-lives. Up volatilities exhibit EGARCH dynamics and their half-lives are similar to those estimated by standard specifications with the exception of NASDAQ which exhibits substantially larger half-lives. For all indices, negative and positive shocks to down volatility exhibit relatively short initial half-lives which progressively get longer. It is interesting to notice that the transition from short to longer half-lives occurs at different rates for the two volatilities. In particular, while it takes around 4 short halvings before positive shocks to down volatility converge to half-lives of the largest eigenvalue it only takes 2 short halvings before negative shocks to down volatility converge to long-run half-lives. These patterns are obviously non-geometric and share some similarities with hyperbolic decays. These findings are in line with Bollerslev *et al.* (2006) *"analysis of several popular continuous-time stochastic volatility models [which] clearly points to the importance of allowing for multiple latent volatility factors for satisfactorily describing the observed volatility asymmetries"*. Furthermore, Litvinova (2003) observed a *"slowly decreasing correlation pattern between the squared returns and lagged returns in 5-minute data [that] exhibit a mixture of geometric and hyperbolic rate of decay"*. Indeed, the UD specification is a two factor model capable of capturing both geometric decays and decays that look more similar to hyperbolic[9]. When compared to the EGARCH, the magnitude of the initial half-lives is striking: 2 days vs. no less than 2 months for S&P500 and DJIA, 2 days vs. no less than 6 weeks for NASDAQ, Nikkei, EuroSTOXX and DAX, 3 days vs. at least 6 weeks for FTSE and CAC and 2 days vs. 1 month for SMI. The relatively low (initial) persistence of down volatility is easily interpretable within the framework of autoregressive processes. There is low persistence if and only if there is low explanatory power which suggests that down volatility is less predictable and therefore more random than was previously thought. Compared to contemporaneous asymmetry specifications standard models induce a false sense of future calmness (activity) when current down volatility is low (high).

5 Predictions and Stability

The IS results of Section 4 show superior performance of the down and up approach to volatility in terms of a statistical likelihood-based measure such as the BIC/SIC. As a robustness check, five years of data have been set aside to evaluate the OOS predictions of the different specifications as well as to conduct stability tests.

From a forecasting perspective it would be desirable to update the models' estimates as new information becomes available or at predetermined time intervals such

[9]Appendix A.3 describes the conditions under which the UD-EGARCH produces non-geometric decays.

as weeks or months. These procedures, based on rolling estimations over the OOS period are particularly time consuming due to the numerical optimizations involved in the estimation of the models. Hence, a simple baseline approach is implemented where the models' parameters are kept fixed at the IS estimates and new information incorporated into the models only through the realizations ϵ_τ as they become observable day-by-day. Since, in principle, rolling windows parameter updates should generate better forecasts, the adopted approach may be viewed as providing a lower bound to the full OOS potential.

Ideally, the models' predictions would be compared with the realizations by means of some measure of distance[10]. Unfortunately, since variances are not directly observable, the researcher may only rely on noisy proxies such as daily squared-returns[11]. The information that may be derived both directly and indirectly from such class of proxies has been at the center of a lively debate in the past. In fact, using squared daily returns as a benchmark the empirical evidence seemed to favor simple squared-returns autoregressions against conditional volatility models, see Cumby *et al.* (1993), Figlewski (1997) and Jorion (1995). Andersen and Bollerslev (1998), on the other hand, showed that even when the true data generating process is known, using poor proxies as a benchmark, results in large Mean-Squared-Errors (MSE) and low R^2 in Mincer-Zarnowitz regressions. As a result, IS model evaluations are conducted primarily in terms of Information Criteria and at times following the Box-Jenkins methodology. Nevertheless, the precision of squared-returns as OOS benchmarks may be improved upon by considering average realizations over K periods as in Ledoit *et al.* (2003).

Specifically, let $t_{U,i}$ and $t_{D,j}$ be the *time stamps* of positive and negative return days: $r_{t_{U,i}} \geq 0$, $\forall i = 1, \ldots, N_U$ and $r_{t_{D,j}} < 0$, $\forall j = 1, \ldots, N_D$. The K period down and up average variance proxies and predictions of model m are then constructed as follows:

$$\widehat{R}^{(K)}_{s,l} \equiv \frac{1}{K}\sum_{k=1}^{K} r^2_{t_{s,(l-1)K+k}} \qquad \text{and} \qquad \widehat{H}^{(K)}_{m,s,l} \equiv \frac{1}{K}\sum_{k=1}^{K} \widehat{h}_{m,s,t_{s,(l-1)K+k}}$$

where $s = \{U, D\}$, $l = 1, \ldots, \lfloor N_U/K \rfloor$ for $s = \{U\}$ and $l = 1, \ldots, \lfloor N_D/K \rfloor$ for $s = \{D\}$ and $\widehat{h}_{m,s,t_{s,(l-1)K+k}}$ is the one-day ahead s variance forecast of model m for

[10]See Patton (2011) for *"[...] sufficient conditions on the functional form of the loss function for the ranking of competing volatility forecasts to be robust to the presence of noise in the volatility proxy [...]"*

[11]Arguments regarding squared daily returns apply *mutatis mutandis* to other proxies based on daily observations such as the daily returns' absolute value.

$(l-1)K+k$. The K period MSE of model m forecasts for s variances is:

$$\widehat{MSE}^{(K)}_{m,s} \equiv \frac{1}{\lfloor N_U/K \rfloor + \lfloor N_D/K \rfloor} \sum_{l=1}^{\lfloor N_s/K \rfloor} \left[\widehat{H}^{(K)}_{m,s,l} - \widehat{R}^{(K)}_{s,l} \right]^2$$

An alternative benchmark may be constructed following the insight from the literature on volatility: superior variance measures may be obtained from conditional volatility models. Hence, I estimate EGARCH and UD-EGARCH specifications only on OOS observations in order to generate quality volatility measures. QML parameter estimates for the EGARCH and the UD-EGARCH, reported in Table 11, are by themselves informative on parameters' stability between IS and OOS. The best OOS specification is selected in terms of BIC/SIC with the aim of obtaining the best statistical description of the data which combines fit and parsimony. Additionally, comparing OOS predictions based on IS estimates with OOS estimates gives a measure of how the IS models perform relative to the full potential they may achieve only when the future becomes history.

5.1 Predictions

Forecast MSE are computed from the the one-step-ahead predictions generated by the IS estimates of EGARCH and UD-EGARCH and OOS filtered variances as well as the K period average realizations. In particular, the periods considered are 1 day, 1 month ($K = 20$), 3 months ($K = 60$) and 6 months ($K = 120$). In using K period averages there is a trade-off between variance and bias. Ideally K should be large to minimize the noise component in the squared-log-returns. However, for large K the models' predictions and the realized measures converge to the unconditional variances giving the misleading impression that the competing models perform equally well. This is not the case for the periods considered: $K = 120$ strikes the largest differences in models' performance signaling both a significant variance reduction and a negligible bias which would make the models look the same (for a detailed discussion see Palandri (2009)). Therefore, throughout the rest of the paper, the empirical findings are presented and discussed for $K = 120$. The results are reported in Tables 12-14.

For all indices considered, the joint MSE of down and up variances is substantially smaller for the UD-EGARCH when compared to the EGARCH. OOS MSE reductions with respect to the standard modeling approach range between 6.39% (CAC) and 76.52% (NASDAQ) with a median value of 21.39% (EuroSTOXX) for squared-logreturn proxies (henceforth SqR) and between 11.49% (CAC) and 44.92% (NASDAQ) with a median value of 21.66% (Nikkei) for OOS filtered proxies (henceforth Filtered).

With respect to the isolated down component of volatility the UD-EGARCH exhibits OOS MSE that are approximately 12% larger than those of the EGARCH for S&P500 and DJIA when the proxy is SqR. For all other indices the UD-EGARCH reduces the OOS MSE from 1.57% (CAC) to 94.10% (NASDAQ) with a median value of 23.84% (FTSE) for SqR. UD-EGARCH reduces reduces the EGARCH OOS MSE for all indices when the proxy is Filtered. Diminutions range from 7.57% (SMI) to 71.17% (NASDAQ) with a median value of 23.24% (FTSE). Such staggering improvements may be reconducted, at least in part, to the sign stability of news impact on down volatility IS and OOS: positive shocks decrease while negative shocks increase down volatility.

With respect to the isolated up component of volatility, in terms of OOS MSE the UD-EGARCH specification does not perform uniformly better than standard EGARCH. Sizable improvements are found for S&P500, DJIA, EuroSTOXX, CAC and SMI which exhibit reductions that range from 28.03% (CAC) to 95.65% (S&P500) with a median of 52.18% (EuroSTOXX) for SqR and from 12.56% (CAC) to 53.21% (S&P500) with a median of 23.87% (SMI) for Filtered. For NASDAQ, Nikkei and DAX the OOS predictions of UD-EGARCH are superior when benchmarked against SqR but inferior when the benchmark is Filtered. FTSE exhibits OOS predictions of UD-EGARCH that are inferior to those of EGARCH by 21.36% for SqR and by 14.83% for Filtered. The underperformance of the nesting UD-EGARCH specification OOS in the latter cases is due to structural changes in the volatility dynamics: the IS α^U coefficients are all relatively large (0.116-0.140) while OOS they are all equal to zero except for FTSE for which it is estimated to be of relatively large magnitude but opposite sign (-0.267).

In the hypothetical scenario in which the UD-specification coincides with the data generating process the above empirical exercise should be expected to highlight uniform improvements in the down and up volatilities with respect to a misspecified model that does not differentiate between the two. However, in more realistic scenarios in which the UD-specification is simply a better approximation to the data generating process, the UD-methodology should not necessarily be expected to deliver improvements in both the down and up dimension. This is a limitation of the approach presented in this paper and it arises from the estimator's objective function. In particular, the concentrated log-likelihood of equation (5) may be rewritten separating negative and positive return days, $L_c = L_{c,D} + L_{c,U}$ where:

$$\begin{aligned} L_{c,D} &= -\frac{1}{2}\sum_{\epsilon_t<0} \ln h_{D,t} + \epsilon_t^2 \cdot h_{D,t}^{-1} \\ L_{c,U} &= -\frac{1}{2}\sum_{\epsilon_t>0} \ln h_{U,t} + \epsilon_t^2 \cdot h_{U,t}^{-1} \end{aligned}$$

Thus, the estimated UD-projection maximizes the sum of $L_{c,D}$ and $L_{c,U}$ which is different from the maximization of $L_{c,D}$ and $L_{c,U}$. As a result, the nesting UD-EGARCH specification implies $L_c(\widehat{\theta}_{\text{UD-EGARCH}}) > L_c(\widehat{\theta}_{\text{EGARCH}})$ but does not guarantee that $L_{c,D}(\widehat{\theta}_{\text{UD-EGARCH}}) > L_{c,D}(\widehat{\theta}_{\text{EGARCH}})$ and $L_{c,U}(\widehat{\theta}_{\text{UD-EGARCH}}) > L_{c,U}(\widehat{\theta}_{\text{EGARCH}})$ hold simultaneously. This characteristic of the estimator is compatible with the OOS findings of FTSE and the triplet NASDAQ, Nikkei, DAX with respect to Filtered.

The EGARCH produces OOS MSE that are substantially larger for down volatility. If the same was true IS, it would have been optimal for the UD-EGARCH to specialize more in describing the down rather than the up dynamics. As a result the losses in the up components would be more than compensated by the gains on the down side, improving the joint IS fit and presumably the joint OOS MSE. Performance differences in Tables 12-14 between the two volatilities could arise from multiple sources the most likely of which are data differences in the IS and OOS (characterized by the crisis period). Nevertheless, it is important to highlight that UD-specifications should not be expected to outperform standard approaches in both the down and up dimensions of volatility. Indeed, the current setting may easily accentuate performance differences in the down and up components in order to maximize joint fit.

5.2 Stability

In the OOS period, BIC/SIC selects the EGARCH specification with $\alpha^U = 0$ for all indices: positive shocks are not found to have any impact on future volatility. The α^D coefficients are all positive and statistically significant. The magnitude of the estimated parameters is in line with the IS results even though, with the exception of DAX, point estimates of α^D are larger OOS than IS.

In the UD-EGARCH specification, equality of α^D in h_U and h_D is BIC/SIC preferred for all indices. Thus, in the OOS period bad news have equal impact on future down and up volatilities. As for the EGARCH specification, the magnitudes of the point estimates of these coefficients are in line with those found IS even though slightly larger. The α^U coefficients are negative and statistically significant in the down volatility of all indices like in the IS period: good news decrease future down volatility. However, the specification with $\alpha^U = 0$ in the up volatility is BIC/SIC preferred for the S&P500, DJIA, NASDAQ, Nikkei, EuroSTOXX and DAX. Therefore, while IS good news increase up volatility they are found to have no effect in the OOS period. For the remaining indices (FTSE, CAC and SMI) the estimated α^U coefficients are negative and significant: good news decrease future up volatility. These findings are in stark contrast with the IS data features.

The triangular structure of the B-matrix with equal lower elements is BIC/SIC preferred for S&P500, DJIA, NASDAQ, Nikkei, EuroSTOXX and DAX. Similarly to the IS, the OOS estimated autoregressive coefficients in the up volatility range from 0.970 to 0.985 while the twin coefficients in the down volatility range from 0.414 to 0.510. For the remaining indices, on the other hand, the OOS results do not mimic the IS findings. CAC and SMI exhibit down volatilities with large autoregressive coefficients, 0.946 and 0.958 respectively, and no loading on lag-one up volatilities. Up volatilities have smaller autoregressive coefficients and statistically significant negative loadings on lag-one down volatilities. OOS the FTSE index exhibits reversal of the IS B-matrix structure. Specifically, in the up volatility the loadings on lag-one down and up volatilities are found to be equal. Down volatility exhibits a larger than unity autoregressive coefficient and a statistically significant negative loading on lag-one up volatility of -0.400.

The next Section provides additional robustness checks, using high-frequency observations and associated methodologies, for the S&P500 index for which I have high-frequency observations.

