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ON THE PERFORMANCE EVALUATION OF A MULTIFREQUENCY-TONE ENVELOPE DETECTOR

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Abstract. The performance of an envelope detector receiver for tone detection applications is evaluated in terms of probability of false alarm and probability of correct detection as functions of the receiver type, signal and noise parameters. The single tone case is considered first. The analysis is then carried out for L (>1) simultaneous tones and it is shown how the results found for the single tone case can be conveniently exploited for evaluating the performance in the more general case. Curves of the probability of detection versus system parameters are also explicitly given for some typical system configurations.

At last some practical considerations are discussed with reference to a prototype implementation of an envelope detector receiver for tone transmissions.

Zusammenfassung. Die Wirkungsweise eines Empfängers für reine Sinustöne nach dem Prinzip der Hüllkurvendetektion wird untersucht anhand der Wahrscheinlichkeit eines falschen Alarms und der Wahrscheinlichkeit der korrekten Meldung in Abhängigkeit vom verwendeten Empfänger sowie von Signal- und Rauschparametern. Zuerst wird der Fall des einzelnen Sinustons betrachtet. Anschließend wird die Analyse für L (>1) gleichzeitig übertragene Sinustöne durchgeführt. Es wird gezeigt, auf welche Weise die Ergebnisse, die bei der Analyse des Einzeltons gefunden wurden, auf den allgemeineren Fall mehrerer Sinustöne ausgedehnt werden können. Für einige typische Systemkonfigurationen wird die Wahrscheinlichkeit der korrekten Meldung in Abhängigkeit von den Systemparametern explizit in Kurvenform angegeben. Abschließend werden einige praktische Überlegungen zur Diskussion gestellt. Sie betreffen die Implementierung des Prototyps eines Empfängers nach dem Hüllkurvendetektionsprinzip für Systeme zur Übertragung von Sinustönen.

Résumé. Les performances d'un récepteur à démodulation d'enveloppe pour la détection de tons sont évaluées à l'aide des probabilités de fausse alarme et de bonne détection en fonction des paramètres du récepteur, du signal et du bruit. On considère d'abord le cas du ton unique, puis on effectue l'analyse pour L (>1) tons transmis simultanément. On démontre que les résultats obtenus pour le cas du ton unique peuvent être judicieusement utilisés pour le cas général. Les courbes de probabilité de détection en fonction des paramètres du système sont données explicitement pour quelques configurations typiques.

Enfin on présente quelques considerations sur la mise en oeuvre d'un prototype d'un récepteur à détection d'enveloppe pour signaux qui utilisent la transmission de tons.

Keywords. Envelope detector, multifrequency transmission, probability of false alarm, probability of detection, single and multiple transmission tones, CCD filter implementation.

1. Introduction

Multifrequency-tone transmission is widely used for automatic dialing and metering information in conventional telephone networks and for mobile communications. The signals consist of bursts of one or more tones chosen from a set of specified frequencies. The receiver has to determine whether any signal is present and, if so, which frequency or frequencies have been actually transmitted. Many approaches have been proposed, including bandpass filtering [1], use of discrete Fourier Transform [2, 3], tone parameter extraction [4] and recently a pattern recognition detection [5]. Among all the proposed techniques one of the most promising and attractive methods from a practical implementation point of view is the envelope detection of the transmitted signal performed for example by multiplying the received waveform with two in-quadrature sinusoids and subsequent lowpass filtering [6] or by using two bandpass filters with a 90° phase shift difference (quadrature filters) [7, 8].

This paper will consider the envelope extraction technique for tone detection and will give some performance criteria for the practical design of a tone receiver by evaluating the expressions of important parameters such as the probability of false alarm and the probability of correct detection. In Section 2 the case of a single tone will be dealt with and in Section 3 the analysis will be extended to the cases of simultaneous presence of tones and the detection of more than one tone. Finally in Section 4 some practical considerations will be discussed with reference to a prototype of an envelope receiver implemented by using bandpass quadrature filters.

2. Statistics for an envelope detector receiver

Consider first the case of detecting only one tone out of M possible transmissions. The general structure of an envelope detector receiver is shown in Fig. 1. The system consists of M envelope detectors, each having a bandwidth W centered at the various specific transmission frequencies. We will suppose that the bandwidth of the detectors is chosen narrow enough so that they do not overlap in frequency. At regularly spaced time intervals

