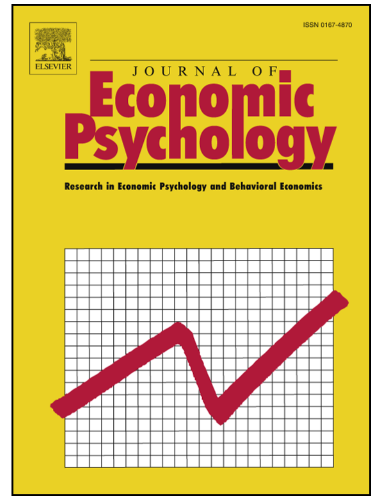

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March 19, 2018

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JEL classification code: D01, D03, D82, D83.

Keywords: persuasion, coarse thinking, peripheral and central route, heuristic and systematic reasoning, information elaboration, arousal.

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In this paper we develop a parsimonious model of decision-making that aims at capturing the determinants of consumers' attitude change in response to persuasive messages that exploit reference cues. Our model is inspired by dual-process theories of information elaboration, but does not introduce purely behavioral traits. The decision-maker receives a message containing an offer of unknown quality together with a reference cue that associates the offer with a category of offers, whose average quality is known. The decision-maker initially exerts little cognitive effort in processing the message, assessing the offer quality on the sole basis of the reference cue (acting as a "coarse thinker"). The decision-maker can then take a decision on the offer without further investigating or she can exert substantial cognitive effort to scrutinize the message carefully and obtain more precise knowledge of the offer quality. The proposed model predicts that the persuader can exploit reference cues to affect attitudes both directly (by inducing acceptance of offers) and indirectly (by inducing low cognitive effort, and hence influencing acceptance). This model matches several predictions of prominent psychological models of attitude change such as the Elaboration Likelihood Model and the Heuristic-Systematic Model.

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1 Introduction

Consider the following situation. A milk producer can bottle milk in either glass or plastic. Glass is more expensive than plastic, and has no impact on milk quality per se. A consumer has to decide whether to buy a bottle of milk, and can choose whether to exert high or low effort in evaluating the quality of the milk offered. Quality is not just milk taste, but expected effects of repeated milk consumption over a longer horizon. If the consumer exerts low effort, then she only relies on the fact that milk is contained in a glass bottle, and she generically thinks of products in glass containers, whose average quality can be higher than that of products in plastic containers. If the consumer exerts high effort, then she reads and understands labels, recovers data from memory or other sources, and elaborates all the information extracted; this is a costly activity, requiring cognitive and search effort, at the end of which she is able to assess the quality of the product. If the milk producer anticipates the behavior of the consumer, then he can use the container – which we call *cue* (or signal) throughout the rest of the paper – to persuade the consumer to buy his product.

In this paper we develop a parsimonious model of persuasion by means of reference cues that aims at capturing the situation described above in a rational and Bayesian framework. Cognitive limitations are modeled as costly information acquisition (Dewatripont and Tirole, 2005) plus coarse reasoning (Mullainathan, 2002), with the decision-maker being perfectly aware of both. From a general perspective, the main contribution of our analysis is the demonstration that introducing these two ingredients into an otherwise standard model of costly communication allows to rationalize the strategic use of cues, even when the target of the communication is a rational and Bayesian decision-maker. From the specific perspective of persuasion activities, our contribution is to show that such setting is rich enough to match several predictions of prominent psychological models of attitude change such as the Elaboration Likelihood Model (Petty and Cacioppo, 1986a) and the Heuristic-Systematic

Model (Eagly and Chaiken, 1993).

We briefly sketch the basic working of our model. The persuader makes an offer to the decision-maker, and provides a reference cue that refers the offer to a category of offers from which the decision-maker can obtain summary information about the expected quality. The decision-maker initially processes all information under *low* elaboration, which leads her to form beliefs on the sole basis of the reference cue. Such elaboration is *automatic*, in the sense that it cannot be avoided by the decision-maker. However, the decision-maker can, at her choice, decide to exert effort in order to acquire and elaborate specific information on the offer quality. In this way available information is processed under *high* elaboration which is a *deliberate* and *controlled* cognitive act.

Low elaboration is not modelled here as bounded reasoning, but as ignorance of which the decision-maker is aware and that she is able to quantify, so that she can form expectations and choose the elaboration level that maximizes expected utility. In short, the decision-maker faces the following trade-off: low elaboration allows to save on effort but it leads to take a decision that, although good on average, is not necessarily the best choice for the current offer. Hence, the decision-maker can well decide not to engage in high elaboration, especially when beliefs about the quality of the current offer are quite extreme (in which case the decision-maker does not expect to learn much from high elaboration). When the persuader anticipates all this, he can make a strategic use of the reference cue.[1]

This sender-receiver setup shares many features with the communication model in Dewatripont and Tirole (2005).[2,3] However, the modeling of the reference cue is different: while

[1] A similar mechanism drives the results in Guo and Zhang (2012), where a single firm that offers products of different quality can choose quality dispersion and prices in order to induce, or prevent, deliberation, which is needed for a consumer to unveil her own valuation of the product (not objective quality).

[2] Other recent contributions considering the costly acquisition of information are Dewatripont (2006), Caillaud and Tirole (2007), Tirole (2009) and Butler et al. (2013).

[3] Brocas and Carrillo (2008) and Brocas (2012) stress that the evidence provided by brain sciences on the multi-system nature of the human brain should be a fundamental source of inspiration for the modeling of decision-making (see Brocas and Carrillo, 2014, for a focused survey).

Dewatripont and Tirole (2005) only consider cues related to the sender's expertise, we consider a reference cue that induces the decision-maker to associate the offer to a category of offers. So, the decision-maker who observes a cue updates her beliefs on the basis of the average quality of offers that belong to the associated category, meaning that she reasons in this respect as a "coarse thinker" (Mullainathan, 2002).[4] Thanks to this our model can account for persuasion activities such as the one considered in the field experiment by Bertrand et al. (2010) – where prospective borrowers are offered loans by mailed advertisement cards with different cues – whereas the model proposed by Dewatripont and Tirole (2005) cannot. Also our model is consistent with how cues are used to influence consumers' perception of quality (see, e.g., Iyer and Kuksov, 2010), and in particular with the fact that very effective cues might have little to do with actual quality of products (Teas and Agarwal, 2000).[5]

Finally, it may be useful to contrast our model with a standard signaling model. Reference cues are actually costly signals, but the information that they convey does not depend exclusively on the behavior of the persuader. In particular, such information is partially exogenous with respect to the specific sender-receiver interaction because the decision-maker reasons coarsely, updating her beliefs on the sole basis of the average quality of offers in the referenced category.[6] So, it is as if the decision-maker ignores not only the quality but also the identity of the current sender of the signal she faces, while she knows the average quality of offers that are accompanied by the same signal. As a result, the behavior of the current sender can have little impact on equilibrium beliefs and the decision-maker never observes out-of-equilibrium signals. Another important difference with a standard signaling model is that in

[4] Other papers considering coarse thinking or thinking by categories are: Mullainathan et al. (2008) on persuasion; Mohlin (2014), Peski (2011) and Fryer and Jackson (2008) on optimal categorization, Ettinger and Jehiel (2010) on coarse understanding of opponents' play, and Mengel (2012) on the evolution of coarse categorization.

[5] This is also suggested by experimental and survey evidence indicating that cues can exert a sizeable impact on consumers' valuation (see, e.g., Sáenz-Navajas et al., 2013 and Woodside, 2012, for wine and food products, respectively).

[6] In Appendix D we provide an example of how to explicitly model the quality of the referenced category and so fully endogenize the informational content of cues.

our model the single-crossing condition does not hold: cues provide no direct benefit to the persuader and they cost the same to all persuader's types. This, however, does not prevent the possibility of separation, thanks to a further crucial difference: the decision-maker has a costly option that provides more information than what can be conveyed through the signal. Indeed, the expected net benefits of sending a given cue can differ across persuader's types, if the cue induces the decision-maker to engage in high elaboration. Perhaps surprisingly, besides standard separation this also allows *reverse* separation to take place, i.e., high quality persuaders sending low cost cues and viceversa, an outcome that is impossible in standard signaling games.[7]

The paper is organized as follows. Section 2 surveys the relevant literature on persuasion and the modeling of scarce cognitive resources. Section 3 presents the model in three steps: the processing of the message (Subsection 3.1), the optimal behavior of the decision-maker (Subsection 3.2), and the optimal behavior of the persuader (Subsection 3.3). Section 4 describes the possible equilibria of the model and discusses why strong enough coarse thinking grants that an equilibrium exists and is unique. Section 5 summarizes the contribution and explores some natural extensions of the model. Finally, the appendices collect proofs and related technical details (Appendix A), the exact conditions for all kinds of persuasion equilibria (Appendix B), examples and results on equilibrium existence and uniqueness (Appendix C), and some further model extensions (Appendix D).

2 Related literature

Several models have been proposed that study how a message can persuade a decision-maker, in both economics and psychology. In economics, we can distinguish between belief-based

[7]Reverse separation should not be confused with counter-signaling (Feltovich et al., 2002), where distant types can be distinguished easily and therefore high types send low signals to separate from medium types (who send high signals to separate from low types) because they are not afraid to be confused with low types.

and preference-based persuasion. Preference-based persuasion is obtained because the message itself impacts on utility and, hence, on behavior (Stigler and Becker, 1977; Becker and Murphy, 1993), as in persuasive advertising (Braithwaite, 1928). Belief-based persuasion affects behavior by changing decision-maker's beliefs, as in informative communication (Stigler, 1961; Telser, 1964) and informative advertising (see, e.g., Bagwell and Ramey, 1993). Information is transferred through signals, that can be costly (Nelson, 1970) or not (Gentzkow and Shapiro, 2006; Kamenica and Gentzkow, 2011). Our model belongs to the class of belief-based persuasion through costly signals.

Among belief-based persuasion models a further distinction can be made between models where agents are perfect Bayesian updaters and models where they are not, e.g., they are constrained by limited memory (Mullainathan, 2002; Shapiro, 2006), they double-count repeated information (De Marzo et al., 2003), they neglect the incentives of the sender (Eyster and Rabin, 2010), or they have limitations in the ability to lie (Glazer and Rubinstein, 2012). Both types of agents can co-exist in the same model (Gabaix and Laibson, 2006). In our model agents are Bayesian, but they are uncertain about the source of the persuasive message.

Economic models of persuasion can also be distinguished on the basis of the nature of the information sent: hard versus soft. Hard information is actually verifiable, while soft information is not. Cheap talk models typically rely on soft information (Crawford and Sobel, 1982) which credibility is increased in the presence of multiple and alternative message dimensions (Chakraborty and Harbaugh, 2014), while models that exploit the strategic use of verifiable information (Milgrom and Roberts, 1986) can consider a verification cost (Caillaud and Tirole, 2007), full and costless verifiability (Glazer and Rubinstein, 2006) or only partial verifiability with the receiver deciding which part to be verified (Glazer and Rubinstein, 2004). In our model the information sent by the persuader is hard.