6 High-Frequency Volatility Specifications

This section investigates whether volatility measures based on high-frequency trades provide stylized facts for contemporaneous asymmetry in line with those that have emerged from the empirical study based on daily observations.

6.1 Predictors Based on Realized Measures

Assuming intradaily log-prices follow a jump diffusion process, the volatility over the day is measured by the quadratic variation. While not directly observable, it may be consistently estimated by realized variance:

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \tag{6}$$

where M is the number of intradaily returns $r_{t,j}$ measured over a given sampling frequency. As the number of intradaily observations M goes to infinity, the estimator in (6) converges[12] to the sum of the continuous volatility path component and the jump contribution to the total daily quadratic variation. To separately measure these two components, I use the Hausman-type testing procedure based on bipower variation introduced by Barndorff-Nielsen and Shephard (2004, 2006). Bipower variation is an estimator of the continuous volatility component defined by:

$$RBV_t = \mu_1^{-2} \left(\frac{M}{M-1} \right) \sum_{j=2}^{M} |r_{t,j-1}| \cdot |r_{t,j}| \tag{7}$$

where $\mu_k = \mathbb{E}\left(|\zeta|^k\right)$ with $\zeta \sim N(0,1)$. Under general conditions[13], the estimator in (7) converges in probability to the integrated variance. The jump detection test statistic is:

$$Z_t = \frac{1 - RBV_t/RV_t}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right) \frac{1}{M} \max\left(1, \frac{RTQ_t}{RBV_t^2}\right)}} \tag{8}$$

where RTQ_t is realized tripower quarticity[14]. Under the null hypothesis of no within-day jumps, the test statistic (8) is asymptotically standard normal distributed. Following the jump detection test, the contribution of the jump component J_t and the continuous component C_t to the overall quadratic variation is defined by:

$$\begin{aligned} J_t &\equiv I(Z_t > \Phi_\alpha) \cdot (RV_t - RBV_t) \\ C_t &\equiv RV_t - J_t \end{aligned}$$

where $I(\cdot)$ denotes the indicator function and Φ_α the appropriate critical value from the standard normal distribution[15]. Having decoupled C_t and J_t it is possible to use

[12] See Andersen and Bollerslev (1998), Comte and Renault (1998), Andersen *et al.* (2001,2003) and Barndorff-Nielsen and Shephard (2001,2002), among others.

[13] See Barndorff-Nielsen, Shephard, and Winkel (2006) and Jacod (2008), among others.

[14] $RTQ_t = M\mu_{4/3}^{-3}\left(\frac{M}{M-2}\right)\sum_{j=3}^{M} |r_{t,j-2}|^{4/3} \cdot |r_{t,j-1}|^{4/3} \cdot |r_{t,j}|^{4/3}$

[15] Corsi *et al.* (2010) propose a different jump testing procedure based on the concept of threshold multipower variation: a combination of the multipower variation of Barndorff-Nielsen and Shephard (2004) and the threshold realized variance of Mancini (2009). Replicating the work of Andersen *et al.* (2007) using threshold multipower variation, Corsi *et al.* (2010) find a larger number of jumps for the same data sample and a positive impact of the latter on future volatility. Preliminary checks based on this approach indicated that the results presented in this paper are robust to the choice of testing procedure.

them as predictors in regressions for future RV and C.

The above set of predictors may be extended by separating the negative and positive returns to construct two distinct series with no gaps: $r_{t,j}^{D}$ and $r_{t,j}^{U}$ with negative- and positive-only returns, respectively, as in Barndorff-Nielsen *et al.* (2010) and Patton and Sheppard (2011). Treating these two series separately, I compute the usual estimates (RV, RBV and RTQ) necessary for the implementation of the test statistic defined in (8). In particular, standard realized variance measures are extended to:

$$RV_t^U = \sum_{j=1}^{M^U} \left(r_{t,j}^U\right)^2 \qquad \text{and} \qquad RV_t^D = \sum_{j=1}^{M^D} \left(r_{t,j}^D\right)^2$$

The joint variance of positive and negative returns gives the overall daily variance: $RV_t^U + RV_t^D = RV_t$. From the series of realized variances, the jump test statistic[16] in (8) generates the series of continuous- (C_t^U and C_t^D) and jump-components (J_t^U and J_t^D):

$$RV_t^U = C_t^U + J_t^U \qquad \text{and} \qquad RV_t^D = C_t^D + J_t^D$$

It should be noticed that J_t^U and J_t^D do not necessarily add up to the standard J_t. They are simply a different set of covariates obtained by performing standard jump-testing procedures to the split series of intradaily returns. Further analysis of these two jump components is beyond the scope of this paper. Since J_t^U and J_t^D are only used as covariates it is not particularly relevant to investigate their fine properties as what really matters is that they are good predictors. It must be emphasized that down and up jumps defined above are intrinsically different from the signed jumps of Patton and Sheppard (2011)[17].

6.2 HAR Specification

The benchmark specification is the log-version of the Heterogeneous AR (HAR) model, originally proposed by Corsi (2009), which consists of volatility components measured over different time horizons. Thanks to the daily, weekly and monthly time scales considered, the HAR specification well approximates the long-memory behavior

[16] Jump-test(s) are performed if within the day there are at least five returns of the same sign. If this is not the case, set $C_t^U = RV_t^U$ and/or $C_t^D = RV_t^D$.

[17] *"[. . .] the variation due to the continuous component can be removed by simply subtracting one [Realized Semivariance] from the other, and the remaining part is what we define as the signed jump variation"*. However, separating intradaily down and up variances seems reasonable only under the assumption that the two may differ, in which case the *signed jump variation* statistic will also pick up differences in the down and up continuous components (a measure of intradaily asymmetry).

of volatility in a simple and parsimonious way.

$$
\begin{aligned}
\ln(X_t) &= \omega + \beta_D \ln(C_{t-1}) + \beta_W \ln(C_{W,t-1}) + \beta_M \ln(C_{M,t-1}) \\
&+ \beta_J \ln(1 + J_{t-1}) + \epsilon_t
\end{aligned} \tag{9}
$$

where $X_t = \{RV_t, C_t\}$ is the left-hand-side variable[18] and $C_{W,t}$ and $C_{M,t}$ are defined as usual: $C_{W,t} = \frac{1}{5}\sum_{j=0}^{4} C_{t-j}$ and $C_{M,t} = \frac{1}{22}\sum_{j=0}^{21} C_{t-j}$. The HAR regression based on intradaily realized measures is given by:

$$
\begin{aligned}
\ln(X_t) &= \omega + \beta_D^U \ln(C_{t-1}^U) + \beta_D^D \ln(C_{t-1}^D) \\
&+ \beta_W^U \ln(C_{W,t-1}^U) + \beta_W^D \ln(C_{W,t-1}^D) \\
&+ \beta_M \ln(C_{M,t-1}) + \beta_J^U \ln(1 + J_{t-1}^U) + \beta_J^D \ln(1 + J_{t-1}^D) + \epsilon_t
\end{aligned} \tag{10}
$$

where $C_{W,t}^U = \frac{1}{5}\sum_{j=0}^{4} C_{t-j}^U$ and $C_{W,t}^D = \frac{1}{22}\sum_{j=0}^{21} C_{t-j}^D$. With the exception of the unique monthly component and different definition of down and up jumps, this specification is the same as that in Patton and Sheppard (2011). Here, distinct down and up monthly measures are not considered due to potential multicollinearity issues. In the data, monthly down and up measures alone exhibit correlation levels of 0.90 which increase to 0.98 with the inclusion of daily and weekly dichotomized regressors.

6.3 UD-HAR Specification

This specification of contemporaneous asymmetry is the counterpart of HAR regressions. Investigating the dynamics of down and up volatilities requires the definition of two sets of variables depending on the sign of the corresponding daily return. Since observability of the up variance excludes observability of the down variance and *vice versa*, I define daily down and up volatilities as latent processes and replace the unobserved components in the daily and weekly regressors with predictions generated by the model itself. Monthly down and up regressors are not used due to borderline multicollinearity. The latent down and up processes are modeled by:

$$
\begin{aligned}
\ln h_{s,t} &= \omega_s + \beta_{s,D}^U \psi_{D,t-1}^U + \beta_{s,D}^D \psi_{D,t-1}^D + \beta_{s,W}^U \psi_{W,t-1}^U + \beta_{s,W}^D \psi_{W,t-1}^D \\
&+ \beta_{s,M} \ln(C_{M,t-1}) + \left[\beta_{s,J}^U d_{t-1} + \beta_{s,J}^D (1 - d_{t-1})\right] \ln(1 + J_{t-1})
\end{aligned} \tag{11}
$$

where $s = \{U, D\}$ and:

$$
\begin{aligned}
\psi_{D,t}^U &= d_t \ln C_t + (1 - d_t) \ln h_{U,t} & \psi_{D,t}^D &= (1 - d_t) \ln C_t + d_t \ln h_{D,t} \\
\psi_{W,t}^U &= \ln\left[\tfrac{1}{5}\textstyle\sum_{j=0}^{4} \exp\left(\psi_{D,t-j}^U\right)\right] & \psi_{W,t}^D &= \ln\left[\tfrac{1}{5}\textstyle\sum_{j=0}^{4} \exp\left(\psi_{D,t-j}^D\right)\right]
\end{aligned}
$$

[18]The study of the dynamics and predictability of the jump component is beyond the scope of this paper. For a convenient reduced form model of jump occurrence and jump size see Andersen *et al.* (2011).

The beta coefficients, one set for $s = \{U\}$ and another for $s = \{D\}$, have superscripts indicating whether they refer to an up (U) or down (D) measure and subscripts indicating the time interval (Day, Week and Month) over which such variable is computed. The same indexing system is used for the regressors. The binary variable d_t equals unity on positive return days and zero otherwise. Coherently with the definition of down and up variance, jumps are classified as down if occurring on negative return days and up otherwise. This specification has built-in a certain degree of persistence for the effects of jumps. In particular, since they enter the latent processes $h_{U,t}$ and $h_{D,t}$, jumps also enter indirectly[19] the daily and weekly components ψ which determine persistence in the HAR. The model's parameters are estimated by non-linear least squares (NLS) with objective function:

$$\sum_{t=1}^{T} \left[\ln X_t - d_t \ln h_{U,t} - (1 - d_t) \ln h_{D,t}\right]^2 \tag{12}$$

where $X_t = \{RV_t, C_t\}$ is the left-hand-side variable that is being modeled.

Observable intradaily measures may be used to replace the model-implied predictions. In fact, although C^U_{t-1} and C^D_{t-1} are intrinsically different from C_{t-1} on negative and positive return days, if the latter are a function of the former the substitution of unobservable with observable variables is perfectly justifiable from a reduced-form modeling perspective. Hence, the down and up processes are modeled by:

$$\begin{aligned} \ln h_{s,t} &= \omega + \beta^U_{s,D} \ln(C^U_{t-1}) + \beta^D_{s,D} \ln(C^D_{t-1}) + \beta^U_{s,W} \ln(C^U_{W,t-1}) + \beta^D_{s,W} \ln(C^D_{W,t-1}) \\ &+ \beta_{s,M} \ln(C_{M,t-1}) + \beta^U_{s,J} \ln(1 + J^U_{t-1}) + \beta^D_{s,J} \ln(1 + J^D_{t-1}) \end{aligned} \tag{13}$$

Since in this setting there are no latent covariates, the model's parameters may be estimated by standard OLS.

6.4 UD-EGARCHX Specification

This is the counterpart of the low-frequency specification (4). The GARCHX class of models consists of GARCH specifications based on volatility measures constructed from high-frequency data and have been studied in Engle (2002), Engle and Gallo (2006) and Shephard and Sheppard (2010), among others. Here, the EGARCHX

[19] A specification in which jumps do not enter the components ψ, while not presented here, has been considered and found to provide very similar qualitative and quantitative results, although BIC/SIC inferior.

specification is extended to the down and up components:

$$\begin{aligned}\begin{bmatrix}\ln h_{U,t}\\ \ln h_{D,t}\end{bmatrix} &= \omega + B\begin{bmatrix}\ln h_{U,t-1}\\ \ln h_{D,t-1}\end{bmatrix} + A\begin{bmatrix}d_{t-1}\ln(C_{t-1})\\ (1-d_{t-1})\ln(C_{t-1})\end{bmatrix}\\ &+ G\begin{bmatrix}d_{t-1}\ln(1+J_{t-1})\\ (1-d_{t-1})\ln(1+J_{t-1})\end{bmatrix}\end{aligned} \tag{14}$$

It should be noticed that while in the UD-EGARCH persistence is solely determined by the B-matrix coefficients, in the UD-EGARCHX the A-matrix parameters also play a role as they determine the loading on past realizations rather then past shocks. In equation (14) it is the G-matrix of jump coefficients that more closely resembles the A-matrix of the low frequency UD-EGARCH. The model's parameters are estimated by NLS with objective function as in equation (12).

The latent down and up processes may be modeled by exploiting the information contained in the intradaily down and up measures:

$$\begin{aligned}\begin{bmatrix}\ln h_{U,t}\\ \ln h_{D,t}\end{bmatrix} &= \omega + B\begin{bmatrix}\ln h_{U,t-1}\\ \ln h_{D,t-1}\end{bmatrix} + A\begin{bmatrix}\ln(C^U_{t-1})\\ \ln(C^D_{t-1})\end{bmatrix}\\ &+ G\begin{bmatrix}\ln(1+J^U_{t-1})\\ \ln(1+J^D_{t-1})\end{bmatrix}\end{aligned} \tag{15}$$

Notice that specification (15) does not nest (14) as it does not distinguish between realizations on negative and positive return days. Persistence of the latent components is jointly determined by the B- and A-matrix coefficients. The unknown parameters are estimated by NLS with objective function as in equation (12).

