

A Robust Spatial Filtering Method for DOA Estimation of Coherent Sources in Impulsive Interference

Marilli Rupi, Panagiotis Tsakalides, Enrico Del Re and Chrysostomos L. Nikias

Università di Firenze, Dipt. Ingegneria Elettronica, Laboratorio Elaborazione Numerica dei Segnali e Telematica; University of Southern California, Dept. Electrical Engineering, Signal & Image Processing Institute

Abstract. Multipath propagation is one of the dominant environmental influences on the performance of wireless communication systems. Meanwhile, heavy-tailed interference from neighboring cells may further degrade communication quality in mobile systems. This paper employs notions of spatial diversity using an array of sensors to address the inter-signal coherence problem. At the same time, the paper proposes the use of fractional lower-order moments to mitigate the effects of the heavy-tailed background noise environment. The improved performance achieved by the combination of these two processing modules is demonstrated via Monte Carlo simulations.

1 Introduction

The rapidly increasing demands for personal mobile communications have created an avid interest in new array processing algorithms and antenna architectures. Our choice for a particular antenna array model depends on its capability to discriminate different incoming signals, which can be generated from various digitally modulated sources. Specifically, in the wireless communications, the transmitted signal arrives to the receiver via multiple paths, due to some physical phenomena such as diffraction and reflection, caused by obstacles present in the line of view between the transmitter and the receiver. Multipath can affect the incoming signal and destroy completely the information sequence. Moreover, relative motion between the transmitter and the receiver further degrade the desired signal, due to an undesired Doppler shift. Finally, coherent interference can arise when "smart"jammers deliberately redirect scaled and delayed replicas of the same signal to the receiver. The array antenna is able to process the incoming signal and improve its quality by applying techniques based on interference cancellation and spatial diversity.

In recent years, considerable effort has been spent in developing high resolution techniques for estimating the direction of arrival of multiple signals using antenna arrays. The eigen-based class of methods has been proven to

be an effective means for this goal, even when the signal sources are partially correlated [1]. However, when some of the signals are perfectly correlated, eigen-based or subspace techniques can not be further used since fall the algorithm hypotheses. Several alternatives have been proposed to overcome this problem, using ideas such as sub-aperture sampling of spatial smoothing which essentially decorrelate the coherent signals [2, 3]. In any case, all these done works dealt with Additive White Gaussian noise.

Although the Gaussian distribution plays a significant role in a mobile communication environment, mainly due to its capability to lead to simple solutions, in certain practical applications it is not an accurate model for the noise. For these reasons we need to use more realistic non-Gaussian models, such as the complex Symmetric Alpha-Stable ($S\alpha S$) distribution, which is a class of heavy-tailed random variables holding, as particular cases, the Gaussian and the Cauchy distributions.

A brief review of the $S\alpha S$ family is undertaken in Section 2. In Section 3, we apply the Spatial Smoothing Scheme to the ROC-MUSIC, introduced in [4] to address the problem of high resolution DOA estimation of narrow-band coherent sources in the presence of impulsive noise. In Section 5, we demonstrate the improved performance of the proposed method via simulation examples. Finally, in Section 6, we summarize the results and present avenues of future research.

2 Symmetric Alpha Stable mathematical analysis

The $S\alpha S$ family of distributions shares many properties with the popular Gaussian random variables, including the stability property and the central limit theorem. The main difference between the two distributions concerns the tail of the stable density, which is heavier than the tail of the Gaussian density. This characteristic makes the $S\alpha S$ variables suitable for modeling signals and noise of an impulsive nature. The impulsive component of the noise has been found to be significant in many communications problems. The atmospheric noise, e.g., may be considered as the result of a large number of independent sources (mainly thunderstorms) which shows such statistical behavior.

