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Signaling with costly acquisition of signals

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ABSTRACT

In this paper we investigate the consequences of introducing a cost to observe the signal in an otherwise standard signaling game. Beyond identifying equilibria, which we contrast with those of a standard signaling game, we study their robustness to two important classes of refinements: acting through restrictions on out-of-equilibrium beliefs and through trembles. Our results suggest that more prominence should be given to the pooling outcome on the minimum signal.

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1. Introduction

Signaling is a pervasive phenomenon in economic interactions, emerging in many situations where there are information asymmetries. Many signaling models have been developed and studied, making the class of signaling games a quite prominent one in economics (see Riley, 2001, for a comprehensive survey). An important characteristic of signaling games is that they typically show many equilibria with rather distinct features: equilibria in which sender's types pool together by sending the same signal and equilibria where sender's types separate from each other by sending different signals.

In applied research, signaling models are often used with the focus on the best separating equilibrium, also called the Riley equilibrium, i.e., the equilibrium where all sender's types separate but they incur the minimum necessary signaling cost to do so. This is in good part due to an important stream of literature that has shown the prominence of the Riley equilibrium when players are supposed to possess a sufficient degree of forward induction: typically, plausible restrictions on out-of-equilibrium beliefs are applied to refine away many equilibria (see Sobel, 2009, for an instructive survey).

Despite the relevance of signaling models, the literature has so far given relatively scarce attention to the possibility that the acquisition of the signal by the receiver might be a deliberate and costly activity. Is the assumption of automatic acquisition of signals innocuous? In this paper we show that it is definitely not so. Indeed, even a very small cost of signal

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acquisition can make a great difference in terms of the robustness (and plausibility) of equilibria. In particular, we show that in the presence of costs to acquire the signal the pooling of sender's types can only happen on the minimum (least costly) signal, while all other pooling outcomes cannot be sustained as equilibria. At the same time, the pooling equilibrium on the minimum signal is not refined away by any restriction on beliefs at out-of-equilibrium signals, so that it becomes at least as prominent as the Riley equilibrium as outcome of a signaling game. These results are driven by the emergence of a strategic complementarity: if the receiver chooses not to incur the acquisition cost, then all sender's types find it optimal to pool on the minimum signal and, at the same time, if the different types of the sender pool on the same signal, then the receiver finds it optimal not to incur the acquisition cost. So, a complementarity naturally arises between the receiver's incentive to *costly* acquire the signal and the sender's incentive to engage in the *costly* signaling activity. This also implies that two deviations are required, starting from a pooling equilibrium on the minimum signal, for a non-equilibrium signal to be observed by the receiver: a deviation by the sender only would not even be noticed by the receiver. This intuitively explains the robustness to restrictions on out-of-equilibrium beliefs.

Since our model is not a pure signaling game, as it also deals with information acquisition by the receiver, we study the robustness of equilibria when out-of-equilibrium play is not due to intentional deviations, but to mistakes that players make when choosing equilibrium strategies. Our main finding is that the pooling outcome on the minimum signal is robust to trembles on players' strategies, since the benefits of changing strategy are negligible when trembles are very small, while both signaling and signal acquisition imply non-negligible costs for players.

Our results suggest that new attention should be given to pooling outcomes, in particular to pooling on the minimum signal. This could have far-reaching implications, especially in the light of the widespread reliance on separating equilibria in applied models.

In this paper we also consider separation outcomes. In particular, we show that the set of separating equilibria is the same when signal acquisition is automatic and when, instead, is deliberate and costly, provided that the acquisition cost is sufficiently small relatively to the value of information. The same result holds when we restrict attention to separating equilibria refined through restrictions on beliefs at out-of-equilibrium signals and trembles on players' strategies.

The paper is organized as follows. In Section 2 we review the literature on costly acquisition of information. In Section 3 we introduce signaling games with costly acquisition of signals by means of an example that is a variant of the classical Spence's signaling model. In Section 4 we define a general class of signaling games with costly acquisition of signals. In Section 5 we give our results on the equilibria of these games, and how they are refined by means of restrictions on beliefs at out-of-equilibrium signals and by means of trembles. In Section 6 we explore the robustness of our results along a number of dimensions. Section 7 summarizes our contribution and provides final remarks on welfare. A formal definition of Perfect Bayes-Nash equilibrium can be found in Appendix A. Proofs are collected in Appendix B.

