



FLORE

Repository istituzionale dell'Università degli Studi di Firenze

Heat and mass transfer coefficients of falling-film absorption on a partially wetted horizontal tube

Questa è la Versione finale referata (Post print/Accepted manuscript) della seguente pubblicazione:

Original Citation:

Heat and mass transfer coefficients of falling-film absorption on a partially wetted horizontal tube / Giannetti, Niccolò; Rocchetti, Andrea; Yamaguchi, Seiichi; Saito, Kiyoshi. - In: INTERNATIONAL JOURNAL OF THERMAL SCIENCES. - ISSN 1290-0729. - ELETTRONICO. - 126:(2018), pp. 56-66. [10.1016/j.ijthermalsci.2017.12.020]

Availability:

This version is available at: 2158/1124113 since: 2018-04-05T22:24:47Z

Published version: DOI: 10.1016/j.ijthermalsci.2017.12.020

Terms of use: Open Access

Upen Access La pubblicazione à resa

La pubblicazione è resa disponibile sotto le norme e i termini della licenza di deposito, secondo quanto stabilito dalla Policy per l'accesso aperto dell'Università degli Studi di Firenze (https://www.sba.unifi.it/upload/policy-oa-2016-1.pdf)

Publisher copyright claim:

Conformità alle politiche dell'editore / Compliance to publisher's policies

Questa versione della pubblicazione è conforme a quanto richiesto dalle politiche dell'editore in materia di copyright. This version of the publication conforms to the publisher's copyright policies.

(Article begins on next page)

Heat and mass transfer coefficients of falling-film absorption on a partially wetted horizontal tube

 Niccolò Giannetti^{1*}, Andrea Rocchetti², Seiichi Yamaguchi¹, Kiyoshi Saito¹
 ¹ Department of Applied Mechanics and Aerospace Engineering, Waseda University 3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan
 ² Department of Industrial Engineering of Florence, University of Florence Via Santa Marta 3, Firenze, 50139, Italy
 ^{*} Corresponding author: niccolo@aoni.waseda.jp

3

10

Abstract- Detailed, reliable, and time-saving methods to predict 11 12 the transfer characteristics of horizontal-tube falling-film 13 absorbers are critical to control system operability, such that 14 it is closer to its technical limitations, and to optimise 15 increasingly complex configurations. In this context, analytical 16 approaches continue to hold their fundamental importance. This study presents an analytical solution of the governing transport 17 equations of film absorption around a partially wetted tube. A 18 19 film stability criterion and a wettability model extend the 20 validity range of the resulting solution and increase its 21 accuracy. Temperature and mass fraction fields are analytically expressed as functions of Prandtl, Schmidt, and Reynolds numbers 22 as well as tube dimensionless diameter and wetting ratio of the 23 24 exchange surface. Inlet conditions are arbitrary. The Lewis 25 number and a dimensionless heat of absorption affect the 26 characteristic equation and the corresponding eigenvalues. and average transfer coefficients 27 Consequently, local are estimated and discussed with reference to the main geometrical 28 29 and operative parameters. Finally, a first comparison with the 30 numerical solution of the problem and experimental data from 31 previous literature is presented to support the simplifying 32 assumptions, which are introduced and as а first model 33 validation.

34	Nomen	clature	68	Greek s	ymbols
35	A, B	Eigenfunction coefficients	69	α	Thermal diffusivity, m ^{2.} s ⁻¹
36	a, b	Power series coefficients	70	β	Contact angle
37	c_p	Isobaric specific heat, J · kg ⁻¹ K ⁻¹	71	ε	Dimensionless tangential position
38	D	Mass diffusivity, m ^{2.} s ⁻¹	72	γ	Dimensionless LiBr mass fraction
39	d	Diameter, m	73	distribut	tion
40	E, H	Single variable exponential functions	74	η	Dimensionless normal position
41	F, G	Eigenfunctions	75	Λ	Normalised heat of absorption
42	g	Gravity, m ⁻ s ⁻²	76	[Λ=h _{abs}	$(\omega_{e}-\omega_{in})$ · $(T_{e}-T_{in})^{-1} \omega_{e}^{-1}c_{p}^{-1}$]
43	h	Specific enthalpy, kJ kg-1	77	λ,φ	Eigenvalues
44	htc	Heat transfer coefficient, kW ⁻ m ⁻² K ⁻¹	78	θ	Dimensionless temperature distribution
45	k	Thermal conductivity, $W^{-}m^{-1}K^{-1}$	79	Г	Mass flow rate per unit length, kg [.] s ⁻¹ m ⁻¹
46	1	Reference axial length, m	80	δ	Film thickness, m
47	L _c	Characteristic length, m [L _c = $v^{2/3}$. g ^{-1/3}]	81	μ	Dynamic viscosity, Pa-s
48	Le	Lewis number [Le= α · D ⁻¹]	82	ρ	Density, kg ⁻ m ⁻³
49	mtc	Mass transfer coefficient, m ⁻ s ⁻¹	83	ω	LiBr mass fraction
50	Nu	Nusselt number [Nu=htc L_c · k^{-1}]	84		
	-		OE	Cubaan	nte
51	Р	Pressure, kPa	85	Subscri	pis
51 52	P Pr	Pressure, kPa Prandtl number [Pr= $\nu \cdot \alpha^{-1}$]	85 86	0	Film breaking condition
					-
52	Pr Q	Prandtl number [Pr= $\nu \cdot \alpha^{-1}$]	86	0	Film breaking condition
52 53	Pr Q	Prandtl number [Pr= $\nu \cdot \alpha^{-1}$] Heat flux, W	86 87 88 89	0 abs	Film breaking condition Absorption Average Bulk value
52 53 54	Pr Q r	Prandtl number [Pr= $\nu \cdot \alpha^{-1}$] Heat flux, W Outer tube radius, m	86 87 88 89 90	0 abs av b c	Film breaking condition Absorption Average Bulk value Cooling water side
52 53 54 55	Pr Q r Re	Prandtl number [Pr= $\nu \cdot \alpha^{-1}$] Heat flux, W Outer tube radius, m Reynolds Number [Re=4 $\Gamma \cdot \mu^{-1}$]	86 87 88 89 90 91	0 abs av b c e	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium
52 53 54 55 56 57	Pr Q r Re S Sc	Prandtl number [Pr= $\nu \cdot \alpha^{-1}$] Heat flux, W Outer tube radius, m Reynolds Number [Re=4 $\Gamma \cdot \mu^{-1}$] Area, m ² Schmidt Number [Sc= $\mu \cdot \rho^{-1}D^{-1}$]	86 87 88 90 91 92	0 abs av b c e g	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global
52 53 54 55 56 57 58	Pr Q r Re S Sc Sh	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$	86 87 88 90 91 92 93	0 abs av b c e g i	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index
52 53 54 55 56 57 58 59	Pr Q r Re S Sc Sh t	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m	86 87 88 90 91 92 93 94	0 abs av b c e g i i f	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface
52 53 54 55 56 57 58 59 60	Pr Q r Re S Sc Sh t T	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m Temperature, K	86 87 88 90 91 92 93 94 95	0 abs av b c e g i i if in	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface Inlet
52 53 54 55 56 57 58 59 60 61	Pr Q r Re S Sc Sh t T u	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m Temperature, K Streamwise Velocity, m · s ⁻¹	86 87 88 90 91 92 93 94	0 abs av b c e g i i f	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface
52 53 54 55 56 57 58 59 60 61 62	Pr Q r Re S Sc Sh t T u v	Prandtl number $[Pr= \nu \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m Temperature, K Streamwise Velocity, m · s ⁻¹	86 87 88 90 91 92 93 94 95 95 96 97	0 abs av b c e g i if in max	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface Inlet Maximum
52 53 54 55 56 57 58 59 60 61	Pr Q r Re S Sc Sh t T u	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m Temperature, K Streamwise Velocity, m · s ⁻¹	86 87 88 90 91 92 93 94 95 95 96 97	0 abs av b c e g i if in max n, m	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface Inlet Maximum Eigenvalue/Eigenfunction indexes
52 53 54 55 56 57 58 59 60 61 62 63	Pr Q r Re S Sc Sh t T u v W	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m Temperature, K Streamwise Velocity, m · s ⁻¹ Normal Velocity, m · s ⁻¹ Transversal extension of the wet part, n Wetting Ratio	86 87 88 90 91 92 93 94 95 96 97 97 "98	0 abs av b c e g i if in max n, m o	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface Inlet Maximum Eigenvalue/Eigenfunction indexes Outlet
52 53 54 55 56 57 58 59 60 61 62 63 64	Pr Q r Re S Sc Sh t T u v W W	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m Temperature, K Streamwise Velocity, m · s ⁻¹ Normal Velocity, m · s ⁻¹ Transversal extension of the wet part, r Wetting Ratio Local tangential position, m	86 87 88 90 91 92 93 94 95 96 97 97 99	0 abs av b c e g i if in max n, m o sat	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface Inlet Maximum Eigenvalue/Eigenfunction indexes Outlet Phases equilibrium
52 53 54 55 56 57 58 59 60 61 62 63 64 65	Pr Q r Re S Sc Sh t T u v W W W W X	Prandtl number $[Pr = v \cdot \alpha^{-1}]$ Heat flux, W Outer tube radius, m Reynolds Number $[Re=4\Gamma \cdot \mu^{-1}]$ Area, m ² Schmidt Number $[Sc=\mu \cdot \rho^{-1}D^{-1}]$ Sherwood Number $[Sh=mtc L_c \cdot D^{-1}]$ Tube wall thickness, m Temperature, K Streamwise Velocity, m · s ⁻¹ Normal Velocity, m · s ⁻¹ Transversal extension of the wet part, n Wetting Ratio Local tangential position, m Local normal position, m	86 87 88 90 91 92 93 94 95 96 97 95 96 97 99 100	0 abs av b c e g i if in max n, m o sat T	Film breaking condition Absorption Average Bulk value Cooling water side Equilibrium Global Power series index Interface Inlet Maximum Eigenvalue/Eigenfunction indexes Outlet Phases equilibrium Temperature