The $S\alpha S$ processes are defined by four parameters: the characteristic component $0 < \alpha \leq 2$ (the smaller the α , the heavier the tails of the density), the skewness parameter $-1 \leq \beta \leq 1$ (the distribution is symmetric when $\beta = 0$), the dispersion $\gamma > 0$ (which plays a similar role of variance for the Gaussian case), and the location parameter $-\infty \leq a \leq \infty$ (which is the mean when $1 < \alpha \leq 2$ and the median when $0 < \alpha < 1$). All these parameters appear in the definition of the *characteristic function*:

$$\varphi(t) = e^{jat - \gamma|t|^\alpha [1 + j\beta \text{sign}(t)\omega(t, \alpha)]} \quad (1)$$

where $\omega(t, \alpha)$ is $\tan \frac{\alpha\pi}{2}$ if $\alpha \neq 1$ and $\frac{2}{\pi} \log|t|$ if $\alpha = 1$, and $\text{sign}(t)$ is $|t|$ if $t \neq 0$ and 0 if $t = 0$ (Gaussian and Cauchy distributions have $\alpha = 2$ and $\alpha = 1$ respectively). The main characteristic associated with $S\alpha S$ processes is the capability to possess finite p th order moments only for $p < \alpha$, so it is clear that for all non Gaussian $S\alpha S$ variables finite second or higher-order statistics do not exist. Hence, it is necessary to define a dual tool, namely the fractional lower-order statistics (FLOS) [5].

3 Problem Statement

We consider a uniformly-spaced linear array antenna consisting of N elements, in which impinge $Q < N$ signals (with λ being the carrier wavelength), that are located at different angles with respect to the orthogonal axis $\{\vartheta_k; k = 1, \dots, Q\}$ and have constant velocities with respect to the receiver $\{\nu_k; k = 1, \dots, Q\}$ corresponding to Doppler frequencies $\{f_k = \nu_k/\lambda; k = 1, \dots, Q\}$. Since we assume the signal bandwidth to be narrow as compared to the inverse of the travel time across the array, it follows that, by using a complex envelop representation, the array output can be expressed as [6]:

$$\mathbf{x}(t) = \mathbf{V}(\boldsymbol{\vartheta})\mathbf{s}(t) + \mathbf{n}(t), \quad (2)$$

where

- $\mathbf{x}(t) = [x_1(t), \dots, x_N(t)]^T$ is the array output vector;
- $\mathbf{s}(t) = [s_1(t), \dots, s_Q(t)]^T = \mathbf{A}\mathbf{f}$ is the signal vector received by the array;
- $\mathbf{A} = \text{diag}(a_1, a_2, \dots, a_N)$ is the $N \times N$ matrix of the attenuation factors;
- $\mathbf{f} = [e^{j2\pi f_1}, e^{j2\pi f_2}, \dots, e^{j2\pi f_N}]^T$ is the Doppler shift vector, and it is a function of the speed velocity and the wavelength λ : $f_k = \nu_k/\lambda$;
- $\mathbf{V}(\boldsymbol{\vartheta})$ is the *space-time steering matrix*, whose r -th column vector $\mathbf{v}(\vartheta_r)$ is $[1, e^{-j2\pi(d/\lambda)\sin\vartheta_r}, \dots, e^{-j(N-1)2\pi(d/\lambda)\sin\vartheta_r}]^T$
- $\mathbf{n}(t) = [n_1(t), \dots, n_N(t)]^T$ is the noise vector, of impulsive nature, assumed to be uncorrelated with the signals.

In this paper, we assume that the Q signal waveforms are fully coherent, phase-delayed amplitude-weighted replicas of one of them. Also, the noise vector $\mathbf{n}(t)$ is a complex isotropic $S\alpha S$ random process with $1 < \alpha \leq 2$ and zero location parameter. The noise is assumed to be independent of the signals and it has covariation matrix $\Gamma_N = \gamma_n \mathbf{I}$.

