Sample rate reduction for spectral estimation performed through digital filtering.

1. - INTRODUCTION.

As it is well known, several methods are available for performing spectral estimation: band-pass analysis, autocorrelation, Fast Fourier Transform.

The method of band-pass analysis and evaluation of power in each band can be more useful in some cases than the others, because it gives as intermediate result also the signal behaviour in each band and may not require operations on long blocks of data (and consequent memory sections) (1) as, for instance, the Fast Fourier method.

In this letter some methods of band-pass analysis and spectral estimation using digital filtering operations are described. These methods have, as peculiar characteristic, high processing efficiency, due to the use of sample reduction criteria.

2. - SPECTRAL ESTIMATION USING A MINIMUM NUMBER OF SAMPLES.

It is well known that a simple way for obtaining an estimation of the amplitude spectral density $|X(f)|$ *of a signal* $x(t)$ *is that of using a bank of non-overlapping band-pass filters, each with a suitably small bandwidth B, covering all the signal spectrum of interest, and of evaluating or measuring the power of the signal in each band of analysis.*

By using digital techniques, such a method implies that a digital filtering of the signal must be performed in each analysis band. If the samples of the filtered signals have the same sampling rate as the samples of the original signal $x(t)$*, improvements in the estimate accuracy (decreasing the bandwidth B and consequently increasing the number of the analysis bands) require longer computational times and larger memory.*

However observing that the spectrum extension of the filtered signals is limited to a bandwidth B, it is possible to reduce their sampling rate and therefore to achieve considerable savings in the required computational time and memory registers. According to this observation a detailed method is described in the following, by using an extension of the sampling theorem to narrow band signals [1]. The presented method leads to a computational structure of a FIR digital filter, which, as it has been shown [2], is highly suitable for sampling rate and computation speed reduction.

Now it is known [1, ch. 7] that a signal $y_i(t)$ *(with spectrum limited to a bandwidth B centered on the frequency* f_i*) set in the form*

$$y_i(t) = Re\{a_i(t)\, e^{j[2\pi f_i t + \theta_i(t)]}\} = Re\{\bar{y}_i(t)\} \tag{1}$$

is completely defined when the two sample sequences $\{a_i(mT')\}$, $\{\theta_i(mT')\}$, *m integer, with sampling interval* $T' \leqslant \frac{1}{B}$ *are known, and its energy* E_i *(from which the power can be obtained) is given by*

$$E_i = \frac{T'}{2} \sum_{m=-\infty}^{+\infty} a_i^2\,(mT') \tag{2}$$

The samples $a_i(mT')$ *may be computed with the following steps. The signal* $y_i(t)$ *can be expressed by the relation*

$$y_i(t) = \int_{-\infty}^{+\infty} X(f)\, H_i(f)\, e^{j2\pi ft}\, df$$

where $H_i(f)$ *is the transfer function of the i-th ideal*

(1) The power estimation can be indeed done by using an accumulator in which the squared values are subsequently sent.

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band-pass filter with bandwidth B centered on f_i, and $y_i(t)$ is given by

$$\overline{y}_i(t) = y_i(t) + j\,y_i'(t) = \int_0^{\infty} 2\,X(f)\,H_i(f)\,e^{j2\pi ft}\,df$$

Therefore $\overline{y}_i(t)$ may be defined by the convolution

$$(3)\qquad \overline{y}_i(t) = \overline{h}_i(t) * x(t) = \int_{-\infty}^{+\infty} \overline{h}_i(\tau)\,x(t-\tau)\,d\tau$$

if we define

$$(4)\qquad \overline{h}_i(t) = \int_0^{\infty} 2\,H_i(f)\,e^{j2\pi ft}\,df$$

which represents the 'complex impulse response' of a linear system (not physically realizable) having a transfer function equal to zero for $f < 0$ and equal to $2\,H_i(f)$ for $f > 0$.