105 * Dimensionless

104 Superscripts

106

107 **1. Introduction**

108 It is not possible to consider heat transfer and mass transfer 109 separately in several technical circumstances and physical 110 processes. Absorption systems, such as chillers, heat amplifiers, 111 and heat transformers, belong to the aforementioned category and 112 represent an opportunity for clean and efficient energy 113 conversion systems (1). The main advantages of these systems 114 include low-grade heat as the main energy source, higher 115 reliability, and environmentally friendly refrigerants. This is 116 accompanied by the possibility of realising the refrigerant 117 pressure jump in a liquid phase. Accordingly, the compressor of 118 a conventional system is substituted with a set of components, 119 such as a solution pump, a generator, an absorber, and a 120 solution heat exchanger, termed as a "thermal compressor". As a 121 downside, this requires a significantly larger exchange surface. 122 In addition, extant studies indicated that the highest amount of 123 irreversibility occurs in an absorber (2) and that global capacity and first law efficiency are limited by the amount of 124 125 refrigerant that is absorbed in this component (3-4). Therefore, 126 the intensification of the absorption process and proper design 127 of an absorber are the critical factors that should be addressed. 128 Conversely, the recent technical development of absorption 129 chillers, heat pumps, and heat transformers corresponds to 130 increasingly complex plant configurations (5-6),and 131 specifically constitutes a step forward with respect to the 132 theoretical background required for an accurate performance prediction, optimisation, and control. In general, the systems 133 134 design approach continues to rely on empirical rules, heuristic 135 correlations, or trial and error procedures on a global and 136 component scale. The correlations rely on large sets of data, in 137 which each set depends on experimental equipment as well as the

138 specific boundary conditions of these measurements. Furthermore, 139 devices that are designed to achieve high performance under 140 nominal conditions may not exhibit a sufficient performance over 141 most of the actual operative range. Similarly, in practice, 142 conditions are transient and change continuously, because they 143 are affected by interrelations with the external environment. 144 Consequently, instantaneous conditions significantly differ from 145 the design point. The construction of reliable and widely 146 applicable theoretical models enables the design, optimisation, 147 and definition of an effective control method without depending 148 on trial and error procedures or empirical rules.

More specifically, horizontal-tube falling-film absorbers can realise high heat and mass transfer rates with compact size and negligible pressure losses. Nevertheless, prior experimental studies on falling film absorption (7-12) report a limited amount of results with high uncertainties and within a relatively narrow range of operative conditions.

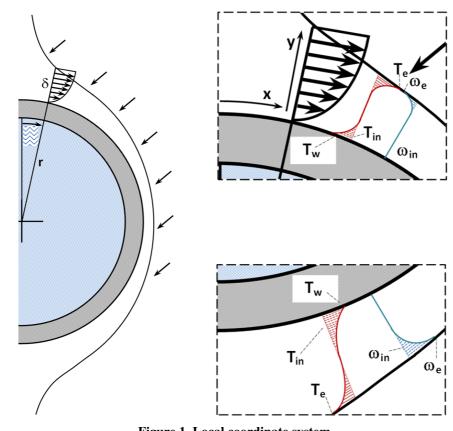
155 Reference (13) numerically discusses a model for film absorption 156 and desorption of a laminar liquid film with constant thickness 157 that flows over a vertical isothermal plate. A similar model was 158 applied by (14) to a horizontal tube heat exchanger. References (15-18) introduce the effects of thickness and velocity 159 distributions around a tube surface via numerical analyses. 160 161 Finally, references (19-25) use the Volume of Fluid technique to 162 examine and extract detailed descriptions of the wavy film dynamics, inter-tube droplets formation, detachment, and impact. 163 164 Numerical analysis and computational fluid dynamics (CFD) are powerful tools that could be very precise when the problem is 165 properly formulated. However, it is necessary to adequately 166 167 consider the time required to reach an accurate solution and the fact that its validity is restricted to the specific case and 168 condition. 169 the selected operative Generalisable design guidelines are not directly provided by specific results as well 170 171 heuristic methods. Given this viewpoint, analytical as 172 approaches continue to maintain their fundamental importance to 173 capture the physics of the problem and generalise the validity 174 of the solution. The main limitations of extant analytical 175 models include the geometry of the solid surface, assumptions of 176 complete wetting, equilibrium of an inlet solution with the 177 refrigerant vapour, uniform velocity profile, and film thickness 178 (26-29). Reference (28) indicated that uniform velocity profile 179 and film thickness are responsible for approximately 20% 180 deviations in the heat and mass transfer coefficients, and they 181 under-predict approximately 40% of the distance required for the 182 development of the thermal boundary layer. Therefore, this study 183 successfully achieves an accurate and widely applicable 184 analytical solution of the governing equations of falling film 185 absorption over a horizontal tube including the effects of 186 thickness variations, incomplete wetting, and the corresponding 187 reduction in transfer interfaces.

188

189 **2. Physical model**

190 The present analysis focuses on an absorptive liquid film flowing 191 over a vertical row of horizontal smooth tubes. Droplet impact 192 and hydrodynamic boundary layer development (19-25, 30) are not 193 discussed herein. Figure 1 schematically illustrates the system 194 under consideration. A single tube at uniform wall-temperature, 195 T_{w} , is considered. A thin film of LiBr-H₂O solution impinges at 196 the top (x=0) and flows viscously down the tube due to gravity as 197 a laminar incompressible liquid. Additionally, absorption can occur at the free-interface of the film based on the thermo-198 199 physical relation between the solution and the vapour. The enthalpy of vapour condensation that is released in the lithium-200 201 bromide/water mixture is rejected to the cooling water flowing 202 inside the tube. Following the development of the thermal 203 boundary layer, the temperature gradient related to the cooling 204 process at the wall also influences the temperature at the 205 interface, and this in turn establishes the equilibrium mass

206 fraction at the vapour pressure within the heat exchanger and 207 consequently controls mass transfer.



209 210

211

208

Figure 1. Local coordinate system

212 In order to reach a closed analytical solution of the governing 213 transport equations, heat and mass transfer processes are 214 considered under the following main assumptions:

215 - The zone of impingement is assumed as a small fraction of the 216 total periphery, and it is assumed that the thermal boundary layer starts its growth from the upper stagnation point $(x \approx 0)$; 217

218 - It is assumed that both the tube circumference and length are 219 large when compared to the film thickness and that the 220 disturbances at the edges of the system can be neglected;

221 - The flow is laminar;

222 - Neither interfacial shear forces with the vapour nor 223 interfacial waves exist;

224 - Thermodynamic equilibrium occurs at the film inlet-interface 225 with the vapour at the heat exchanger pressure, and thus mass 226 transfer occurs without any resistance;

227 - Thermo-physical solution properties are similar to those of an 228 ideal mixture and remain constant along the film thickness and 229 around the tube. As a corollary, natural and Marangoni convection 230 are not considered;

231 - Heat transfer to the vapour environment is neglected;

232 - The variation of the mass flowrate due to the absorbed vapour233 is negligible;

- According to the thin film approximation introduced by (27),
body fitted coordinates (x along the tube surface and y normal to
it at any point) are used because the film thickness is low when
compared to the tube diameter.

A curvilinear coordinate transformation is adopted to map the 238 239 flow domain of the physical space to a simple rectangular domain 240 (16). The dimensionless variables considered in the 241 circumferential and radial directions correspond to $\mathcal{E}=x/\pi r$ and $\eta=y/\delta$, 242 respectively. Tangential (eq. 1) and normal (eq. 2) velocity components based on the Nusselt integral solution of the boundary 243 244 layer momentum and continuity equations with constant properties 245 form (see, for instance, references 13-18) are employed under the 246 assumption that the momentum transfer of the fluid is dominated 247 by viscous forces in the absence of inertia and pressure forces.

249
$$u = \frac{\rho g \delta^2}{2\mu} \sin \pi \varepsilon \left(2\eta - \eta^2 \right)$$
(1)

250
$$v = -\frac{\rho g \delta^2 \eta^2}{2\mu r} \left[\frac{1}{\pi} \frac{d\delta}{dx} \sin \pi \varepsilon + \delta \left(1 - \frac{\eta}{3} \right) \cos \pi \varepsilon \right]$$
(2)

251

248

252 Accordingly, once the film mass flowrate per unit length of the 253 tube is known, the corresponding film thickness is given by eq. 254 3.

$$\delta = \left(\frac{3\mu\Gamma}{\rho^2 g\sin\pi\varepsilon}\right)^{\frac{1}{3}}$$

A small thermal resistance is associated with a thinner film at 256 257 low specific mass flowrates, and thus moving the operability of 258 falling film absorbers to a low Reynolds number is attractive in 259 increasing the performance of absorption systems and reducing 260 their overall size. However, it is necessary to consider the 261 reduction in the contact area due to partial wetting as a 262 critical related issue. In these operative conditions, specifically at a low film Reynolds number (Re=4 Γ/μ) and while 263 264 employing liquids with high surface tension (i.e., low Weber 265 numbers), it is not possible to consider the assumptions of a 266 film with uniform thickness and complete wetting of the transfer 267 surface as even approximately rigorous. This leads to an 268 unacceptable inaccuracy of simulation results (i.e., the 269 obtained trend of the predicted heat transfer coefficient itself 270 disagrees with measurements (31)). Furthermore, it is recognised 271 that partial wetting occurs even at typical operative conditions. 272 Among the previously proposed models, the effect of the amount 273 of wetted surface is not assessed or is merely assumed as a 274 fixed value imposed on the calculation (15, 32) albeit with a 275 few exceptions (9, 33-35). Moreover, related experimental data 276 and visual descriptions by digital image processing are also 277 extremely limited in terms of the number of studies that report 278 the same as well as in the range of conditions that is covered 279 (36-39). Nevertheless, the role of wettability is recognised as 280 а dominant factor in determining the efficiency of the 281 absorption process. Therefore, both a criterion of stability of 282 the uniform film to identify the minimum flow rate to ensure a 283 complete wetting of the surface and a method to estimate the wetted area after the film breakage should be included to 284 285 enhance the model capability to predict the performance of these 286 devices.