A large body of psychological evidence suggests that persuasion activities exploit the fact that individuals have two distinct ways of processing information when they receive a message and have to take decisions based on it (Chaiken and Trope, 1999). Theories in cognitive and social psychology that refer to this idea are typically labelled as dual process theories (Evans, 2003), and the two ways of processing information are called System 1 and System 2. Kahneman (2003) refers to System 1 and System 2 as, respectively, *intuition* and *reasoning*. Dual process theories have been applied to explain human behaviors in different setups (Gawronski and Creighton, 2013): persuasion, attitude-behavior relations, prejudice and stereotyping, impression formation, dispositional attribution.

There are two workhorse models of persuasion in psychology that rely on dual information processing. One is the Elaboration Likelihood Model (ELM) (Petty and Cacioppo, 1986a), where the decision-maker can use the "central route" or the "peripheral route". These two routes can be understood as an approximation of a continuum of elaboration intensities which a subject can use when processing information: the higher an individual's cognitive effort, the more likely that she processes all relevant information. At the extremum characterized by highest level of elaboration individuals use all available information and integrate it with already stored information. On the contrary, at the extremum characterized by lowest level of elaboration individuals minimally scrutinize relevant information, extensively using short-cuts to process information. The other model is the Heuristic-Systematic Model (HSM) (Chaiken et al., 1989), where the decision-maker can use "systematic elaboration" or "heuristic elaboration". The basic idea is very similar to that of the ELM: a fully systematic processing of information requires effort and considers all relevant data, while a purely heuristic processing requires minimal effort and considers only a small amount of data. Our paper provides a theoretical bridge between these two workhorse models of persuasion and the belief-based models of persuasion mentioned above, showing that we can retain the focus on Bayesian agents if we allow for some degree of coarse thinking.

From a broader perspective, our paper also belongs to the small but growing body of literature that tries to model the scarcity of human cognitive resources in a tractable and meaningful way, with the central idea that such a scarcity generates an allocation problem that the decision-maker solves rationally. Even if there are notable exceptions (see, e.g., Shugan, 1980, for an analytical measure of the cost of comparing alternatives), only recently there has been an upsurge of interest in the subject. One important paper in this regard is Dewatripont and Tirole (2005), that we have already discussed in the Introduction. Another interesting attempt is the model of decision-making proposed by Dickhaut et al. (2009), where the informativeness of a signal (about the payoff associated with different options) increases in the effort that the decision-maker puts in the observation of the signal. Our model is similar in the modeling of cognitive costs, but adds the possibility of the strategic use of the signal by the sender and the explicit modeling of coarse reasoning under low elaboration. An alternative modelization which is explicitly based on dual process theories of decision-making is provided by Achtziger and Alós-Ferrer (2014) where decisions are the result of two interacting decision processes. One process is controlled, slow and cognitively costly, and it produces a Bayesian updating of beliefs. The second process is automatic, fast and cognitively cheap, and it produces a sort of reinforcement learning that relies on past performance. [8] Our model is similar in spirit, but differs in the focus: while Achtziger and Alós-Ferrer (2014) explore the timing of decisions, we study the outcome in a persuasion setup. In the literature on k-level reasoning, Alaoui and Penta (2015a) introduce costs to access higher levels of reasoning, and perform a cost-benefit analysis that allows to endogenize the level of reasoning; pursuing this line, Alaoui and Penta (2015b) test this model in the lab and find evidence that supports the idea that players weigh the value of thinking deeper against the cost of reasoning. Our model resembles theirs in that there are explicit cognitive costs but in our model the costly activity is only about information processing.

[8] See also Alos-Ferrer (2015) for an application to self-control and Alos-Ferrer and Ritschel (2015) for further experimental evidence.

3 The model

3.1 Message processing: High and low elaboration

The decision-maker (DM) faces a two-part message (q, r) associated with an offer which she has to decide upon. Part $r \in \{x, y\}$ of the message is a cue that references the offer to a category of offers that is either X or Y. Each of these categories contains a finite number of offers, respectively N_x and N_y, that share the same cue, denoted with x and y, respectively. Part $q \in \{G, B\}$ of the message contains the actual quality of the offer: $q = G$ stands for good quality, $q = B$ for bad quality.

Whenever DM is aware of a message (q, r), she immediately processes it under low elaboration, observing r but not q. Then, DM has the option to pay c_e and engage in high elaboration, acquiring the knowledge of q. We denote with $e \in \{H, L\}$ the elaboration effort exerted by DM, where H stands for "engage in high elaboration" and L stands for "stay with low elaboration".

Part r of the message is informative in the sense that it refers to a category of objects of which DM knows the average quality. More precisely, DM has a prior $\mu_0 \in [0, 1]$ that the offer is of quality G which is updated to μ_r when cue r is observed, with μ_r being the expected quality of offers in category r. This models the fact that under low elaboration DM is a coarse reasoner (Fryer and Jackson, 2008; Mullainathan et al., 2008). We denote with χ the strength of DM's coarse thinking. We assume that χ ranges from 0 to ∞, where χ close to 0 means that low elaboration allows in any case a good level of elaboration, making DM uncertain between the current offer and only a few other offers, while χ very high means that thinking is very coarse when DM resorts to low elaboration, and hence the number of offers among which she is unable to distinguish is very large. Formally, we assume that N_x is non-decreasing in χ and $N_x \to \infty$ when $\chi \to \infty$, and analogously for N_y.

Figure 1 is a graphical representation of the different information on quality that can be drawn from the same message depending on the elaboration level chosen by DM. This modeling of message processing embeds the following fundamental feature of the ELM and the HSM:

Remark **1.** *The decision-maker interprets the persuasive message differently under high elaboration and under low elaboration (Petty and Cacioppo, 1986a; Eagly and Chaiken, 1993).*

It should be stressed that the interpretation of the persuasive message is different even in the case low and high elaboration lead to the same behavior: under high elaboration DM is less uncertain about quality, and hence has more extreme beliefs, than if she sticks to low elaboration.

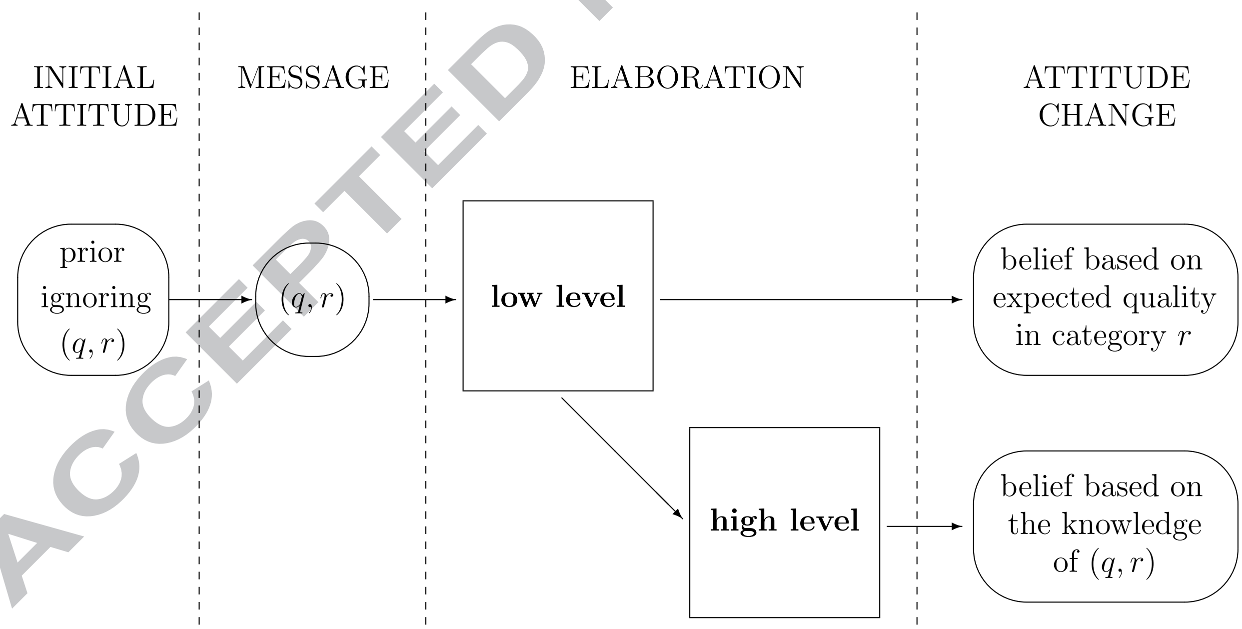

Figure 1: A graphical representation of message processing. Two elaboration levels are possible: high elaboration H which allows to observe (q, r) and low elaboration L which allows to observe only r.

3.2 Decision-maker behavior: Optimal elaboration and decision

After having processed the message, DM has to decide whether to accept the offer, a case denoted with Y, or reject it, denoted with N. The utility obtained by DM depends on the actual quality of the offer and the exerted level of cognitive effort. If DM accepts the offer when $q = G$, then she obtains $\gamma U_G > 0$. If instead she accepts the offer when $q = B$, then she obtains $\gamma U_B < 0$. Here, $\gamma > 0$ is a parameter that represents the motivation of DM, i.e., her perception of how much stake is involved in the offer. If DM rejects the offer, then she obtains a null utility independently of q. In any case, if DM exerts $e = H$, then she also has to bear the elaboration cost c_e.[9]

From the structure of DM's utility immediately follows that, if DM exerts $e = H$, then she finds it profitable to choose Y in case $q = G$ and N in case $q = B$. We indicate such behavior with HYN. We denote behaviors that make use of low elaboration with LY and LN, respectively leading to acceptance and rejection.

Given a belief μ_r, the choice of LY leads to an expected utility of $\mu_r \gamma U_G - (1-\mu_r)\gamma|U_B|$, the choice of HYN leads to an expected utility of $\mu_r \gamma U_G - c_e$, and the choice of LN leads to an expected utility of 0. So, DM's expected utility is maximized by HYN when $\frac{c_e}{\gamma U_G} \leq \mu_r \leq 1 - \frac{c_e}{\gamma |U_B|}$, by LY when $\mu_r \geq \max\left\{1 - \frac{c_e}{\gamma|U_B|}, \frac{|U_B|}{U_G + |U_B|}\right\}$, and by LN when $\mu_r \leq \min\left\{\frac{c_e}{\gamma U_G}, \frac{|U_B|}{U_G+|U_B|}\right\}$.[10] The optimal behavior by DM as a function of μ_r is summarized in Figure 2. This leads to the following result (of which we omit the trivial proof):

PROPOSITION **1** (Decision-maker's optimal behavior).

