Signal Processing 2 (1980) 283–287

SHORT COMMUNICATION

A PRACTICAL METHOD OF COEFFICIENT COMPUTATION FOR A MICROPROCESSOR-CONTROLLED VARIABLE RECURSIVE DIGITAL FILTER

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Received 21 September 1979
Revised 10 December 1979

Abstract. The synthesis of general-type recursive digital filters from analog prototypes may require complicated computations not suitable for filter structures where the coefficients are to be evaluated by a microprocessor control unit. A general synthesis procedure suitable for the application to such structures is described and some design examples are reported.

Zusammenfassung. Die Synthese von rekursiven digitalen Filtern allgemeiner Form aus analogen Vorbildern, kann komplexe Berechnungen erfordern, die ungeeignet für die Filterstrukturen sind, in denen die Evaluation der Koeffizienten durch einen Mikroprocessor vorgenommen wird. Ein allgemeiner Syntheseverfahren wird vorgeschlagen, dass in deratigen Strukturen angewendet werden kann. Einige praktische Resultate sind vorgestellt.

Résumé. La synthèse des filtres numériques récursifs généralisés à partir de prototypes analogiques peut demander des calculs compliqués, inadaptés aux structures des filtres pour lesquelles les coefficients doivent être évalués par une unité de contrôle à microprocesseur. Une procédure appropriée de synthèse générale pour application à ces structures est décrite et quelques exemples d'élaboration sont donnés.

Keywords. Recursive digital filters, variable filters, microprocessors, signal processing.

1. Introduction

The implementation of variable, i.e. simply programmable as well as adaptive, digital filters has become a much more feasible technical solution following the introduction and recent improvements in microprocessors. However, in many cases the use of a microprocessor for implementing the arithmetic operations of the digital filters is not convenient or is precluded due to speed requirements on the filter arithmetic and/or because, in an adaptive context, the microprocessor has to implement the chosen adaptation algorithm. In such situations a suitable structure is that shown in Fig. 1, where the microprocessor computes the programmable filter coefficients when necessary (continuous diagram) or updates the adaptive filter weights according to the chosen modification algorithm (dashed diagram). The actual filtering operations are performed by a dedicated hardware logic accepting the external coefficients from the microprocessor and meeting the filtering computation speed requirements. Furthermore this solution for recursive filters can benefit from the present availability of integrated circuits realizing second-order digital filter sections, for example the TMC 539N. For this and other reasons (mainly because they are less sensitive to the roundoff errors caused by a finite-precision arithmetic implementation

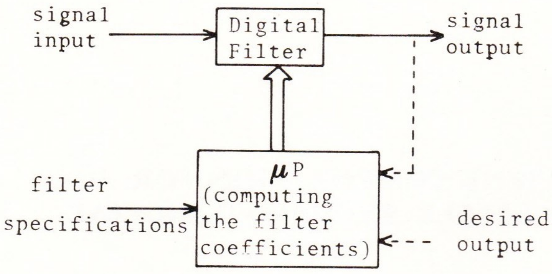

Fig. 1. Microprocessor-controlled variable digital filter.

[1]) second-order digital sections are generally the basic blocks of any (cascaded, parallel or mixed) realization of recursive digital filters. Thus the microprocessor in Fig. 1 has to evaluate the coefficients of each second-order digital section, the number of sections being dependent upon the desired order of the filter (with a possible first-order section for odd-order overall filters).

A possible approach to this problem is to fix a suitable analog prototype filter (for example a Butterworth or Chebyshev lowpass prototype of convenient order with the cutoff frequency normalized to unity) factored into second-order sections in the s domain. Each of these second-order analog prototypes is then transformed by the appropriate bilinear transformation [1] to obtain the coefficients of the desired lowpass, highpass, bandpass or bandstop digital filter. This procedure is straightforward for the design of lowpass and highpass digital filters, because each second-order analog prototype gives rise to a second-order digital section, thus directly matching the digital filter structure as a combination of second-order blocks.

In the design of bandpass and bandstop digital filters, however, the application of the appropriate transformation changes second-order sections in the s domain into fourth-order ones in the z domain. Thus this step must be followed by a further factorization of every fourth-order section into two second-order ones. This operation usually requires the determination of the roots of the numerator and denominator polynomials by one of the available iteration methods or by the exact algebraic algorithms known for fourth-order polynomials [2]. The first method needs many program steps, is time-consuming and generally requires double-precision arithmetic to get satisfying results, thus being feasible only on mini or larger computers. The second method involves operations such as square and cubic roots and, even if the former can be acceptable in a microprocessor-based system, the latter is still too heavy to implement. Thus a different approach for the determination of the digital second-order sections is called for in the design of bandpass and bandstop filters.