4 Subspace Methods based on FLOS

We start our analysis by first considering the case of independent or uncorrelated incoming signals. In impulsive noise environments, the concept of the covariation matrix has been introduced in [4] to characterize the correlation properties of the signal noise field. In addition, application of eigen-decomposition methods to the covariation matrix resulted to robust DOA

estimates of independent signal sources in $S\alpha S$ noise.

The covariation matrix, Γ_X , of the observation vector process $\mathbf{x}(t)$ is defined as the matrix whose elements are the covariations $[x_i(t), x_j(t)]_\alpha$ of the components of $\mathbf{x}(t)$ [4]. We obtain the following expression for the covariations of the sensor measurements:

$$[x_i(t), x_j(t)]_\alpha = \sum_{k=1}^Q v_i(\vartheta_k) v_j(\vartheta_k)^{<\alpha-1>} \gamma_{s_k} + \gamma_n \delta_{i,j} \quad i, j = 1, \dots, N. \quad (3)$$

where we define the signed power operation as $a^\alpha = |a|^\alpha \text{sign}(a)$ if a is real or $a^\alpha = |a|^{\alpha-1} a^*$ if a is complex. In matrix form, (3) gives the following expression for the covariation matrix of the observation vector

$$\Gamma_X \triangleq [\mathbf{x}(t), \mathbf{x}(t)]_\alpha = \mathbf{V}(\boldsymbol{\vartheta}) \Gamma_S \mathbf{V}^{<\alpha-1>}(\boldsymbol{\vartheta}) + \gamma_n \mathbf{I}, \quad (4)$$

where Γ_S is the covariation matrix of the incident signals and the (i, j) th element of matrix $\mathbf{V}^{<\alpha-1>}(\boldsymbol{\vartheta})$ is $|\mathbf{V}(\boldsymbol{\vartheta})]_{j,i}|^{\alpha-2} [\mathbf{V}(\boldsymbol{\vartheta})]_{j,i}^* = [\mathbf{V}(\boldsymbol{\vartheta})]_{j,i}^*$, and thus, the covariation matrix can be written as

$$\Gamma_X \triangleq \mathbf{V}(\boldsymbol{\vartheta}) \Gamma_S \mathbf{V}(\boldsymbol{\vartheta})^H + \gamma_n \mathbf{I} \quad (5)$$

Clearly, when $\alpha = 2$, i.e., for Gaussian distributed noise, the expression for the covariation matrix is identical to the well-known expression for the covariation matrix: $\mathbf{R}_X = \mathbf{V}(\boldsymbol{\vartheta}) \Sigma \mathbf{V}^H(\boldsymbol{\vartheta}) + \sigma^2 \mathbf{I}$, where Σ is the signal covariation matrix and σ^2 is the variance of the Gaussian noise variable. Observing (5), we conclude that standard subspace techniques can be applied to the covariation of the observation vector to extract the bearing information. In practice, we have to estimate the covariation matrix from a finite number of array sensor measurements. Besides, in a real communication model, the statistics of the transmitted signal $s(t)$ are not alpha-stable. Hence, in the following, we will assume $s(t)$ to be a white, zero-mean, random process. In this contest, we define the fractional lower-order cross correlation of two random processes $x_i(t)$ and $x_j(t)$ as:

$$<x_i(t), x_j(t)>_p = E[x_i(t) x_j(t)^{<p-1>}] \quad (6)$$

According to (6), we can see the presence of the operator expectation, which is a linear operator, and the introduction of the fractional p th power, which is, conversely, not linear. Even in presence of independent sources, it is not possible to find a closed-form expression suitable for this problem, that could be used for write the covariation of random variables, not necessarily only $S\alpha S$.