7 High-Frequency Data

In this part of the paper the focus is on down and up volatility patterns from high-frequency observations. The data consists of S&P500 tick-by-tick trades from TickData beginning January 1985 and ending December 2005. Following Andersen et al. (2007), the sampling frequency is lowered to mitigate market microstructure contamination. Investigating the impact of microstructure effects for one-, two-, five-, ten- and fifteen-minute returns it is found that the latter strikes the best balance[20]

[20]From signature plots: shorter time intervals lead to underestimation of the total quadratic variation. Preliminary studies of down and up volatility dynamics based on five-minute returns, conducted as a robustness check, confirmed the findings based on fifteen-minute returns.

between the desire for finely sampled observations and unconditional unbiasedness of the variance estimates. Hence, fifteen-minute prices, corresponding to 25 non-overlapping returns per day, are used throughout this section.

8 High-Frequency Findings

8.1 Predictors Based on Realized Measures

Results of standard HAR regressions for RV and C, reported in Table 15, parallel those of Andersen *et al.* (2011) who find jump coefficients to be negative but not statistically significant. Also, BIC/SIC selects the specification with equal loadings on the weekly and monthly components.

Estimation results for the UD-HAR specification (with unobserved components replaced by predictions generated by the model) are reported in Table 15. Specifications with RV and C as LHS variables exhibit the same pattern. With respect to the daily component, if from negative-return days it has the same impact on down and up volatilities (β_D^D) while from positive-return days it has a larger effect on down volatility (β_D^U). The β_W^U coefficients are not statistically significant and their elimination is BIC/SIC preferred, suggesting that down and up volatilities are not tied to weekly up movements. On the other hand, weekly down movements do have a statistically significant and equal effect on current down and up volatilities. The coefficient of the monthly component in the up volatility is BIC/SIC equal to the weekly component as in the standard HAR specification. Instead, in the down volatility the loading on the monthly component is significantly smaller which could be indicative of a somewhat shorter memory. Estimated jump coefficients are stable across RV and C specifications and qualitatively very similar to those estimated using low-frequency observations and reported in Table 4. In particular, jumps on negative-return days have the same impact on future down and up volatilities while jumps occurring on positive-return days have no effect on up volatility but a sizable negative effect on down volatility. In other words, the arrival of good news in the form of a jump is found to reduce the magnitude of future down movements.

Estimates reported in the bottom panels of Table 15 parallel very closely the estimates based on low-frequency observations of Table 4. Specifically, the triangular structure of the B-matrix: up volatility exhibits a large autoregressive coefficient but no loading on past down volatility and down volatility with equal and smaller loadings on past down and up volatilities. However, it is worth recalling that here the triangular structure of B does not imply that up volatility follows its own trajectory. The realizations of down volatility have the same impact on future down and up volatilities (α^D) while the realizations of up volatility are found to have a larger impact on future down volatility (α^U). The estimated jump coefficients reproduce the pattern found for UD-EGARCH on daily data and UD-HAR on realized measures. Jumps on negative-return days increase future down and up volatilities equally while jumps on positive-return days reduce future down volatility and are neutral to up volatility.

8.2 Predictors Based on Intradaily Measures

Results for HAR regressions using intradaily down and up measures of C and J are reported in Table 16. In-Sample, the set of intradaily regressors provides a superior fit as measured by the log-likelihood and information criteria. Daily RV and C are found not to react to up intradaily C and intradaily down and up jumps are statistically significant with equal but opposite effects.

The intradaily down and up regressors provide a superior IS fit, as seen in Table 16. Down and up estimated jump coefficients remain substantially unchanged whether the regressors are daily or intradaily. In particular, up jumps have no impact on future up volatility but decrease future down volatility. Instead, down jumps are once again found to increase future down and up volatilities equally. Unchanged is also the finding that weekly down volatility equally increases future down and up volatilities. Main differences for this specification with respect to its counterpart based on daily information only is that here daily up volatility does not predict down volatility while weekly up volatility is found to increase down volatility.

The UD-EGARCHX specification with intradaily regressors exhibits an inferior IS fit than that of the UD-EGARCHX based on daily regressors. Nevertheless, the estimated coefficients are quite similar across specifications with the only exception being the α coefficients: equal impact of down C in daily as opposed to equal impact of up C in intradaily. The structure of the matrix of coefficients of the jump is the same as that found for all other specifications highlighting a high degree of robustness of the findings pertaining to the jump components.

8.3 Persistence

Persistence of the down and up volatilities modeled and estimated on realized measures is described by the half-lives of negative and positive shocks to volatility and jump processes as reported in Table 17. In order to compute half-lives, as described in Section 4.2, the models object of study need to be fully dynamic. Without specifying additional dynamics for the regressors the half-lives may be readily computed for the specifications with C as LHS variable which make use of daily information only. Hence, I look at the constrained specifications of Panels B and D in Table 15 treating the jumps as i.i.d. processes.

In the HAR specification there are no half-lives for jumps as they have not been found to be statistically significant. Furthermore, since this specification does not differentiate between down and up quantities the two exhibit a common pattern. Specifically, a first halving of approximately 1 week followed by successive halvings occurring approximately every 7 weeks.

The half-lives of shocks to C and J on negative return days for down and up volatilities in the UD-HAR are similar. The only difference is that the third halving of jumps occurs 1 week earlier in the down volatility. Shocks on positive return days, instead, are found to be more persistent in the up volatilities. In particular, shocks to C on positive return days have second and third half-lives that are 1 and 3 weeks shorter, respectively, for down volatility. The differences between down and up volatilities are even more marked when considering the half-lives of jumps on positive return days: 1 week vs. 1 day, 5 weeks vs. 1 day and 7 weeks vs. 2 weeks for the first three halvings in down and up volatilities.

In the UD-EGARCHX, the half lives of shocks to up volatility are the same across shock type: C or J, on negative or positive return days. Down volatilities exhibit half-lives equal to up volatility after the first halving with the exception of jumps on positive return days. Shocks to C have first half-life of approximately 2 weeks shorter for down volatility. 3 weeks shorter for jumps on negative return days. Jumps on positive return days have initial half lives of 1 day, 2 days and 3 weeks in down volatility as opposed to approximately 5 weeks in up volatility.

8.4 Stability

The IS results of Tables 15 and 16 show superior performance of the down and up approach to volatility in terms BIC/SIC. Furthermore, they clearly outline a ranking which sees the UD-EGARCHX specification as providing the best data description, followed by the UD-HAR. To assess the stability of these findings, I use the five years of data that have been set aside to evaluate the OOS performance of the different specifications. The model's predictions are compared to the realized measures RV

and C in terms of MSE. The same baseline approach of Section 5 is followed keeping parameters fixed at the IS estimates and incorporating new information only through the realizations as they become available day-by-day. Even though RV and C are more precise measure of daily variance than squared-returns they are still proxies[21]. Hence, K period average RV, C and predictions are considered for $K = 1, 20, 60, 120$. Tables 18 and 19 confirm the IS findings for the down and up specifications. Regardless of the information set used, whether daily or intradaily, the UD-EGARCHX performs better than the down and up HAR generalization UD-HAR.

Tables 18 and 19 highlight stabilized MSEs for $K = 60$ and $K = 120$. Since as K gets larger the models' predictions and proxies converge to the long-run variance, empirically $K = 60$ seems to strike the best balance in the trade-off between variance and bias and is the measure I refer to in the following analysis.

From the OOS joint MSE, the down and up specifications outperform the standard HAR. In particular, the UD-EGARCHX exhibits MSE reductions slightly above 30%. The UD-HAR specification, on the other hand, does not exhibit substantial improvements. In more detail, the down and up specifications outperform the standard HAR with respect to the down side of volatility but underperform with respect to the up side. This is in line with the low-frequency findings which highlight MSE gains for the down volatilities of all indices considered and losses for the up volatilities of NASDAQ, Nikkei, DAX and FTSE. Hence, while down volatilities exhibit stable dynamic structures that may be exploited OOS, up volatilities exhibit less stable structures with sign changes leading to inferior OOS performances. The use of intradaily information reduces the OOS MSE for both specifications with respect to standard HAR. In particular, UD-HAR presents joint reductions of 10.91% and 15.45% for RV and C predictions, respectively. The UD-EGARCHX exhibits OOS joint MSE reductions of approximately 40.00% with respect to standard HAR.

Separating the two volatilities and down from up jumps leads to the results reported in Table 16. In Panels B and C it is clear that down jumps increase both down and up volatility and that their impact on the two is not statistically different, as suggested by the Wald tests. Up jumps are not found to have statistically significant effects on up volatility but are found to be the only covariate decreasing down volatility. The patterns outlined by the various jump components fit remarkably well with those found at low-frequencies for the α parameters. This parallelism seems to

[21]In the specific case, using fifteen-minute returns, RV is constructed from 25 observations. Thus, although a good proxy, RV will contain measurement errors that may be partially offset when considering average RV over various days.

be due the fact that the UD-EGARCH, as well as the standard EGARCH, produces quite good one-step-ahead predictions of the continuous component of the conditional variance. Hence, the terms multiplying the α coefficients are equal to the daily shock standardized by the continuous component only. It follows that in the occurrence of a jump the standardized shocks substantially reflect the jump itself. In other words, the forcing variables of the process are unit-variance random draws from a stationary continuous distribution mixed with the distribution of jumps standardized by the continuous components.

9 Conclusions

Using daily returns from nine major world market indices this paper studies contemporaneous asymmetry. With a simple and straightforward extension of the EGARCH model I find substantial and not previously documented differences in the persistence levels of down and up volatilities and the way they react to good and bad news. The empirical evidence obtained effectively suggests the following stylized facts. Up volatility evolves based on its own past history alone and may be more persistent than estimated by standard models. Down volatility exhibits a persistent component equal to that of up volatility coupled with a fast mean-reverting component. Their combination determines decaying patterns of shocks to down volatility with half lives that are initially very short and subsequently equal to those of the up volatility. This implies that down volatility is less predictable and more random than otherwise concluded from standard models that do not contemplate contemporaneous asymmetry. These findings are confirmed by robustness checks on the S&P500 index where numerous specifications with observed and unobserved components have been estimated for daily measures of variance computed from high-frequency returns. The empirical analysis has also outlined that, in contrast to its lower persistence, down volatility exhibits dynamic relations that are considerably more stable over time than those found for the more persistent up volatility. The OOS exercise has highlighted that for all indices considered investors with long positions, for whom risk is associated with down movements, would have obtained significant improvements by the separate modeling of down and up volatilities.

Further investigations of the potential gains that may derive from the implementation of weighted objective functions favoring this or that type of volatility depending on the researcher's interest, specification of a down and up data generating process with clear separation of mean and variance and the study of the theoretical properties of down and up jumps, as defined in the paper, are all issues that await future research.

A Appendix

A.1 Ergodicity Conditions

From equation (2), the standardized residuals ϵ_{t-1} in equation (4) are equal to the primitive shocks z_{t-1}. Thus, the UD-EGARCH may be rewritten as:

$$\begin{bmatrix} \ln h_{U,t} \\ \ln h_{D,t} \end{bmatrix} = \omega + B \begin{bmatrix} \ln h_{U,t-1} \\ \ln h_{D,t-1} \end{bmatrix} + A \begin{bmatrix} d_{t-1} \cdot |z_{t-1}| \\ (1 - d_{t-1}) \cdot |z_{t-1}| \end{bmatrix}$$

or with a more compact notation:

$$x_t = \omega + Bx_{t-1} + A\psi_{t-1} \tag{16}$$

with $x_t = (\ln h_{U,t}\ ,\ \ln h_{D,t})$ and $\psi_t = (d_t\ ,\ 1 - d_t) \cdot |z_t|$. Letting $\psi \equiv \psi_t - \mathbb{E}[\psi_t]$ and $\eta_t \equiv \psi_t - \psi$, then:

$$x_t = (\omega + A\psi) + Bx_{t-1} + A \cdot \eta_{t-1} \tag{17}$$

The skeleton of the Markov chain has a unique fixed point at $x^* = (I - B)^{-1}(\omega + A\psi)$. With the subscript n denoting the mapping of the skeleton:

$$x_n = (\omega + A\psi) + Bx_{n-1} \tag{18}$$

Equation (17) defines a two-dimensional VAR(1) with innovations η_{t-1}. Hence, the UD-EGARCH specification is uniformly ergodic if and only if the eigenvalues of B are inside the unit circle.