[9]In this paper we stick to the interpretation that Y means acceptance and N means rejection. However we stress that other interpretations are possible. For instance, if the offer is a consumer good to be bought, then Y could mean to buy a large quantity of the good and N to buy a small quantity; also, Y could mean to pay a high price per unit while N to pay a low price per unit. Both cases can easily be accommodated by the model since what really matters for the results is that Y is preferred when quality is G and N is preferred when quality is B. We observe that U_G and U_B can be interpreted as the relative gains of choosing Y over N when, respectively, quality is G and quality is B.

[10]Note that HYN turns out to be optimal for some intermediate range of values of μ_r only if $c_e \leq \frac{U_G|U_B|}{U_G+|U_B|}$, although when equality holds there is just one value of p for which HYN is optimal and for that value DM is indifferent between HYN, LN, and LY.

Let μ_r be DM's belief about q after the automatic low elaboration of (q, r). Then, DM's optimal behavior is:

- *HYN, when μ_r is not too extreme and c_e is sufficiently low;*
- *LY, when μ_r is sufficiently high;*
- *LN, when μ_r is sufficiently low.*

(a) when $c_e < \frac{U_G|U_B|}{U_G+|U_B|}$.

(b) when $c_e > \frac{U_G|U_B|}{\gamma(U_G+|U_B|)}$.

Figure 2: DM's optimal behavior as a function of μ.

So, the more extreme the beliefs induced by the reference cue, the more likely that DM does not engage in high elaboration and takes action relying on the information provided by the cue. Moreover, the effects of a larger γ capture the following feature of the ELM and the HSM:

Remark **2.** *The recourse to high elaboration is more likely if DM's motivation (or need for cognition) is higher, even if motivation does not affect the relative desirability of good versus bad offers (Petty and Cacioppo, 1979, 1981; Cacioppo et al., 1983; Petty et al., 1981).*

To see why, consider an increase from γ' to γ''. This triggers two distinct effects, which both make the recourse to high elaboration more likely. First, H becomes possibly optimal (for some belief) for a wider range of values of U_G and U_B, since $\gamma' U_G|U_B|/(U_G + |U_B|) < \gamma'' U_G|U_B|/(U_G + |U_B|)$ (see footnote 10). Second, H becomes optimal for a wider range of beliefs on quality, including now beliefs that are a bit more extreme, since $c_e/(\gamma' U_G) > c_e/(\gamma'' U_G)$ and $1 - c_e/(\gamma'|U_B|) < 1 - c_e/(\gamma'|U_B|)$. It should be noted that there are no effects on the likelihood of acceptance of the offer under L, since the threshold $|U_B|/(U_G + |U_B|)$ is unaffected by γ. Hence, motivation modifies the likelihood of acceptance only through its impact on the likelihood of high elaboration.

Finally, the effects of a smaller c_e are similar to those of a larger γ but capture a different feature of the ELM and the HSM:

Remark **3.** *A greater ability to think and focus on the content of the message makes it more likely that the decision-maker recurs to high elaboration (Petty et al., 1976; Petty and Cacioppo, 1986b).*

It is worth stressing that, although the effects in terms of attitude change induced by a smaller c_e are qualitatively the same as those induced by a larger γ, the model does not predict observational equivalence between a reduction in c_e and an increase in γ. Indeed, a given individual has a fixed c_e for a fixed complexity of the message, while she typically has a value of γ that depends on the nature and context of the offer. So, it remains possible to separate the effects of c_e from the effects of γ by observing the same DM receiving messages of fixed complexity but concerning offers towards which DM has different intensities of motivation.

3.3 Persuader behavior: Strategic use of reference cues

In the initial stage of the game the source of the offer is determined: either from the persuader (P) or from someone else. The number of offers coming from P is denoted with N_P, while the number of offers coming from someone else is denoted with N_{-P}. More precisely, $N_{-P}(x)$ denotes the number of offers in category X not coming from P, and similarly $N_{-P}(y)$ denotes the number of offers in category Y not coming from P, with $N_{-P}(x) + N_{-P}(y) = N_{-P}$. Therefore, $N_x = N_{-P}(x) + N_P$ and $N_y = N_{-P}(y)$ if P's offers have cue x, while $N_x = N_{-P}(x)$

and $N_y = N_{-P}(y) + N_P$ if P's offers have cue y. Assuming that offers are equally likely, we have that $\alpha_x = N_{-P}(x)/(N_{-P} + N_P)$ is the probability that the offer does not come from P and belongs to the category X, $\alpha_y = N_{-P}(y)/(N_{-P} + N_P)$ is the probability that the offer does not come from P and belongs to the category Y, and $\alpha_P = N_P/(N_{-P} + N_P)$ is the probability that the offer comes from P. As the same offer may be made repeatedly, parameters α_x, α_y, and α_P are basically frequencies of occurrence. Importantly, N_P does not depend on the strength of DM's coarse thinking χ. Therefore, α_P is non-increasing in χ, $\alpha_P \to 0$ when $\chi \to \infty$, both α_x and α_y are non-decreasing in χ and $\alpha_x + \alpha_y \to 1$ when $\chi \to \infty$.

We denote with $\beta_x \in (0, 1)$ the probability that $q = G$ conditional on the offer not coming from P and r being equal to x. Similarly, we denote with $\beta_y \in (0, 1)$ the probability that $q = G$ conditional on the offer not coming from P and r being equal to y. Hence, parameters β_x and β_y represent the fraction of good quality offers in, respectively, category X and Y, when the behavior of P is not taken into account. We assume that $\beta_x > \beta_y$, i.e., on average x refers to a higher quality than y.[11]

Instead, if the offer comes from P, then the game unfolds as follows: $q = G$ with probability α_G while $q = B$ with probability $\alpha_B = 1 - \alpha_G$, and then P chooses $r \in \{x, y\}$. The cost for P of choosing reference cue r is c_r. Since $\beta_x > \beta_y$, to rule out uninteresting cases we assume that $c_x > c_y$; also, c_y is normalized to zero. Quality q is known to P, so a strategy for P is a function $\rho : \{G, B\} \longrightarrow \{x, y\}$ indicating which reference cue is chosen conditionally on the quality of the offer. The utility for P is $V > c_x$ in case DM accepts the offer while it is 0 if DM rejects the offer. In any case, the cost c_r must be borne.[12]

The choices that maximize P's utility are easily established, since P obtains V only if

[11]An example showing how to endogenize β_x and β_y is provided in Appendix D.3.

[12]The zero utility obtained by P when DM chooses N should be seen as a normalization, so to be consistent with different interpretations of Y and N; for instance, if N means to buy at a low price or a small quantity, then P would earn a low but possibly positive profit.

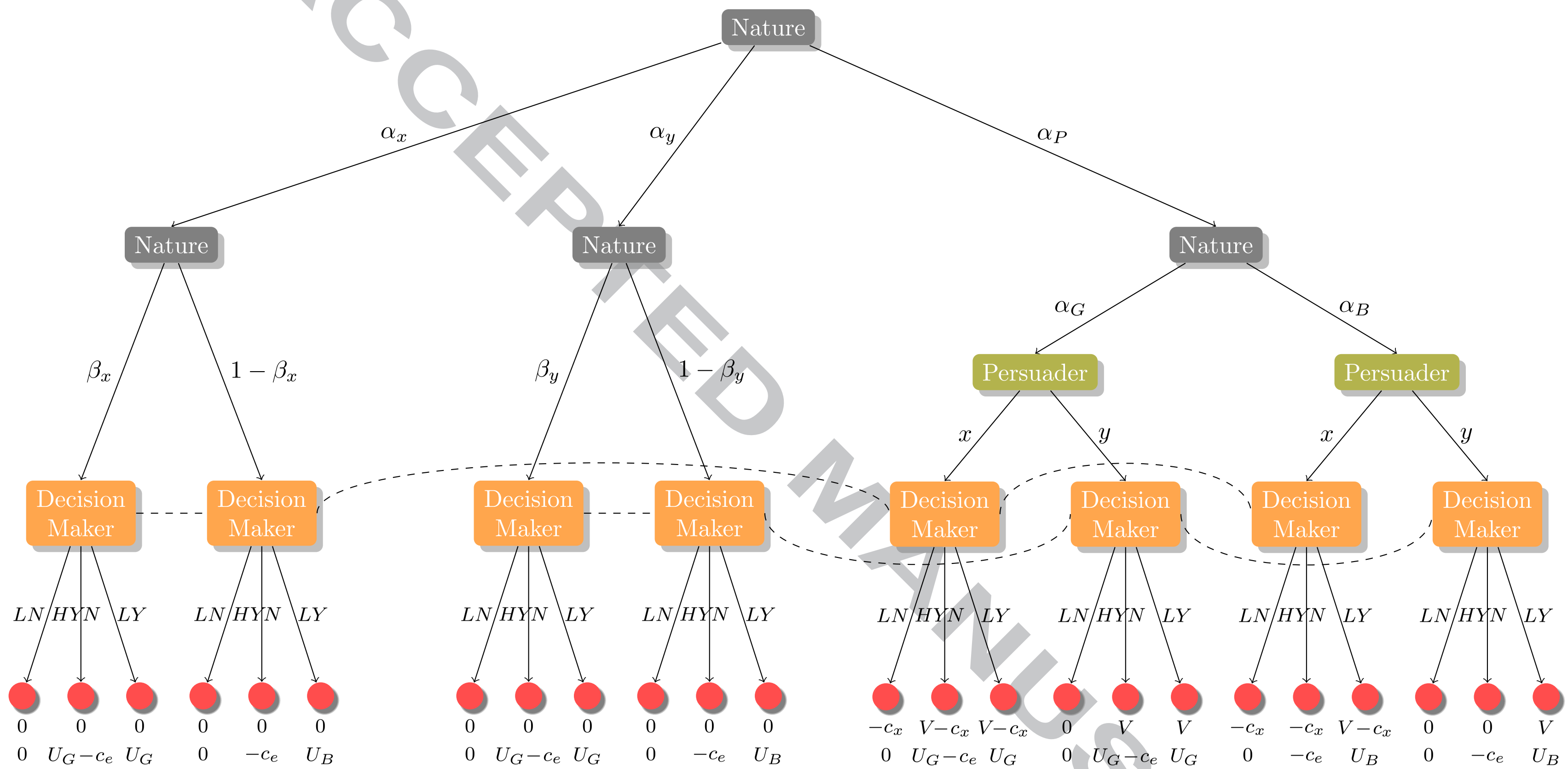

Figure 3: The game tree. In the left and the central branch DM faces an offer that does not come from P. In the left branch the offer is characterized by cue x and has probability β_x to be of quality G. In the central branch the offer is characterized by cue y and has probability β_y to be of quality G. Since in these parts of the game P is not involved, his payoff is set equal to 0. In the right branch DM faces an offer made by P, which has probability α_G to be of quality G. DM has two information sets each of which encompasses nodes from all three branches; one set is associated with reference cue x and the other set with reference cue y.