2. Related literature

The idea that the acquisition of information is a strategic choice which comes at a cost is receiving increasing attention in economics. Since the seminal contribution by Grossman and Stiglitz (1980), where it is shown how costly information can impair the efficient functioning of markets, several models with this feature have been investigated, but just a few of them are closely related to our model. In fact, most of these models do not consider a sender-receiver setup, and those who consider a signaling framework do not explore the possibility that information acquisition involves the signal itself.^{1,2}

To the best of our knowledge, the only papers considering costly acquisition of information in signaling games are Bilancini and Boncinelli (2016b), who study the case where the receiver is an analogical reasoner, and Bilancini and Boncinelli (2016a), who consider the case of a receiver that has to incur a cognitive cost to fully and precisely elaborate information on sender's type. An important difference between the models in Bilancini and Boncinelli (2016a,b) and the one developed in the present paper is that in the formers the acquisition cost is paid to acquire hard information on the state of the world, while in the present paper the cost is paid to acquire the soft information embodied by the signal.

A paper more closely related to ours is Dewatripont and Tirole (2005) which develops a theory of costly communication where both the sender and the receiver have to incur a cost in order to communicate.³ The model can be seen as a standard cheap talk model, with the additional feature that in order for communication to occur both the sender and the receiver have to incur a cost.⁴ Due to this, a form of strategic complementarity arises – that is similar to the one emerging in our model – which gives rise to a robust babbling equilibrium where the message sent by the sender contains no information and

¹ See for instance the literature on strategic pricing and information acquisition of product quality from a third party started with Bester and Ritzberger (2001) (see also Gertz, 2014 and Martin, 2015).

² Costly acquisition of information has been recently studied in a number of different settings: costly cognition (Gabaix et al., 2006; Caplin and Dean, 2014); strategic interaction to influence third parties (Liu, 2011; Brocas et al., 2012) or the subsequent outcome (Morath and Münster, 2013; Colombo et al., 2014); auctions (Shi, 2012); voting (Oliveros, 2013); experts and audit structure (Argenziano et al., 2014; Menichini and Simmons, 2014).

³ We observe that, while costs are paid by both the sender and the receiver, communication remains one-sided. For a recent contribution where communication is two-sided and costless, see Eső and Wallace (2014).

⁴ The informativeness of communication in a cheap talk setting has been recently studied by Chen and Gordon (2015), who generalize the comparative statics analysis in the seminal paper of Crawford and Sobel (1982), and by Shimizu and Ishida (2015), who show that the extent of communication is severely limited as the receiver becomes more informed.

the receiver does not acquire it. The main difference concerning the setting is that Dewatripont and Tirole (2005) consider signals that have all the same cost, while in our model higher signals are more costly. Besides considering a different setting, we also carry out a different analysis that focuses on the robustness of equilibria to refinements acting through restrictions on beliefs at out-of-equilibrium signals and through refinements that are based on players' trembles. Intuitively, our results about the robustness of pooling equilibria on the minimum signal extend in the setting of Dewatripont and Tirole (2005) to babbling equilibria. However, the fact that different signals entail different costs for the sender is crucial for the elimination of pooling equilibria that are not on the minimum signal.

In an apparently very different setting, Solan and Yariv (2004) obtain results that partly overlap with ours. They consider games with espionage: starting from a setting where each of two players has to make a decision without knowing the choice made by the opponent, they endow one of the players with the possibility to observe the opponent's action at a cost prior to taking his own decision. Among other things, they show that an equilibrium of the original game where the opponent plays a pure strategy will be maintained in case the possibility of espionage is given to the other player, who however prefers to save espionage costs since in equilibrium there is no information to be acquired. Such result is basically the same as observing in our setting that the pooling profile on the minimum signal is equilibrium for any level of the acquisition cost.⁵ Differently from Solan and Yariv (2004), we focus on a specific class of games, i.e., signaling games. This leads us to compare the case of positive acquisition costs with the case with zero acquisition costs, which can be seen as the standard signaling game; on the contrary, Solan and Yariv (2004)'s basis for comparison is the case of infinite costs, when no espionage occurs. This allows us to show that pooling profiles which are not on the minimum signal cease to be equilibria when we introduce costly acquisition of signals. Furthermore, our analysis explores the robustness of the pooling profile on the minimum signal to refinements acting through restrictions on beliefs at out-of-equilibrium signals, and to refinements acting through players' trembles as well, whereas Solan and Yariv (2004) only consider espionage itself as a potential refinement tool (for games where originally there is no espionage).