A.2 Identification Conditions

Let $h_{U,t}$ and $h_{D,t}$ be the optimized conditional down and up variances and define:

$$\tilde{h}_t \equiv \begin{bmatrix} \kappa_U \cdot h_{U,t} \\ \kappa_D \cdot h_{D,t} \end{bmatrix}$$

from which it follows that:

$$\tilde{x}_t = \ln \kappa + x_t \tag{19}$$

with $\kappa = (\kappa_U\ ,\ \kappa_D)$ and $\tilde{x}_t = \left(\ln \tilde{h}_{U,t}\ ,\ \ln \tilde{h}_{D,t}\right)$. Substituting equation (19) in (16) gives:

$$\begin{aligned} \tilde{x}_t &= [\omega + \ln \kappa - B \ln \kappa] + B\tilde{x}_{t-1} + A\psi_{t-1} \\ &= \tilde{\omega} + B\tilde{x}_{t-1} + A\psi_{t-1} \end{aligned} \tag{20}$$

The vector ψ_{t-1} is equal to:

$$
\begin{aligned}
\psi_{t-1} &= \begin{bmatrix} d_{t-1}|\epsilon_{t-1}|\kappa_U^{1/2}\tilde{h}_{U,t-1}^{-1/2} \\ (1-d_{t-1})|\epsilon_{t-1}|\kappa_D^{1/2}\tilde{h}_{D,t-1}^{-1/2} \end{bmatrix} \\
&= \begin{bmatrix} \kappa_U^{1/2} & 0 \\ 0 & \kappa_D^{1/2} \end{bmatrix} \cdot \tilde{\psi}_{t-1} \qquad (21)
\end{aligned}
$$

Substituting equation (21) in (20) yields:

$$
\begin{aligned}
\tilde{x}_t &= \tilde{\omega} + B\tilde{x}_{t-1} + A \begin{bmatrix} \kappa_U^{1/2} & 0 \\ 0 & \kappa_D^{1/2} \end{bmatrix} \cdot \tilde{\psi}_{t-1} \\
&= \tilde{\omega} + B\tilde{x}_{t-1} + \tilde{A}\tilde{\psi}_{t-1} \qquad (22)
\end{aligned}
$$

Therefore, the rescaling of the down and up conditional variances by the κ_D and κ_U factors, respectively, results in processes that are still UD-EGARCH as may be seen from equation (22). The concentrated Gaussian log-likelihood may be rewritten, consistently with the model, in terms of $\tilde{h}_{U,t}$ and $\tilde{h}_{D,t}$:

$$
\begin{aligned}
L_c &= -\frac{1}{2}\sum_{t=1}^{T} d_t[\ln \tilde{h}_{U,t} + \epsilon_t^2\tilde{h}_{U,t}^{-1}] + (1-d_t)[\ln \tilde{h}_{D,t} + \epsilon_t^2\tilde{h}_{D,t}^{-1}] \\
&= -\frac{1}{2}\sum_{t=1}^{T} d_t[\ln \kappa_U + \ln h_{U,t} + \kappa_U^{-1}y_t^2h_{U,t}^{-1}] + (1-d_t)[\ln \kappa_D + \ln h_{D,t} + \kappa_D^{-1}y_t^2h_{D,t}^{-1}]
\end{aligned}
$$

The first order conditions with respect to κ_U and κ_D are:

$$
\begin{aligned}
\sum_{t=1}^{T} d_t\left(\kappa_U^{-1} - \kappa_U^{-2}\epsilon_t^2h_{U,t}^{-1}\right) &= 0 \\
\sum_{t=1}^{T}(1-d_t)\left(\kappa_D^{-1} - \kappa_D^{-2}\epsilon_t^2h_{D,t}^{-1}\right) &= 0
\end{aligned}
$$

However, since the QML estimator maximizes L_c and $h_{U,t}$ and $h_{D,t}$ are the optimized down and up variances, it must be that $\kappa_U = \kappa_D = 1$. Imposing these conditions and solving yields:

$$
\frac{\sum_{t=1}^{T} d_t\epsilon_t^2h_{U,t}^{-1}}{\sum_{t=1}^{T} d_t} = 1 \qquad \text{and} \qquad \frac{\sum_{t=1}^{T}(1-d_t)\epsilon_t^2h_{D,t}^{-1}}{\sum_{t=1}^{T}(1-d_t)} = 1
$$

which are the sample counterparts of the conditions in (3). Hence, both positive and negative components of the residuals have unit variance.

A.3 Decaying Patterns

From equation (17), a shock at $t = 0$ impacts the elements of x_0 through the matrix of coefficients A while in the subsequent n periods the evolution of x_n and its mean reversion are described by equation (18). Consider the spectral decomposition $C\Lambda C^{-1}$ of the matrix of coefficients B:

$$\begin{aligned} x_n &= (\omega + A\psi) + C\Lambda C^{-1} x_{n-1} \\ C^{-1} x_n &= C^{-1}(\omega + A\psi) + \Lambda C^{-1} x_{n-1} \end{aligned}$$

Letting $\mathring{\omega} = C^{-1}(\omega + A\psi)$ and $\mathring{x}_n = C^{-1} x_n$ gives:

$$\mathring{x}_n = \mathring{\omega} + \Lambda \mathring{x}_{n-1}$$

and therefore:

$$\begin{aligned} \mathring{x}_{1,n} &= \mathring{\omega}_1 + \lambda_1 \mathring{x}_{1,n-1} \\ \mathring{x}_{2,n} &= \mathring{\omega}_2 + \lambda_2 \mathring{x}_{2,n-1} \end{aligned}$$

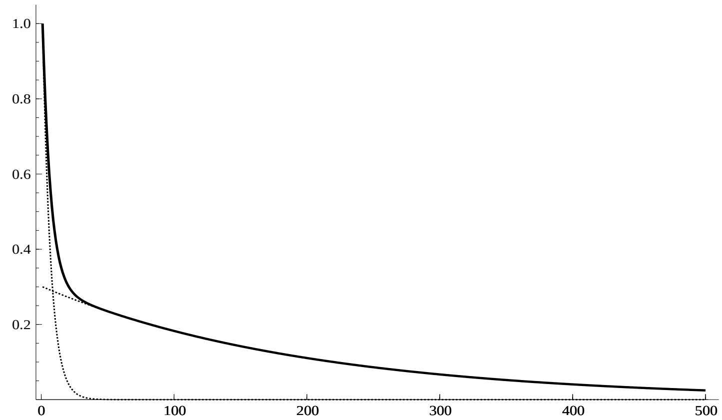
For $\lambda_1 \neq \lambda_2$ the components $\mathring{x}_1$ and $\mathring{x}_2$ exhibit different persistence and therefore different geometrically decaying rates. The elements of x_n are a linear combination of those of $\mathring{x}_n$:

$$\begin{aligned} x_{1,n} &= c_{11} \mathring{x}_{1,n} + c_{12} \mathring{x}_{2,n} \\ x_{2,n} &= c_{21} \mathring{x}_{1,n} + c_{22} \mathring{x}_{2,n} \end{aligned}$$

where c_{ij} are the elements of the eigenvector matrix C. In turn, this produces richer persistence and decaying patterns for the quantities of interest. To exemplify one of the possible decaying patterns of the UD-EGARCH, consider the case in which one of the elements of $\mathring{x}$ is very persistent while the other is significantly less so. Furthermore, let the loadings on the persistent element (c_{11} and/or c_{21}) be significantly smaller than the loadings on the transitory element (c_{12} and/or c_{22}). Then, the process will initially exhibit low persistence due to its small-eigenvalue component vanishing relatively quickly. Subsequently, the process will exhibit near-full loading on the large eigenvalue component and therefore exhibit high persistence as depicted by the simulation in Figure 1.

The dynamics of the UD-EGARCH decaying patterns are carried over to the autocovariances of the process. In particular, while the variance processes rotated by the matrix of eigenvectors will exhibit geometrically decaying autocovariances, the

Figure 1: Decay: $\lambda_1 = 0.995$, $\lambda_2 = 0.85$, $c_{11} = 0.3$ and $c_{12} = 0.7$. Solid line: compound process x with fast initial decay and slower decay at larger lags. Dotted lines: constituting elements with geometric decay matching either the initial or the final shape of the x process.



autocovariances of the processes themselves will exhibit combined decaying patters which may result in overall patterns that are more similar to an hyperbolic than an geometric decay.

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Table 1: Winsorization of Low-Frequency Returns

The table reports the 0.1-th (Lower) and 99.9-th (Upper) Winsorization percentiles. The third column (W/T) reports the ratio of Winsorized observations to the total number of observations.

	Lower	Upper	W/T
S&P500	-6.801%	+6.325%	16/7815
DJIA	-6.578%	+4.810%	14/6857
NASDAQ	-8.850%	+7.579%	16/8030
Nikkei	-7.141%	+7.274%	16/7840
EuroSTOXX	-7.355%	+7.078%	12/6396
DAX	-7.270%	+6.979%	16/7997
FTSE	-5.885%	+5.904%	14/7026
CAC	-7.678%	+8.327%	12/6139
SMI	-6.794%	+6.079%	12/5863

Table 2: In- and Out-Of-Sample Distribution of Winsorized Low-Frequency Returns

The table reports the distribution of extreme observations. Returns are categorized as extreme if they are smaller than the 0.1-th Winsorization percentile, reported in the Lower column, or greater than the 99.9-th Winsorization percentile, reported in the Upper column. The IS columns contain the permillage of extreme returns observed In-Sample. The OOS columns contain the permillage of extreme returns observed Out-Of-Sample.

	Lower		Upper	
	IS	OOS	IS	OOS
S&P500	0.610	3.175	0.152	5.556
DJIA	1.251	0.000	0.893	1.587
NASDAQ	0.739	2.381	0.886	1.587
Nikkei	0.456	3.968	0.912	1.587
EuroSTOXX	0.584	2.381	0.195	3.968
DAX	0.891	1.587	0.594	3.175
FTSE	0.694	2.381	0.347	3.968
CAC	0.615	2.381	0.000	4.762
SMI	1.086	0.794	0.652	2.381

Table 3: Specifications Considered in Model Selection Search

The table lists the exclusion and equality constraints that have been considered for the UD-EGARCH and UD-GTARCH specifications. B-matrix constraints comprise: no dependence of down (up) volatility on lagged up (down) volatility and equal dependence of down (up) volatility on lagged down and up volatilities. A-matrix constraints comprise: no good news effects and good (bad) news having equal effects on down and up volatilities.

	B-matrix:	A-matrix:
No Constraints:		
	$\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}$	$\begin{bmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}$
One Constraint:		
	$\begin{bmatrix}\beta_{11} & 0\\ \beta_{21} & \beta_{22}\end{bmatrix}$ $\begin{bmatrix}\beta_{11} & \beta_{12}\\ 0 & \beta_{22}\end{bmatrix}$ $\begin{bmatrix}\boldsymbol{\beta_{11}} & \boldsymbol{\beta_{11}}\\ \beta_{21} & \beta_{22}\end{bmatrix}$ $\begin{bmatrix}\beta_{11} & \beta_{12}\\ \boldsymbol{\beta_{21}} & \boldsymbol{\beta_{21}}\end{bmatrix}$	$\begin{bmatrix}0 & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}$ $\begin{bmatrix}\alpha_{11} & \alpha_{12}\\ 0 & \alpha_{22}\end{bmatrix}$ $\begin{bmatrix}\boldsymbol{a_{11}} & \alpha_{12}\\ \boldsymbol{a_{11}} & \alpha_{22}\end{bmatrix}$ $\begin{bmatrix}\alpha_{11} & \boldsymbol{a_{12}}\\ \alpha_{21} & \boldsymbol{a_{12}}\end{bmatrix}$
Two Constraints:		
	$\begin{bmatrix}\beta_{11} & 0\\ 0 & \beta_{22}\end{bmatrix}$ $\begin{bmatrix}\beta_{11} & 0\\ \boldsymbol{\beta_{21}} & \boldsymbol{\beta_{21}}\end{bmatrix}$ $\begin{bmatrix}\boldsymbol{\beta_{11}} & \boldsymbol{\beta_{11}}\\ 0 & \beta_{22}\end{bmatrix}$	$\begin{bmatrix}0 & \boldsymbol{a_{12}}\\ \alpha_{21} & \boldsymbol{a_{12}}\end{bmatrix}$ $\begin{bmatrix}\alpha_{11} & \boldsymbol{a_{12}}\\ 0 & \boldsymbol{a_{12}}\end{bmatrix}$ $\begin{bmatrix}\boldsymbol{a_{11}} & \boldsymbol{a_{12}}\\ \boldsymbol{a_{11}} & \boldsymbol{a_{12}}\end{bmatrix}$
Three Constraints:		
	$\begin{bmatrix}\boldsymbol{\beta_{11}} & 0\\ 0 & \boldsymbol{\beta_{11}}\end{bmatrix}$	$\begin{bmatrix}0 & \boldsymbol{a_{12}}\\ 0 & \boldsymbol{a_{12}}\end{bmatrix}$

Table 4: In-Sample Low-Frequency Volatility Estimates

The table reports estimation results for the EGARCH and the UD-EGARCH. The first column reports standard EGARCH estimates. The second column reports the UNCONSTRAINED UD-EGARCH estimates of the UP and DOWN components. The third column reports UD-EGARCH estimates of the UP and DOWN components with parametric constraints selected by BIC/SIC. With respect to equation (4): ω, β^U,β^D, α^U, α^D in the UP (DOWN) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}), respectively. CONSTRAINED coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the UP and DOWN components are equal. The row before last of each panel reports the concentrated log-likelihood value of the estimated models with ^{a} indicating the best BIC/SIC specification between EGARCH and UNCONSTRAINED UD-EGARCH and ^{b} indicating the best BIC/SIC specification of the three. Last row of each box reports the number of observations.

	EGARCH		UNCONSTRAINED					CONSTRAINED				
			UP		DOWN		Δ	UP		DOWN		Δ
S&P500												
ω	-0.084	(0.011)***	-0.079	(0.017)***	0.042	(0.048)		-0.093	(0.011)***	0.076	(0.036)**	
β^U	0.987	(0.003)***	1.029	(0.057)***	0.446	(0.122)***	[0.000]	0.987	(0.003)***	**0.459**	(0.021)***	
β^D			-0.046	(0.065)	0.477	(0.128)***	[0.000]			**0.459**	(0.021)***	
α^U	0.049	(0.016)***	0.029	(0.038)	-0.300	(0.064)***	[0.000]	0.059	(0.016)***	-0.337	(0.058)***	[0.000]
α^D	0.165	(0.017)***	0.175	(0.019)***	0.224	(0.061)***	[0.386]	**0.175**	(0.017)***	**0.175**	(0.017)***	
log-lik	-2564.955		-2497.725					-2499.301				
BIC/SIC	5165.062		5083.330^a					5060.118^b				
obs.	6555		6555					6555				
Dow Jones Industrial Average												
ω	-0.083	(0.013)***	-0.098	(0.019)***	0.024	(0.057)		-0.092	(0.012)***	0.080	(0.037)	
β^U	0.984	(0.004)***	0.942	(0.057)***	0.417	(0.140)***	[0.000]	0.985	(0.003)***	**0.466**	(0.025)***	
β^D			0.050	(0.063)	0.522	(0.143)***	[0.001]			**0.466**	(0.025)***	
α^U	0.059	(0.019)***	0.104	(0.042)**	-0.231	(0.076)***	[0.000]	0.078	(0.018)***	-0.272	(0.061)***	[0.000]
α^D	0.153	(0.020)***	0.138	(0.025)***	0.254	(0.076)***	[0.162]	**0.159**	(0.018)***	**0.159**	(0.018)***	
log-lik	-2320.086		-2269.318					-2272.451				
BIC/SIC	4674.692		4624.936^a					4605.312^b				
obs.	5597		5597					5597				
NASDAQ												
ω	-0.157	(0.013)***	-0.152	(0.018)***	0.061	(0.047)		-0.144	(0.011)***	0.090	(0.043)**	
β^U	0.982	(0.003)***	0.970	(0.030)***	0.362	(0.072)***	[0.000]	0.989	(0.002)***	**0.462**	(0.012)***	
β^D			0.022	(0.035)	0.568	(0.079)***	[0.000]			**0.462**	(0.012)***	
α^U	0.146	(0.018)***	0.152	(0.029)***	-0.210	(0.060)***	[0.000]	0.136	(0.015)***	-0.223	(0.058)***	[0.000]
α^D	0.261	(0.018)***	0.226	(0.021)***	0.373	(0.052)***	[0.008]	0.236	(0.017)***	0.386	(0.053)***	[0.005]
log-lik	-3117.654		-2986.110					-2988.072				
BIC/SIC	6270.589		6060.423^a					6046.706^b				
obs.	6770		6770					6770				

Table 5: In-Sample Low-Frequency Volatility Estimates

The table reports estimation results for the EGARCH and the UD-EGARCH. The first column reports standard EGARCH estimates. The second column reports the UNCONSTRAINED UD-EGARCH estimates of the UP and DOWN components. The third column reports UD-EGARCH estimates of the UP and DOWN components with parametric constraints selected by BIC/SIC. With respect to equation (4): ω, β^U,β^D, α^U, α^D in the UP (DOWN) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}), respectively. CONSTRAINED coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the UP and DOWN components are equal. The row before last of each panel reports the concentrated log-likelihood value of the estimated models with ^{a} indicating the best BIC/SIC specification between EGARCH and UNCONSTRAINED UD-EGARCH and ^{b} indicating the best BIC/SIC specification of the three. Last row of each box reports the number of observations.