DM reacts with Y, while P incurs the cost of reference c_r independently of DM's choices. So, P wants the offer to be accepted by DM, but while it is enough that DM plays HYN when $q = G$, when $q = B$ it is necessary that DM plays LY. Also, everything else being equal, P strictly prefers to send cue y, since it costs less than cue x. This leads to the following proposition summarizing P's best replies to the optimal choices by DM, as described in Proposition 1 (the proof can be found in Appendix A):

PROPOSITION **2** (Persuader's optimal behavior).
P's optimal behavior is to send:

- *cue y, when DM reacts in the same way to x and y;*
- *cue x, when DM reacts to x with LY and to y with LN;*
- *cue x if $q = G$ and cue y if $q = B$, when DM reacts to x with HYN and to y with LN;*
- *cue x if $q = B$ and cue y if $q = G$, when DM reacts to x with LY and to y with HYN.*

The last two bullets of Proposition 2 show that the model is consistent with another feature of the ELM and HSM:

REMARK **4.** *The persuader can use cues to affect the decision-maker's choice of the elaboration level and, through it, the decision-maker's attitudes (Petty and Cacioppo, 1986b; Chaiken and Trope, 1999).*

As an example, suppose that upon observation of cue x DM forms the posterior belief $\mu_x > 1 - c_e/(\gamma|U_B|)$. So, when DM sees x, she decides to *accept the offer without engaging in high elaboration*. Hence, if $\mu_y < |U_B|/(U_G + |U_B|)$ and $q = B$, P can send x instead of cue y and so doing he can change the attitude of DM from *reject* to *accept* and *prevent high elaboration*.

Finally, it is important to stress that P's choice of reference cue can affect the expected quality of offers from categories X and Y when DM takes P's behavior into account. Applying Bayes' rule, DM's expected quality of an offer showing reference cue r is given by:

$$\hat{\beta}_r = \frac{\alpha_r \beta_r + \alpha_P \mathbb{I}_r(G)}{\alpha_r + \alpha_P(\alpha_G \mathbb{I}_r(G) + \alpha_B \mathbb{I}_r(B))} \tag{1}$$

where $\mathbb{I}_r(q)$ is an indicator function that takes value 1 if the persuader of type q sends cue r and 0 otherwise. So, the exact value taken by $\hat{\beta}_r$ depends on P's behavior. The different possible values of $\hat{\beta}_r$ are listed in Appendix A.

4 Persuasion equilibria and coarse thinking

Since α_x, α_y, β_x, β_y are assumed to be strictly comprised between 0 and 1, DM faces each combination of offer quality and reference cue with positive probability. Therefore, every Bayes-Nash equilibrium, to which we refer as *persuasion equilibrium*, is also sequential, and, hence, weak perfect Bayesian.[13]

When does a persuasion equilibrium exist? When is it unique? It turns out that the strength of DM's coarse thinking χ affects both existence and uniqueness of persuasion equilibria. This is summarized by the following proposition, whose proof can be found in Appendix A:

PROPOSITION **3** (Existence and uniqueness of persuasion equilibrium).
If χ is large enough, then almost always an equilibrium exists and is unique.

To see why a strong coarse thinking guarantees both existence and uniqueness of a persuasion equilibrium it is first useful to observe that a strong enough dependency of $\hat{\beta}_x$ and $\hat{\beta}_y$ on P's

[13]This is because there are no out-of-equilibrium information sets, and hence no out-of-equilibrium beliefs. We also note that such an absence makes it impossible to refine the set of equilibria by applying criteria that rule out some scarcely plausible out-of-equilibrium beliefs (such as, e.g., the Intuitive Criterion or D1).

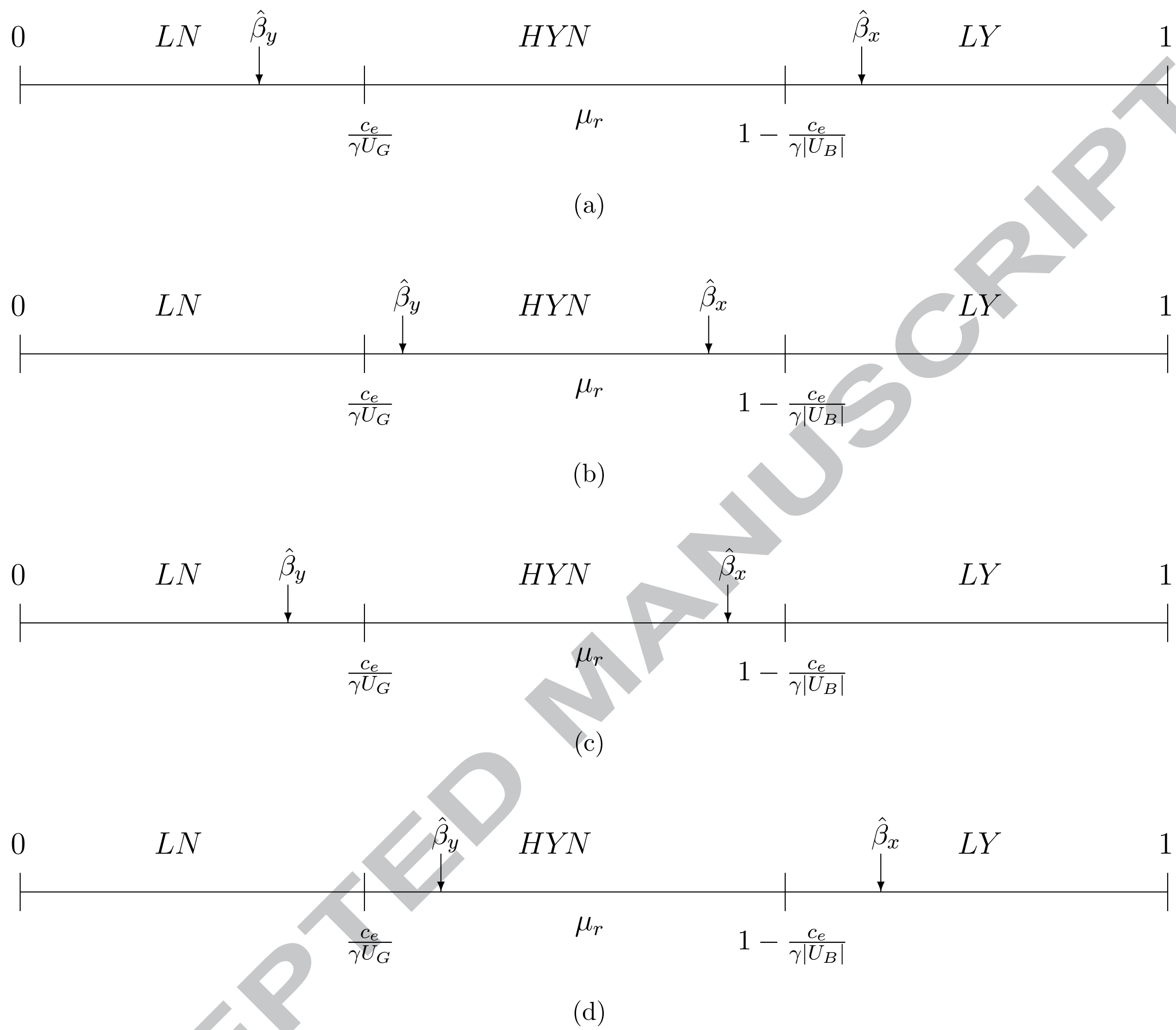

Figure 4: Panel (4a) shows a pooling equilibrium where sending cue x is optimal for both G and B. Panel (4b) shows a pooling equilibrium where sending cue y is optimal for both G and B. Panel (4c) shows a separating equilibrium where sending cue x is optimal for G, and sending cue y is optimal for B. Panel (4d) shows a separating equilibrium where sending cue y is optimal for G, and sending cue x is optimal for B.

choices is necessary for both inexistence and multiplicity (see an example in Appendix C). However, the dependency of DM's posteriors on P's behavior decreases as coarse thinking gets stronger because the impact of P's behavior becomes less relevant for the expected quality associated with each reference cue.

As it is typical of communication models, there are two main kinds of equilibria: *pooling* equilibria where P always sends x or always sends y, and *separating* equilibria where P sends x when the offer is of good quality and y when the offer is of bad quality, or viceversa. All four kinds of equilibria can occur, depending on the model parameters. We do not give here the exact conditions on parameters that make a particular kind of equilibrium possible (we refer the interested reader to Appendix B). Instead, we give an example for each kind of equilibrium, providing a graphical illustration in terms of DM's beliefs and optimal behavior.

Figure 4 illustrates the four kinds of equilibria, one per panel. In all panels the cost of elaboration is low enough for HYN to be DM's optimal behavior for an intermediate range of beliefs. Panel 4a represents a pooling equilibrium where P always sends the costly cue x. The belief associated with cue x is high enough, so that DM chooses LY, while the belief associated with y is quite low, so that DM chooses LN.

Panel 4b represents a pooling equilibrium where P always sends the cheap cue y. Here DM's beliefs are sufficiently similar for both x and y so that DM chooses HYN for all cues. In this equilibrium, P cannot do much to influence DM's attitude: actual quality always determine acceptance.

Panel 4c represents a separating equilibrium where P sends the costly cue x when the offer is of good quality while he sends the cheap cue y when the offer is of bad quality. The belief associated with x is intermediate so that DM chooses HYN and, at the same time, the belief associated with cue y is so low that DM chooses LN. In this equilibrium P is able to persuade DM to accept, but only if his offer is of good quality. This is similar to what happens in a separating equilibrium of a standard signaling model: the good type sends the high signal and the bad type sends the low one.

Finally, the alternative kind of separating equilibrium is depicted in panel 4d, where the belief associated with x is so high that DM chooses LY and the belief associated with cue y is not too low so that DM chooses HYN. In this equilibrium P is able to persuade DM to accept, even when his offer is of bad quality. Moreover, and somewhat surprisingly, this is attained with P's types that separate in *reverse* with respect to what happens in a separating equilibrium of a standard signaling model: the bad type sends the high signal while the good type sends the low one.

We conclude by noting that successful persuasion under low elaboration is not equivalent to successful persuasion under high elaboration, in line with the indications coming from the ELM and the HSM:

Remark **5.** *Persuasion under high elaboration is stabler than persuasion under low elaboration (Petty and Cacioppo, 1986b; Haugtvedt and Petty, 1992).*

Attitude change is less stable under low elaboration because the posterior beliefs obtained without engaging in high elaboration are more sensitive to new information regarding the offer since the DM is more uncertain about q. Indeed, if the DM receives additional and unexpected information about the offer, after the choice of the elaboration level, but prior to the choice to accept or reject the offer, then such additional information is more likely to affect her beliefs under low elaboration.