(3)

287 To consider the effect of partial wetting, after the thermo-288 physical properties of the solution are given, the extension of 289 the range affected by the phenomenon is identified by the 290 critical condition for a uniform film in terms of minimum wetting 291 rate Γ_0 that corresponds to a critical Reynolds number Re_0 . The 292 latter can be experimentally measured (37-41) or analytically 293 estimated for a surface with generic inclination (42-43) once the 294 characteristic contact angle that is representative of the 295 affinity of the solid-liquid interaction is known. Among the various available methods (44-47), the principle of minimising 296 297 the energy contained in a given stream wise length of the 298 falling film is hereby used to assess the stability of the 299 uniform configuration (eq. 4) and to provide an estimate (eq. 5) 300 of the rivulet wetting ability (42-43) given the assumption of a 301 rivulet cross-section geometry. The value of the dimensionless group $(Re_0 \cdot We_0^3)^{1/15}$ in (43) is directly proportional to the 302 dimensionless critical thickness δ_0^* that is defined in (42) (eq. 303 304 4). Therefore, equation 4 represents the flow regime transition 305 between a uniform film and a rivulet flow configuration with 306 circular cross-section shape and contact angle β_{ϵ} which 307 partially wets the solid surface. This is obtained from the 308 condition of equivalent kinetic plus surface tension energy, and 309 flowrate of the two regimes, when the stable condition of the 310 rivulet is identified through the principle of minimum energy. 311

312
$$\delta_0^{*5} + (1 - \cos \beta) - G(\beta) \delta_0^{*3} = 0$$
 , $\delta_0^* = \left(\frac{\rho^3 g^2}{15\mu^2 \sigma}\right)^{1/5} \delta_0$ (4)

313 Equation 5 corresponds to the minimisation of the energy 314 contents of a given stream-wise length of the rivulet, with 315 respect to the geometrical parameter that defines its wetting 316 ability WR (the ratio of the base of the rivulet w to the total 317 axial length l taken as a reference).

319
$$WR = \left\{ \frac{1}{15} \frac{\rho g}{\sigma} \frac{\psi(\beta)}{\sin \beta} \left[\frac{\beta}{\sin \beta} - \cos \beta \right]^{-1} \right\}^{\frac{3}{5}} \frac{\sin \beta}{\gamma(\beta)} \operatorname{Re}$$
(5)

321 where $G(\beta)$, $\psi(\beta)$, and $\gamma(\beta)$ denote geometrical functions of the contact 322 angle β between the liquid-gas interface of the rivulet (further 323 details are given in reference 43). When *WR* is used to estimate 324 the wetting ability of the film along the absorber tube, its 325 value corresponds to the ratio of the wetted portion *w* to the 326 tube unit length *l* (Figure 2).

327 For lower solution flowrates, methods based on the principle of 328 minimum energy (eq. 5) as well as experiments (37-41, 43) are in 329 agreement with a linearised wettability model (eq. 6) relative to 330 the film Reynolds number, which gives zero wetting when Reynolds 331 number is zero, and complete wetting at $Re=Re_0$.

(6)

332

$$333 \qquad WR = \frac{\text{Re}}{\text{Re}_0}$$

334

335 Therefore, δ_0^* can be evaluated from eq. 4, once the value of the 336 characteristic contact angle of the liquid-solid pair is known. 337 Afterwards, using the Nusselt velocity profile for a vertical 338 falling film, the film thickness can be directly related to the 339 film Reynolds number ($Re=4\Gamma/\mu$) and the critical Reynolds number 340 at which the film breaking occurs Re_0 is calculated as in eq. 7.

341
$$\operatorname{Re}_{0} = \left(\frac{3^{5} g \mu^{4}}{4^{5} 15^{3} \rho \sigma^{3}}\right)^{-\frac{1}{5}} \delta_{0}^{*^{3}}$$
 (7)

342 The approach aims at estimating the wetted exchange area on an 343 average basis while not targeting a local description of the 344 complex film hydrodynamics. Furthermore, a closed solution 345 requires considering WR as an independent function of the angular 346 position on the tube surface. Accordingly, the film thickness 347 distribution (eq. 9) is adjusted to assure the consistency 348 between uniform and partial wetting configurations (eq. 8) by 349 using a modified form of the Nusselt equation (as in (32)). 350

351

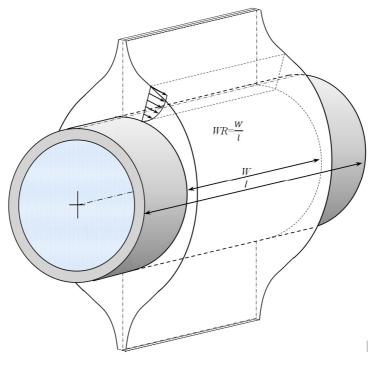
352
$$\frac{\Gamma}{2WR} = \int_{0}^{\delta} \rho u(y) dy = \frac{1}{3} \frac{\rho^2 g \sin \beta}{\mu} \delta^3$$
(8)

353
$$\delta = \left(\frac{3\mu\Gamma}{WR\rho^2 g\sin\pi\varepsilon}\right)^{\frac{1}{3}}$$
(9)

354

355 To the authors' knowledge, a direct validation of eq. 9 has not 356 been achieved in previous literature and further research efforts 357 in this regard are needed.

358



359 360

Figure 2. A physical model of film partial wetting

361

362 Heat and mass transfer characteristics of the system under 363 analysis are studied with reference to eq.s 10 and 11. This two-364 dimensional form of the energy and species transport equations is 365 written for a steady flow with constant properties without 366 internal heat generation and viscous dissipation and neglecting 367 diffusion terms in the flowing direction (see, for instance, 15-368 16, 27).

369

$$370 \qquad \frac{\partial T}{\partial \varepsilon} = \frac{\pi r \alpha}{u \delta^2} \frac{\partial^2 T}{\partial \eta^2} + \left(\frac{\eta}{\delta} \frac{d\delta}{d\varepsilon} - \frac{\pi r v}{u \delta}\right) \frac{\partial T}{\partial \eta}$$
(10)

371
$$\frac{\partial \omega}{\partial \varepsilon} = \frac{\pi r D}{u \delta^2} \frac{\partial^2 \omega}{\partial \eta^2} + \left(\frac{\eta}{\delta} \frac{d\delta}{d\varepsilon} - \frac{\pi r v}{u\delta}\right) \frac{\partial \omega}{\partial \eta}$$
(11)

372

373 Where,

374

375
$$\frac{d\delta}{d\varepsilon} = -\left(\frac{\mu\Gamma\pi^3}{9WR\rho^2 g}\right)^{\frac{1}{3}} \frac{1}{\sin^{\frac{1}{3}}\pi\varepsilon} \frac{1}{\tan\pi\varepsilon}$$
(12)

376

377 It is shown that eq. 13 is generally applicable for the velocity 378 distribution expressed in eq. 1 and eq. 2.

379

$$380 \quad \left(\frac{\eta}{\delta}\frac{d\delta}{d\varepsilon} - \frac{\pi rv}{u\delta}\right) = 0 \tag{13}$$

381

382 As a result, the simplified expression is obtained as follows: 383

384
$$\frac{\partial T}{\partial \varepsilon} = \frac{\pi r \alpha}{u \delta^2} \frac{\partial^2 T}{\partial \eta^2}$$
(14)

$$385 \qquad \frac{\partial \omega}{\partial \varepsilon} = \frac{\pi r D}{u \delta^2} \frac{\partial^2 \omega}{\partial \eta^2} \tag{15}$$

386

387 An analytical solution of the coupled set of equations is 388 approached with the final aim of obtaining Nusselt and Sherwood 389 number expressions in terms of the operative parameters, 390 geometrical features, and boundary conditions.

391 It is advantageous for the solution of the problem to use a 392 dimensionless form of the variables T and ω as defined by eqs. 16-393 17 where T_e and ω_e are defined in (28). These values are, 394 respectively, the equilibrium temperature of the solution at LiBr 395 mass fraction ω_n and the equilibrium LiBr mass fraction of the 396 solution at temperature T_{in} , namely, the temperature and the mass 397 fraction reached if thermodynamic equilibrium is obtained without 398 changes in mass fraction and temperature.