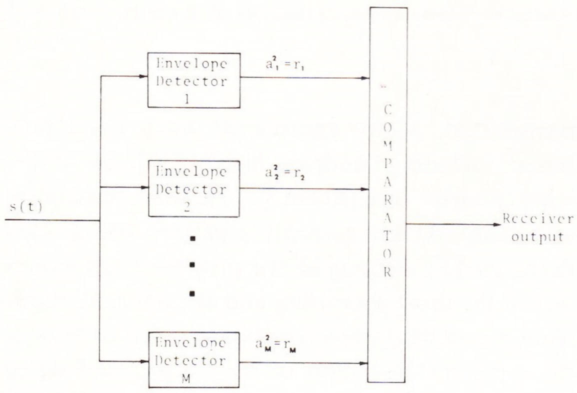

Fig. 1. General structure of an envelope detector receiver for M possible transmission tones.

the system determines the maximum envelope (say the jth envelope) and compares its value to a specific threshold. If the maximum value is greater than the threshold, the jth tone is declared to be present, otherwise no tone is assumed to have been transmitted. In the following we will consider the above receiver structure, with no particular assumptions about the actual technique used for the envelope detection implementation. Implementation considerations are deferred to Section 4.

We will suppose now that at most only one tone is present at any time. The analysis and results will be successively extended to situations where more than one tone are simultaneously present. Furthermore, we will consider that at any instant the decision will only depend on the present M-tuple of the envelopes and not on previous envelope values, i.e. no smoothing is assumed for the envelope samples. The analysis can be readily extended to systems using some smoothing on independent envelope samples.

The envelope samples a_i, $i=1, 2, \ldots, M$, should be evaluated as the square roots of the sum of the squares of two suitable quantities (e.g. the in-phase and quadrature components of the received signal in the ith band). For a simpler implementation on a microprocessor unit as outlined in Section 4, it is however advantageous to consider the squares r_i (Fig. 1) of the envelope samples a_i for saving the square root operation in the decision process.

The performance of a tone receiver is evaluated by considering the probability of detecting a tone when only noise is present (probability of false alarm) and the probability that a tone, when transmitted, is correctly detected. These probabilities are considered in the following two sections. A detailed derivation of their expressions is given in the Appendix.

The noise is supposed to be an additive stationary Gaussian zero-mean white process, having a two-sided power spectral density $\frac{1}{2}n_0$. Therefore

$$N = n_0 W \tag{1}$$

is the noise power in each of the M detector bands.

Of course the white Gaussian noise is not the only type of noise that may corrupt the received signal. In typical applications of tone transmissions, such as in mobile communications and telephone networks, for example, the impulsive noise is of relevant importance. However we will consider only the white Gaussian case for the noise because it gives sufficient characterization for the mutual performance comparison of different systems also in presence of different types of noise and because the obtained analytical results will appear as generalization of previously reported analyses that were developed only with the white Gaussian noise assumption.

2.1. Probability of false alarm

The expression of the probability P_F that a tone will be detected when only noise is received is a function of the threshold T that must be exceeded by the receiver output of Fig. 1 for detecting the presence of a tone. It is given by (see the Appendix)

$$P_F = 1 - (1 - e^{-T/2N})^M. \tag{2}$$

For an allowed maximum probability of false alarm P_F^*, from (2) the minimum value for the threshold T results to be

$$T/N = -2 \log[1 - (1 - P_F^*)^{1/M}]. \tag{3}$$

Considering that $P_F^*/M \ll 1$, (3) can be approximated by

$$T/N \simeq 2 \log(M/P_F^*) \tag{4}$$

which gives a simple relationship for the receiver threshold in function of the noise power N in each detector passband and the acceptable probability of false alarm P_F^*. The receiver has to know or estimate the value of N and then adjust the threshold according to eqn. (4). Fig. 2 shows on a semilogarithmic plot the linear dependence of the normalized threshold $\lambda^2 = T/N$ upon P_F^* parametrized for some values of M.