3. A motivating example

Consider the following simple variant of the classical model by Spence (1973). There is one employer E that wants to hire a worker W . There are two types of workers, distinguished by their productivity $\theta \in \{1, 2\}$, which is a worker's private information; E has a prior $0 < p < 1$ that W is highly productive, i.e., that $\theta = 2$. Technology and market conditions are such that E 's net profits are given by $\theta - w$ if a worker is hired, with w the wage paid to the hired worker and θ his productivity, while otherwise profits are 0.

Moreover, W can acquire education by incurring a cost that is type-dependent. In particular, suppose that W comes from a foreign country and that he has to move to E 's country in order to be hired. Suppose also that W can only acquire education in the foreign country, and that the only available alternatives are a good school G and a bad school B , which are not previously known to E . For the prospective worker of type θ , the cost of attending G is $2/\theta$ and the cost of attending school B is $1/\theta$. So, attending school G is more costly than attending school B , and it is relatively more so for the low type (i.e., $\theta = 1$). This provides W with a costly signal $x \in \{G, B\}$ that potentially allows W 's types to separate.

So far, there is no substantial difference from Spence's model. However, what if E , in order to assess the quality of the schooling signal x sent by W , has to actively acquire the information on what school W has attended in the country he comes from, and what attendance costs have been paid? This information can well not come for free and our point is that this can make the difference. We observe that the costs of acquiring such information can be interpreted as due to the material and the cognitive effort which is necessary to retrieve and elaborate the relevant data on x . On the material side, E might have to search and collect information on G and B , and maybe also pay to translate documents that would be otherwise inaccessible. On the cognitive side, E might have to exert effort to elaborate the collected information in order to establish that one school is G with costs 2θ and the other is B with costs θ and to assign the signal x to either G or B . If no (material or cognitive) effort is exerted, then schools are fundamentally indistinguishable to E ; in this case, it is reasonable that E does not condition her decision on x . To model this, suppose that E has to pay a cost $c > 0$ to acquire the signal x sent by W . In particular, if E does not incur the cost c , then signal x remains unknown to E .

Consider now the following situation: W chooses B independently of his type, i.e., $x(1) = x(2) = B$, and E decides not to acquire the signal x . It is easy to check that this is an equilibrium in the present example (whatever is the value for E of knowing W 's types): both types of W strictly lose by switching to the more costly G as this does not grant a different wage, and E strictly loses by acquiring the signal because it costs c and provides no new information. We observe that such an equilibrium is very similar to the pooling equilibrium with lowest signal that emerges in Spence's model. However, there is an important difference: because of acquisition costs such pooling equilibrium is much more robust to refinements than the one in Spence's model. Firstly, we note that restrictions on beliefs at out-of-equilibrium signals are ineffective to refine away this equilibrium, since E does not acquire any signal, and hence W cannot use out-of-equilibrium signals to communicate with E ; indeed, a deviation by W only would not even be noticed by E . In particular, even if W 's high type deviates from $x(2) = B$ to $x(2) = G$, when E decides not to acquire the signal there is no way to let E know – or even imagine – about such a deviation. So, arguments based on the reasonableness of out-of-equilibrium beliefs cannot refine away this pooling

⁵ We wish to thank a referee for pointing us the relationship of our contribution with this paper.

equilibrium. Secondly, this pooling equilibrium is robust even when deviations from equilibrium behavior is not intentional but the result of mistakes. Indeed, when the probability of mistakes becomes sufficiently low, the possible benefit of changing strategy is not worth its cost for both players, since choosing G is more costly than choosing B for W , and acquiring the signal is costly for E as well.