In this paper a suitable procedure for the determination of the second-order sections for bandpass and bandstop filters requiring at most square root operations will be described and explicit general formulas for all filter types (lowpass, highpass, bandpass and bandstop) will be calculated for a rapid and easy design reference in the case of a Butterworth model.

The following algorithm for the bandpass and bandstop filters is based on the results already reported in a letter by Saraga [3], where there was described a method of performing a lowpass-bandpass transformation in the s domain that, starting from one second-order lowpass prototype, determines two second-order bandpass sections through an appropriate polynomial factorization requiring only algebraic operations on real numbers. Once the two bandpass sections in the s domain are obtained by this method, it is then possible to apply the standard lowpass-lowpass bilinear transformation to synthesize the two second-order digital bandpass sections in the z domain.

Moreover Saraga's procedure, though developed for a lowpass-bandpass transformation, can be readily adapted also to the factorization of a lowpass-bandstop transformation. Thus from a second-order lowpass prototype two second-order bandstop sections can be determined in the s domain and then transformed as before into the z domain.

2. Procedure for coefficient evaluation

In the following the steps for the evaluation of the coefficients of each second-order digital section according to the outlined procedure will be described. The coefficient expressions (Tables 1 and 2) will be explicitly given for the lowpass, highpass, bandpass and bandstop filter design, all starting as an example from a second-order analog Butterworth prototype of the form

$$H_a(s)=\frac{1}{s^2+as+1}. \tag{1}$$

For the lowpass and highpass digital filter design each section as (1) is transformed into a $H(z)$ of the form

$$H(z)=G\frac{1+L_1z^{-1}+L_2z^{-2}}{1+M_1z^{-1}+M_2z^{-2}} \tag{2}$$

by the following steps:

(a) Evaluate the warping factor [1]

$$K=\mathrm{tg}(\pi f_c), \tag{3}$$

f_c being the desired digital cutoff frequency (normalized, as every digital frequency in the following, to the sampling frequency). It is worth noting that the microprocessor computation of tg(·) is not difficult because it can be easily performed by table look-up memories or by an external unit (like AM 9511) added to the microprocessor structure for floating-point arithmetic.

(b) Choose the appropriate entries in Table 1 for the expressions of the coefficients in (2).

For the bandpass and bandstop designs each analog section (1) is transformed into two cascaded digital sections of the form

$$H_A(z)H_B(z)=G_A\frac{1+L_{1A}z^{-1}+L_{2A}z^{-2}}{1+M_{1A}z^{-1}+M_{2A}z^{-2}}\times G_B\frac{1+L_{1B}z^{-1}+L_{2B}z^{-2}}{1+M_{1B}z^{-1}+M_{2B}z^{-2}} \tag{4}$$

by the following steps:

(a) Given the two desired digital cutoff frequencies f_1 and f_2, $f_1<f_2$, evaluate the following (real) quantities

$$\Omega_0=\tfrac{1}{2}[\mathrm{tg}(\pi f_2)+\mathrm{tg}(\pi f_1)], \tag{5a}$$

$$\Omega_r=\tfrac{1}{2}[\mathrm{tg}(\pi f_2)-\mathrm{tg}(\pi f_1)], \tag{5b}$$

$$\eta=\Omega_r/\Omega_0, \tag{5c}$$

$$D=\sqrt{2(1+\eta^2)\left[1-\sqrt{1-\left(\frac{\eta a}{1+\eta^2}\right)^2}\right]}, \tag{5d}$$

$$\Omega_{ra}=\Omega_0\left[\frac{\eta a}{D}+\sqrt{\left(\frac{\eta a}{D}\right)^2-1}\right], \tag{5e}$$

$$\Omega_{rb}=\Omega_0\left[\frac{\eta a}{D}-\sqrt{\left(\frac{\eta a}{D}\right)^2-1}\right]. \tag{5f}$$

(b) Knowing the previous quantities, choose the appropriate entries in Table 2 for the expressions of the coefficients in (4).

Table 1

Coefficients of the lowpass and highpass second-order digital section

	L_1	L_2	M_1	M_2	G
Low pass	+2	1	$\frac{2(K^2-1)}{K^2+aK+1}$	$\frac{K^2-aK+1}{K^2+aK+1}$	$\frac{K^2}{K^2+aK+1}$
High pass	−2	1	$\frac{2(K^2-1)}{K^2+aK+1}$	$\frac{K^2-aK+1}{K^2+aK+1}$	$\frac{1}{K^2+aK+1}$

Table 2
Coefficients of the two bandpass and bandstop second-order digital sections