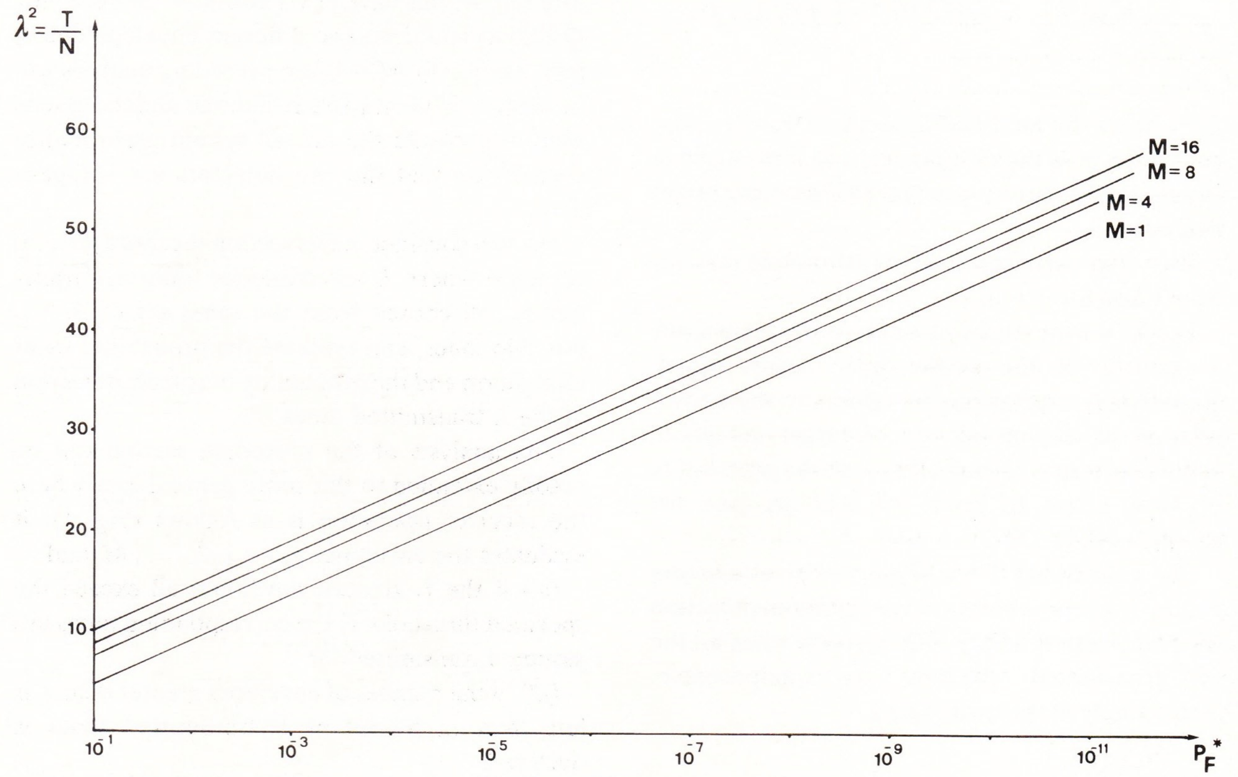

Fig. 2. Plot of the normalized threshold $\lambda^2 = T/N$ versus the allowed probability of false alarm P_F^*.

2.2. Probability of detection of a single transmitted tone

In order to evaluate the probability of detection, the jth tone is supposed to be transmitted and received corrupted by an additive noise having the same statistical properties as before. Each of the possible tones is supposed to be received with the same amplitude S. In the Appendix it is shown that the probability $P_D(1, M)$ of the jth transmitted tone being correctly detected is given by

$$P_D(1, M) = \frac{1}{M}\sum_{m=0}^{M-1}(-1)^m \times \binom{M}{m+1}\exp\left(-\frac{m}{2(m+1)}\frac{S^2}{N}\right) \times Q\left(\frac{1}{\sqrt{m+1}}\frac{S}{\sqrt{N}}, \sqrt{m+1}\sqrt{\frac{T}{N}}\right) \tag{5}$$

where

$$Q(a, b) \triangleq \int_b^\infty x\exp\{-\tfrac{1}{2}(a^2+x^2)\}I_0(ax)\,dx, \tag{6}$$

$I_0(\cdot)$ being the modified Bessel function of order zero. $Q(a, b)$ is the Q function [11], introduced in the classical paper by Marcum [10] on radar target detection.

Two particular cases are worth noting, i.e. for $M = 1$ and for $T = 0$.

For $M = 1$ the situation is exactly the same as for a receiver of the single radar return signal. Accordingly expression (5) reduces to the known relation of the probability of target detection, while expression (2) coincides with the probability of false alarm in radar applications [see for example 10, p. 158].

For a threshold $T = 0$, $P_D(1, M)$ gives also the values of the probability of the optimum detection for noncoherent M-ary FSK systems when all the M transmitted symbols are equiprobable. Accordingly in this case, being

$$Q(a, 0) = 1$$

the expression of $P_D(1, M)$ given by (5) coincides with the results already reported in the literature [12].

Therefore the expression (5) appears as the generalization of known relations previously derived for radar applications and for digital FSK transmissions.

The behaviour of $P_D(1, M)$ is shown in Fig. 3 to 6 as a function of the signal-to-noise ratio $\gamma = S/\sqrt{N}$ and the square root of the normalized threshold $\lambda = \sqrt{T/N}$ for some values of M encountered in many practical applications.

3. Multitone case

In some cases the information transmitted is coded by means of the simultaneous presence of more than one tone. For example, in the Touch-Tone signaling two frequencies are transmitted, each of them being chosen from a different set of four [1]. In this case, if we consider the receiving system as split into two different envelope detectors, each with $M = 4$, the preceding analysis can be directly applied to each detector and the results used to evaluate the overall system performance considering that the two detectors are independent.