399

400
$$\theta(\varepsilon,\eta) = \frac{T(\varepsilon,\eta) - T_w}{T_e - T_w}$$
(16)

401
$$\gamma(\varepsilon,\eta) = \frac{\omega(\varepsilon,\eta) - \omega_{in}}{\omega_e - \omega_{in}}$$
 (17)

402

403 Accordingly, $T_e - T_W$ represents the level of sub-cooling of the wall 404 while $\omega_e - \omega_m$ embodies the driving force for vapour diffusion at the 405 inlet of the calculation domain. The dimensionless tube diameter 406 $d^* = 2\pi r/L_c$ is defined as the ratio of the tube circumference to the 407 characteristic length L_c , which is expressed in eq. 18 as follows 408 (17):

409

410
$$L_c = \left(\frac{\mu^2}{\rho^2 g}\right)^{\frac{1}{3}}$$
 (18)

411

412 Finally, non-constant terms of eq.s 14 and 15 are developed and 413 dimensionless variables and parameters are used to express energy 414 and species transport equations in eq. 19 and eq. 20, 415 respectively, in which the independent variables are separated 416 between the sides of the equations as follows:

418
$$\frac{1}{d^* \sin^{\frac{1}{3}} \pi \varepsilon} \left(\frac{3 \operatorname{Re}}{4WR}\right)^{\frac{4}{3}} \frac{\partial \theta}{\partial \varepsilon} = \frac{1}{\operatorname{Pr}(2\eta - \eta^2)} \frac{\partial^2 \theta}{\partial \eta^2}$$
(19)

419
$$\frac{1}{d^* \sin^{\frac{1}{3}} \pi \varepsilon} \left(\frac{3 \operatorname{Re}}{4WR}\right)^{\frac{4}{3}} \frac{\partial \gamma}{\partial \varepsilon} = \frac{1}{Sc(2\eta - \eta^2)} \frac{\partial^2 \gamma}{\partial \eta^2}$$
(20)

421 The solution is approached with the following boundary and inlet 422 conditions: solution temperature and mass fraction at the 423 distributor or, by assuming that complete mixing occurs, the bulk 424 values of the solution coming from the previous tube ($x \approx 0$ and 425 $0 < y < \delta$; $T = T_{in}$, $\theta(0,\eta) = \theta_{in}$; $\omega = \omega_{in}$, $\gamma(0,\eta) = 0$), at the tube wall constant temperature and non-permeability to species are assured (y=0; 426 $T=T_w$, $\theta(\varepsilon,0)=0$; $\partial \omega \partial \gamma = 0$, $\partial \gamma / \partial \eta |_w = 0$), and at the phase interface 427 428 $(y=\delta, T=T_{sat}(\omega_{if},P), \omega=\omega_{if})$ phase equilibrium is established.

430
$$\left. \frac{\partial \theta}{\partial \eta} \right|_{if} = \frac{\Lambda}{Le} \frac{\partial \gamma}{\partial \eta} \right|_{if}$$
 (21)

431

429

432 Equation 21 constitutes a rearrangement of Fick's law of 433 diffusion and Fourier law that assures that the heat produced by 434 absorption at the film interface is conducted through the film 435 towards the tube surface. Where, the following expression holds 436 and defines the normalised heat of absorption (28):

437

438
$$\Lambda = -\frac{h_{abs}\left(\omega_{e} - \omega_{W}\right)}{\omega_{e}c_{p}\left(T_{e} - T_{W}\right)}$$
(22)

439

440 Additionally, with respect to the vapour pressure equilibrium at 441 the interface, a linear relation (as in (28)) between temperature 442 and mass fraction at the film interface is employed. Accordingly, 443 in terms of the dimensionless variables at a constant pressure, 444 the relation expressed by eq. 23 is obtained.

445

$$446 \qquad \gamma_{if} = 1 - \theta_{if} \tag{23}$$

447 Equation 23 was found in good agreement for a wide range of 448 operative conditions of $LiBr-H_2O$ solution and a thermodynamic 449 justification (although it limited to electrolytic solutions) was 450 presented in reference (48).

451

452 **3. Solution method**

453 The dependent functions (eq.s 24-25) are assumed as a infinite 454 series of products of a number of eigenfunctions in which each is 455 dependent on a single variable as shown in (12) and (13). 456

457
$$\theta(\varepsilon,\eta) = \sum_{n=1}^{\infty} A_n F_n(\eta) E_n(\varepsilon)$$
(24)

458
$$\gamma(\varepsilon,\eta) = 1 - \sum_{n=1}^{\infty} B_n G_n(\eta) H_n(\varepsilon)$$
 (25)

459

460 The application of this method results in four ordinary 461 differential equations as follows: 462

463
$$\frac{1}{d^* \sin^{\frac{1}{3}} \pi \varepsilon} \left(\frac{3 \operatorname{Re}}{4WR}\right)^{\frac{4}{3}} \frac{E_n'}{E_n} = \frac{1}{\operatorname{Pr}(2\eta - \eta^2)} \frac{F_n''}{F_n} = -\lambda_n^2$$
(25)

464
$$\frac{1}{d^* \sin^{\frac{1}{3}} \pi \varepsilon} \left(\frac{3 \operatorname{Re}}{4WR}\right)^{\frac{4}{3}} \frac{H_n'}{H_n} = \frac{1}{Sc(2\eta - \eta^2)} \frac{G_n''}{G_n} = -\phi_n^2$$
(27)

465

466 The general solutions of the left members of both eq. 26 and eq. 467 27 are as follows:

468

$$469 \qquad E_n(\varepsilon) = e^{-\lambda_n^2 d^* \left(\frac{4WR}{3Re}\right)^{\frac{4}{3}} \int_0^{\varepsilon} \sin^{\frac{1}{3}} \pi \varepsilon d\varepsilon}$$

$$470 \qquad H_n(\varepsilon) = e^{-\phi_n^2 d^* \left(\frac{4WR}{3Re}\right)^{\frac{4}{3}} \int_0^{\varepsilon} \sin^{\frac{1}{3}} \pi \varepsilon d\varepsilon}$$

$$(28)$$

471

472 Where λ_n and ϕ_n denote the eigenvalues corresponding to the 473 eigenfunctions F_n and G_n , respectively. Additionally, for the 474 linear equilibrium condition at the interface (eq. 23) that 475 should be satisfied for every ε , it is necessary for every *n* that 476 $\lambda_n = \phi_n$. The boundary conditions at the wall require $F_n(0) = 0$ and 477 $G_n'(0) = 0$, while eq. 30 and eq. 31 are obtained at the interface. 478

479
$$A_n F_n(1) = B_n G_n(1)$$
 (30)

480
$$A_n F_n'(1) = -\frac{\Lambda}{Le} B_n G_n'(1)$$
 (31)

482 Equation 30 and eq. 31 represent two homogeneous equations for A_n 483 and B_n , and thus a non-null solution is reached given the 484 condition that the determinant equals zero.

485

486
$$\frac{F_n'(1)}{F_n(1)} = -\frac{\Lambda}{Le} \frac{G_n'(1)}{G_n(1)}$$
 (32)

487

488 Equation 32 represents the characteristic equation to determine 489 the eigenvalues λ_n when the solution for F_n and G_n is determined. 490 The power series solutions for the right-side members of eq. 26 491 and eq. 27 are expressed as follows:

492

493
$$F_n(\eta) = \sum_{i=0}^{\infty} a_{n,i} \eta^i$$
(33)

494
$$G_n(\eta) = \sum_{i=0}^{\infty} b_{n,i} \eta^i$$
 (34)

495

496 The boundary conditions at the wall $F_n(0)=0$ and $G_n'(0)=0$, namely 497 constant temperature and non-permeability to species , are used 498 to calculate the coefficients $a_{n,i}$ and $b_{n,i}$ by the recursive 499 relations represented by eq. 35 and eq. 36, respectively.

500

501
$$a_{n,0} = 0, a_{n,1} = 1, a_{n,2} = 0, a_{n,3} = 0, a_{n,i} = \frac{\lambda_n^2 \Pr(a_{n,i-4} - 2a_{n,i-3})}{i(i-1)}, i \ge 4$$
 (35)

502
$$b_{n,0} = 1, b_{n,1} = 0, b_{n,2} = 0, b_{n,3} = -\frac{\lambda_n^2}{3},$$

503 $b_{n,i} = \frac{\lambda_n^2 Sc(b_{n,i-4} - 2b_{n,i-3})}{i(i-1)}, i \ge 4$
(36)

505 The coefficients A_n and B_n are determined by using a Sturm-506 Liouville orthogonality condition at the inlet and the boundary 507 conditions at the interface. The solution method follows the 508 procedure presented in (28) although the inlet temperature value 509 in this case is different from the constant value at the wall. 510 Equations 37 and 38 are expressed by multiplying the right-side members of eq. 26 and eq. 27 by the eigenfunctions F_m and G_m , 511 respectively, in the specified order and integrating with respect 512 513 to η . This is expressed as follows:

515
$$\lambda_n^2 \Pr \int_0^1 (2\eta - \eta^2) F_m F_n d\eta = -\int_0^1 F_m F_n \, "d\eta = F_m(0) F_n \, '(0) - F_m(1) F_n \, '(1) + \int_0^1 F_m \, 'F_n \, 'd\eta$$
(37)

516
$$\lambda_n^2 Sc \int_0^1 (2\eta - \eta^2) G_m G_n d\eta = -\int_0^1 G_m G_n \, "d\eta = G_m(0) G_n \, '(0) - G_m(1) G_n \, '(1) + \int_0^1 G_m \, 'G_n \, 'd\eta$$
(38)

517

518 The corresponding equations (obtained by proceeding in the same 519 way for eigenvalues and eigenfunctions with index m) are 520 subtracted and the boundary conditions expressed in eq. 30 and eq. 521 31 are used to yield eq. 39 and eq. 40 as follows: 522

523
$$\Pr\left(\lambda_n^2 - \lambda_m^2\right) \int_0^1 \left(2\eta - \eta^2\right) F_n F_m d\eta = F_n(1) F_m'(1) - F_m(1) F_n'(1)$$
(39)

524
$$Sc\left(\lambda_{n}^{2}-\lambda_{m}^{2}\right)\int_{0}^{1}\left(2\eta-\eta^{2}\right)G_{n}G_{m}d\eta=G_{n}(1)G_{m}'(1)-G_{m}(1)G_{n}'(1)$$
(40)