	EGARCH		UNCONSTRAINED					CONSTRAINED				
			UP		DOWN		Δ	UP		DOWN		Δ
Nikkei												
ω	-0.156	(0.013)***	-0.141	(0.014)***	-0.105	(0.035)***		-0.151	(0.013)***	-0.091	(0.041)**	
β^U	0.979	(0.003)***	1.039	(0.060)***	0.369	(0.106)***	[0.000]	0.983	(0.003)***	**0.484**	(0.014)***	
β^D			-0.058	(0.062)	0.596	(0.103)***	[0.000]			**0.484**	(0.014)***	
α^U	0.117	(0.019)***	0.092	(0.031)***	-0.098	(0.048)**	[0.000]	0.121	(0.019)***	-0.125	(0.053)**	[0.000]
α^D	0.300	(0.021)***	0.292	(0.028)***	0.441	(0.052)***	[0.000]	0.282	(0.022)***	0.449	(0.056)***	[0.004]
log-lik	-3672.331		-3621.803					-3624.809				
BIC/SIC	7379.829		7331.524^a					7319.952^b				
obs.	6580		6580					6580				
EuroSTOXX												
ω	-0.104	(0.018)***	-0.062	(0.045)	0.135	(0.081)*		-0.125	(0.012)***	0.146	(0.041)***	
β^U	0.979	(0.005)***	1.103	(0.096)***	0.450	(0.169)***	[0.000]	0.979	(0.003)***	**0.439**	(0.015)***	
β^D			-0.155	(0.120)	0.414	(0.207)**	[0.000]			**0.439**	(0.015)***	
α^U	0.061	(0.029)**	0.011	(0.042)	-0.223	(0.083)***	[0.005]	0.084	(0.018)***	-0.254	(0.070)***	[0.000]
α^D	0.215	(0.030)***	0.256	(0.029)***	0.260	(0.079)***	[0.947]	**0.246**	(0.022)***	**0.246**	(0.022)***	
log-lik	-2567.974		-2501.073					-2503.920				
BIC/SIC	5170.124		5087.586^a					5067.648^b				
obs.	5136		5136					5136				
DAX												
ω	-0.131	(0.014)***	-0.084	(0.036)**	0.063	(0.063)		-0.149	(0.015)***	0.071	(0.038)*	
β^U	0.979	(0.004)***	1.173	(0.131)***	0.629	(0.207)***	[0.000]	0.979	(0.004)***	**0.474**	(0.018)***	
β^D			-0.203	(0.136)	0.317	(0.207)	[0.000]			**0.474**	(0.018)***	
α^U	0.114	(0.022)***	0.048	(0.041)	-0.177	(0.071)**	[0.002]	0.140	(0.022)***	-0.208	(0.063)***	[0.000]
α^D	0.235	(0.022)***	0.261	(0.028)***	0.305	(0.067)***	[0.395]	**0.254**	(0.023)***	**0.254**	(0.023)***	
log-lik	-3989.576		-3932.700					-3937.655				
BIC/SIC	8014.413		7953.554^a					7937.018^b				
obs.	6737		6737					6737				

Table 6: In-Sample Low-Frequency Volatility Estimates

The table reports estimation results for the EGARCH and the UD-EGARCH. The first column reports standard EGARCH estimates. The second column reports the Unconstrained UD-EGARCH estimates of the Up and Down components. The third column reports UD-EGARCH estimates of the Up and Down components with parametric constraints selected by BIC/SIC. With respect to equation (4): ω, β^U,β^D, α^U, α^D in the Up (Down) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}), respectively. Constrained coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the Up and Down components are equal. The row before last of each panel reports the concentrated log-likelihood value of the estimated models with ^{a} indicating the best BIC/SIC specification between EGARCH and Unconstrained UD-EGARCH and ^{b} indicating the best BIC/SIC specification of the three. Last row of each box reports the number of observations.

	EGARCH		Unconstrained					Constrained				
			Up		Down		Δ	Up		Down		Δ
FTSE												
ω	-0.122	(0.013)***	-0.151	(0.022)***	-0.059	(0.038)		-0.135	(0.013)***	0.012	(0.034)	
β^U	0.983	(0.003)***	0.912	(0.054)***	0.191	(0.096)**	[0.000]	0.983	(0.003)***	**0.478**	(0.019)***	
β^D			0.078	(0.058)	0.776	(0.101)***	[0.000]			**0.478**	(0.019)***	
α^U	0.100	(0.018)***	0.155	(0.035)***	-0.025	(0.049)	[0.005]	0.116	(0.019)***	-0.058	(0.050)	[0.001]
α^D	0.198	(0.018)***	0.190	(0.027)***	0.249	(0.046)***	[0.319]	**0.214**	(0.020)***	**0.214**	(0.020)***	
log-lik	-2182.067		-2156.974					-2161.599				
BIC/SIC	4398.773^a		4400.545					4383.816^b				
obs.	5766		5766					5766				
CAC												
ω	-0.098	(0.015)***	-0.105	(0.035)***	0.024	(0.084)		-0.104	(0.015)***	0.067	(0.050)	
β^U	0.979	(0.005)***	0.975	(0.093)***	0.288	(0.241)	[0.003]	0.979	(0.005)***	**0.448**	(0.027)***	
β^D			0.005	(0.113)	0.625	(0.286)**	[0.022]			**0.448**	(0.027)***	
α^U	0.065	(0.024)***	0.071	(0.038)*	-0.077	(0.087)	[0.112]	0.068	(0.023)***	-0.102	(0.081)	[0.032]
α^D	0.206	(0.024)***	0.215	(0.025)***	0.225	(0.086)***	[0.913]	**0.217**	(0.025)***	**0.217**	(0.025)***	
log-lik	-3174.565		-3156.442					-3156.977				
BIC/SIC	6383.100^a		6397.811					6373.403^b				
obs.	4879		4879					4879				
Swiss Market Index												
ω	-0.135	(0.021)***	-0.159	(0.024)***	-0.065	(0.068)		-0.153	(0.020)***	-0.045	(0.067)	
β^U	0.961	(0.008)***	0.946	(0.057)***	0.280	(0.133)**	[0.000]	0.969	(0.006)***	**0.410**	(0.029)***	
β^D			0.031	(0.078)	0.566	(0.164)***	[0.001]			**0.410**	(0.029)***	
α^U	0.078	(0.030)**	0.122	(0.039)***	-0.078	(0.082)	[0.026]	0.109	(0.028)***	-0.081	(0.084)	[0.023]
α^D	0.269	(0.032)***	0.270	(0.045)***	0.450	(0.109)***	[0.071]	0.282	(0.031)***	0.480	(0.093)***	[0.032]
log-lik	-2130.939		-2080.162					-2081.034				
BIC/SIC	4295.616		4244.669^a					4229.544^b				
obs.	4603		4603					4603				

Table 7: In-Sample Low-Frequency Volatility Estimates

The table reports estimation results for the TGARCH and the UD-TGARCH of El Babsiri and Zakoian (2001). The first column reports standard TGARCH estimates. The second column reports the Unconstrained UD-TGARCH estimates of the Up and Down components. The third column reports UD-TGARCH estimates of the Up and Down components with parametric constraints selected by BIC/SIC. With respect to equation (4): ω, β^U,β^D, α^U, α^D in the Up (Down) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}), respectively. Constrained coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the Up and Down components are equal. The row before last of each panel reports the concentrated log-likelihood value of the estimated models with ^{a} indicating the best BIC/SIC specification between TGARCH and Unconstrained UD-TGARCH and ^{b} indicating the best BIC/SIC specification of the three. Last row of each box reports the number of observations.

	TGARCH		Unconstrained					Constrained				
			Up		Down		Δ	Up		Down		Δ
S&P500												
ω	0.012	(0.004***	0.007	(0.005)	0.043	(0.044)		0.007	(0.003)**	0.042	(0.049)	
β^U	0.943	(0.009)***	0.950	(0.062)***	0.377	(0.210)*	[0.008]	0.950	(0.008)***	**0.443**	(0.034)***	
β^D			0.000	(0.067)	0.507	(0.205)**	[0.016]			**0.443**	(0.034)***	
α^U	0.025	(0.010)**	0.034	(0.012)***	0.000	(0.053)	[0.537]	0.034	(0.009)***	0.000	(0.055)	[0.547]
α^D	0.090	(0.013)***	0.073	(0.020)***	0.230	(0.071)***	[0.033]	0.074	(0.011)***	0.024	(0.066)***	[0.015]
log-lik	-2564.850		-2519.494					-2519.858				
BIC/SIC	5164.852		5126.868^{a}					5110.020^{b}				
obs.	6555		6555					6555				
Dow Jones Industrial Average												
ω	0.016	(0.005)***	0.006	(0.008)	0.045	(0.056)		0.009	(0.004)**	0.037	(0.065)	
β^U	0.941	(0.012)***	0.917	(0.055)***	0.360	(0.201)*	[0.012]	0.951	(0.009)***	**0.449**	(0.043)***	
β^D			0.038	(0.055)	0.526	(0.192)***	[0.019]			**0.449**	(0.043)***	
α^U	0.029	(0.012)**	0.043	(0.015)***	0.000	(0.045)	[0.414]	0.038	(0.010)***	0.000	(0.047)	[0.442]
α^D	0.084	(0.015)***	0.052	(0.016)***	0.236	(0.080)***	[0.027]	0.063	(0.011)***	0.244	(0.075)***	[0.019]
log-lik	-2319.932		-2274.629					-2275.441				
BIC/SIC	4674.384		4635.559^{a}					4619.922^{b}				
obs.	5597		5597					5597				
NASDAQ												
ω	0.019	(0.005)***	0.006	(0.007)	0.054	(0.040)		0.008	(0.004)**	0.061	(0.048)	
β^U	0.896	(0.014)***	0.926	(0.057)***	0.358	(0.205)*	[0.007]	0.911	(0.013)***	**0.452**	(0.038)***	
β^D			0.000	(0.056)	0.543	(0.175)***	[0.003]			**0.452**	(0.038)***	
α^U	0.077	(0.015)***	0.073	(0.018)***	0.000	(0.072)	[0.323]	0.084	(0.015)***	0.000	(0.070)	[0.247]
α^D	0.145	(0.020)***	0.088	(0.021)***	0.274	(0.080)***	[0.027]	0.108	(0.017)***	0.282	(0.077)***	[0.030]
log-lik	-3108.376		-2992.653					-2995.974				
BIC/SIC	6252.033		6073.509^{a}					6062.510^{b}				
obs.	6770		6770					6770				

Table 8: In-Sample Low-Frequency Volatility Estimates

The table reports estimation results for the TGARCH and the UD-TGARCH of El Babsiri and Zakoian (2001). The first column reports standard TGARCH estimates. The second column reports the Unconstrained UD-TGARCH estimates of the Up and Down components. The third column reports UD-TGARCH estimates of the Up and Down components with parametric constraints selected by BIC/SIC. With respect to equation (4): ω, β^U,β^D, α^U, α^D in the Up (Down) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}), respectively. Constrained coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the Up and Down components are equal. The row before last of each panel reports the concentrated log-likelihood value of the estimated models with ^{a} indicating the best BIC/SIC specification between TGARCH and Unconstrained UD-TGARCH and ^{b} indicating the best BIC/SIC specification of the three. Last row of each box reports the number of observations.