5 Discussion

In this paper we have framed persuasion activities within a sender-receiver model where the sender wants to persuade the receiver to accept his offer. Both agents are rational and Bayesian, but the decision-maker has to pay a cognitive cost to extract all information from the message sent by the persuader. The novelty of our approach is the consideration of two distinct intensities of the elaboration of information: an initial automatic low elaboration that requires little cognitive effort but entails coarse thinking – i.e., the decision-maker heuristically relies on category-wide information to assess the quality of the offer – and deliberative high elaboration that is more costly in terms of cognitive resources but provides precise information on the quality of the offer. This setup has allowed us to endow the persuader with a novel strategic tool of persuasion – i.e., reference cues – that we model as references to categories of objects. Despite its simplicity, the proposed model has proved rich enough to rationalize a number of well documented findings from cognitive and social psychology of persuasion. Further, the model lends itself to a number of straightforward extensions that can expand its scope. In what follows we provide a non-exhaustive list.

The decision-maker might not be constrained to choose sharply between high and low elaboration, but able to select an elaboration intensity out of an elaboration continuum (Petty and Cacioppo, 1986b). The model presented in this paper can be easily adapted to this idea: engaging in high elaboration, the decision-maker does not directly observe q but extracts a signal σ which conveys information on q with a precision $p(e)$ that depends on the elaboration intensity $e \geq 0$ that is chosen. We observe that relaxation of the hypothesis of just two elaboration levels gives an extra role to U_G and U_B: under HYN the optimal elaboration intensity increases, decreases, or is constant in expected quality depending on whether $U_G > |U_B|$, $U_G < |U_B|$, or $U_G = |U_B|$. This is because the nature of the stake – i.e., whether it is most important to get the good offer or to avoid the bad one – determines whether direct information on quality and expected quality are complements or substitutes.

Prejudice can affect the elaboration level as well as induce biased elaboration (Petty and Cacioppo, 1986b; Petty et al., 1999). Prejudice can be formalized by allowing biased priors about the relative size of offer categories, i.e., α_x, α_y, the average quality in each category, i.e., β_x and β_y, and the likelihood that the persuader has a good quality offer, i.e., α_G. Any such bias leads to the calculation of expected qualities $\tilde{\beta}_x$ and $\tilde{\beta}_y$ which are biased with respect to the correct ones, i.e., $\hat{\beta}_x$ and $\hat{\beta}_y$. The decision to engage in high elaboration would then be affected by prejudice, being based on $\tilde{\beta}_x$ and $\tilde{\beta}_y$. Further, prejudice can have effects even *after* high elaboration whenever effortful elaboration does not guarantee perfect knowledge of the offer quality, as discussed above.

The persuader can send arousing and other mood-affecting cues to induce low elaboration (Petty and Cacioppo, 1986b; Sanbonmatsu and Kardes, 1988). The model can accommodate this as follows. Consider, besides x and y, a further characteristic of the offer: a mood cue $m \in \{a, n\}$, where $m = a$ means that the cue induces arousal and $m = n$ that it does not. In this setup there are four offer categories identified by the pairs (x, a), (y, a), (x, n), and (y, n). In addition to the cost of the reference cue (x or y), sending the mood cue a costs $c_a > 0$, while n costs nothing. Under low elaboration, the decision-maker observes both r and m, and if $m = a$ the cost of engaging in high elaboration increases from c_e to $c_e^a > c_e$. So, the persuader can use the mood cue together with a reference cue in order to induce low elaboration and, thanks to this, possibly obtain the acceptance of his offer.

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Appendices

Preliminarily, we denote with $\hat{\beta}_r(\rho_G, \rho_B)$ the belief on quality conditional on cue $r \in \{x, y\}$ given P's behavior $\rho(G) \in \{x, y\}$, $\rho(B) \in \{x, y\}$. Such beliefs are derived by means of the Bayes rule. Also, we denote a strategy for DM with function $\delta : \{x, y\} \rightarrow \{LY, LN, HYN\}$ (we neglect behaviors which make use of high elaboration but do not react optimally to the knowledge of actual quality).

A Proofs of main results and related technical details

A.1 Beliefs on quality conditional on cue given P's behavior

From (1), it directly follows that:

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=y) = \beta_x \tag{2}$$

$$\hat{\beta}_x(\rho(G)=y, \rho(B)=x) = \frac{\alpha_x \beta_x}{\alpha_x + \alpha_P \alpha_B} \tag{3}$$

$$\hat{\beta}_x(\rho(G)=x, \rho(B)=x) = \frac{\alpha_x \beta_x + \alpha_P \alpha_G}{\alpha_x + \alpha_P} \tag{4}$$

$$\hat{\beta}_x(\rho(G)=x, \rho(B)=y) = \frac{\alpha_x \beta_x + \alpha_P \alpha_G}{\alpha_x + \alpha_P \alpha_G} \tag{5}$$

$$\hat{\beta}_y(\rho(G)=y, \rho(B)=y) = \frac{\alpha_y \beta_y + \alpha_P \alpha_G}{\alpha_y + \alpha_P} \tag{6}$$

$$\hat{\beta}_y(\rho(G)=y, \rho(B)=x) = \frac{\alpha_y \beta_y + \alpha_P \alpha_G}{\alpha_y + \alpha_P \alpha_G} \tag{7}$$

$$\hat{\beta}_y(\rho(G)=x, \rho(B)=x) = \beta_y \tag{8}$$

$$\hat{\beta}_y(\rho(G)=x, \rho(B)=y) = \frac{\alpha_y \beta_y}{\alpha_y + \alpha_P \alpha_B} \tag{9}$$

A.2 Proof of Proposition 2

Proof. Suppose that DM chooses $\delta(x) = \delta(y)$. If $\delta(x) = \delta(y) = LY$ or $\delta(x) = \delta(y) = HYN$ and P is of type G, then P's utility is $V - c_r$. If instead $\delta(x) = \delta(y) = LN$ or $\delta(x) = \delta(y) = HYN$ and P is of type B, then P's utility is $-c_r$. Since $0 = c_y < c_x$, $r = y$ is optimal for P independently of his type.

Suppose that DM chooses $\delta(x) = LY$ and $\delta(y) = LN$. P's utility is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for P independently of his type.

Suppose that DM chooses $\delta(x) = HYN$ and $\delta(y) = LN$. If P is of type G then his utility is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for type G. If P is of type B then his utility is equal to $-c_x$, if $r = x$, and to 0, if $r = y$. Since $0 < c_x$, $r = y$ is optimal for type B.

Suppose that DM chooses $\delta(x) = LY$ and $\delta(y) = HYN$. If P is of type G then his utility is equal to $V - c_x$, if $r = x$, and to V, if $r = y$. Since $0 = c_y < c_x$, $r = y$ is optimal for type G. If P is of type B then his utility is equal to $V - c_x > 0$, if $r = x$, and to 0, if $r = y$. Hence, $r = x$ is optimal for type B. □

A.3 Proof of Proposition 3

Proof. We observe that $\hat{\beta}_x(\rho(G), \rho(B))$ converges to β_x and $\hat{\beta}_y(\rho(G), \rho(B))$ converges to β_y irrespective of ρ when the degree of coarse thinking grows larger and larger, i.e., χ tends to infinity. This is evident when looking at (2), (3), (4), (5), (6), (7), (8) and (9).

We assume that β_x and β_y are interior points to the intervals of beliefs that determine DM's best choices according to Proposition 1. More precisely, β_x and β_y are both different from $c_e/\gamma U_G$ and $1 - c_e/\gamma|U_B|$ if $c_e < \gamma U_G|U_B|/\gamma(U_G + |U_B|)$, and different from $|U_B|/(U_G + |U_B|)$ if $c_e \geq U_G|U_B|/\gamma(U_G + |U_B|)$. By so doing we are neglecting values that have measure zero in the parameters space.

We now build a profile that we then check to be an equilibrium. We set $\delta(x)$ and $\delta(y)$ equal to the best action by DM against a belief equal to β_x and β_y, respectively, as shown by Proposition 1. We set $\rho(G)$ and $\rho(B)$ equal to the best action by P conditional on G and B, respectively, against $\delta(x)$ and $\delta(y)$, as shown by Proposition 2.

By construction, in the above profile P is best replying to δ, while DM is best replying given β_x and β_y, which are not equilibrium beliefs. However, we can choose χ high enough that, whatever ρ is chosen by P, $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$ are arbitrarily close to β_x and β_y, respectively. This means that DM is best replying even against $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$, since β_x and

β_y are interior points to the intervals of beliefs that determine DM's best choices.

We have just proved equilibrium existence. To understand that such equilibrium is unique, we simply observe that, when χ is large enough, the best reply by DM is uniquely determined whatever strategy is chosen by P, and P's best reply against such an optimal behavior by DM is uniquely determined as well. $\square$

B Persuasion equilibria

Preliminarily, let $\delta : \{x, y\} \rightarrow \{LY, HYN, LN\}$ be the function describing DM's behavior conditional upon the observation of the reference cue.

B.1 Pooling equilibria: High and low signals

We say that an equilibrium is pooling with high signal if $\rho(G) = \rho(B) = x$. Note that the rejection of any offer associated with cue y, i.e., $\delta(y) = LN$, and the acceptance of any offer associated with cue x, i.e., $\delta(x) = LY$, is the only behavior by DM that can sustain a pooling equilibrium with high signal. Indeed, in such a case P can have his offer accepted only by using the reference cue x, and this independently of whether he is of type G or of type B. We also note that a pooling equilibrium with high signal can exist for low elaboration costs – so that HYN is optimal for intermediate values of expected quality – and for high elaboration costs – so high that HYN is never optimal. The following proposition provides the conditions for the existence of a pooling equilibrium with high signal.

Proposition 4 (Pooling equilibrium with high signal). *The profile (ρ, δ) such that $\rho(G) = \rho(B) = x$, $\delta(x) = LY$, $\delta(y) = LN$ is an equilibrium if and only if:*

(4.1) $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{\gamma |U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$;

(4.2) $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$.

There is no other equilibrium profile (ρ, δ) such that $\rho(G) = \rho(B) = x$, whatever the values of $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$.

Proof. The last claim of the proposition follows directly from Proposition 2: for $r = x$ to be P's optimal choice independently of his type, DM's choice must be such that $\delta(x) = LY$ and $\delta(y) = LN$.

So, let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that

4.1 must hold for DM to find $\delta(x) = LY$ optimal and that 4.2 must hold for DM to find $\delta(y) = LN$ optimal.

Suppose now that 4.1 and 4.2 hold. Then, from Proposition 1 follows that $\delta(x) = LY$ and $\delta(y) = LN$ is optimal for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. □

We say that an equilibrium is pooling with low signal if $\rho(G) = \rho(B) = y$. The existence of such an equilibrium depends on the convenience for DM to behave in the same way when observing x or y, i.e., $\delta(x) = \delta(y)$. Indeed, when this occurs, P clearly finds it optimal to choose y irrespectively of whether he is of type G or of type B, since DM's behavior is not affected by the choice of the reference cue, and y costs less than x. We note that there are variants of this type of equilibrium, depending on the behavior held by DM. If elaboration costs are so large that HYN is never optimal then there are two cases: either $\delta(x) = \delta(y) = LN$ or $\delta(x) = \delta(y) = LY$. Otherwise, if elaboration costs are not so large, then there is also a third possibility, namely that $\delta(x) = \delta(y) = HYN$. The following proposition provides the conditions for the existence of a pooling equilibrium with low signal for each of the three variants.