We will consider in this section the more general situation where L simultaneous tones are transmitted, all chosen from the same set of $M \geq L$ possible tones, and evaluate the probability P_F of false alarm and the probability of correct detection of the L transmitted tones.

The analysis of the preceding section can be readily extended to this more general case where the receiver operation is as follows (Fig. 1): it evaluates the envelopes r_i, $i = 1, 2, \ldots, M$, and

(a′) if the L greatest envelopes all exceed the specified threshold T, the corresponding tones are assumed transmitted, or

(a″) if the number of envelopes greater than T is less than L, no set of L transmitted tones is declared.

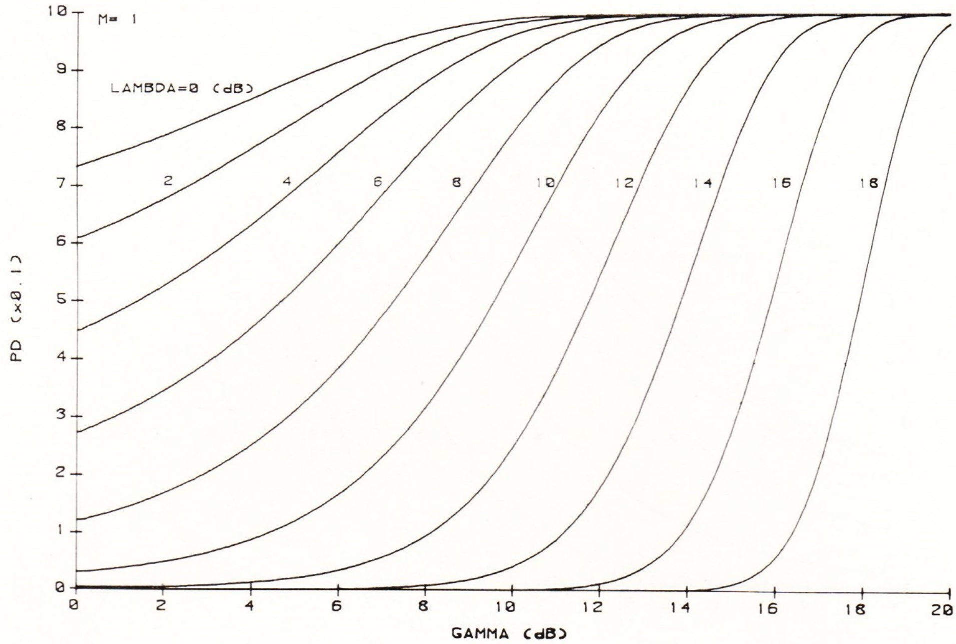

Fig. 3. Plot of the probability of detection of a transmitted tone versus $\gamma = S/\sqrt{N}$ (dB) and $\lambda = \sqrt{T/N}$ (dB) for $M = 1$.

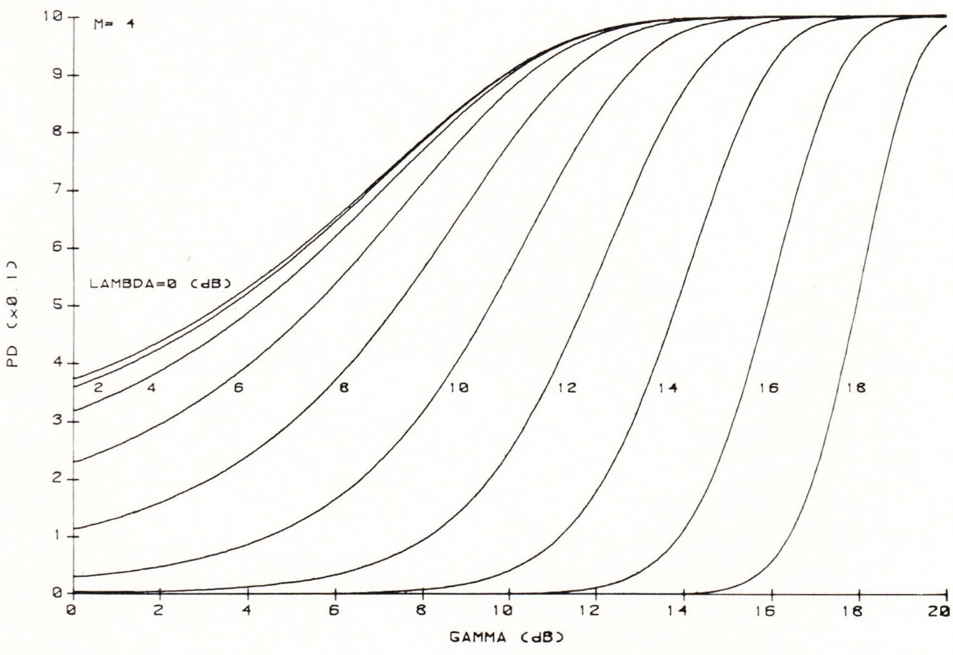

Fig. 4. As in Fig. 3 for $M = 4$.