525

526 The coupling between the previous two conditions is established 527 by using eq. 30 and eq. 31 as follows:

528

529
$$F_{n}(1)F_{m}'(1) - F_{m}(1)F_{n}'(1) = -\frac{\Lambda}{Le} \frac{B_{n}B_{m}}{A_{n}A_{m}} [G_{n}(1)G_{m}'(1) - G_{m}(1)G_{n}'(1)]$$
(41)

530

531 Equation 41 enables the combination of eq. 39 and eq. 40 as 532 follows:

534
$$Sc(\lambda_{n}^{2} - \lambda_{m}^{2})\int_{0}^{1} (2\eta - \eta^{2}) (\Pr LeA_{n}A_{m}F_{n}F_{m} + Sc\Lambda B_{n}B_{m}G_{n}G_{m}) d\eta = 0$$
(42)

535

536 This directly implies,

537

538
$$\int_{0}^{1} (2\eta - \eta^{2}) (\Pr LeA_{n}A_{m}F_{n}F_{m} + ScAB_{n}B_{m}G_{n}G_{m}) d\eta \begin{cases} = 0, n \neq m \\ \neq 0, n = m \end{cases}$$
(43)

539

540 The boundary conditions of constant temperature and mass fraction 541 are used over the entire film thickness at the inlet of the 542 calculation domain as follows:

543

544
$$\sum_{n=1}^{\infty} A_n F_n(\eta) = \theta_{in}$$
(44)

545
$$\sum_{n=1}^{\infty} B_n G_n(\eta) = 1$$
 (45)

546

547 The summation of the integrals is simplified as follows: 548

549
$$\sum_{n=1}^{\infty} \int_{0}^{1} (2\eta - \eta^2) (\Pr LeA_n A_m F_n F_m + ScAB_n B_m G_n G_m) d\eta = \int_{0}^{1} (2\eta - \eta^2) (\Pr Le\theta_{in} A_m F_m + ScAB_m G_m) d\eta$$

550

551 According to eq. 43, the first relation between A_n and B_n can be 552 obtained in eq. 44, while the second relation is expressed by 553 either eq. 30 or eq. 31.

(46)

554

555
$$\int_{0}^{1} (2\eta - \eta^{2}) (\Pr LeA_{n}^{2}F_{n}^{2} + ScAB_{n}^{2}G_{n}^{2}) d\eta = \int_{0}^{1} (2\eta - \eta^{2}) (\Pr Le\theta_{in}A_{n}F_{n} + ScAB_{n}G_{n}) d\eta$$
(47)

556

557 Finally, A_n and B_n are solved for as follows:

559
$$A_n = B_n \frac{G_n(1)}{F_n(1)}$$
 (48)

560
$$B_{n} = \frac{\int_{0}^{1} (2\eta - \eta^{2}) \left(\Pr Le\theta_{in} \frac{G_{n}(1)}{F_{n}(1)} F_{n}(\eta) + Sc\Lambda G_{n}(\eta) \right) d\eta}{\int_{0}^{1} (2\eta - \eta^{2}) \left(\Pr Le \frac{G_{n}^{2}(1)}{F_{n}^{2}(1)} F_{n}^{2}(\eta) + Sc\Lambda G_{n}^{2}(\eta) \right) d\eta}$$
(49)

562 As a result, temperature and mass fraction fields are expressed 563 in eq.s 50 and 51.

564

565
$$T(\varepsilon,\eta) = T_W + (T_e - T_W) \sum_{n=1}^{\infty} \left[A_n \sum_{i=0}^{\infty} (a_{n,i}\eta^i) e^{-\lambda_n^2 d^* \left(\frac{4WR}{3Re}\right)^{4/3} \int_0^{\varepsilon} \sin^{1/3} \pi \varepsilon d\varepsilon} \right]$$
(50)

566
$$\omega(\varepsilon,\eta) = \omega_e + (\omega_{in} - \omega_e) \sum_{n=1}^{\infty} \left[B_n \sum_{i=0}^{\infty} (b_{n,i}\eta^i) e^{-\lambda_n^2 d^* \left(\frac{4WR}{3Re}\right)^{\frac{4}{3}} \int_{0}^{\varepsilon} \sin^{\frac{1}{3}} \pi \varepsilon d\varepsilon} \right]$$
(51)

567

568 **4. Results**

569 The following analysis is performed for a set of representative 570 operative conditions of the absorber in a cooling system (Table 571 1) and LiBr-H₂O solution properties (49) are calculated for the 572 values of temperature, pressure, and mass fraction. Subsequently, 573 the main influential dimensionless parameters are calculated and 574 listed in Table 2.

- 575
- 576

Table 1. Operative conditions

$T_{in}(^{o}C)$	$T_W(^{o}C)$	ω_{in} (%)	P (kPa)	r (m)	β Ref. (39)
40	32	60	1.0	0.0090	32 °

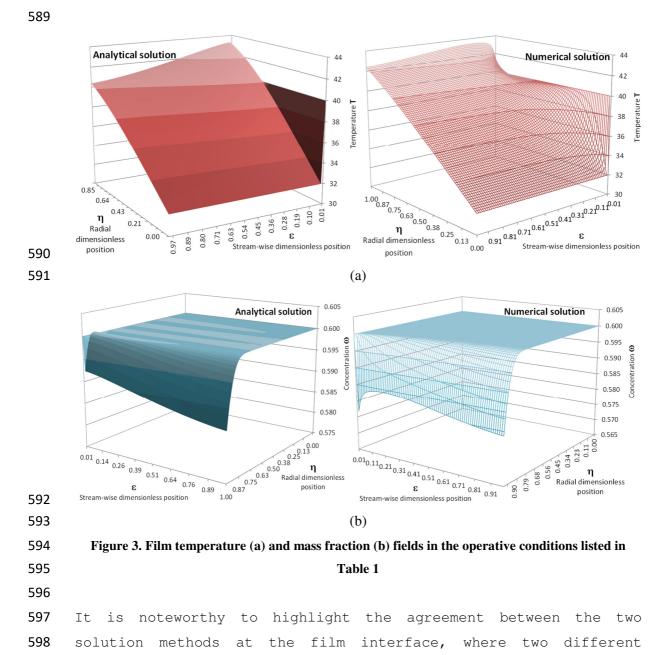
5	7	7

578

Table 2. Operative dimensionless parameters

Le	Λ	Sc	Pr	d*	Re	Re ₀
110.8	5.515	2567	23.17	568.4	42.95	95.00

580 Figures 3(a) and 3(b) compare temperature and mass fraction 581 fields, respectively, obtained with the first as 9 582 eigenvalues/eigenfunctions of the present analytical solution 583 (Table 3) to the corresponding numerical solutions of energy and 584 species transport equations. Both fields indicate good agreement. 585 However, the temperature distribution specifically appears as a 586 rough approximation at the entrance region in proximity to the wall $(\mathcal{E}\sim 0)$, where the highest deviations with respect to the 587 numerical results are observed. 588



599 equilibrium relations are employed. Equation 23 is used for writing the analytical solution, whereas, the thermo-physical 600 601 properties from (49) are used when numerically solving eq.s 10 and 11. A larger number of eigenvalues and terms representing the 602 603 eigenfunctions F_n and G_n are considered, and it is possible to 604 model the entrance region with increased accuracy. However, in 605 the case of a subcooled or superheated inlet solution, given the 606 very small values of the coefficient B_n for eigenvalues higher 607 than 9 (Table 3), which goes beyond the number of significant 608 figures available on the calculation platform, this creates 609 instability of the analytical solution away from the wall and 610 specifically close to the film interface (η =1).

611 The temperature field close to the tube surface obtained with the 612 first 14 eigenvalues (Table 4) is compared to the corresponding 613 numerical solution in Figure 4. It is observed that this enables 614 the analytical solution to model the gradual transition of the 615 temperature distribution at the entrance region in proximity to 616 the wall. Hence, the heat transfer at the tube surface is 617 estimated by considering 14 eigenvalues as listed in Table 3.

618

	-	_	
n	λ_n	A _n	B _n
1	0.0418	0.129	1.34
2	0.116	0.133	-0.551
3	0.189	0.154	0.369
4	0.259	0.176	-0.275
5	0.326	0.168	0.196
6	0.392	0.113	-0.121
7	0.462	0.0536	0.0610
8	0.533	0.0194	-0.0243
9	0.607	0.00328	0.00440
10	2.26	1.28	-9.00E-45
11	3.06	-0.368	-1.00E-70
12	3.91	1.26	-3.00E-45
13	4.72	-0.504	-1.00E-107
14	5.53	1.27	-8.00E-121

Table 3. Eigenvalues and eigenfunction coefficients

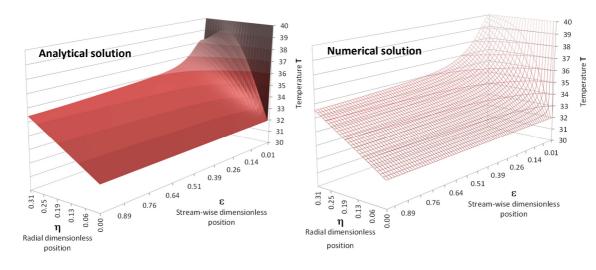


Figure 4. Film temperature field in proximity to the tube wall

623

624 5. Heat and mass transfer coefficients

625 It is assumed that the reduction of the surface in the vapour 626 absorption is represented by the values of WR, and thus the local 627 heat and mass transfer coefficient (*htc* and *mtc*) are defined by 628 eq. 52 and eq. 53, respectively, and by eq. 54 and eq. 55, 629 respectively, with respect to the dimensionless parameters (i.e., 630 Nusselt and Sherwood Numbers).