	L_{1A}	L_{2A}	M_{1A}	M_{2A}	G_A
Band pass	0	1	$\frac{2(\Omega_{ra}^2-1)}{\Omega_{ra}^2+D\Omega_{ra}+1}$	$\frac{\Omega_{ra}^2-D\Omega_{ra}+1}{\Omega_{ra}^2+D\Omega_{ra}+1}$	$\frac{2\Omega_r}{\Omega_{ra}^2+D\Omega_{ra}+1}$
	0	1	$\frac{2(\Omega_{rb}^2-1)}{\Omega_{rb}^2+D\Omega_{rb}+1}$	$\frac{\Omega_{rb}^2-D\Omega_{rb}+1}{\Omega_{rb}^2+D\Omega_{rb}+1}$	$\frac{2\Omega_r}{\Omega_{rb}^2+D\Omega_{rb}+1}$
	L_{1B}	L_{2B}	M_{1B}	M_{2B}	G_B
	L_{1A}	L_{2A}	M_{1A}	M_{2A}	G_A
Band stop	$\frac{2(\Omega_o^2-1)}{\Omega_o^2+1}$	1	$\frac{2(\Omega_{ra}^2-1)}{\Omega_{ra}^2+D\Omega_{ra}+1}$	$\frac{\Omega_{ra}^2-D\Omega_{ra}+1}{\Omega_{ra}^2+D\Omega_{ra}+1}$	$\frac{\Omega_o^2+1}{\Omega_{ra}^2+D\Omega_{ra}+1}$
	$\frac{2(\Omega_o^2-1)}{\Omega_o^2+1}$	1	$\frac{2(\Omega_{rb}^2-1)}{\Omega_{rb}^2+D\Omega_{rb}+1}$	$\frac{\Omega_{rb}^2-D\Omega_{rb}+1}{\Omega_{rb}^2+D\Omega_{rb}+1}$	$\frac{\Omega_o^2+1}{\Omega_{rb}^2+D\Omega_{rb}+1}$
	L_{1B}	L_{2B}	M_{1B}	M_{2B}	G_B

3. Results and conclusions

The above design procedures have been successfully applied to the synthesis of eighth-order Butterworth digital filters using the structure shown in Fig. 1. Specifically, the implementation with a Rockwell 6502 microprocessor having an external unit (AM 9511) for floating point arithmetic, gave the following measured computation times for the coefficients for all the four second-order digital sections: lowpass 16ms, highpass 14ms, bandpass 32ms and bandstop 33ms. As an example, the amplitude response of an actually implemented bandpass filter with cutoff frequencies $f_1=0.22$ and $f_2=0.28$ is shown in Fig. 2.

With minor modifications only in the coefficient expressions, filters different from the Butterworth model can be easily synthesized.

In conclusion the outlined design procedures appear to be well suited to the implementation of microprocessor-controlled variable recursive digital filters in a structure such as that shown in Fig. 1.

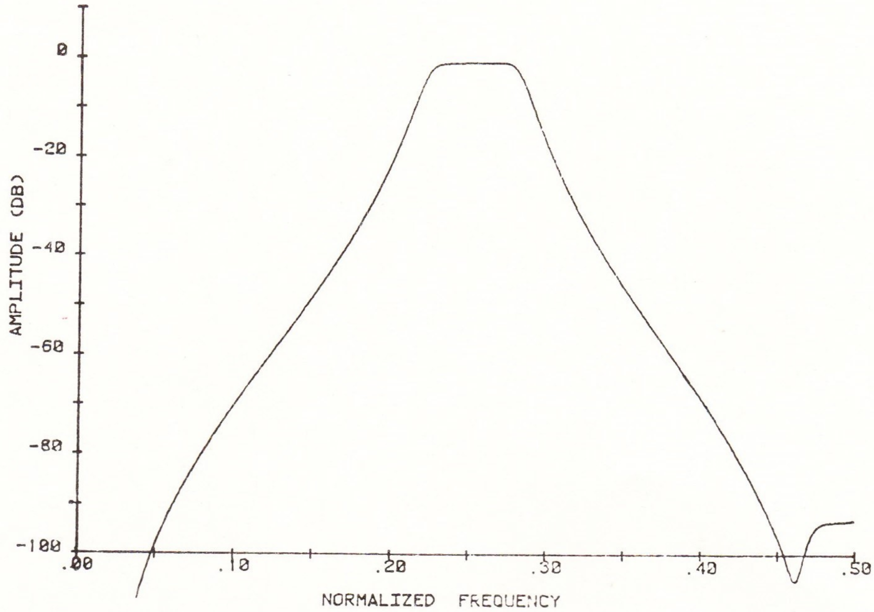

Fig. 2. Example of a designed eight-order bandpass filter (Butterworth prototype) with $f_1 = 0.22$ and $f_2 = 0.28$.

Acknowledgment

The authors wish to thank Profs. V. Cappellini and A. G. Constantinides for useful discussions on this subject.

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