If the signal $x(t)$ is sampled with a sampling period T, the relation (3) becomes [3]

$$(5)\qquad \overline{y}_i(nT) = \sum_{k=-N}^{N} \overline{h}_i(kT)\,x(nT - kT)$$

which has the form of a FIR non-recursive digital filter.

required to determine the R sequences defined by (2)

$$(6)\qquad \overline{y}_i(mT') = \sum_{k=-N}^{N} \overline{h}_i(kT)\,x(mT' - kT)$$

$$m \text{ integer},\ i = 1, 2, \ldots R$$

For an « on-line » computation (Fig. 1) of the samples $\overline{y}_i(mT')$ it is necessary in the time interval T' to perform (6) R times, one for each analysis band, and therefore $R\,D_c \leqslant T'$, D_c being the time required to perform the computation defined by (6). If we choose $T' = 2RT$ (which verifies $T' \leqslant 1/B$), the above inequality becomes

$$(7)\qquad D_c \leqslant 2T$$

and also

$$(7a)\qquad D_c \leqslant \frac{1}{f_M}$$

If condition (7) or (7a) is verified, the samples $\overline{y}_i(mT')$ of all R analysis bands may be computed by an « on-line » processing, regardless how large the number R is chosen, so being possible to improve the precision and resolution of the spectral estimation.

It is also possible to show that computation (6), which is of a complex type, though easily performed

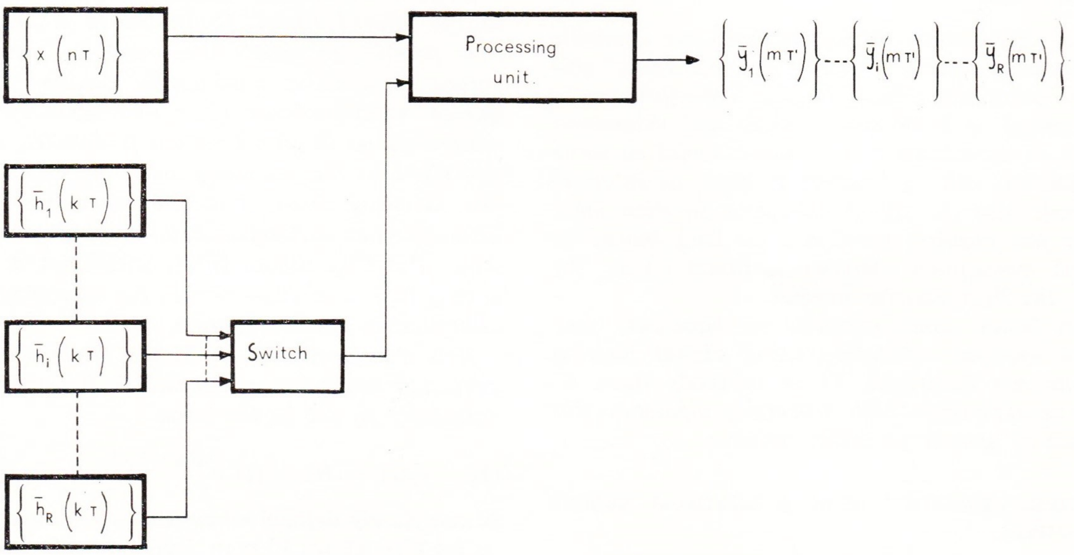

Fig. 1. - Block diagram of the processing system performing the spectral estimation with sample rate reduction.

Every computation according to (5), giving the real part $y_i(nT)$ and the imaginary part $y_i'(nT)$, allows finally to determine $a_i^2(nT) = y_i^2(nT) + y_i'^2(nT)$, where $T \leqslant 1/2 f_M$, f_M being the maximum frequency of $X(f)$. At this point it is important to observe that according to formula (2), not all $a_i(nT)$ are necessary, but for each analysis band it is sufficient to determine the $a_i(mT')$ with a sampling interval $T' \leqslant 1/B$, i.e. it is sufficient to perform the computation (5) once every T' seconds, instead of every T seconds. Therefore if the spectral analysis is carried out with R analysis bands (where $RB = f_M$), it is by present processors, may be avoided through two computations with real coefficents. Indeed expressing $\overline{h}_i(t) = h_i(t) + j\,h_i'(t)$, two relations of the type (6) can be obtained giving the real data $y_i(mT')$ by $h_i(kT)$ and the imaginary data $y_i'(mT')$ by $h_i'(kT)$.