632
$$htc = WR \frac{k \frac{\partial T}{\partial y}\Big|_{W}}{T_{av} - T_{W}}$$
(52)

633
$$mtc = -WR \frac{D}{\omega_{if}} \frac{\frac{\partial \omega}{\partial y}\Big|_{if}}{\omega_{W} - \omega_{if}}$$
(53)

$$634 \qquad Nu(\varepsilon) = \left(\frac{4}{3} \frac{WR^4 \sin \pi\varepsilon}{Re}\right)^{\frac{1}{3}} \frac{\sum_{n=1}^{\infty} \left[\frac{G_n(1)}{F_n(1)} B_n a_{n,1} e^{-\lambda_n^2 d^* \left(\frac{4WR}{3Re}\right)^{\frac{4}{3}} \int_0^{\varepsilon} \sin^{\frac{1}{3}} \pi\varepsilon d\varepsilon}\right]}{\sum_{n=1}^{\infty} \left[\frac{G_n(1)}{F_n(1)} B_n \sum_{i=0}^{\infty} \left(\frac{a_{n,i}}{i+1}\right) e^{-\lambda_n^2 d^* \left(\frac{4WR}{3Re}\right)^{\frac{4}{3}} \int_0^{\varepsilon} \sin^{\frac{1}{3}} \pi\varepsilon d\varepsilon}\right]}$$
(54)

$$Sh(\varepsilon) = \left(\frac{4}{3}\frac{WR^{4}\sin\pi\varepsilon}{Re}\right)^{\frac{1}{3}} \left[B_{n}\sum_{i=1}^{\infty}\left(ib_{n,i}\right)e^{-\lambda_{n}^{2}d^{*}\left(\frac{4WR}{3Re}\right)^{\frac{4}{3}}\int_{0}^{\varepsilon}\sin^{\frac{1}{3}}\pi\varepsilon d\varepsilon}}\right] \frac{\sum_{n=1}^{\infty}\left[B_{n}\sum_{i=1}^{\infty}\left(ib_{n,i}\right)e^{-\lambda_{n}^{2}d^{*}\left(\frac{4WR}{3Re}\right)^{\frac{4}{3}}\int_{0}^{\varepsilon}\sin^{\frac{1}{3}}\pi\varepsilon d\varepsilon}}{\sum_{n=1}^{\infty}\left[B_{n}\sum_{i=1}^{\infty}\left(ib_{n,i}\right)e^{-\lambda_{n}^{2}d^{*}\left(\frac{4WR}{3Re}\right)^{\frac{4}{3}}\int_{0}^{\varepsilon}\sin^{\frac{1}{3}}\pi\varepsilon d\varepsilon}}\right]}{\sum_{n=1}^{\infty}\left[B_{n}\sum_{i=1}^{\infty}\left(b_{n,i}\right)e^{-\lambda_{n}^{2}d^{*}\left(\frac{4WR}{3Re}\right)^{\frac{4}{3}}\int_{0}^{\varepsilon}\sin^{\frac{1}{3}}\pi\varepsilon d\varepsilon}}\right]}$$

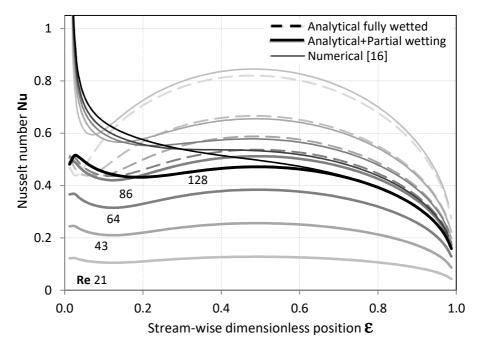
$$(55)$$

638 The denominators of these last two expressions represent the driving potentials for heat transfer and that for mass transfer, 639 640 respectively; in the analytical formulation of the Nusselt 641 number, corresponding to the temperature difference between the 642 bulk value of the liquid film and the solid wall; in the expression of the Sherwood number, the difference between the 643 644 mass fraction at the interface and at the tube wall. On the 645 right-side of the expressions, the numerators include terms corresponding to the temperature gradient at the tube wall and 646 the mass fraction gradient at the film interface. Hence, the 647 648 factors on the extreme left-side embody the products of the 649 active extension of the film interface and the inverse of the 650 variation of the film thickness while normalised with respect to 651 the characteristic length L_{C} .

652 First, the inferences of the main parameters are locally 653 examined for the reference conditions of the absorber as listed 654 in Table 1, and the results obtained are compared while considering the effect of partial wetting (continuous lines) 655 656 with the solution obtained when the effect is ignored (dashed 657 lines). Figure 5 describes the local Nusselt number distribution 658 along the tube surface. The large temperature difference between 659 the tube wall and the impinging solution at the entrance region 660 is responsible for a local peak in the Nusselt number. 661 Additionally, a local maximum that is positioned in proximity of the vertical part of the tube $(\mathcal{E}\sim 0.5)$ is ascribed to the minimum 662 663 film thickness. Conversely, in the second half of the tube, the

664 thickening of the film is associated to a decreasing trend of 665 the Nusselt number. It is also stated that higher local 666 flowrates extend the region affected by the development of the 667 thermal boundary-layer and are responsible for moving the first 668 local minimum of the heat transfer coefficient to higher stream-669 wise positions. This trend matches the trend presented in extant 670 studies when the governing equations of horizontal tube falling 671 film absorption are numerically solved (16), and the highest 672 deviation occurs in proximity of the inlet of the calculation 673 domain in which the temperature gradient is steeper due to the 674 boundary condition of constant tube wall temperature. The 675 discrepancy between the analytical solution and the numerical 676 solution of the governing equations (eqs. 10-11) increases when 677 the solution flowrate increases. The remaining deviations are 678 related to the assumption of a linear equilibrium-relationship 679 at the interface.

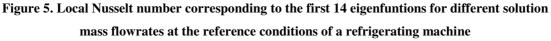








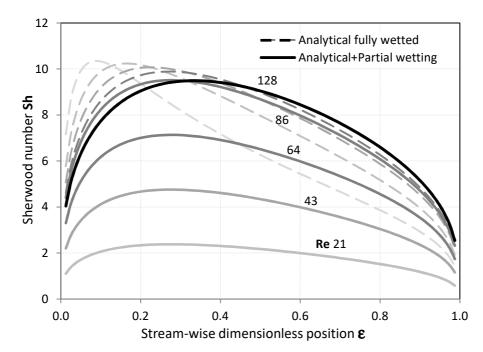
684

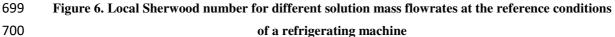


685 Figure 5 shows a comparison of continuous and dashed lines of 686 corresponding colours and highlights that low Reynolds 687 conditions are associated with a globally higher heat transfer 688 rate if partial wetting is overlooked while a gradual reduction 689 in the heat transfer coefficient that is mainly related to the 690 decreasing wetting ability of the solution is experimentally 691 observed (7-11).

692 In figure 6, the mass transfer at the film interface is locally 693 considered in terms of Sherwood number and indicates a maximum 694 value that grows and moves forward when the solution flowrate 695 increases in the partial wetting region (as shown by the 696 continuous lines).

697





701

698

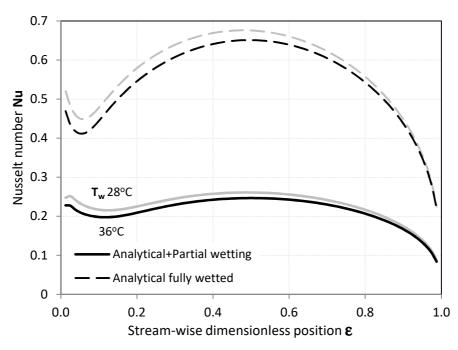
702 Table displays the eigenvalues respective 4 and their 703 eigenfunctions coefficients for two different temperatures at 704 the tube wall of the absorber. A change in this parameter causes 705 the eigenvalues from the characteristic equation (eq. 32) and 706 eigenfunctions coefficients to assume different values.

n		λ_n	A _n	B _n		λ_n	A _n	B _n
1		0.0424	0.103	1.35		0.0409	0.171	1.33
2		0.118	0.112	-0.571		0.114	0.162	-0.517
3		0.191	0.144	0.407		0.186	0.156	0.310
4		0.262	0.199	-0.337		0.256	0.137	-0.196
5		0.327	0.231	0.271		0.325	0.0971	0.113
6		0.391	0.168	-0.185		0.395	0.0477	-0.0497
7	8°C	0.459	0.0819	0.105	6°C	0.465	0.00537	0.00543
8	T _w 28	0.531	0.0368	-0.0554	T _w 36 °C	0.537	-0.0205	0.0207
9		0.605	0.0177	0.0296	Ľ	0.609	-0.0340	-0.0345
10		2.23	1.40	-1.E-43		2.30	0.991	-3.E-46
11		3.03	-0.282	-9.E-70		3.10	-0.419	-1.E-71
12		3.83	1.24	-4.E-89		3.90	1.06	-8.E-91
13		4.63	-0.316	-4.E-106		4.70	-0.597	-4.E-107
14		5.45	1.20	-2.E-119		5.51332	1.18	-2.E-120

708 Table 4. Eigenvalues and coefficients with wall temperatures corresponding to 28 °C and 36 °C

710 As a rule, a lower wall temperature enhances heat and mass 711 transfer both locally (Figures 7-8) and globally.

712



713

714 Figure 7. Local Nusselt number for different Tw at reference conditions of a refrigerating machine

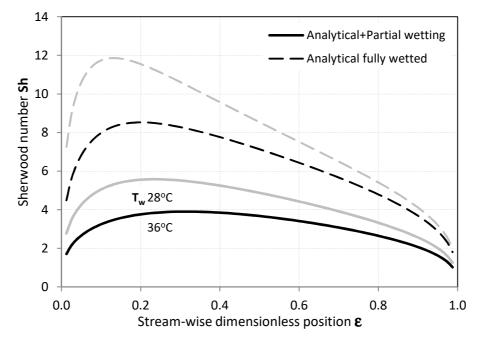


Figure 8. Local Sherwood number for different tube T_w at reference conditions of a refrigerating
 machine

716

720 The wall temperature affects the Sherwood number through the 721 interfacial temperature and consequently changes the interface 722 mass fraction due to the equilibrium hypothesis. Therefore, a 723 lower heat sink temperature can significantly enhance the system 724 capacity by increasing the amount of refrigerant that steadily 725 circulates within the system for a specific solution flowrate. 726 A local analysis further suggests (50) that a lower tube radius 727 globally increases heat and mass transfer coefficients although 728 it reduces the heat flux per unit length due to a lower heat 729 transfer surface. Accordingly, the best selection of the tube 730 size results from a compromise between the conflicting effects. 731 The local values of *htc* and *mtc* around the tube are averaged to 732 perform a global analysis for the absorber tube in a wide range 733 of flowrates. Figures 9 and 10 show that heat and mass transfer 734 coefficients are maximised at a certain solution mass flowrate 735 based on the extension of the region affected by partial wetting.