Under the hypothesis of uncorrelated noise and signals, independent with each other and zero-mean, we obtain that

$$\begin{aligned} \langle x_i(t), x_j(t) \rangle_p &= \sum_{k=1}^Q v_i(\vartheta_k) v_j(\vartheta_k) \langle^{p-1} \rangle E[s_k(t) s_k(t) \langle^{p-1} \rangle] + \\ &+ E \left[f \left(\sum_{k=1}^Q v_{r_i}(\vartheta_k) s_k, \sum_{k=1}^Q v_{r_j}(\vartheta_k) s_k^a s_k^b, n_{\alpha_j}^c \right) \right] + \gamma_{n_\alpha} \delta_{i,j} \end{aligned} \quad (7)$$

where $f(x)$ is a function of x , which is given by all the contributions containing the fractional products between the signal $s_k(t)$ and the noise source n_{α_j} . The elements a, b, c are three fractional values strictly less than $p-1$, with the constraint $b \neq 0, c \neq 1$. Observing (7), we see that the use of FLOS for the non- $S\alpha S$ noise case, has caused the introduction of cross-terms involving correlation functions between the signal and noise components. The extent(?) to which the existence of these cross-terms affects the performance of eigen-based methods is studied in the simulation section.

4.1 Subspace Methods based on FLOS & Spatial Smoothing

Second-order or fractional lower-order methods for bearing estimation require a non coherent environment in which all Q sources are statistically independent. In a realistic mobile communication scenario, this hypothesis does not hold, due to the reflection of the transmitted signal which arrives at the receiver via correlated multi-paths. Furthermore, the relatively motion between receiver and transmitter makes the Doppler shift another important factor to consider during a wireless communication. Hence, Fading and Doppler, adversely, affect the performance of standard bearing estimation methods and create the need to introduce methods that can address the signal coherence problem.

For dealing with coherence phenomena in an impulsive noise background, we introduce a *Spatial Smoothing* version of the FLOS-based subspace algorithm. The basic principle states to divide the array of dimension $N > Q$ into uniformly overlapping sub-arrays of dimension $P > Q$, in such a way that each one shares with an adjacent sub-array all but one of its sensors. Calling as the r th sub-array the one consisting of the elements $(r, r+1, \dots, r+P-1)$, we can write the covariation of the i th with the j th elements of this sub-array as

$$\langle x_i(t), x_j(t) \rangle_p = E \left[\left(\sum_{k=1}^Q v_{r_i}(\vartheta_k) s_k + n_{\alpha_i} \right) \left(\sum_{h=1}^Q v_{r_j}(\vartheta_h) s_h + n_{\alpha_j} \right) \langle^{p-1} \rangle \right] \quad (8)$$

for each $i, j = 1, \dots, P$ and for all sub-arrays $r = 1, \dots, N - P + 1$. This expectation, this time, has inside the cross terms between $s_k s_h = c_k c_h s(t)^2$, and all the values which contain fractional powers of the product between

the noise and the signal. Due to the linearity property with respect to the first term, and the non-linearity of the p th fractional power, we can write

$$\begin{aligned} \langle x_i(t), x_j(t) \rangle_p &= \sum_{h,k=1}^Q v_{r_i}(\vartheta_k) v_{r_j}(\vartheta_h) \langle^{p-1} \rangle E[s_k(t) s_h(t) \langle^{p-1} \rangle] + \\ &+ \gamma_{n_\alpha} \delta_{i,j} + \Phi_{i,j} \\ \Phi_{i,j} &= E \left[g \left(\sum_{k=1}^Q v_{r_i}(\vartheta_k) s_k, \sum_{h=1}^Q v_{r_j}(\vartheta_h) s_h^a s_h^b, n_{\alpha_j}^c \right) \right] \end{aligned} \quad (9)$$

with $g(x)$ a function of the variable x . The term $\Phi_{i,j}$ is what we call *corruption factor*, and is a finite quantity, since all the fractional powers (a, b, c, d) are strictly less than $p-1$ and $(n_{\alpha_i}, n_{\alpha_j})$ are $S\alpha S$ noise components that have finite moments of order $p < \alpha$. Then, it is possible to find an equivalent expression for the FLOS matrix of the r th sub-array