	TGARCH	Unconstrained			Constrained		
		Up	Down	Δ	Up	Down	Δ
Nikkei							
ω	0.020 (0.004)***	0.013 (0.005)**	0.029 (0.035)***		0.014 (0.004)***	0.023 (0.038)	
β^U	0.898 (0.011)***	0.911 (0.078)***	0.330 (0.153)**	[0.000]	0.910 (0.012)***	**0.442** (0.028)***	
β^D		0.000 (0.077)	0.549 (0.149)***	[0.000]		**0.442** (0.028)***	
α^U	0.060 (0.012)***	0.073 (0.023)***	0.000 (0.035)	[0.098]	0.070 (0.014)***	0.000 (0.037)	[0.089]
α^D	0.163 (0.020)***	0.128 (0.028)***	0.297 (0.067)***	[0.021]	0.130 (0.020)***	0.312 (0.066)***	[0.011]
log-lik	-3679.446	-3629.342			-3630.835		
BIC/SIC	7394.059	7346.602^a			7332.004^b		
obs.	6580	6580			6580		
EuroSTOXX							
ω	0.024 (0.011)**	0.013 (0.021)	0.090 (0.108)		0.014 (0.007)**	0.111 (0.116)	
β^U	0.918 (0.023)***	0.926 (0.107)***	0.252 (0.526)	[0.218]	0.923 (0.021)***	**0.435** (0.085)***	
β^D		0.000 (0.111)	0.621 (0.540)	[0.269]		**0.435** (0.085)***	
α^U	0.033 (0.020)	0.048 (0.027)*	0.000 (0.122)	[0.706]	0.047 (0.021)**	0.000 (0.140)	[0.749]
α^D	0.123 (0.032)***	0.104 (0.025)***	0.210 (0.158)	[0.518]	0.109 (0.026)***	0.224 (0.171)	[0.518]
log-lik	-2555.974	-2507.757			-2509.370		
BIC/SIC	5146.124	5100.954^a			5087.092^b		
obs.	5136	5136			5136		
DAX							
ω	0.026 (0.008)***	0.018 (0.008)**	0.044 (0.061)		0.020 (0.007)***	0.032 (0.080)	
β^U	0.904 (0.015)***	0.915 (0.090)***	0.316 (0.288)	[0.044]	0.911 (0.016)***	**0.463** (0.052)***	
β^D		0.000 (0.083)	0.589 (0.261)**	[0.029]		**0.463** (0.052)***	
α^U	0.059 (0.016)***	0.070 (0.026)***	0.000 (0.072)	[0.376]	0.070 (0.017)***	0.000 (0.076)	[0.000]
α^D	0.129 (0.020)***	0.100 (0.020)***	0.223 (0.097)**	[0.223]	0.105 (0.018)***	0.230 (0.097)**	
log-lik	-3988.522	-3943.265			-3944.868		
BIC/SIC	8012.305	7974.684^a			7960.259^b		
obs.	6737	6737			6737		

Table 9: In-Sample Low-Frequency Volatility Estimates

The table reports estimation results for the TGARCH and the UD-TGARCH of El Babsiri and Zakoian (2001). The first column reports standard TGARCH estimates. The second column reports the UNCONSTRAINED UD-TGARCH estimates of the UP and DOWN components. The third column reports UD-TGARCH estimates of the UP and DOWN components with parametric constraints selected by BIC/SIC. With respect to equation (4): ω, β^U,β^D, α^U, α^D in the UP (DOWN) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}), respectively. CONSTRAINED coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the UP and DOWN components are equal. The row before last of each panel reports the concentrated log-likelihood value of the estimated models with [a] indicating the best BIC/SIC specification between TGARCH and UNCONSTRAINED UD-TGARCH and [b] indicating the best BIC/SIC specification of the three. Last row of each box reports the number of observations.

	TGARCH		UNCONSTRAINED					CONSTRAINED				
			UP		DOWN		Δ	UP		DOWN		Δ
FTSE												
ω	0.017	(0.005)***	0.011	(0.006)*	0.026	(0.023)		0.017	(0.006)***	0.020	(0.052)	
β^U	0.920	(0.010)***	0.916	(0.048)***	0.172	(0.116)	[0.000]	0.908	(0.011)***	**0.486**	(0.037)***	
β^D			0.009	(0.044)	0.754	(0.102)***	[0.000]			**0.486**	(0.037)***	
α^U	0.051	(0.010)***	0.066	(0.018)***	0.000	(0.025)	[0.038]	0.068	(0.013)***	0.000	(0.031)	[0.042]
α^D	0.107	(0.012)***	0.083	(0.015)***	0.162	(0.036)***	[0.049]	**0.110**	(0.014)***	**0.110**	(0.014)***	
log-lik	-2183.823		-2152.266					-2164.151				
BIC/SIC	4402.285		4391.129[a]					4397.580[b]				
obs.	5766		5766					5766				
CAC												
ω	0.027	(0.009)***	0.015	(0.017)	0.072	(0.061)		0.022	(0.007)***	0.091	(0.107)	
β^U	0.921	(0.016)***	0.935	(0.071)***	0.130	(0.224)	[0.001]	0.918	(0.015)***	**0.456**	(0.053)***	
β^D			0.000	(0.080)	0.762	(0.243)***	[0.004]			**0.456**	(0.053)***	
α^U	0.032	(0.016)**	0.038	(0.017)**	0.000	(0.067)	[0.583]	0.043	(0.016)***	0.000	(0.088)	[0.627]
α^D	0.112	(0.020)***	0.092	(0.020)***	0.159	(0.076)**	[0.408]	**0.114**	(0.020)***	**0.114**	(0.020)***	
log-lik	-3171.215		-3153.291					-3158.330				
BIC/SIC	6376.703[a]		6391.509					6384.602[b]				
obs.	4879		4879					4879				
Swiss Market Index												
ω	0.045	(0.013)***	0.020	(0.021)	0.129	(0.060)**		0.023	(0.010)**	0.145	(0.078)*	
β^U	0.877	(0.027)***	0.899	(0.076)***	0.219	(0.208)	[0.002]	0.896	(0.022)***	**0.391**	(0.052)***	
β^D			0.000	(0.083)	0.571	(0.223)**	[0.017]			**0.391**	(0.052)***	
α^U	0.044	(0.023)*	0.072	(0.034)**	0.000	(0.052)	[0.266]	0.067	(0.026)***	0.000	(0.052)	[0.278]
α^D	0.159	(0.034)***	0.125	(0.033)***	0.300	(0.128)**	[0.189]	0.129	(0.027)***	0.335	(0.134)**	[0.141]
log-lik	-2119.366		-2070.241					-2072.226				
BIC/SIC	4272.470		4224.827[a]					4211.928[b]				
obs.	4603		4603					4603				

Table 10: Half-Lives of Volatility Shocks from In-Sample Estimates

The table reports the half-lives of volatility shocks for the EGARCH and Constrained UD-EGARCH specifications. Columns of the left and right panels indicate the elapsed times, expressed in number of days, between the (**n-1**)-th **n**-th halving. The rows in each panel report the following: Up$^+$ the half-lives of positive shocks in up volatilities, Down$^+$ the half-lives of positive shocks in down volatilities, Up$^-$ the half-lives of negative shocks in up volatilities and Down$^-$ the half-lives of negative shocks in down volatilities.

n:	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**n:**	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**
S&P500							**Dow Jones Industrial Average**						
EGARCH	53	53	53	53	53	53	EGARCH	44	44	44	44	44	44
Up$^+$	55	53	54	53	54	53	Up$^+$	47	47	47	47	46	47
Down$^+$	2	1	2	37	54	53	Down$^+$	2	1	1	2	26	47
Up$^-$	52	53	53	53	54	53	Up$^-$	46	46	47	46	47	46
Down$^-$	2	9	52	53	53	54	Down$^-$	2	7	46	46	46	47
NASDAQ							**Nikkei**						
EGARCH	39	39	39	39	39	39	EGARCH	33	33	33	33	33	33
Up$^+$	62	61	62	61	62	61	Up$^+$	42	40	41	41	40	41
Down$^+$	2	1	1	1	1	6	Down$^+$	2	1	6	40	41	40
Up$^-$	60	60	61	61	61	62	Up$^-$	39	40	40	40	41	40
Down$^-$	2	2	42	60	61	61	Down$^-$	2	16	39	40	40	41
EuroSTOXX							**DAX**						
EGARCH	33	33	33	33	33	33	EGARCH	32	32	32	32	32	32
Up$^+$	33	33	32	33	32	32	Up$^+$	33	32	32	32	32	32
Down$^+$	2	1	4	32	32	33	Down$^+$	2	1	1	2	18	32
Up$^-$	31	32	32	32	32	33	Up$^-$	31	32	32	32	32	32
Down$^-$	2	14	32	32	32	32	Down$^-$	2	16	31	32	31	32
FTSE							**CAC**						
EGARCH	40	40	40	40	40	40	EGARCH	32	32	32	32	32	32
Up$^+$	40	40	39	40	39	40	Up$^+$	33	32	33	32	32	32
Down$^+$	2	1	2	27	40	39	Down$^+$	2	3	28	33	32	32
Up$^-$	39	39	39	39	40	39	Up$^-$	31	32	32	32	32	32
Down$^-$	4	34	39	39	39	39	Down$^-$	4	27	32	31	32	32
Swiss Market Index													
EGARCH	18	18	18	18	18	18							
Up$^+$	23	22	22	22	22	22							
Down$^+$	2	1	3	20	22	21							
Up$^-$	21	22	21	22	22	22							
Down$^-$	2	2	19	21	22	22							

Table 11: Out-Of-Sample Low-Frequency Volatility Estimates

The table reports estimation results for the EGARCH and the UD-EGARCH. The first column reports standard EGARCH estimates with parametric constraints selected by BIC/SIC. The second column reports UD-EGARCH estimates of the UP and DOWN components with parametric constraints selected by BIC/SIC. With respect to equation (4): ω, β^U,β^D, α^U, α^D in the UP (DOWN) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}), respectively. CONSTRAINED coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. The last row of each panel reports the concentrated log-likelihood value of the estimated models with ^{b} indicating the best BIC/SIC specification. For each index the total number of observations is 1260.

	EGARCH		UD-EGARCH Up		UD-EGARCH Down		EGARCH		UD-EGARCH Up		UD-EGARCH Down	
S&P500							**Dow Jones Industrial Average**					
ω	-0.100	(0.013)***	-0.121	(0.015)***	0.210	(0.075)***	-0.071	(0.014)***	-0.076	(0.016)***		
β^U	0.979	(0.005)***	0.977	(0.006)***	**0.448**	(0.028)***	0.987	(0.005)***	0.985	(0.005)***	**0.414**	(0.036)***
β^D					**0.448**	(0.028)***					**0.414**	(0.036)***
α^U					-0.240	(0.117)**					-0.185	(0.107)*
α^D	0.277	(0.036)***	**0.325**	(0.045)***	**0.325**	(0.045)***	0.167	(0.032)***	**0.180**	(0.038)***	**0.180**	(0.038)***
log-lik	-757.848		-738.774				-351.944		-341.982			
BIC/SIC	1537.113		1520.381^b				725.305		719.658^b			
NASDAQ							**Nikkei**					
ω	-0.087	(0.011)***	-0.101	(0.012)***	0.313	(0.076)***	-0.089	(0.011)***	-0.096	(0.012)***	0.242	(0.076)***
β^U	0.972	(0.007)***	0.970	(0.007)***	**0.426**	(0.030)***	0.972	(0.006)***	0.973	(0.007)***	**0.426**	(0.031)***
β^D					**0.426**	(0.030)***					**0.426**	(0.031)***
α^U					-0.242	(0.106)**					-0.230	(0.101)**
α^D	0.276	(0.033)***	**0.330**	(0.042)***	**0.330**	(0.042)***	0.277	(0.032)***	**0.305**	(0.039)***	**0.305**	(0.039)***
log-lik	-1016.271		-991.324				-1073.477		-1059.796			
BIC/SIC	2053.959		2025.481^b				2168.371		2162.425^b			
EuroSTOXX							**DAX**					
ω	-0.106	(0.011)***	-0.105	(0.014)***			-0.089	(0.011)***	-0.093	(0.013)***	0.205	(0.081)**
β^U	0.976	(0.006)***	0.978	(0.008)***	**0.510**	(0.030)***	0.972	(0.007)***	0.973	(0.007)***	**0.455**	(0.034)***
β^D					**0.510**	(0.030)***					**0.455**	(0.034)***
α^U					-0.250	(0.099)**					-0.285	(0.117)**
α^D	0.314	(0.033)***	**0.312**	(0.043)***	**0.312**	(0.043)***	0.279	(0.033)***	**0.300**	(0.042)***	**0.300**	(0.042)***
log-lik	-1055.091		-1047.429				-1005.059		-993.495			
BIC/SIC	2131.599		2130.552^b				2031.535		2029.832^b			
FTSE							**CAC**					
ω	-0.097	(0.010)***	-0.132	(0.045)***	-0.125	(0.036)***	-0.102	(0.011)***	-0.006	(0.021)	0.002	(0.021)
β^U	0.978	(0.005)***	**0.475**	(0.013)***	-0.400	(0.047)***	0.972	(0.007)***	0.991	(0.002)***		
β^D			**0.475**	(0.013)***	1.360	(0.041)***			-0.038	(0.008)***	0.946	(0.010)***
α^U			-0.267	(0.062)***	-0.194	(0.037)***			**-0.194**	(0.040)***	**-0.194**	(0.040)***
α^D	0.270	(0.029)***	**0.261**	(0.040)***	**0.261**	(0.040)***	0.314	(0.033)***	**0.327**	(0.036)***	**0.327**	(0.036)***
log-lik	-881.216		-860.407				-1072.552		-1054.479			
BIC/SIC	1783.849		1777.925^b				2166.521		2158.930^b			
Swiss Market Index												
ω	-0.160	(0.013)***	-0.050	(0.022)**	-0.046	(0.022)**						
β^U	0.970	(0.007)***	0.940	(0.002)***								
β^D			-0.031	(0.007)***	0.958	(0.009)***						
α^U			**-0.136**	(0.037)***	**-0.136**	(0.037)***						
α^D	0.312	(0.035)***	**0.323**	(0.038)***	**0.323**	(0.038)***						
log-lik	-765.830		-743.477									
BIC/SIC	1553.077		1536.926^b									

Table 12: Out-Of-Sample Performance of Low-Frequency Volatility Specifications

The table reports the OOS performance of the EGARCH and UD-EGARCH specifications of Table 4. Daily predictions are generated by keeping parameters fixed at the IS estimates for both specifications. OOS Up and Down variances are Filtered from the BIC/SIC preferred specifications of Table 11 and approximated by K period averages of squared-log-returns SqR for $K = 1, 20, 60, 120$. Mse measures of distance between predictions and filtered values are reported in the first column for Up variances and in the second column for Down variances. The third column reports Mse for the standard Joint measure of variance. Δ reports the percentage differences in Mse of UD-EGARCH with respect to EGARCH.