Proposition **5** (Pooling equilibrium with low signal)**.**

5.1 *The profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = LN$ is an equilibrium if and only if:*

5.1.1 $\hat{\beta}_x(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$*;*

5.1.2 $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$.

5.2 *The profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = HYN$ is an equilibrium if and only if:*

5.2.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|\gamma U_B|}$*;*

5.2.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|\gamma U_B|}$.

5.3 The profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, $\delta(x) = \delta(y) = LY$ is an equilibrium if and only if:

5.3.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|\gamma U_B|}$ and $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$;

5.3.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{|\gamma U_B|}$ and $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$.

There is no other equilibrium profile (ρ, δ) such that $\rho(G) = \rho(B) = y$, whatever the values of $\hat{\beta}_x(\rho(G), \rho(B))$ and $\hat{\beta}_y(\rho(G), \rho(B))$.

Proof. The last claim of the proposition follows directly from Proposition 2: for $r = y$ to be P's optimal choice independently of his type, DM's choice must be such that $\delta(x) = \delta(y)$.

Suppose (ρ, δ) of 5.1 be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 5.1.1 and 5.1.2 must hold for to find $\delta(x) = \delta(y) = LN$ optimal.

Suppose now that 5.1.1 and 5.1.2. Then, from Proposition 1 follows that $\delta(x) = \delta(y) = LN$ is optimal for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium.

Claims 5.2 and 5.3 can be proved with analogous arguments. □

B.2 Separating equilibrium: High quality going with high signal

We say that an equilibrium is separating with high quality going with high signal if $\rho(G) = x$ and $\rho(B) = y$. Note that this behavior by P can form an equilibrium only if DM chooses $\delta(x) = HYN$ and $\delta(y) = LN$. In such a case, the persuader of type G finds it optimal to incur the cost of sending x, since this leads his offer to be accepted, while the persuader of type B prefers to save on costs and send reference cue y, since in no case his offer will be accepted. Note also that for this equilibrium to exist elaboration costs must be low enough so that DM actually best replies with HYN for intermediate values of μ.

Proposition **6** (Separating equilibrium with signaling).

The profile (ρ, δ) such that $\rho(G) = x$, $\rho(B) = y$, $\delta(x) = HYN$, $\delta(y) = LN$ is an equilibrium if and only if:

6.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{|\gamma U_B|}$*;*

6.2 $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq \frac{|U_B|}{U_G + |U_B|}$*.*

Proof. Let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 6.1 must hold for DM to find $\delta(x) = HYN$ optimal and that 6.2 must hold for DM to find $\delta(y) = LN$ optimal.

Suppose now that 6.1 and 6.2 hold. Then, from Proposition 1 follows that $\delta(x) = HYN$ and $\delta(y) = LN$ is an optimal choice for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. □

B.3 Separating equilibrium: High quality going with low signal

We say that an equilibrium is separating with high quality going with low signal if $\rho(G) = y$ and $\rho(B) = x$. Note that this behavior by P can form an equilibrium only if DM chooses $\delta(y) = HYN$ and $\delta(x) = LY$. Indeed, in such a case the persuader of type G finds it optimal to save on costs and send y, since this leads nevertheless his offer to be accepted, while the persuader of type B finds it optimal to pay the cost of sending x, since this is the only way to have his offer accepted. Note also that, as for the other separating equilibrium, in order for this equilibrium to exist elaboration costs must be low enough so that DM actually best replies with HYN for intermediate values of μ.

Proposition **7** (Separating equilibrium with reverse-signaling)**.**
The profile (ρ, δ) *such that* $\rho(G) = y$*,* $\rho(B) = x$*,* $\delta(x) = LY$*,* $\delta(y) = HYN$ *is an equilibrium if and only if:*

7.1 $\hat{\beta}_x(\rho(G), \rho(B)) \geq 1 - \frac{c_e}{\gamma |U_B|}$ *and* $\hat{\beta}_x(\rho(G), \rho(B)) \geq \frac{|U_B|}{U_G + |U_B|}$*;*

7.2 $\hat{\beta}_y(\rho(G), \rho(B)) \geq \frac{c_e}{\gamma U_G}$ *and* $\hat{\beta}_y(\rho(G), \rho(B)) \leq 1 - \frac{c_e}{\gamma |U_B|}$*.*

Proof. Let (ρ, δ) be an equilibrium. Note that, along the equilibrium path, $\mu = \hat{\beta}_x(\rho(G), \rho(B))$ if DM sees $r = x$ and $\mu = \hat{\beta}_y(\rho(G), \rho(B))$ if DM sees $r = y$. Hence, from Proposition 1 follows that 7.1

must hold for DM to find $\delta(x) = LY$ optimal and that 7.2 must hold for DM to find $\delta(y) = HYN$ optimal.

Suppose now that 7.1 and 7.2 hold. Then, from Proposition 1 follows that $\delta(x) = LY$ and $\delta(y) = HYN$ is an optimal choice for DM. Hence, by Proposition 2 we can conclude that the profile (ρ, δ) is an equilibrium. $\square$

C Multiplicity and existence of persuasion equilibria: Examples and results

C.1 Multiple equilibria can coexist

An example of multiplicity of equilibria is depicted in Figure 5, where the following two equilibria coexist: a pooling equilibrium with high signal and a separating equilibrium with high quality going with low signal.

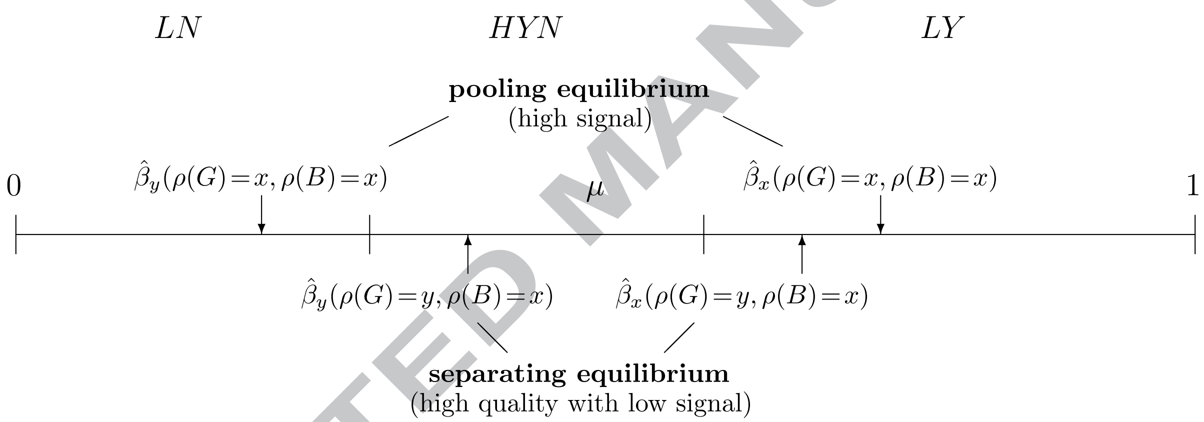

Figure 5: An example of the coexistence of two equilibria, for $c_e < \frac{U_G|U_B|}{\gamma(U_G+|U_B|)}$.

Looking at Figure 5 we see that, when both types of P choose cue x, then the expected quality $\hat{\beta}_x$ is high and DM's best reply is LY, while the expected quality $\hat{\beta}_y$ is so low that DM's best reply is LY; this justifies the pooling equilibrium where both type G and type B choose the high signal x. At the same time, if type G switches from x to y, it may happen that $\hat{\beta}_x$ remain high enough to have that DM best replies with LY, and $\hat{\beta}_y$ raises entering the region where DM best replies with HYN; this is what occurs in the case represented in the figure, and it justifies the reverse-signaling equilibrium.

We note, however, that not all types of equilibria can coexist. The following proposition lists the possible cases of coexistence of equilibria:

Proposition **8** (Equilibrium multiplicity and coexistence).

Multiple equilibria can exist, but only in the following pairs:

- *a pooling where* $\rho(G)=\rho(B)=x$ *and a separating where* $\rho(G)=y$ *and* $\rho(B)=x$*;*
- *a pooling where* $\rho(G)=\rho(B)=y$ *and a separating where* $\rho(G)=x$ *and* $\rho(B)=y$*;*
- *a pooling where* $\rho(G)=\rho(B)=x$ *and a pooling where* $\rho(G)=\rho(B)=y$*.*

The proof of Proposition 8 can be found in Subsection C.3. Here we provide the intuition why a reverse-signaling equilibrium – where high quality goes with low signal – cannot coexist with a signaling equilibrium – where high quality goes with high signal. In a signaling equilibrium type G sends cue x and type B sends cue y, and for B to send cue y DM's belief conditional on cue x must be low enough not to induce DM to play LY – otherwise type B would find it profitable to deviate from sending cue y to sending cue x. On the contrary, in a reverse-signaling equilibrium type G sends cue y and type B sends cue x, but for B to send cue x DM's belief conditional on cue x must be high enough to have DM play LY – so that B actually has the offer accepted if he sends cue x. These two conditions are incompatible because having G sending y and B sending x decreases the belief conditional on x with respect to having G sending x and B sending y, so that if beliefs are low enough to sustain a signaling equilibrium then they cannot be high enough to sustain a reverse-signaling equilibrium.

C.2 An equilibrium may fail to exist

Another possible occurrence is that no equilibrium exists in pure strategies. An example of equilibrium inexistence is depicted in Figure 6, where it can be easily checked that for any given behavior by P the best reply by DM is such that at least one type of persuader strictly gains by deviating.