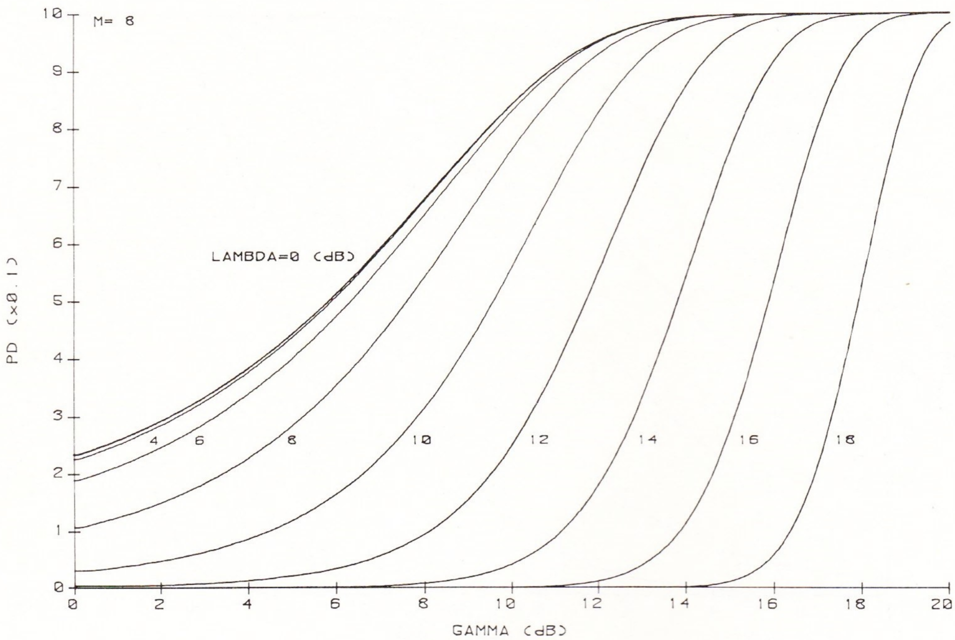

Fig. 5. As in Fig. 3 for $M = 8$.

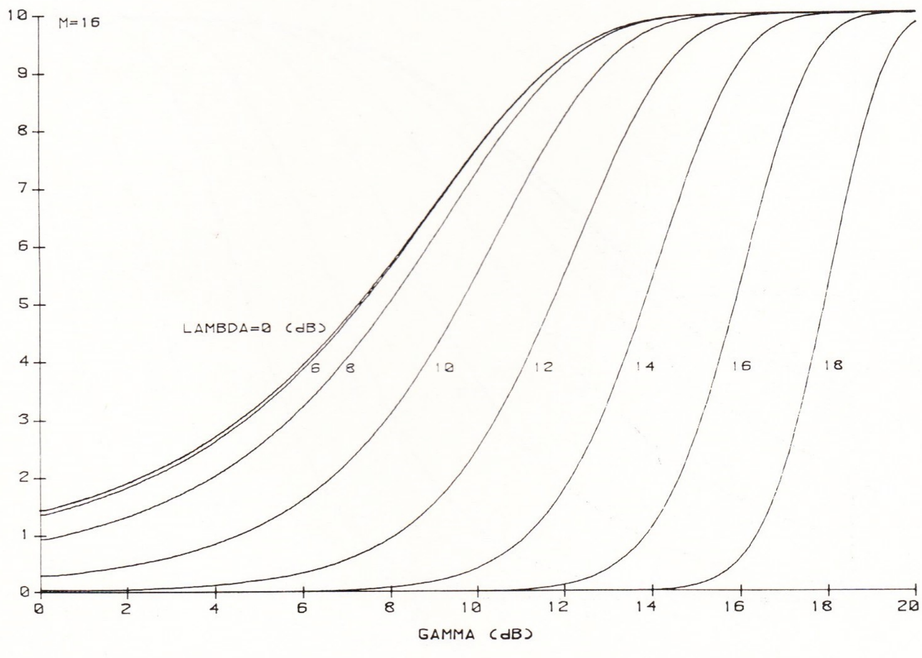

Fig. 6. As in Fig. 3 for $M = 16$.