	EGARCH			UD-EGARCH			Δ		
	Up	Down	Joint	Up	Down	Joint	Up	Down	Joint
S&P500									
Filtered	1.255	2.579	3.834	0.588	2.072	2.660	−53.21%	−19.70%	−30.67%
1-SqR	9.251	13.282	22.533	9.088	13.077	22.165	−1.76%	−1.54%	−1.63%
20-SqR	0.664	1.205	1.869	0.526	1.334	1.860	−20.83%	+10.74%	−0.48%
60-SqR	0.196	0.989	1.185	0.016	1.141	1.156	−92.04%	+15.29%	−2.46%
120-SqR	0.186	0.608	0.794	0.008	0.680	0.688	−95.65%	+11.87%	−13.28%
Dow Jones Industrial Average									
Filtered	0.100	0.062	0.162	0.081	0.047	0.128	−18.12%	−25.02%	−20.77%
1-SqR	2.192	1.527	3.719	2.200	1.540	3.740	+0.35%	+0.88%	+0.56%
20-SqR	0.139	0.061	0.200	0.124	0.074	0.198	−10.70%	+20.67%	−1.09%
60-SqR	0.089	0.014	0.103	0.073	0.017	0.089	−18.38%	+19.89%	−13.18%
120-SqR	0.050	0.013	0.063	0.033	0.014	0.047	−34.70%	+12.09%	−25.14%
NASDAQ									
Filtered	0.564	2.293	2.857	0.912	0.661	1.573	+61.75%	−71.17%	−44.92%
1-SqR	12.522	23.605	36.127	13.110	23.849	36.959	+4.70%	+1.03%	+2.30%
20-SqR	0.676	1.429	2.105	0.856	0.941	1.797	−34.11%	+26.57%	−14.62%
60-SqR	0.108	0.579	0.688	0.119	0.249	0.368	−57.00%	+9.94%	−46.46%
120-SqR	0.036	0.343	0.380	0.002	0.087	0.089	−74.67%	−94.10%	−76.52%

Table 13: Out-Of-Sample Performance of Low-Frequency Volatility Specifications

The table reports the OOS performance of the EGARCH and UD-EGARCH specifications of Table 5. Daily predictions are generated by keeping parameters fixed at the IS estimates for both specifications. OOS Up and Down variances are Filtered from the BIC/SIC preferred specifications of Table 11 and approximated by K period averages of squared-log-returns SqR for $K = 1, 20, 60, 120$. Mse measures of distance between predictions and filtered values are reported in the first column for Up variances and in the second column for Down variances. The third column reports Mse for the standard Joint measure of variance. Δ reports the percentage differences in Mse of UD-EGARCH with respect to EGARCH.

	EGARCH			UD-EGARCH			Δ		
	Up	Down	Joint	Up	Down	Joint	Up	Down	Joint
Nikkei									
Filtered	0.399	1.410	1.809	0.482	0.935	1.417	+20.94%	−33.71%	−21.66%
1-SqR	9.486	20.247	29.733	10.116	20.613	30.729	+6.64%	+1.81%	+3.35%
20-SqR	0.355	1.525	1.880	0.365	1.196	1.561	+2.72%	−21.58%	−16.99%
60-SqR	0.135	0.659	0.794	0.113	0.363	0.476	−16.54%	−44.85%	−40.03%
120-SqR	0.079	0.426	0.506	0.049	0.213	0.262	−37.47%	−50.09%	−48.11%
EuroSTOXX									
Filtered	0.641	3.951	4.592	0.466	2.924	3.389	−27.35%	−26.01%	−26.19%
1-SqR	14.980	19.673	34.652	15.203	19.463	34.667	+1.49%	−1.06%	+0.04%
20-SqR	0.358	2.799	3.157	0.345	2.588	2.933	−3.79%	−7.51%	−7.09%
60-SqR	0.129	1.136	1.265	0.097	0.966	1.063	−25.11%	−14.92%	−15.97%
120-SqR	0.116	0.703	0.819	0.056	0.588	0.644	−52.18%	−16.29%	−21.39%
DAX									
Filtered	0.403	2.172	2.575	0.539	1.056	1.595	+33.59%	−51.39%	−38.07%
1-SqR	12.551	18.126	30.677	12.922	17.898	30.820	+2.96%	−1.26%	+0.47%
20-SqR	0.138	2.043	2.181	0.150	1.764	1.914	+8.96%	−13.64%	−12.22%
60-SqR	0.050	0.688	0.739	0.062	0.551	0.613	+23.85%	−20.01%	−17.02%
120-SqR	0.040	0.519	0.559	0.037	0.420	0.458	−6.00%	−19.02%	−18.09%

Table 14: Out-Of-Sample Performance of Low-Frequency Volatility Specifications

The table reports the OOS performance of the EGARCH and UD-EGARCH specifications of Table 6. Daily predictions are generated by keeping parameters fixed at the IS estimates for both specifications. OOS Up and Down variances are Filtered from the BIC/SIC preferred specifications of Table 11 and approximated by K period averages of squared-log-returns SqR for $K = 1, 20, 60, 120$. Mse measures of distance between predictions and filtered values are reported in the first column for Up variances and in the second column for Down variances. The third column reports Mse for the standard Joint measure of variance. Δ reports the percentage differences in Mse of UD-EGARCH with respect to EGARCH.

	EGARCH			UD-EGARCH			Δ		
	Up	Down	Joint	Up	Down	Joint	Up	Down	Joint
FTSE									
Filtered	0.673	2.530	3.203	0.772	1.942	2.715	+14.83%	−23.24%	−15.24%
1-SqR	7.316	10.122	17.438	7.399	9.980	17.378	+1.12%	−1.40%	−0.34%
20-SqR	0.301	1.213	1.514	0.306	1.114	1.420	+1.62%	−8.15%	−6.21%
60-SqR	0.088	0.526	0.613	0.102	0.413	0.515	+16.18%	−21.42%	−16.05%
120-SqR	0.073	0.346	0.419	0.088	0.264	0.352	+21.36%	−23.84%	−15.99%
CAC									
Filtered	3.889	2.663	6.552	3.401	2.399	5.800	−12.56%	−9.92%	−11.49%
1-SqR	25.249	18.951	44.200	25.275	19.040	44.315	+0.10%	+0.47%	+0.26%
20-SqR	0.970	2.405	3.375	1.002	2.511	3.512	+3.31%	+4.40%	+4.09%
60-SqR	0.209	1.059	1.268	0.127	1.065	1.193	−39.11%	+0.56%	−5.98%
120-SqR	0.135	0.606	0.741	0.097	0.596	0.693	−28.03%	−1.57%	−6.39%
SMI									
Filtered	1.674	1.058	2.732	1.275	0.978	2.252	−23.87%	−7.57%	−17.56%
1-SqR	6.899	8.125	15.025	6.847	8.280	15.126	−0.76%	+1.90%	+0.68%
20-SqR	0.584	1.195	1.779	0.294	0.915	1.209	−49.67%	−23.45%	−32.05%
60-SqR	0.245	0.357	0.603	0.050	0.232	0.282	−79.74%	−35.16%	−53.30%
120-SqR	0.186	0.251	0.437	0.042	0.161	0.203	−77.51%	−35.72%	−53.52%

Table 15: In-Sample High-Frequency Volatility Estimates with Daily Components

The table reports **S&P500** estimation results for daily measures computed from high-frequency observations. HAR indicates the specification of equation (9), UD-HAR that of equation (11) and UD-GARCHX that of equation(14). With respect to equation (14): ω, β^U,β^D, α^U, α^D, γ^U, γ^D in the Up (Down) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12}, γ_{11}, γ_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}, γ_{21}, γ_{22}), respectively. Constrained coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the Up and Down components are equal. The last row of each panel reports the concentrated log-likelihood value of the estimated models with the letters ^{a} through ^{f} indicating the BIC/SIC ranking of the different specifications, with ^{a} indicating the best.

	Unconstrained				Constrained		
	HAR	UD-HAR Up	UD-HAR Down	Δ	HAR	UD-HAR Up	UD-HAR Down
Panel A: RV on C and J							
ω	-0.135 (0.017)***	-0.195 (0.026)***	0.010 (0.024)		-0.140 (0.016)***	-0.205 (0.021)***	0.003 (0.021)
β_D^U	0.191 (0.020)***	0.178 (0.035)***	0.275 (0.037)***	[0.052]	0.189 (0.020)***	0.183 (0.030)***	0.303 (0.029)***
β_D^D		0.256 (0.038)***	0.195 (0.041)***	[0.279]		**0.225** (0.026)***	**0.225** (0.026)***
β_W^U	0.336 (0.034)***	0.014 (0.080)	0.072 (0.064)	[0.595]	**0.354** (0.012)***		
β_W^D		0.228 (0.065)***	0.291 (0.059)***	[0.510]		**0.271** (0.021)***	**0.271** (0.021)***
β_M	0.374 (0.031)***	0.277 (0.052)***	0.087 (0.048)*	[0.017]	**0.354** (0.012)***	**0.271** (0.021)***	0.119 (0.032)***
β_J^U	-0.063 (0.065)	-0.141 (0.082)*	-0.323 (0.115)***	[0.200]			-0.317 (0.115)***
β_J^D		0.391 (0.104)***	0.411 (0.163)**	[0.916]		**0.392** (0.094)***	**0.392** (0.094)***
log-lik	-736.095	-665.692			-736.976	-668.312^b	
BIC/SIC	1531.710^f	1464.248^d			1498.864^e	1411.360^b	
Panel B: C on C and J							
ω	-0.237 (0.017)***	-0.289 (0.028)***	-0.028 (0.025)		-0.241 (0.017)***	-0.298 (0.022)***	-0.037 (0.022)*
β_D^U	0.214 (0.021)***	0.183 (0.036)***	0.260 (0.037)***	[0.121]	0.215 (0.020)***	0.187 (0.030)***	0.293 (0.029)***
β_D^D		0.271 (0.038)***	0.233 (0.040)***	[0.488]		**0.260** (0.025)***	**0.260** (0.025)***
β_W^U	0.338 (0.033)***	0.021 (0.075)	0.071 (0.060)	[0.623]	**0.336** (0.012)***		
β_W^D		0.241 (0.060)***	0.261 (0.055)***	[0.820]		**0.239** (0.020)***	**0.239** (0.020)***
β_M	0.337 (0.031)***	0.211 (0.049)*	0.104 (0.044)**	[0.159]	**0.336** (0.012)***	**0.239** (0.020)***	0.133 (0.032)***
β_J^U	-0.071 (0.067)	-0.146 (0.075)*	-0.357 (0.127)***	[0.157]			-0.351 (0.127)***
β_J^D		0.346 (0.121)***	0.308 (0.160)*	[0.856]		**0.322** (0.099)***	**0.322** (0.099)***
log-lik	-698.599	-617.042			-699.216	-619.793	
BIC/SIC	1438.718^f	1366.948^d			1423.344^e	1314.322^b	
		UD-GARCHX Up	UD-GARCHX Down	Δ		UD-GARCHX Up	UD-GARCHX Down
Panel C: RV on C and J							
ω		-0.031 (0.024)	0.179 (0.043)***			0.029 (0.005)***	0.130 (0.020)***
β^U		0.806 (0.084)***	0.524 (0.136)***	[0.048]		0.805 (0.013)***	**0.362** (0.012)***
β^D		0.000 (0.093)	0.179 (0.136)	[0.212]			**0.362** (0.012)***
α^U		0.154 (0.020)***	0.309 (0.027)***	[0.000]		0.152 (0.013)***	0.286 (0.021)***
α^D		0.197 (0.016)***	0.233 (0.030)***	[0.307]		**0.203** (0.013)***	**0.203** (0.013)***
γ^U		0.044 (0.065)	-0.405 (0.124)***	[0.002]			-0.333 (0.117)***
γ^D		0.341 (0.071)***	0.303 (0.170)**	[0.838]		**0.340** (0.063)***	**0.340** (0.063)***
log-lik		-655.864				-658.042	
BIC/SIC		1427.984^c				1390.820^a	
Panel D: C on C and J							
ω		-0.060 (0.030)**	0.180 (0.052)***			-0.061 (0.007)***	0.134 (0.021)***
β^U		0.786 (0.089)***	0.497 (0.143)***	[0.061]		0.779 (0.014)***	**0.361** (0.013)***
β^D		0.000 (0.095)	0.207 (0.139)	[0.168]			**0.361** (0.013)***
α^U		0.170 (0.020)***	0.321 (0.027)***	[0.000]		0.172 (0.014)***	0.300 (0.022)***
α^D		0.207 (0.018)***	0.251 (0.030)***	[0.214]		**0.219** (0.014)***	**0.219** (0.014)***
γ^U		0.027 (0.062)	-0.376 (0.139)***	[0.010]			-0.327 (0.133)**
γ^D		0.238 (0.081)***	0.343 (0.169)**	[0.591]		**0.269** (0.069)***	**0.269** (0.069)***
log-lik		-606.134				-608.250	
BIC/SIC		1328.524^c				1291.236^a	

Table 16: In-Sample High-Frequency Volatility Estimates with Intra-Daily Components

The table reports **S&P500** estimation results for daily measures computed from high-frequency observations. HAR indicates the specification of equation (10), UD-HAR that of equation (13) and UD-GARCHX that of equation(15). With respect to equation (15): ω, β^U,β^D, α^U, α^D, γ^U, γ^D in the Up (Down) column indicate ω_1, β_{11}, β_{12}, α_{11}, α_{12}, γ_{11}, γ_{12} (ω_2, β_{21}, β_{22}, α_{21}, α_{22}, γ_{21}, γ_{22}), respectively. Constrained coefficients are reported in **bold**. Standard errors are reported in parentheses with ***, ** and * indicating significance at 1%, 5% and 10% respectively. In Δ are reported the p-values for the Wald test that the coefficients of the Up and Down components are equal. The last row of each panel reports the concentrated log-likelihood value of the estimated models with the letters [a] through [f] indicating the BIC/SIC ranking of the different specifications, with [a] indicating the best.