There is another important difference. In this example the pooling outcome associated with the lowest signal pooling equilibrium is the *only* pooling outcome that can be sustained in equilibrium, which is in contrast with Spence's model where multiple pooling outcomes are possible in equilibrium. This happens without any additional refinement of equilibria. To see why it is so, consider the case where both types of W pool on G , i.e., $x(1) = x(2) = G$. Given this behavior by W , E finds it strictly profitable *not to incur* the acquisition cost, as acquiring the signal provides no new information. But if E does not acquire the signal x , then the choice of $x(1) = x(2) = G$ cannot be sustained in equilibrium since each of W 's type would strictly gain by switching from G to B , as this allows to save on the cost of signaling without adversely affecting E 's beliefs.

4. The model

We now introduce the more general game of *signaling with costly acquisition of signals* (SCAS). There is one sender S and one receiver R (sometimes referred to as "he" and "she", respectively). The sender S observes his own type $t \in T$, with T a finite set of cardinality n , and then chooses a signal $x \in X = \mathbb{R}_+$.⁶ The receiver R can exert costly effort and acquire the signal x , or save on effort and observe nothing. We denote with $s \in \{s_1, s_2\}$ such a choice, where s_1 means that x is not acquired and s_2 means that x is acquired and effort is exerted.⁷ In any case, then R has to choose an action $y \in Y = \mathbb{R}$. The prior beliefs held by R on T are given by $p = (p_1, \dots, p_n) \in \Delta T$, where p_t denotes the probability that S is of type $t \in T$, and ΔT denotes the unit $(n - 1)$ -simplex. The posterior beliefs held by R , once a signal x has been sent and an information acquisition choice s has been made, are denoted by $\beta(x, s) = (\beta_1(x, s), \dots, \beta_n(x, s)) \in \Delta T$, where each $\beta_t(x, s)$ is the probability that S is of type t conditional on (x, s) . Consistently with the idea that no signal is observed in case $s = s_1$, we assume that $\beta(x, s_1) = \beta(x', s_1)$ for all $x, x' \in X$.

Utility for S is $U : T \times X \times Y \rightarrow \mathbb{R}$, and utility for R is $V : T \times X \times Y \times \{s_1, s_2\} \rightarrow \mathbb{R}$. We assume that U is strictly increasing in y , and strictly decreasing in x . We also assume that V exhibits a fixed positive cost of acquiring the signal, so that the notation can be simplified as follows: $V(t, x, y, s_1) = v(t, x, y)$ and $V(t, x, y, s_2) = v(t, x, y) - c$, for all $t \in T, x \in X, y \in Y$. Moreover, we assume that $\arg \max_{y \in Y} \sum_{t \in T} \beta_t v(t, x, y)$ is a singleton, for all $\beta \in \Delta T$, and all $x \in X$. Finally, the single-crossing property is assumed to hold: $U(t, x, y) \leq U(t, x', y')$, with $x' > x$, implies that $U(t', x, y) < U(t', x', y')$ for all $t' > t$ and $y, y' \in Y$.

We limit our analysis to pure strategies. A strategy for S is a function $\mu : T \rightarrow X$; we denote with \mathcal{M} the set of all possible μ . A strategy for R is a pair (s, α) where $s \in \{s_1, s_2\}$ and $\alpha : X \times \{s_1, s_2\} \rightarrow Y$ is a function such that $\alpha(x, s_1) = \alpha(x', s_1)$ for all $x, x' \in X$, i.e., R 's action is unconditional on x whenever $s = s_1$ is chosen; we denote with \mathcal{A} the set of all such functions.⁸

Lastly, in order to better contrast our results with the results on standard signaling games we find it useful to define a class of games that are equivalent to standard signaling games and that can be obtained from a SCAS game. This can be done, starting from a SCAS game, by forcing R to play $s = s_2$. We refer to such game as *signaling with forced acquisition of signals* (SFAS). Note that a SFAS game with utilities U and v is actually the standard signaling game – i.e., with no costs to acquire the signal – that can be obtained from a SCAS game with utilities U and v and any acquisition cost $c > 0$.