$$\Gamma_{X_P}^{(r)} \triangleq \mathbf{V}_P(\vartheta) \mathbf{D}^{r-1} \Gamma_S \mathbf{D}^{r-1 \langle^{p-1} \rangle} \mathbf{V}_P^{\langle^{p-1} \rangle}(\vartheta) + \gamma_{n_\alpha} \mathbf{I}_P + \Phi_P^{(r)} \quad (11)$$

where $\mathbf{V}_P(\vartheta)$ is the set of steering vectors for a sub-array of length P and \mathbf{D} is a diagonal matrix whose l th element is equal to $e^{-j2\pi(d/\lambda)\sin(\vartheta_l)}$. The (i, j) th element of $\mathbf{V}_P^{\langle^{p-1} \rangle}(\vartheta)$ is the (j, i) th element of $\mathbf{V}_P(\vartheta)$ to the signed power of $\langle^{p-1} \rangle$. Since the elements of $\mathbf{V}_P(\vartheta)$ and \mathbf{D}^{r-1} have unit magnitude, it follows that

$$\Gamma_{X_P}^{(r)} = \mathbf{V}_P(\vartheta) \mathbf{D}^{r-1} \Gamma_S \mathbf{D}^{r-1H} \mathbf{V}_P(\vartheta)^H + \gamma_{n_\alpha} \mathbf{I}_P + \Phi_P^{(r)} \quad (12)$$

The effect of this corruption matrix is that of a correlated noise field which will adversely affect the performance of the method. However, as we demonstrate in the simulation section, the advantage to use the FLOS formulation in the presence of heavy-tailed environment noise outweighs the negative effect of the induced corruption factors.

The FLOS matrix is evaluated as the average of the sub-array FLOS matrices.

$$\bar{\Gamma}_X = \frac{1}{N - P + 1} \sum_{r=1}^{N-P+1} \Gamma_{X_P}^{(r)} \quad (13)$$

Using (12) and (13) we can write the FLOS matrix as

$$\bar{\Gamma}_X = \mathbf{V}_P(\vartheta) \bar{\Gamma}_S \mathbf{V}_P(\vartheta)^H + \bar{\Gamma}_{n_\alpha} + \frac{1}{N - P + 1} \sum_{r=1}^{N-P+1} \Phi_P^{(r)} \quad (14)$$

where

$$\tilde{\Gamma}_S \triangleq \frac{1}{N-P+1} \sum_{r=1}^{N-P+1} \mathbf{D}^{r-1} \Gamma_S \mathbf{D}^{r-1H} \quad (15)$$

$$\tilde{\Gamma}_{n_\alpha} \triangleq \frac{1}{N-P+1} \sum_{r=1}^{N-P+1} \gamma_{n_\alpha} \mathbf{I}_P \quad (16)$$

As a result of the spatial smoothing operation the matrix in (15) is non singular [2]. Hence, we can apply an eigen-decomposition of $\tilde{\Gamma}_X$ to achieve robust DOA estimation in a fading Doppler environment with heavy-tailed noise. However, we have to consider the effect of the corruption matrix Φ . This matrix corrupts the eigen-based methods in such a way that the resulting eigenvectors of $\tilde{\Gamma}_X$ can not perfectly span the two original orthogonal signal and signal-plus-noise sub-spaces. In other words, we can say that the corruption matrix is in effect a result of an induced colored noise component which, as we demonstrate from the experimental results, is not disruptive to the algorithms in terms of their performance.