	Unconstrained							Constrained					
	HAR		UD-HAR					HAR		UD-HAR			
			Up		Down		Δ			Up		Down	
Panel A: RV on C and J													
ω	0.233	(0.031)***	0.058	(0.040)	0.443	(0.047)***		0.228	(0.031)***	0.037	(0.034)	0.439	(0.046)***
β_D^U	0.023	(0.016)	0.047	(0.021)**	0.001	(0.025)	[0.153]			0.058	(0.019)***		
β_D^D	0.112	(0.013)***	0.077	(0.018)***	0.145	(0.020)***	[0.011]	0.113	(0.013)***	0.073	(0.017)***	0.149	(0.019)***
β_W^U	0.114	(0.034)***	0.052	(0.048)	0.168	(0.046)***	[0.080]	0.133	(0.030)***			0.174	(0.040)***
β_W^D	0.264	(0.025)***	0.261	(0.035)***	0.297	(0.036)***	[0.478]	0.268	(0.025)***	**0.283**	(0.024)***	**0.283**	(0.024)***
β_M	0.374	(0.033)***	0.459	(0.044)***	0.267	(0.047)***	[0.003]	0.373	(0.033)***	0.481	(0.032)***	0.271	(0.045)***
β_J^U	-0.224	(0.092)**	-0.088	(0.108)	-0.360	(0.153)**	[0.146]	**-0.235**	(0.067)***			-0.355	(0.152)**
β_J^D	0.236	(0.103)**	0.259	(0.123)**	0.222	(0.168)	[0.856]	**0.235**	(0.067)***	**0.245**	(0.102)**	**0.245**	(0.102)**
log-lik	-721.304		-655.864					-722.385		-657.081			
BIC/SIC	1509.040[f]		1444.592[d]					1494.594[e]		1405.506[b]			
Panel B: C on C and J													
ω	0.150	(0.031)***	-0.045	(0.041)	0.385	(0.047)***		0.145	(0.031)***	-0.075	(0.035)**	0.371	(0.046)***
β_D^U	0.030	(0.016)*	0.041	(0.020)**	0.025	(0.025)	[0.607]			0.057	(0.020)***		
β_D^D	0.127	(0.013)***	0.105	(0.018)***	0.145	(0.020)***	[0.130]	0.127	(0.013)***	0.096	(0.018)***	0.153	(0.019)***
β_W^U	0.123	(0.033)***	0.088	(0.048)*	0.143	(0.046)***	[0.409]	0.148	(0.031)***			0.176	(0.040)***
β_W^D	0.258	(0.025)***	0.239	(0.034)***	0.311	(0.036)***	[0.148]	0.263	(0.025)***	**0.284**	(0.025)***	**0.284**	(0.025)***
β_M	0.336	(0.033)***	0.390	(0.045)***	0.262	(0.047)***	[0.048]	0.335	(0.033)***	0.426	(0.032)***	0.272	(0.045)***
β_J^U	-0.179	(0.095)*	-0.016	(0.103)	-0.357	(0.167)**	[0.083]	**-0.216**	(0.068)***			-0.358	(0.152)**
β_J^D	0.245	(0.103)**	0.311	(0.119)***	0.183	(0.172)	[0.540]	**0.216**	(0.068)***	**0.255**	(0.102)**	**0.255**	(0.102)**
log-lik	-687.687		-607.269					-689.649		-611.115			
BIC/SIC	1441.806[f]		1347.402[d]					1429.122[e]		1313.574[b]			
			UD-GARCHX							UD-GARCHX			
			Up		Down		Δ			Up		Down	
Panel C: RV on C and J													
ω			0.110	(0.028)***	0.318	(0.039)***				0.111	(0.010)***	0.349	(0.026)***
β^U			0.822	(0.061)***	0.272	(0.104)***	[0.000]			0.824	(0.012)***	**0.351**	(0.013)***
β^D			0.000	(0.073)	0.441	(0.105)***	[0.000]					**0.351**	(0.013)***
α^U			0.061	(0.010)***	0.046	(0.020)**	[0.523]			**0.057**	(0.009)***	**0.057**	(0.009)***
α^D			0.098	(0.015)***	0.202	(0.018)***	[0.000]			0.100	(0.009)***	0.204	(0.017)***
γ^U			0.054	(0.072)	-0.346	(0.136)**	[0.012]					-0.344	(0.139)**
γ^D			0.252	(0.079)***	0.186	(0.135)	[0.689]			**0.235**	(0.064)***	**0.235**	(0.064)***
log-lik			-663.828							-664.793			
BIC/SIC			1443.912[c]							1404.322[a]			
Panel D: C on C and J													
ω			0.091	(0.032)***	0.321	(0.044)***				0.092	(0.010)***	0.363	(0.026)***
β^U			0.800	(0.066)***	0.247	(0.109)**	[0.000]			0.800	(0.014)***	**0.354**	(0.013)***
β^D			0.000	(0.073)	0.467	(0.106)***	[0.000]					**0.354**	(0.013)***
α^U			0.069	(0.011)***	0.059	(0.020)***	[0.648]			**0.066**	(0.009)***	**0.066**	(0.009)***
α^D			0.103	(0.015)***	0.202	(0.018)***	[0.000]			0.105	(0.010)***	0.205	(0.018)***
γ^U			0.036	(0.074)	-0.387	(0.140)***	[0.011]					-0.382	(0.146)***
γ^D			0.206	(0.080)**	0.120	(0.136)	[0.608]			**0.180**	(0.066)***	**0.180**	(0.066)***
log-lik			-615.477							-616.334			
BIC/SIC			1347.210[c]							1307.404[a]			

Table 17: Half-Lives of Volatility Shocks from In-Sample Estimates

The table reports **S&P500** the half-lives of volatility shocks for the fully dynamic specifications of the continuous component C. HAR indicates the specification of equation (9) , UD-HAR that of equation (11) and UD-GARCHX that of equation(14). All parameter estimates are Constrained and correspond to those in Table 15. Columns of the left and right panels indicate the elapsed times, expressed in number of days, between the (**n-1**)-th **n**-th halving. The rows in each panel report the following: $\text{Up}^{(C,+)}$ the half-lives of positive shocks to the continuous component C in up volatilities, $\text{Down}^{(C,+)}$ the half-lives of positive shocks to the continuous component C in down volatilities, $\text{Up}^{(C,-)}$ the half-lives of negative shocks to the continuous component C in up volatilities, $\text{Down}^{(C,-)}$ the half-lives of negative shocks to the continuous component C in down volatilities, $\text{Up}^{(J,+)}$ the half-lives of positive shocks to the jump component J in up volatilities, $\text{Down}^{(J,+)}$ the half-lives of positive shocks to the jump component J in down volatilities, $\text{Up}^{(J,-)}$ the half-lives of negative shocks to the jump component J in up volatilities and $\text{Down}^{(J,-)}$ the half-lives of negative shocks to the jump component J in down volatilities.

n:	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**	**n:**	**1st**	**2nd**	**3rd**	**4th**	**5th**	**6th**
HAR													
Up(C,+)	6	38	35	36	35	35							
Down(C,+)	6	38	35	36	35	35							
Up(C,−)	6	38	35	36	35	35							
Down(C,−)	6	38	35	36	35	35							
Up(J,+)	-	-	-	-	-	-							
Down(J,+)	-	-	-	-	-	-							
Up(J,−)	-	-	-	-	-	-							
Down(J,−)	-	-	-	-	-	-							
UD-HAR							**UD-GARCHX**						
Up(C,+)	2	9	39	34	33	33	Up(C,+)	29	27	27	27	27	27
Down(C,+)	2	4	23	33	33	34	Down(C,+)	21	27	27	27	27	27
Up(C,−)	2	5	29	34	33	34	Up(C,−)	28	27	27	27	27	27
Down(C,−)	2	5	28	34	33	34	Down(C,−)	16	26	27	27	27	27
Up(J,+)	6	26	34	33	34	33	Up(J,+)	29	27	27	27	27	27
Down(J,+)	1	1	8	34	33	34	Down(J,+)	1	2	17	27	27	27
Up(J,−)	2	5	31	33	34	33	Up(J,−)	29	27	27	27	27	27
Down(J,−)	2	5	26	34	33	33	Down(J,−)	12	27	27	27	27	27

Table 18: Out-Of-Sample Performance of High-Frequency Volatility Specifications

The table reports **S&P500** results for the OOS performance of the specifications of Tables 15 and 16. In Panel A and Panel B: HAR of equation (9) and UD-HAR of equation (11). In Panel C and Panel D: HAR of equation (10) and UD-HAR of equation (13). Daily predictions are generated by keeping parameters fixed at the IS estimates for all specifications. OOS Up and Down variances are measured from high-frequency observations Hf over periods of length $K = 1, 20, 60, 120$. Mse measures of distance between predictions and realizations are reported in the first column for Up variances and in the second column for Down variances. The third column reports Mse for the standard Joint measure of variance. Δ reports the percentage differences in Mse of UD-HAR with respect to HAR.

	HAR			UD-HAR			Δ		
	Up	Down	Joint	Up	Down	Joint	Up	Down	Joint
Panel A: RV on C and J									
1-Hf	0.679	0.823	1.502	0.705	0.763	1.468	+3.82%	−7.27%	−2.26%
20-Hf	0.078	0.179	0.256	0.113	0.140	0.253	+44.91%	−21.39%	−1.30%
60-Hf	0.045	0.107	0.152	0.072	0.081	0.153	+60.55%	−24.35%	+0.62%
120-Hf	0.037	0.086	0.123	0.060	0.063	0.124	+62.98%	−26.19%	+0.69%
Panel B: C on C and J									
1-Hf	0.615	0.863	1.478	0.645	0.786	1.431	+4.85%	−8.88%	−3.17%
20-Hf	0.061	0.216	0.277	0.100	0.161	0.261	+63.73%	−25.24%	−5.58%
60-Hf	0.038	0.137	0.175	0.070	0.097	0.167	+84.94%	−29.28%	−4.48%
120-Hf	0.031	0.112	0.144	0.059	0.078	0.137	+87.58%	−30.50%	−4.84%
Panel C: RV on intradaily C and J									
1-Hf	0.672	0.811	1.483	0.722	0.726	1.448	+7.43%	−10.48%	−2.37%
20-Hf	0.078	0.185	0.263	0.117	0.122	0.239	+49.72%	−33.81%	−8.99%
60-Hf	0.045	0.111	0.156	0.079	0.060	0.139	+75.12%	−45.64%	−10.91%
120-Hf	0.036	0.089	0.125	0.065	0.046	0.112	+79.16%	−47.96%	−11.09%
Panel D: C on intradaily C and J									
1-Hf	0.612	0.855	1.467	0.661	0.748	1.409	+8.04%	−12.54%	−3.95%
20-Hf	0.062	0.222	0.284	0.108	0.139	0.246	+74.67%	−37.48%	−13.16%
60-Hf	0.039	0.141	0.180	0.080	0.072	0.152	+106.83%	−48.81%	−15.45%
120-Hf	0.031	0.116	0.147	0.065	0.058	0.123	+111.10%	−49.94%	−16.02%

Table 19: Out-Of-Sample Performance of High-Frequency Volatility Specifications

The table reports **S&P500** results for the OOS performance of the specifications of Tables 15 and 16. In Panel A and Panel B: HAR of equation (9) and UD-GARCHX of equation (14). In Panel C and Panel D: HAR of equation (10) and UD-GARCHX of equation (15). Daily predictions are generated by keeping parameters fixed at the IS estimates for all specifications. OOS Up and Down variances are measured from high-frequency observations Hf over periods of length $K = 1, 20, 60, 120$. Mse measures of distance between predictions and realizations are reported in the first column for Up variances and in the second column for Down variances. The third column reports Mse for the standard Joint measure of variance. Δ reports the percentage differences in Mse of UD-GARCHX with respect to HAR.

	HAR			UD-GARCHX			Δ		
	Up	Down	Joint	Up	Down	Joint	Up	Down	Joint
Panel A: RV on C and J									
1-Hf	0.679	0.823	1.502	0.674	0.729	1.403	−0.86%	−11.40%	−6.63%
20-Hf	0.078	0.179	0.256	0.091	0.106	0.197	+17.62%	−40.73%	−23.05%
60-Hf	0.045	0.107	0.152	0.057	0.046	0.104	+28.27%	−56.87%	−31.83%
120-Hf	0.037	0.086	0.123	0.050	0.037	0.087	+34.53%	−56.67%	−29.18%
Panel B: C on C and J									
1-Hf	0.615	0.863	1.478	0.620	0.734	1.354	+0.89%	−14.93%	−8.35%
20-Hf	0.061	0.216	0.277	0.086	0.120	0.206	+40.46%	−44.34%	−25.60%
60-Hf	0.038	0.137	0.175	0.061	0.057	0.118	+59.44%	−58.24%	−32.70%
120-Hf	0.031	0.112	0.144	0.051	0.047	0.099	+64.54%	−57.84%	−31.24%
Panel C: RV on intradaily C and J									
1-Hf	0.672	0.811	1.483	0.675	0.678	1.354	+0.49%	−16.36%	−8.73%
20-Hf	0.078	0.185	0.263	0.089	0.094	0.184	+14.61%	−48.91%	−30.04%
60-Hf	0.045	0.111	0.156	0.053	0.039	0.092	+18.36%	−64.71%	−40.82%
120-Hf	0.036	0.089	0.125	0.044	0.030	0.074	+21.71%	−66.58%	−40.98%
Panel D: C on intradaily C and J									
1-Hf	0.612	0.855	1.467	0.624	0.692	1.316	+1.93%	−19.01%	−10.27%
20-Hf	0.062	0.222	0.284	0.085	0.109	0.194	+38.08%	−51.09%	−31.75%
60-Hf	0.039	0.141	0.180	0.059	0.049	0.107	+52.08%	−65.47%	−40.27%
120-Hf	0.031	0.116	0.147	0.048	0.039	0.087	+ 55.35%	−66.25%	−40.63%