Looking at Figure 6 we see that both the separating profile where high quality goes with low signal and the pooling profile with high signal cannot be equilibria because DM best replies with HYN to cue x; in such a case, indeed, the persuader of type B would prefer to send cue y and save on costs, since he will never see his offer accepted. Similarly, both the pooling profile with low

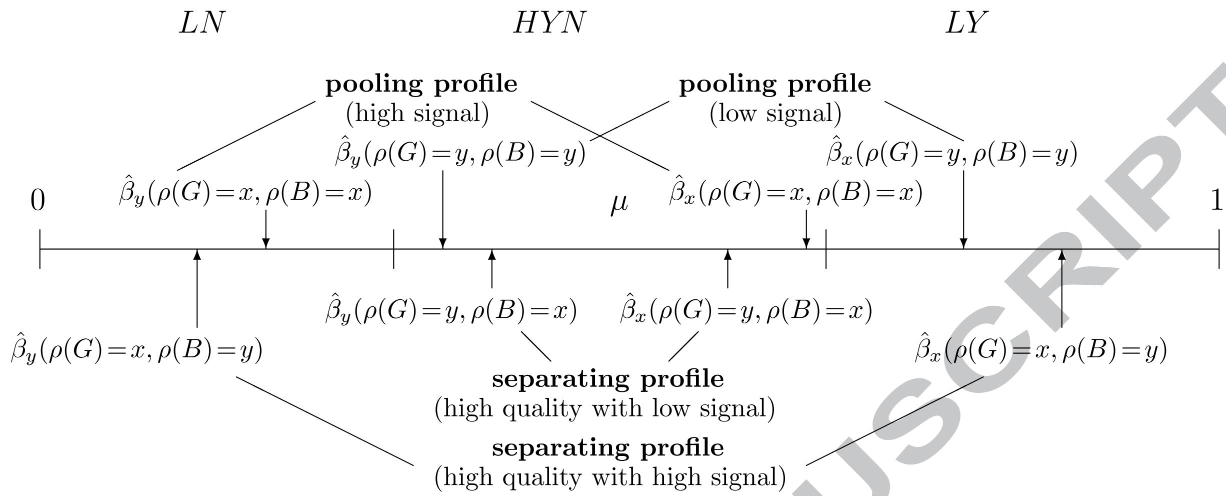

Figure 6: An example where no equilibrium exists, for $c_e < \frac{U_G|U_B|}{\gamma(U_G+|U_B|)}$.

signal and the separating profile with high quality going with high signal cannot be equilibria as well, because DM best replies with LY to cue x, and hence the persuader of type B would prefer to send cue x (so to have his offer accepted) instead of y (being his offer rejected with such a cue). We remark again that the example depicted in the figure – and more in general the possibility that no equilibrium exists – is due to the fact that $\hat{\beta}_x$ and $\hat{\beta}_y$ move along the segment as types G and B change the choice of cues, thus changing also the optimal behavior of DM.

Actually, an equilibrium exists if mixed strategies are considered. To be convinced of this, start considering the pooling profile where both G and B choose signal x; as already remarked, this is not an equilibrium because type B has a profitable deviation from x to y. Let us now suppose that type B chooses a mixed strategy where the probability of playing y progressively increases starting from zero. Consequently, $\hat{\beta}_y$ decreases and hence remains in the region where DM best replies with LN, while $\hat{\beta}_x$ progressively raises until it reaches the point where DM is indifferent between HYN and LY, and hence she can optimally randomize between HYN and LY. In particular, she can randomize with probabilities that make type B indifferent between x and y. We have hence found a mixed strategy equilibrium where type G chooses x, type B randomizes between y and x, and DM chooses LN if y and randomizes between HYN and LY if x. Starting from the pooling equilibrium where both G and B choose signal y, and following a similar reasoning, we can find another mixed strategy equilibrium with the main difference that type G chooses y and DM chooses HYN if y.

C.3 Proof of Proposition 8

Preliminarily, we give the following straightforward results which will be used in the subsequent proof. For α_x, α_y, α_G, β_x, and β_y strictly comprised between 0 and 1, the following inequalities necessarily hold:

$$\hat{\beta}_x(\rho(G)=y,\rho(B)=x) < \hat{\beta}_x(\rho(G)=y,\rho(B)=y) \tag{10}$$

$$\hat{\beta}_x(\rho(G)=y,\rho(B)=x) < \hat{\beta}_x(\rho(G)=x,\rho(B)=x) \tag{11}$$

$$\hat{\beta}_x(\rho(G)=y,\rho(B)=y) < \hat{\beta}_x(\rho(G)=x,\rho(B)=y) \tag{12}$$

$$\hat{\beta}_x(\rho(G)=x,\rho(B)=x) < \hat{\beta}_x(\rho(G)=x,\rho(B)=y) \tag{13}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=y) < \hat{\beta}_y(\rho(G)=x,\rho(B)=x) \tag{14}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=y) < \hat{\beta}_y(\rho(G)=y,\rho(B)=y) \tag{15}$$

$$\hat{\beta}_y(\rho(G)=y,\rho(B)=y) < \hat{\beta}_y(\rho(G)=y,\rho(B)=x) \tag{16}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=x) < \hat{\beta}_y(\rho(G)=y,\rho(B)=x) \tag{17}$$

The following is a check that the above inequalities indeed hold:

$$\hat{\beta}_x(\rho(G)=y,\rho(B)=x) = \frac{\alpha_x\beta_x}{\alpha_x+\alpha_P\alpha_B} = \frac{\beta_x}{1+1-\frac{\alpha_P\alpha_B}{\alpha_x}} < \beta_x = \hat{\beta}_x(\rho(G)=y,\rho(B)=y) \tag{18}$$

$$\begin{aligned}\hat{\beta}_x(\rho(G)=y,\rho(B)=x) = \frac{\alpha_x\beta_x}{\alpha_x+\alpha_P\alpha_B} < \frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_B+\alpha_P\alpha_G} = \\ = \frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P} = \hat{\beta}_x(\rho(G)=x,\rho(B)=x)\end{aligned} \tag{19}$$

$$\hat{\beta}_x(\rho(G)=y,\rho(B)=y) = \beta_x = \beta_x\left(\frac{\alpha_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G}\right) = \frac{\alpha_x\beta_x+\alpha_P\alpha_G\beta_x}{\alpha_x+\alpha_P\alpha_G} < \\ < \frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G} = \hat{\beta}_x(\rho(G)=x,\rho(B)=y) \tag{20}$$

$$\hat{\beta}_x(\rho(G)=x,\rho(B)=x) = \frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P} = \frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G+\alpha_P\alpha_B} < \\ < \frac{\alpha_x\beta_x+\alpha_P\alpha_G}{\alpha_x+\alpha_P\alpha_G} = \hat{\beta}_x(\rho(G)=x,\rho(B)=y) \tag{21}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=y) = \frac{\alpha_y\beta_y}{\alpha_y+\alpha_P\alpha_B} = \frac{\beta_y}{1+\frac{\alpha_P\alpha_B}{\alpha_y}} < \beta_y = \hat{\beta}_y(\rho(G)=x,\rho(B)=x) \tag{22}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=y) = \frac{\alpha_y\beta_y}{\alpha_y+\alpha_P\alpha_B} < \frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_B+\alpha_P\alpha_G} = \\ = \frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P} = \hat{\beta}_y(\rho(G)=y,\rho(B)=y) \tag{23}$$

$$\hat{\beta}_y(\rho(G)=y,\rho(B)=y) = \frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P} = \frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G+\alpha_P\alpha_B} \\ < \frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G} = \hat{\beta}_y(\rho(G)=y,\rho(B)=x) \tag{24}$$

$$\hat{\beta}_y(\rho(G)=x,\rho(B)=x) = \beta_y = \beta_y\left(\frac{\alpha_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G}\right) = \frac{\alpha_y\beta_y+\alpha_P\alpha_G\beta_y}{\alpha_y+\alpha_P\alpha_G} < \\ < \frac{\alpha_y\beta_y+\alpha_P\alpha_G}{\alpha_y+\alpha_P\alpha_G} = \hat{\beta}_y(\rho(G)=y,\rho(B)=x) \tag{25}$$

We note that inequality (18) follows from $(\alpha_P\alpha_B)/\alpha_x > 0$, inequality (19) from $\alpha_P\alpha_G > 0$ and (3) being strictly lower than 1, inequalities (20) and (25) from $\alpha_P\alpha_G\beta_x < \alpha_P\alpha_G$, inequalities (21) and (24) from $\alpha_P\alpha_B > 0$, inequality (22) from $(\alpha_P\alpha_B)/\alpha_y > 0$, and inequality (23) from $\alpha_P\alpha_G > 0$ and (9) being strictly lower than 1.

Proof. From (10) follows that condition 7.1 is incompatible with conditions 5.1.1 and 5.2.1. Moreover, from (16) follows that condition 7.2 is incompatible with condition 5.3.2. This proves that a separating equilibrium where $\rho(G) = y$ and $\rho(B) = x$ and a pooling equilibrium where $\rho(G) = \rho(B) = y$ cannot coexist.

From (13) follows that condition 6.1 is incompatible with condition 4.1. This proves that a separating equilibrium where $\rho(G) = x$ and $\rho(B) = y$ and a pooling equilibrium where $\rho(G) = \rho(B) = x$ cannot coexist.

From (10) and (12) follows that $\hat{\beta}_x(\rho(G) = y, \rho(B) = x) < \hat{\beta}_x(\rho(G) = x, \rho(B) = y)$, which in turn implies that condition 7.1 is incompatible with condition 6.1. This proves that the two types of separating equilibria cannot coexist.

To see that the two types of pooling can coexists suppose that $|U_B|/(U_G + |U_B|) < 1$. If β_x is close enough to 1 so that 5.3.1 is satisfied and that β_y is close enough to 0 so that condition 4.2 is satisfied. For α_G large enough, also 4.1 is satisfied. To have also 5.3.2 satisfied it is enough to have α_P and α_G sufficiently large.

To see that a pooling equilibrium where $\rho(G) = \rho(B) = x$ and a separating equilibrium where $\rho(G) = y$ and $\rho(B) = x$ can coexist, note that from $\hat{\beta}_x(\rho(G) = y, \rho(B) = x) < \hat{\beta}_x(\rho(G) = x, \rho(B) = y)$ follows that condition 7.1 implies condition 4.1. Moreover, by (17) we can set c_e, U_G and U_B such that $U_G \hat{\beta}_y(\rho(G) = x, \rho(B) = x) \leq c_e \leq U_G \hat{\beta}_y(\rho(G) = y, \rho(B) = x)$, and $\hat{\beta}_y(\rho(G) = x, \rho(B) = x) \leq |U_B|/(U_G + |U_B|) \leq \hat{\beta}_y(\rho(G) = y, \rho(B) = x)$, so that both 7.2 and 4.2 are satisfied.

A similar argument can be applied to show that a pooling equilibrium where $\rho(G) = \rho(B) = y$ and a separating equilibrium where $\rho(G) = x$ and $\rho(B) = y$ can coexist. □

D Model extensions

D.1 Many offer qualities

Consider the case where the quality of offers is not limited to two levels, G and B, but can take many possible values. Let us index qualities on the interval of the real line $[\underline{q}, \overline{q}]$, where $\underline{q} > 0$ is minimum quality and $\overline{q}$ is maximum quality. The quality of the offers not coming from P is determined according to the cumulative distribution F^N in case N is chosen and according to the cumulative distribution F^P in case P is chosen. The values of βs are modified accordingly.

DM's utility is given by $U(q)$, which is strictly increasing in q and takes both positive and negative values over $[\underline{q}, \overline{q}]$. In particular, there exists $\tilde{q}$ such that $U(\tilde{q}) = 0$. DM would like to accept any offer of quality $q \geq \tilde{q}$ and reject any offer of quality $q \leq \tilde{q}$. Hence, optimal choice by DM is still described by Proposition 1, where U_G and U_B are replaced by, respectively, $\widetilde{U}_G = \int_{\tilde{q}}^{\overline{q}} U(q)\mathrm{d}F(q)$ and $\widetilde{U}_B = \int_{\underline{q}}^{\tilde{q}} U(q)\mathrm{d}F(q)$, with $F = \alpha_P F^P + (1 - \alpha_P)F^N$.