Given that only noise is received, the probability of false alarm is

$$P_F = \text{Prob}\{\text{at least } L \text{ envelopes} > T\}$$
$$= \text{Prob}\Big[\bigcup_{i=L}^{M} \{i \text{ envelopes} > T \text{ and } M-i \text{ envelopes} < T\}\Big]$$
$$= \sum_{i=L}^{M} \text{Prob}\{i \text{ envelopes} > T \text{ and } M-i \text{ envelopes} < T\}.$$

Now because, from (A2),

$$\text{Prob}\{i \text{ envelopes} > T \text{and } M-i \text{ envelopes} < T\} = \binom{M}{i} e^{-iT/2N}(1-e^{-T/2N})^{M-i}$$

we obtain

$$P_F = \sum_{i=L}^{M} \binom{M}{i} e^{-iT/2N}(1-e^{-T/2N})^{M-i} \qquad (7)$$

which is a generalization of the expression given in the preceding section.

Let us now turn to the case of transmission of L tones, say the first L tones, and determine the probability $P_D(L, M)$ of correct detection of the L transmitted tones. It is given by

$$P_D(L, M) = \text{Prob}\{r_j > r_i \text{ and } r_j > T,\ j = 1, \ldots, L,\ i = L+1, \ldots, M\}$$
$$= \prod_{j=1}^{L} \text{Prob}\{r_j > r_i \text{ and } r_j > T,\ i = L+1, \ldots, M\}.$$

Recalling (A13), this expression can be written through the probabilities of detection of the previous section as

$$P_D(L, M) = \prod_{j=1}^{L} P_D(1, M-L+1) = [P_D(1, M-L+1)]^L \qquad (8)$$

which gives a simple way of determining through (5) the probability of detection of L transmitted tones chosen from a set of M possible frequencies.

4. Implementation considerations

A convenient practical implementation of an envelope detection receiver for multifrequency-tone communications can be realized by using bandpass quadrature filters. This solution has the advantage of avoiding any demodulation process with respect to the more conventional technique of multiplying the received waveform with in-quadrature sinusoids and subsequent lowpass filtering. The former solution is suitable for an implementation with digital circuits and with charge-coupled devices (CCD's).

In [13] a prototype of a multifrequency-tone receiver using CCD's is described and some preliminary experimental results are reported. This multifrequency-tone receiver is based on the implementation of M couples of filters $H_i(f)$ and $\hat{H}_i(f)$, $i = 1, 2, \ldots, M$. The filters $H_i(f)$ are nonoverlapping bandpass filters of bandwidth W. The center frequency of each of them coincides with the nominal frequency of a different transmission tone. The filters $\hat{H}_i(f)$ are defined as

$$\hat{H}_i(f) = -j \,\text{sgn}\, f\, H_i(f). \qquad (9)$$

Therefore $H_i(f)$ and $\hat{H}_i(f)$ are two bandpass quadrature filters. If they are driven by a common input signal, i.e. the received waveform, their respective outputs $x_i(t)$, $\hat{x}_i(t)$ form a Hilbert transform pair. Hence the ith envelope square r_i of Fig. 1 at a given instant of time t_0 may be evaluated as [11]

$$r_i(t_0) = x_i^2(t_0) + \hat{x}_i^2(t_0). \qquad (10)$$

Because the envelope is a lowpass signal of bandwidth $\frac{1}{2}W$, the operations involved in (10) for the envelope square evaluation have to be performed only once every $1/W$ seconds. This is particularly advantageous if we use finite impulse response (FIR) digital or discrete filters to implement the transfer functions $H_i(f)$ and $\hat{H}_i(f)$ [8]. The filtering operations to get the output samples of $x_i(t)$ and $\hat{x}_i(t)$ from the received waveform may thus be performed only once every $1/W$ seconds. This condition permits to time-multiplexing a single filtering hardware unit among all the filters to be implemented. In [13] a receiver prototype of this kind is described and its block diagram is shown in Fig. 7. The system is based on the use of a

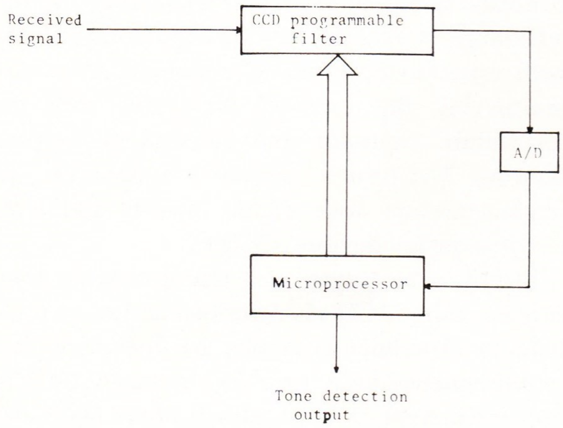

Fig. 7. Block diagram of the prototype of a multifrequency-tone envelope receiver.