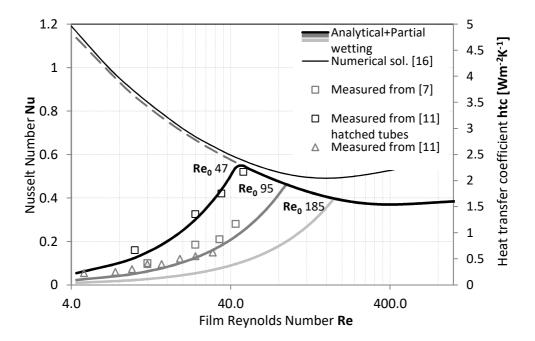
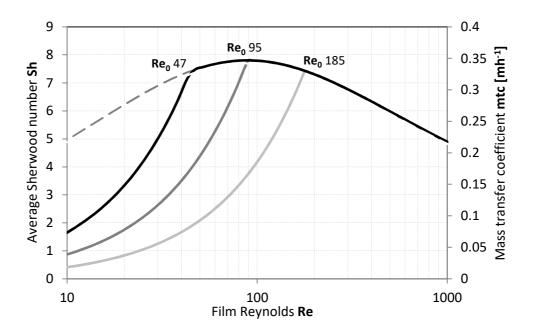


Figure 9. Global Nusselt Number for different wetting behaviours at the reference conditions of a
 refrigerating machine

737

741 The wettability of $LiBr-H_2O$ solution (eq. 5) is increased if 742 tensioactive substances are added to the mixture to decrease the 743 surface tension σ at the vapour-liquid interface or if the solid 744 surface is properly treated (11) to lower the contact angle eta at 745 the solid liquid interface. This stabilises thinner uniform films (eq. 4) and moves the occurrence of the film breaking at a 746 747 lower Reynolds number Re_0 . In contrast, if the affinity between 748 the tube surface and the solution worsens, dry patches also appear at higher Reynolds numbers due to impurities or surface 749 750 roughness. These two cases are qualitatively represented by the 751 lines labelled as Re_0 47 (the simulations are performed by considering $\beta' = \beta/2$) and Re_0 185 ($\beta'' = 2\beta$) in figures 9 and 10, 752 753 respectively, while Re_0 95 represents the case of smooth tubes 754 at reference conditions for a Lithium-Bromide refrigeration 755 machine (Table 1). The dashed line and thin continuous line 756 represent the analytical solution and the numerical results 757 obtained, respectively, when partial wetting (WR=1) over the 758 entire range of operative conditions is neglected.

759 Generally, it is highlighted that both heat and mass transfer 760 are critically improved by improving solution wettability. In 761 the case in which a partial wetting model is not included, the simulated heat transfer coefficients follow an increasing trend 762 763 to decrease the solution mass flowrates. However, this behaviour 764 is in disagreement with all the experimental results indicated in 765 previous studies (5-11). This indicates the necessity to consider 766 partial wetting phenomena in the standard operative range of 767 absorbers operating in real plants.



769

768

770 Figure 10. Global Sherwood number for different wetting behaviours at reference conditions of a

refrigerating machine

- 771
 - 772

773 **7. Conclusions**

774 The presented model for laminar falling film absorption over a 775 horizontal cooled tube considers the cylindrical shape of the 776 tube, the effect of partial wetting, thickness variation of the 777 film flowing around the tube, and arbitrarily selected inlet 778 conditions. A simplified linear model for partial wetting is 779 included to extend the validity of the obtained expressions when 780 complete wetting is not considered as a valid assumption. The 781 model provides detailed information to locally and globally 782 characterise heat and mass transfer of falling film absorbers.
783 The effects related to partial wetting and the main geometrical
784 and operative parameters are investigated to extract general
785 guidelines to optimise the aforementioned devices.

786 Low Reynolds conditions are associated with a globally higher 787 heat transfer rate when partial wetting is overlooked. 788 Conversely, a gradual reduction in the heat transfer coefficient 789 that was mainly related to the decreasing wetting ability of the 790 solution was experimentally observed in previous studies. In 791 general, the results highlight that both heat and mass transfer 792 are critically improved by improving solution wettability.

793 The study indicates the possibility of an optimal tube radius 794 from a compromise between lower heat flux per unit length and 795 higher heat and mass transfer coefficients.

796 Average heat and mass transfer coefficients around the tube are 797 analysed in a wide range of flowrates and show that heat and 798 mass transfer coefficients are maximised at a certain solution 799 mass flowrate based on the extension of the region affected by 800 partial wetting.

801 Given the observed qualitative and quantitative agreements, it is 802 possible to employ the model as a computationally light and 803 accurate module in component and system simulations to design and 804 control actual systems.

805

806 References

807 (1) A. Lubis, J. Jeong, K. Saito, N. Giannetti, H. Yabase, M.I. Alhamid, Nasruddin, (2016),
808 Solar-assisted single-double-effect absorption chiller for use in Asian tropical climates,
809 Renewable Energy, Vol. 99, pp. 825–835.

810 (2) N. Giannetti, A. Rocchetti, K. Saito, (2016), Thermodynamic optimization of three811 thermal irreversible systems, International Journal of Heat and Technology, Vol. 34, pp. S83–
812 S90.

813 (3) N. Giannetti, A. Rocchetti, A. Lubis, K. Saito, S. Yamaguchi, (2016), Entropy
814 parameters for falling film absorber optimization, Applied Thermal Engineering, Vol. 93, pp.
815 750–762.

816 (4) N. Giannetti, A. Rocchetti, K. Saito, S. Yamaguchi, (2015), Irreversibility analysis of
817 falling film absorption over a cooled horizontal tube, International Journal of Heat and Mass
818 Transfer, Vol. 88, pp. 755–765.

K. Saito, N. Inoue, Y. Nakagawa, Y. Fukusumi, H. Yamada, T. Irie, (2015),
Experimental and numerical performance evaluation of double-lift absorption heat transformer,
Science and Technology for the Built Environment, Vol. 21, pp. 312–322.

822 (6) A. Lubis, N. Giannetti, S. Yamaguchi, K. Saito, N. Inoue, (2017), Experimental
823 performance of a double-lift absorption heat transformer for manufacturing-process steam
824 generation, Energy Conversion and Management, Vol. 148, pp. 267-278.

(7) L. Hoffmann, I. Greiter, A. Wagner, V. Weiss, G. Alefeld, (1996), Experimental
investigation of heat transfer in a horizontal tube falling film absorber with aqueous solutions of
LiBr with and without surfactants, International Journal of Refrigeration, Vol. 19 (5), pp. 331–
341.

829 (8) I. Kyung, K.E. Herold, Y.T. Kang, (2007), Experimental verification of H₂O/LiBr
830 absorber bundle performance with smooth horizontal tubes, International Journal of
831 Refrigeration, Vol. 30 (4), pp. 582–590.

832 (9) V.M. Soto Francés, J.M. Pinazo Ojer, (2003), Validation of a model for the absorption
833 process of H₂O(vap) by a LiBr(aq) in a horizontal tube bundle using a multi-factorial analysis,
834 International Journal of Heat and Mass Transfer, Vol. 46, pp. 3299–3312.

835 (10) S.M. Deng, W.B. Ma, (1999), Experimental studies on the characteristics of an absorber
836 using LiBr/H₂O solution as working fluid, International Journal of Refrigeration, Vol. 22, pp.
837 293–301.

838 (11) C.W. Park, S.S. Kim, H.C. Cho, Y.T. Kang, (2003), Experimental correlation of falling
839 film absorption heat transfer on micro-scale hatched tubes, International Journal of
840 Refrigeration, Vol. 26 (7), pp.758–763.

841 (12) S.S. Kim, C.W. Park, H.C. Cho, Y.T. Kang, (2003), The effect of micro-scale surface
842 treatment on heat and mass transfer performance for a falling film H2O/LiBr absorber,
843 International Journal of Refrigeration, Vol. 26 (5), pp. 575–585.

844 (13) M. Mittermaier, P. Schulze, F. Ziegler, (2014), A numerical model for combined heat
845 and mass transfer in a laminar liquid falling film with simplified hydrodynamics, International
846 Journal of Heat and Mass Transfer, Vol. 70, pp. 990–1002.

847 (14) J.W. Andberg, G.C. Vliet, (1987), A simplified model for absorption of vapors into
848 liquid films flowing over cooled horizontal tubes, ASHRAE Trans, Vol. 93, pp. 2454–66.