5 Experimental Results

In this section, we show the results on the resolution capability of ROC-MUSIC Smoothing versus ROC-MUSIC, MUSIC and MUSIC Smoothing as a function of the noise characteristic exponent α and the angle separation. The array is linear with eight sensors spaced half wavelength apart, with overlapped sub-array of dimension five. Four signals impinge on the array from directions $\vartheta = [30^\circ, -40^\circ, 60^\circ, -15^\circ]$. As constant power attenuation factors, we used $\{-3dB, 0dB, -2dB, -6dB\}$ for the four paths. The noise follows the isotropic stable distribution. The transmitted waveform is a QPSK signal, filtered with a squared root raised cosine. The received signal is filtered with the same matched squared root raised cosine filter. The Doppler effect is due to the different motions of incoming signals. Here we considered velocities of $70Km/h$, $60Km/h$, $50Km/h$ and $40Km/h$, carrier frequency of $900MHz$ with relative shift Doppler given as ν_i/λ , $i = 1, \dots, Q$.

Since the alpha stable family for $\alpha < 2$ determines processes with infinite variance, we cannot define an ensemble signal-to-noise ratio. But for finite-sample realizations, we can define the *Effective SNR (ESNR)* to be the ratio of the signal power over the noise power:

$$ESNR = 10 \log \left(\frac{\sum_{t=1}^M |s(t)|^2}{\sum_{t=1}^M |n(t)|^2} \right). \quad (17)$$

The parameters $p_1, p_2 < \alpha$ in the estimation of the covariation matrix [4] were set equal to $p_1 = 1.1$ and $p_2 = 1.2$. For all these simulations, the number of signals is assumed to be known. In every experiment, we perform 10 Monte

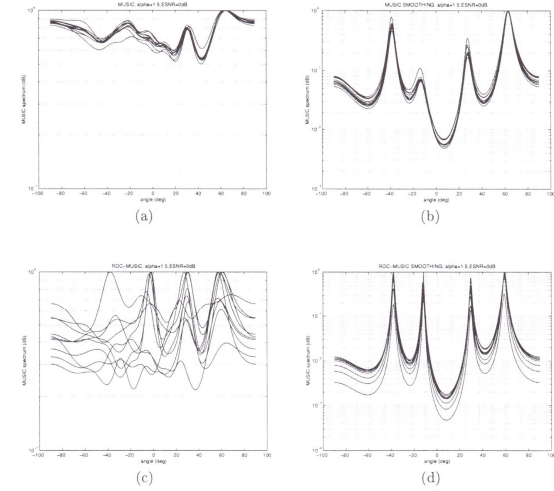


Fig. 1. (a) MUSIC, (b) MUSIC Smoothing, (c) ROC-MUSIC and (d) ROC-MUSIC Smoothing for $\alpha = 1.5$ and $ESNR = 0dB$.

Carlo runs of 10000 snapshots available to the algorithms. As we can see in Figure 1, for a fairly impulsive noise environment case ($\alpha = 1.5$, $ESNR = 0dB$), the MUSIC Smoothing method exhibits low-resolution performance and cannot resolve all four moving signals. Besides, even when the statistic behavior of the noise is close to Gaussian ($\alpha = 1.85$), the MUSIC Smoothing method still cannot clearly resolve all four multipaths (cf. Fig. 2(d)).

On the other hand, the ROC-MUSIC Smoothing method exhibits higher stable resolution capability, showing better performance for non-Gaussian additive noise environments ($\alpha = 1.5$, cf. Fig 1(d)), and at the same time, performing well in quasi-Gaussian interference ($\alpha = 1.85$, cf. Fig. 2).

In Fig. 3 we demonstrate the algorithmic performance with respect to

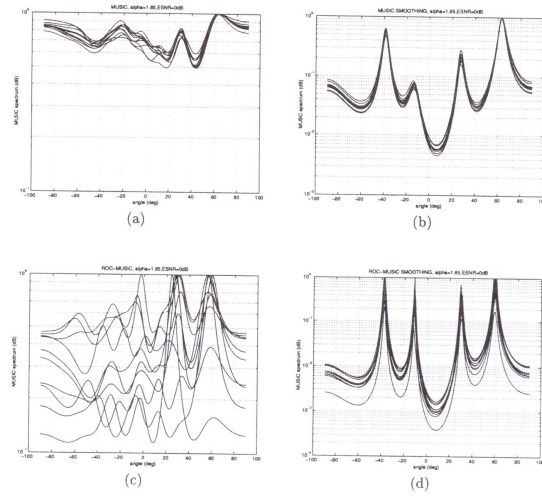


Fig. 2. (a) MUSIC, (b) MUSIC Smoothing, (c) ROC-MUSIC and (d) ROC-MUSIC Smoothing for $\alpha = 1.85$ and $ESNR = 0dB$.