Further, persuaders of types $q \geq \tilde{q}$ find it optimal to behave like G in the model of Section 3, while persuaders of types $q \leq \tilde{q}$ find it optimal to behave like B. So, Proposition 2 still describes the optimal choice by P conditionally on the potentially optimal behavior by DM, where however $\rho(G)$ and $\rho(B)$ are interpreted as referring to, respectively, types in $[\tilde{q}, \overline{q}]$ and types in $[\underline{q}, \tilde{q}]$, and where again U_G and U_B are replaced by $\widetilde{U}_G$ and $\widetilde{U}_B$.

As a consequence, the substance of the findings reported in Proposition 3 through 7 remains true when we allow for many different qualities of the offer.

D.2 Many offer categories and reference cues

Consider the case where the categories of objects known by DM, and to which the offer might be referred to, are not just x and y, but possibly a large number. Let Z be the set of natural numbers $\{1, 2, \ldots, n\}$ indexing the different offer categories, with $n \geq 2$ and $z \in Z$ denoting the generic reference cue. Suppose also that both β_z, which denotes the average quality for category z, and c_z, which denotes the cost of sending a cue referring to category z, are strictly increasing in z. Finally, to rule out uninteresting cases let also $c_z < V$ for all $z \in Z$.

We note that the optimal choice by DM is still described by Proposition 1. However, to describe the optimal choice by P conditionally on the potentially optimal behavior by DM one needs to generalize Proposition 2 to the case of many offer categories. We do not enter the details of this, since the forces driving the choice by P remain the same. Indeed, P will choose a reference cue that leads his offer to be accepted, if such a cue exists. We note that, as in the basic setup, type G of P has more chances to attain this objective, since for him it is enough to choose a cue that induces HYN as best reply by DM, while type B of P needs a cue that leads to immediate acceptance – i.e., that leads DM to choose LY. Moreover, among cues that lead to the same best reply by DM, P will choose the one with the minimum index, since that cue is the least costly. The difference with respect to the setup of Section 3 is the larger number of choices that are available to P, and consequently the larger number of strategies for DM, since a strategy for her is now $\delta : Z \rightarrow \{LN, HYN, LY\}$. A proposition describing the best reply behavior by P should consider all possible behaviors by DM, and hence it would consist of many cases, whose explicit listing would add little to intuition.

From the detailed description of the potentially optimal choices by DM and P – i.e., the counterparts of Propositions 1 and 2 in this setup – one can obtain results that are in line with Proposition 4 through 7. To see that pooling and separating equilibria can emerge with many categories as well, it is enough to think of the equilibria described for the model of Section 3 and add some further categories whose average quality induces the same best reply by DM as done by x or y, but with the associated reference cues being more costly, so that the additional categories are never chosen by P. Moreover, it is easy to understand that as coarse thinking becomes stronger and stronger, existence and uniqueness of the equilibrium are almost always ensured, similarly to what happens in Proposition 3.

Even if all types of equilibria remain possible in the presence of more than two categories, we remark that separation with low types sending the a high reference cue becomes the more likely outcome as the number of categories increases and average qualities are more spread all over $[0, 1]$.

To show this formally, let us introduce a measure of how categories are densely distributed in terms of their average qualities. More precisely, given $0 \leq \beta_1 < \beta_2 < \ldots < \beta_n \leq 1$, we define ξ to be equal to the largest difference of two consecutive numbers in the above sequence. In other words, ξ is the minimum length that an interval of average qualities has to have to be sure that it contains at least the average quality of one category in Z. So, the lower ξ is, the more densely distributed average qualities are.

We are now ready to state the following result:

PROPOSITION **9** (Separation with many offer categories)**.**
Suppose that $c_e < U_G|U_B|/\gamma(U_G + |U_B|)$ and $\xi < \min\{1 - c_e/\gamma|U_B| - c_e/\gamma U_G, c_e/\gamma|U_B|\}$. If coarse thinking is strong enough, then almost always there exists a profile (δ, ρ) that is the unique equilibrium and such that $\hat{\beta}_{\rho(G)} < \hat{\beta}_{\rho(B)}$, $\delta(\rho(G)) = HYN$, and $\delta(\rho(B)) = LY$.

Proof. Since $c_e < U_G|U_B|/\gamma(U_G+|U_B|)$, then there exists an interval of beliefs against which HYN is the best reply by DM. Moreover, since $\xi < \min\{1 - c_e/\gamma|U_B| - c_e/\gamma U_G, c_e/\gamma|U_B|\}$, we are sure that there exist at least one category whose average quality induces HYN as best reply, and at least one category whose average quality induces LY as best reply. We denote with z_{min}^{HYN} the category with the minimum index among those which are best replied with HYN, and we define z_{min}^{LY} analogously. We suppose that z_{min}^{HYN} and z_{min}^{HYN} are interior points in the intervals of beliefs that are best replied, respectively, with HYN and LY. By so doing we are neglecting values that have measure zero in the parameters space.

We now build a profile that we then check to be an equilibrium. For every $z \in Z$, we set $\rho(z)$ as the best action by DM against β_z, as shown by Proposition 1. Then we set $\delta(G) = z_{min}^{HYN}$ and $\delta(B) = z_{min}^{LY}$. We note that such δ selects for both G and B the least costly cue that allows them to have their offer accepted by DM. This shows the optimality of P's strategy against ρ. To understand that ρ is optimal against δ, it is enough to observe that, when coarse thinking is strong enough, $\hat{\beta}_{z_{min}^{HYN}}$ and $\hat{\beta}_{z_{min}^{LY}}$ are arbitrarily close to, respectively, $\beta_{z_{min}^{HYN}}$ and $\beta_{z_{min}^{LY}}$, and hence induce the same best action by DM, since $\beta_{z_{min}^{HYN}}$ and $\beta_{z_{min}^{LY}}$ are interior points in the intervals of best reply behavior by DM.

Finally, uniqueness follows exactly for the same argument shown in Proposition 3, which hence we do not repeat. □

D.3 Cues with fully endogenous quality

Proposition 7 tells us that a reverse-signaling equilibrium can arise in our model, and Proposition 9 states that this type of equilibrium is the only type that is possible when there are many categories and average qualities are sufficiently densely distributed. The combination of these results suggests that reverse-signaling may be a quite relevant case. However, the assumed partial exogeneity of expected average quality that is associated with each reference cue can raise doubts about the relevance of reverse-signaling. Indeed, in the model of Section 3 (and similarly in the model of Subsection D.2) the average quality of offers not coming from P, i.e., β_x and β_y, is exogenous and, in particular, one category of offers is of better average quality than the other by assumption, i.e., $\beta_x > \beta_y$. We observe that in a reverse-signaling equilibrium the average quality remains higher in category x than in category y, despite the fact that when P is called to play he chooses cue y if his type is G and cue x if his type is B. In order for this to occur, the decisions influencing the quality of offers in category x and y that are not made by P should be sustained by sound explanations. In the following we explore one particular explanation: for a subset of offers in each category, cues x and y denote characteristics which provide an intrinsic utility to DM, so that for such offers the choice between x and y directly affects the quality of the offer.

Consider the model presented in Section 3, but modified as follows. Initially, either an offerer O or a persuader P are randomly selected with probability α_O and $\alpha_P = 1 - \alpha_O$, respectively. If P is chosen then the game unfolds as in the original model. Instead, if O is chosen then the game unfolds as follows. Firstly, a type for O is drawn, either the type A – who is endowed with advanced technology – or the type S – who is endowed with standard technology; types are selected with probability α_A and α_S, respectively. Secondly, O chooses between x and y. Lastly, DM observes the reference cue without knowing whether the offer comes from O or from P, and has to decide on both elaboration level and reaction.

The two types of player O are characterized by different technologies that induce different costs to choose x. In particular, the advanced technology allows type A to incur a lower cost than type S to employ x, i.e., $c_x^A < c_x^S$. We also assume that $V - c_x^A > 0$ and $V - c_x^S < 0$, meaning that type A prefers x, while type S prefers y. Moreover, the offer made by O is such that the cue has an intrinsic value for DM, meaning that x qualifies the offer as good, while y qualifies the offer as bad. In other words, the offer is of good quality if x is chosen, while it is of bad quality if y is chosen. Finally, a strategy by O – which we denote by ω – is a choice between x and y as a function of the type, i.e., $\omega : \{A, S\} \to \{x, y\}$.

The following proposition shows that an equilibrium with reverse separation is possible.

Proposition **10** (Reverse-signaling with endogenous quality of cues)**.**
There exist values for c_e, α_0 and α_A such that the following profile (ω, ρ, δ) is an equilibrium: $\omega(A) = x$, $\omega(S) = y$, $\rho(G) = y$, $\rho(B) = x$, $\delta(x) = LY$, $\delta(y) = HYN$.

Proof. By Proposition 1 we know that if $c_e \leq \frac{U_G|U_B|}{U_G+|U_B|}$ then there exists some value for the belief on quality such that HYN is best reply for DM, and clearly for a belief on quality that is high enough LY is best reply.

We observe that $\hat{\beta}_x = \frac{\alpha_O\alpha_A}{\alpha_O\alpha_A+\alpha_P\alpha_B}$ and $\hat{\beta}_y = \frac{\alpha_P\alpha_G}{\alpha_P\alpha_G+\alpha_O\alpha_S}$. By inspection, we see that $\hat{\beta}_x$ approaches 1 if we choose α_O high enough. Given all other parameters, we pick a value of α_O such that LY is best reply against the corresponding value of $\hat{\beta}_x$. Now consider $\hat{\beta}_y$. If $\hat{\beta}_y$ is such that the best reply by DM is LY, we can raise α_O until $\hat{\beta}_y$ decreases enough to take a value such that DM finds it optimal to reply with HYN. If, instead, $\hat{\beta}_y$ is such that the best reply by DM is LN – remembering that $\alpha_A + \alpha_S = 1$ – we can raise α_A until $\hat{\beta}_y$ increases enough to take a value such that DM finds it optimal to reply with HYN. In both cases, $\hat{\beta}_x$ gets higher, and hence LY remains best reply when x is observed.

Therefore, by choosing c_e low enough and by an appropriate choice of α_O and α_A we can obtain that DM has no incentive to deviate from the considered strategy profile. The optimality checks for O and P are straightforward, and hence omitted. □

Highlights

- we frame persuasion activities aimed at attitude change in a sender-receiver model of communication
- the model of information processing is inspired by dual-process theories of information elaboration
- the decision-maker's cognitive limitations are modeled as coarse thinking, which can be avoided if cognitive costs are incurred
- the decision-maker rationally chooses between high and low elaboration effort
- the model matches several predictions of the Elaboration Likelihood Model and the Heuristic-Systematic Model