CCD programmable transversal filter with 32 taps as the multiplexed hardware filtering unit, externally controlled by a microprocessor. The microprocessor supplies in time sequence the $2M$ (prestored) sets of 32 coefficients that synthesize the specific filtering masks and receives the outputs of the filters after analog-to-digital conversion. After evaluating the M envelope squares r_i according to (10), the microprocessor implements the decision algorithm previously defined and detects the presence or absence of tones.

Presently experimental tests are carrying out to estimate the limits and the performance of this prototype. The preliminary results reported in [13] refer to an 8 kHz sampling frequency for the signal samples stored in the CCD tapped delay line, to $M=4$ transmission tones and to $W=500$ Hz bandwidth for the filters.

5. Conclusions

The performance of an envelope detector for a multifrequency-tone receiver has been evaluated, considering in particular the expressions of the probability of false alarm and the probability of correct detection as functions of the receiver parameters. It was also shown that the evaluation of the envelope detector performance for L simultaneous transmitted tones can be conveniently derived using the results found for the case of a single transmitted tone.

At last some practical implementation considerations have been discussed with reference to a multifrequency-tone detector prototype presently under test. The main features of this prototype are: the use of quadrature filters to extract the tone envelope, the multiplexed use of a single CCD tapped delay line to implement all the required filters and the use of a microprocessor that controls the filtering operations and implements the decision algorithm.

Appendix

We will derive here the analytical expressions of the probability of false alarm and of correct detection for the tone receiver of Fig. 1. The transmitted tone, when present, is chosen from a set of M frequencies and is corrupted by an additive white Gaussian zero-mean process having a noise power N in each detector band.

Probability of false alarm P_F for a single transmitted tone

P_F is the probability that a tone will be detected when only noise is received. The probability density function (pdf) of the envelope squares r_i of Fig. 1 is given by [9]

$$p_{r_i|n}(x) = \frac{1}{2N} e^{-x/2N}, \quad x \geq 0, i = 1, \ldots, M \tag{A1}$$

and the corresponding cumulative distribution function (cdf) by

$$P_{r_i|n}(x) = 1 - e^{-x/2N}, \quad x \geq 0, i = 1, \ldots, M. \tag{A2}$$

Let R be the maximum of the envelope samples r_i at the decision instant

$$R \triangleq \max\{r_i, i = 1, \ldots, M\}. \tag{A3}$$

The r_i, $i = 1, \ldots, M$, are independent because of noise assumptions and since the detector bands are non-overlapping. Therefore the cdf for the random variable R, when no transmitted tone is assumed, is given by

$$\begin{aligned} P_{R|n}(r) &= \text{Prob}\{r_i < r, \forall i\} \\ &= \prod_{i=1}^{M} P_{r_i|n}(r) \\ &= (1 - e^{-r/2N})^M, \quad r \geq 0 \end{aligned} \tag{A4}$$

and its pdf by

$$p_{R|n}(r) = \frac{M}{2N} e^{-r/2N}(1 - e^{-r/2N})^{M-1}, \quad r \geq 0 \tag{A5}$$

The probability of false alarm P_F given in (2) follows immediately from (A4), since it can be written as

$$P_F = 1 - P_{R|n}(T) \tag{A6}$$

in function of the threshold T that must be exceeded by the receiver output R for detecting the presence of a tone.

Output statistics and probability of detection of a single transmitted tone

In order to evaluate the probability of detection, the jth tone is supposed to be transmitted while being received with an amplitude S and corrupted by an additive noise having the same statistical properties as defined before.

The pdf and cdf of each envelope detector output other than the jth output are still given by the previous expressions, i.e.

$$p_{r_i|s}(x) = \frac{1}{2N} e^{-x/2N}, \quad x \geq 0, i = 1, \ldots, M, i \neq j \tag{A7}$$

and

$$P_{r_i|s}(x) = 1 - e^{-x/2N}, \quad x \geq 0, i = 1, \ldots, M, i \neq j. \tag{A8}$$

It can be shown [9] that the pdf of the jth output is given by

$$p_{r_j|s}(x) = \frac{1}{2N} \exp\left(-\frac{x + S^2}{2N}\right) I_0\left(\frac{\sqrt{x}S}{N}\right), \quad x \geq 0 \tag{A9}$$

where $I_0(\cdot)$ is the modified Bessel function of order zero.