- 849 (15) V.D. Papaefthimiou, I.P. Koronaki, D.C. Karampinos, E.D. Rogdakis, (2012), A novel
 850 approach for modelling LiBr–H₂O falling film absorption on cooled horizontal bundle of tubes,
 851 International Journal of Refrigeration, Vol. 35 (4), pp. 1115–1122.
- 852 (16) F. Babadi, B. Farhanieh, (2005), Characteristics of heat and mass transfer in vapor
 853 absorption of falling film flow on a horizontal tube, International Communications in Heat and
 854 Mass Transfer, Vol. 32 (9), pp.1253–1265.
- 855 (17) G. Kocamustafaogullari, I.Y. Chen, (1988), Falling film heat transfer analysis on a bank
 856 of horizontal tube evaporator, AIChE Journal, Vol. 34 (9), pp. 1539–1549.
- 857 (18) L. Harikrishnan, S. Tiwari, M.P. Maiya, (2011), Numerical study of heat and mass
 858 transfer characteristics on a falling film horizontal tubular absorber for R-134a-DMAC,
 859 International Journal of Thermal Sciences, Vol. 50 (2), pp. 149–159.
- 860 (19) V. Subramaniam, S. Garimella, (2014), Numerical study of heat and mass transfer in
 861 lithium bromide-water falling films and droplet, International Journal of Refrigeration, Vol. 40,
 862 211–226.
- 863 (20) V. Subramaniam, S.rinivas Garimella, (2009), From measurements of hydrodynamics
 864 to computation of species transport in falling films, International Journal of Refrigeration, Vol.
 865 32 (4), pp. 607–626.
- 866 (21) Q. Qiu, C. Meng, S. Quan, W. Wang, (2017), 3-D simulation of flow behaviour and
 867 film distribution outside a horizontal tube, International Journal of Heat and Mass Transfer, Vol.
 868 107, pp. 1028-1034.
- 869 (22) S.M. Hosseinnia, M. Naghashzadegan, R. Kouhikamali, (2016), CFD simulation of
 870 adiabatic water vapor absorption in large drops of water–LiBr solution, Applied Thermal
 871 Engineering, Vol. 102, pp. 17-29.
- Y. Zhou, Z. Cai, Z. Ning, M. Bi, (2017), Numerical simulation of double-phase coupled
 heat transfer process of horizontal-tube falling film evaporation, Applied Thermal Engineering,
 Vol. 118, pp. 33-40.
- 875 (24) S.M. Hosseinnia, M. Naghashzadegan, R. Kouhikamali, (2017), CFD simulation of
 876 water vapor absorption in laminar falling film solution of water-LiBr Drop and jet modes,
 877 Applied Thermal Engineering, Vol. 115, pp. 860-873.
- 878 (25) G. Ji, J. Wu, Y. Chen, G. Ji, (2017), Asymmetric distribution of falling film solution
 879 flowing on hydrophilic horizontal round tube, International Journal of Refrigeration, Vol. 78, pp.
 880 83-92.

881 (26) N.I. Grigor'eva, V.E. Nakoryakov, (1977), Exact solution of a combined Heat- and
882 Mass- transfer problem during film absorption, Inzhernerno-Fizicheskii Zhurnal, Vol. 33 (5), pp.
883 893–898.

884 (27) S. K. Choudhury, A. Nishiguchi, D. Hisajima, T. Fukushima, T. ohuchi, S. Sakaguchi,
885 (1993), Absorption of vapors into liquid films flowing over cooled horizontal tubes, ASHRAE
886 Transaction, Vol. 99 (2), pp. 81–89.

887 (28) G. Grossman, (1983), Simultaneous heat and mass transfer in film absorption under
888 laminar flow, International Journal of Heat and Mass Transfer, Vol. 26 (3), pp. 357–371.

889 (29) T. Meyer, F. Ziegler, (2014), Analytical solution for combined heat and mass transfer in
890 laminar falling film absorption using first type boundary conditions at the interface,
891 International Journal of Heat and Mass Transfer, Vol. 73, pp. 141–151.

892 (30) M.J. Kirby, H. Perez-Blanco, (1994), A Design Model for Horizontal Tube
893 Water/Lithium Bromide Absorbers, Heat pump and refrigeration systems design, analysis, and
894 applications, ASME-PUBLICATIONS- AES, Vol. 32, pp. 1-10.

- 895 (31) J. Wu, Z. Yi, Y. Chen, R. Cao, C. Dong, S. Yuan, (2015), Enhanced heat and mass
 896 transfer in alternating structure of tubes and longitudinal trough mesh packing in lithium
 897 bromide solution absorber, International Journal of Refrigeration, Vol. 53, pp. 34-41.
- 898 (32) S. Jeong, S. Garimella, (2002), Falling-film and droplet mode heat and mass transfer in
 899 a horizontal tube LiBr/water absorber, International Journal of Heat and Mass Transfer, Vol. 45
 900 (7), pp. 1445–1458.
- 901 (33) V. M. Soto Francés, J. M. Pinazo Ojer, Experimental study about heat and mass transfer
 902 during absorption of water by an aqueous lithium bromide solution, International Proceedings of
 903 the ASME-ZSITS International Thermal Science Seminar, Bled (Slovenia), 11–14 June, (2000),
 904 pp. 535–542.
- 905 (34) N. Giannetti, A. Rocchetti, S. Yamaguchi, K. Saito, (2017), Analytical solution of film
 906 mass-transfer on a partially wetted absorber tube, International Journal of Thermal Sciences,
 907 Vol. 118, pp. 176-186.
- 908 (35) Y. Chen, R. Cao, J. Wu, Z. Yi, G. Ji, (2016), Alternate heat and mass transfer
 909 absorption performances on staggered tube bundle with M–W corrugated mesh guider inserts,
 910 International Journal of Refrigeration, Vol. 66, pp. 10-20.
- 911 (36) R.H. Wassenaar, (1996), Measured and predicted effect of flowrate and tube spacing on
 912 horizontal tube absorber performance, International Journal of Refrigeration, Vol. 19 (5), pp.
 913 347–355.

914 (37) Y. Lazcano-Véliz, J. Siqueiros, D. Juárez-Romero, L.I. Morales, J. Torres-Merino,
915 (2014), Analysis of effective wetting area of a horizontal generator for an absorption heat
916 transformer, Applied Thermal Engineering, Vol. 62 (2), pp. 845–849.

917 (38) K.S. Lee, B. Köroğlu, C. Park, (2012), Experimental investigation of capillary-assisted
918 solution wetting and heat transfer using a micro-scale, porous-layer coating on horizontal-tube,
919 falling-film heat exchanger, International Journal of Refrigeration, Vol. 35 (4), pp. 1176–1187.

920 (39) B. Köroğlu, K.S. Lee, C. Park, (2013), Nano/micro-scale surface modifications using
921 copper oxidation for enhancement of surface wetting and falling-film heat transfer, International
922 Journal of Heat and Mass Transfer, Vol. 62, pp. 794–804.

923 (40) D. M. Maron, G. Ingel, N. Brauner, (1982), Wettability and break-up of thin films on
924 inclined surfaces with continuous and intermittent feed, Desalination, Vol. 42, pp. 87–96.

925 (41) J. Tang, Z. Lu, B. Yu-Chi, S. Lin, (1991), Minimum Wetting Rate of Film Flow on
926 Solid Surface, Proceedings of the 18th International Congress of Refrigeration, Vol. 2, pp. 519–
927 523, Montreal.

928 (42) J. Mikielewicz, J. R. Moszynski, (1976), Minimum thickness of a liquid film flowing
929 vertically down a solid surface, International Journal of Heat and Mass Transfer, Vol. 19, pp.
930 771–776.

931 (43) N. Giannetti, S. Yamaguchi, K. Saito, (2016), Wetting behaviour of a liquid film on an
932 internally-cooled desiccant contactor, International Journal of Heat and Mass Transfer, Vol. 101,
933 pp. 958–969.

- 934 (44) C.S. Yih, (1963), Stability of liquid flow down an inclined plane, Phyics of Fluids, Vol.
 935 6, pp. 321-334.
- 936 (45) D.E. Hartley, W. Murgatroyd, (1964), Criteria for the break-up of thin liquid layers
 937 flowing isothermally over solid surfaces, International Journal of Heat and Mass Transfer, Vol.
 938 7, pp. 1003-1015.
- 939 (46) A.B. Ponter, G.A. Davies, T.K. Ross, P.G. Thornley, (1967), The influence of mass
 940 transfer on liquid film breakdown, International Journal of Heat and Mass Transfer, Vol. 10, pp.
 941 349-359.
- 942 (47) N. Giannetti, D. Kunita, S. Yamaguchi, K. Saito, (2018), Annular flow stability within
 943 small-sized channels, International Journal of Heat and Mass Transfer, Vol. 116, pp. 1153-1162.
- 944 (48) G. Grossmann, (1982), Simultaneous heat and mass transfer in absorption/desorption of
 945 gases in laminar liquid films, Proc. A.I.Ch.E. Winter and Annual Meeting, Orlando, Florida.
- 946 (49) (1990), properties of lithium bromide-water solutions at high temperatures and947 condensations Part I. Thermal Conductivity, ASHRAE Trans, Vol. 96.

- 948 (50) N. Giannetti, A. Rocchetti, K. Saito, S. Yamaguchi, (2016), Analytical description of
- 949 falling film absorption, Proceedings of the 8th Asian Conference on Refrigeration and Air
- 950 Conditioning, May 15th -17th, Taipei, Taiwan.
- 951 <u>https://www.scopus.com/inward/record.uri?eid=2-s2.0-</u>
- **952** <u>84988972662&partnerID=40&md5=72097627e4b8617ef7168d45b30fd3c1</u>