the spatial angle separation of the two closely spaced incoming signals from directions $\theta = [15^\circ, -40^\circ, 40^\circ, -30^\circ]$, for $ESNR = -8$ dB and $\alpha = 1.5$. As expected, the resolution capability improves with increased angle separation between the two paths. But the ROC-MUSIC Smoothing algorithm requires a lower angle separation threshold than the MUSIC Smoothing algorithm and it is able to resolve the two paths from -40° and -30° , unlike the MUSIC Smoothing algorithm. In Fig. 3, a 20 antenna array with sub array length of 12 has been considered for better resolution.

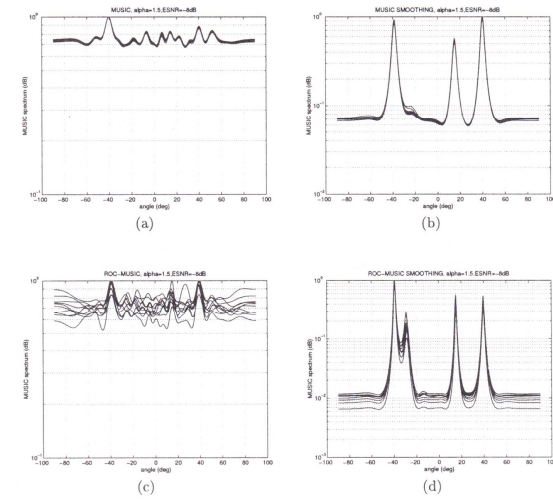


Fig. 3. (a) Music, (b) MUSIC Smoothing, (c) ROC-MUSIC and (d) ROC-MUSIC Smoothing for $\alpha = 1.5$ and $ESNR$ of $-8dB$.

6 Conclusion

Conventional high-resolution eigen-decomposition techniques perform poorly in coherent receiving environments. Spatial Smoothing methods proposed in the past address the signal coherency problem but fail to operate reliably in a non-Gaussian noise. The method proposed in this paper is able to overcome degradation in the performance due to both multipath and impulsive noise environments. The new algorithm is based in spatial Smoothing of the covariance matrix of an antenna array and it is shown to exhibit better high-resolution performance in a wide range of noise environments without considerably increasing the complexity of the system. Several limitations need

to be addressed in the future, including unequally spaced non-linear arrays, and correlated additive noise structures.

References

1. B.C. Ng, M.H. Er, and C. Kot. A music approach for estimation of directions of arrival of multiple narrowband and broadband sources. *Signal Processing*, November 1994.
2. T.-J. Shan, M. Wax, and T. Kailath. On spatial smoothing for direction-of-arrival estimation of coherent signals. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, August 1985.
3. T. Kailath V. U. Reddy, A. Paulray. Performance analysis of optimum beam-former in the presence of correlated sources and its behaviour under spatial smoothing. *IEEE Trans. on Acoustic, Speech and Signal Processing*, July 1987.
4. P. Tsakalides and C.L. Nikias. The robust covariation-based music (roc-music) algorithm for bearing estimation in impulsive noise environments. *IEEE Trans. on Signal Processing*, July 1996.
5. M. Shao C.L. Nikias. *Signal Processing with Alpha-Stable Distributions and Applications*. John Wiley & Sons, 1995.
6. R.T. Compton Jr. *Adaptive antennas*. Prentice-Hall, 1988.