(2) The coefficients $\overline{h}_i(kT)$ of formulas (5) and (6) may be derived but are not generally equal to the samples of $\overline{h}_i(t)$ given by (4) with sampling time T - e.g. they may be modified according to the « window » design method of FIR non-recursive digital filters [4]. Further it is to observe that the complex samples $\overline{y}_i(mT')$ allow to determine also the phase samples $\theta_i(mT')$, which with the amplitude samples $a_i(mT')$ completely define the continuous signal $y_i(t)$, which can be therefore exactly recovered.

If D_r is the time required to perform anyone of the above two real computations, an « on-line » processing of the data sets $y_i(mT')$, $y'_i(mT')$ for every number R of the analysis bands is possible if

$$D_r \leqslant \frac{1}{2 f_M} \qquad (7b)$$

The described method (3) *has been tested through a computer simulation using the addition of many sine waves of unity amplitude and weighted amplitude as the input signal $x(t)$. Each analysis band (R = 8) covered one line of the spectrum of $x(t)$. A FIR non-recursive digital filtering (with N = 30) according to the two above defined real computations was employed, modifying the factors $h_i(kT)$ and $h'_i(kT)$ according to the « window » design method* [4], *using a Lanczos type window to get a more efficient filter frequency response.*

A satisfactory agreement (maximum error 0.25%) *resulted between the obtained spectral estimation and the theoretical one, processing* 839 *samples of $x(t)$ and obtaining* 46 *samples for $y_i(t)$ and $y'_i(t)$ for each analysis band.*

3. - Conclusions.

The described technique requires practically a single digital filtering structure with different memory sections for $\overline{h}_i(kT)$ — or equivalently for $h_i(kT)$ and $h'_i(kT)$ — and results therefore more economic than the standard band-pass analysis techniques (4) *and has the peculiar characteristic of practical interest ta give a minimum number of samples from which the spectral estimation is obtained. This attractive result is essentially achieved through a suitable decimation of the samples of the filtered signals* (5).

The described techniques are particularly interesting for short-time spectral estimation of the audio signal, vibrations and biomedical signals, and for processing of satellite telemetry data.

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REFERENCES

[1] C. Atzeni, V. Cappellini: *Elementi di Teoria delle Comunicazioni.* Ed. G. & G., Florence, 1969.

[2] R. W. Schafer, L. R. Rabiner: *A Digital Signal Processing Approach to Interpolation.* « Proc. IEEE », vol. 61, n. 6, June 1973.

[3] V. Cappellini, T. D'Amico: *Alcuni filtri numerici ottenuti dall'elaborazione dell'integrale di convoluzione e loro applicazione all'analisi spettrale.* « Alta Frequenza », vol. 36, n. 9, p. 835-840, September 1967.

[4] V. Cappellini, T. D'Amico, P. L. Emiliani, F. Lotti, F. Pieralli: *Una classe di filtri numerici con applicazioni all'elaborazione dł segnali da veicoli spaziali.* « Alta Frequenza », vol. 37, p. 834-844, September 1968.

(3) The method includes as a particular case also the spectral estimation in a low-pass band.

(4) It is indeed important to observe that from the filtered outputs $y_i(mT)$ and $y'_i(mT')$ it is possible with the synthesis relation [1, ch. 7] to obtain again the original signal samples in each band and through *D/A* converter the analog signal in each band as by any standard band-pass analysis method. This can be useful when, apart the short-time spectral estimation 'on-line' required, one wishes in subsequent times to recover the original signal in each band from the memorized $y_i(mT')$ and $y'_i(mT')$ values.

(5) An extension of this method to proportional bandwidth analysis is possible and will be described in a next paper.