Recalling for the subsequent developments the definition (6) of the Q function, it is easy to see that the cdf of the jth output can be expressed by the Q function as

$$\begin{aligned} P_{r_j|s}(x) &= \int_0^x p_{r_j|s}(z)\, dz \\ &= 1 - Q\left(\frac{S}{\sqrt{N}}, \sqrt{\frac{x}{N}}\right), \quad x \geq 0. \end{aligned} \tag{A10}$$

For independent detector outputs r_i, the cdf of their maximum value R when a tone is transmitted is

$$\begin{aligned} P_{R|s}(r) &= (1 - e^{-r/2N})^{M-1} \\ &\quad \times \left[1 - Q\left(\frac{S}{\sqrt{N}}, \sqrt{\frac{r}{N}}\right)\right], \quad r \geq 0 \end{aligned} \tag{A11}$$

and, by differentiating, its pdf is

$$\begin{aligned} p_{R|s}(r) &= \frac{1}{2N} e^{-r/2N}(1 - e^{-r/2N})^{M-2} \\ &\quad \times \left\{(M-1)\left[1 - Q\left(\frac{S}{\sqrt{N}}, \sqrt{\frac{r}{N}}\right)\right]\right. \\ &\quad \left. + (1 - e^{-r/2N})\, e^{-S^2/2N} I_0\left(\frac{S\sqrt{r}}{N}\right)\right\}, \\ &\quad r \geq 0. \end{aligned} \tag{A12}$$

The probability $P_D(1, M)$ of the jth transmitted tone being correctly detected is given by

$$P_D(1, M) = \text{Prob}\{r_j > r_i \ \forall i \neq j \text{ and } r_j > T\}$$
$$= \int_T^\infty \text{Prob}\{R_j < r_j\} p_{r_j|s}(r_j)\, dr_j \quad \text{(A13)}$$

where, by definition,

$$R_j \triangleq \max_{i \neq j}\{r_i\} \quad \text{(A14)}$$

and T is the specified receiver threshold.

The cdf of the random variable R_j is given by the expression (A4) substituting $M-1$ for M. Thus we obtain by using eq. (A9)

$$P_D(1, M) = \int_T^\infty (1 - e^{-r_j/2N})^{M-1} \frac{1}{2N} \times e^{-(r_j+S^2)/2N} I_0\left(\frac{\sqrt{r_j}S}{N}\right) dr_j \quad \text{(A15)}$$

Expanding the power of the binomial and using the Q function, this expression can be written finally in the form of eq. (5) given in the text.

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References

[1] B. Gopinath and R.P. Kurshan, "A touch-tone receiver-generator with digital channel filters", *BSTJ*, Vol. 55, No. 4, April 1976, pp. 455–467.
[2] L.C. Palmer, "Coarse frequency estimation using the discrete Fourier transform", *IEEE Trans. Information Theory*, Vol. IT-20, No. 1, Jan. 1974, pp. 104–109.
[3] D.C. Rife and R.R. Boorstyn, "Single-tone parameter estimation from discrete-time observations", *IEEE Trans. Information Theory*, Vol. IT-20, No. 5, Sept. 1974, pp. 591–598.
[4] J.N. Denenberg, "Spectral moment estimator: a new approach to tone detection", *BSTJ*, Vol. 55, No. 2, Febr. 1976, pp. 143–155.
[5] J.G. Gander, "A pattern recognition approach to tone detection", *Signal Processing*, Vol. 1, No. 1, Jan. 1979, pp. 65–81.
[6] C.R. Baugh, "Design and performance of a digital multi-frequency receiver", *IEEE Trans. Communications*, Vol. COM-25, No. 6, June 1977, pp. 608–615.
[7] F.G. Braun, "Non-recursive digital filters for detecting multifrequency code signals", *IEEE Trans. Acoustics, Speech and Signal Processing*, Vol. ASSP-23, No. 3, June 1975, pp. 250–256.
[8] E. Del Re, "Bandpass signal filtering and reconstruction through minimum-sampling-rate digital processing", *Alta Frequenza*, Vol. XLVII, No. 9, Sept. 1978, pp. 675–678.
[9] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, New York, 1965.
[10] J.K. Marcum, "A statistical theory of target detection by pulsed radar", *IRE Trans. Information Theory*, Vol. IT-6, April 1960, pp. 59–268.
[11] M. Schwartz, W. R. Bennett and S. Stein, *Communication Systems and Techniques*, McGraw-Hill, New York, 1966.
[12] E. Arthurs and H. Dym, "On the optimum detection of digital signals in the presence of white Gaussian noise – a geometric interpretation and a study of three basic data transmission systems", *IRE Trans. Communication Systems*, Vol. CS-10, Dec. 1962, pp. 336–372.
[13] G.D. Cain, V. Cappellini, E. Del Re and L. Fini, "CCD multifrequency-tone receiver with coefficient compensation of charge-transfer loss", in: *Proc. EUSIPCO 80*, Lausanne, 16–19 Sept. 1980.