5. Results

5.1. Equilibria

As a solution concept we focus on Perfect Bayes-Nash equilibria (see Appendix A for a formal definition applied to SCAS games and SFAS games). A *pooling equilibrium* is a profile $(\mu, (s, \alpha))$ such that $\mu(t) = \mu(t')$ for all $t, t' \in T$. A *pooling equilibrium on the minimum signal* is a profile $(\mu, (s, \alpha))$ such that $\mu(t) = 0$ for all $t \in T$. A (fully or partially) *separating equilibrium* is a profile $(\mu, (s, \alpha))$ such that $\mu(t) \neq \mu(t')$ for some $t, t' \in T$.

In a SFAS game, given a sender's strategy μ , we define the *value of acquiring information on the signal* as:

$$v(\mu) = \max_{\alpha \in \mathcal{A}} \sum_{t \in T} p_t v(t, \mu(t), \alpha(\mu(t), s_2)) - \max_{\alpha \in \mathcal{A}} \sum_{t \in T} p_t v(t, \mu(t), \alpha(\mu(t), s_1)). \quad (1)$$

⁶ We have chosen to use continuous action spaces because this is the most common setting for signaling models. We stress that our results might be stated, with few straightforward adjustments, using discrete action spaces too.

⁷ This labeling owes to the classification of elaboration processes as "System 1", or S_1 , which is fast, cheap and intuitive, and "System 2", or S_2 , which is slow, costly and analytical (see, e.g., Kahneman, 2003). We stress this interpretation based on cognitive effort because we think that it applies to many relevant cases of signal acquisition. Of course, other interpretations are possible where the cost of acquiring the signal is due to non-cognitive factors.

⁸ Here we adopt a specification of strategies for R that is more compact but perhaps less straightforward than, e.g., $\alpha(s, \alpha^{s_1}, \alpha^{s_2})$ where $\alpha^{s_i} \in Y$ is the action chosen if $s = s_i$ and $\alpha^{s_2} : X \rightarrow Y$ is the action chosen if $s = s_2$. We stress that the two formulations are equivalent in the sense that we can go from one to the other by coalescing (Thompson, 1952) the information set that R reaches when $s = s_1$ with the one where R chooses s , although in this game there is an infinite number of information sets. For more details see Elmes and Reny (1994) and the discussion in Alós-Ferrer and Ritzberger (2013, example 9).

The set of Perfect Bayes-Nash equilibria of a SCAS game is in general different from the set of Perfect Bayes-Nash equilibria of a typical signaling game (a SFAS game in our setting). Proposition 1 below states that pooling equilibria on the minimum signal have a particular role in SCAS games because, while they also exist in ordinary SFAS games, they are the only pooling equilibria in SCAS games. Also, Proposition 1 states that the set of separating equilibria of the SCAS game is included in the set of separating equilibria of the SFAS game: indeed, a separating equilibrium of the SCAS game is a separating equilibrium of the SFAS game with the additional requirement that the cost to acquire the signal is not larger than the value of acquiring information on the signal.

Proposition 1. Consider a SCAS game and its associated SFAS game.

- (a) A pooling equilibrium on the minimum signal exists both in the SCAS game and in the SFAS game; moreover, in the SCAS game no pooling equilibrium exists which is not on the minimum signal.
- (b) If $(\mu, (s, \alpha))$ is a separating equilibrium in the SCAS game, then it is also a separating equilibrium in the SFAS game; if $(\mu, (s_2, \alpha))$ is a separating equilibrium in the SFAS game, then it is a separating equilibrium in the SCAS game if and only if $c \leq v(\mu)$.

We observe that it follows from standard results on signaling games without acquisition costs that pooling equilibria can exist in the SFAS game where the signal chosen by all S 's types is not the minimum. Therefore, in a sense the set of separating equilibria of a SCAS game converges to that of the associated SFAS game as c tends to 0, whereas the same is not true of the set of pooling equilibria. We also observe that as c gets larger and larger, the set of equilibria of a SCAS game shrinks. Indeed, a separating profile $(\mu, (s_2, \alpha))$ which is equilibrium of a SCAS game ceases to be such when c exceeds its value $v(\mu)$ of acquiring information on the signal. Further, since the value of acquiring information on the signal only depends on its informativeness about sender's types, all separating equilibria with the same value cease to exist at the same time. To fix ideas, consider a signaling model with only two types of senders: the entire set of separating equilibria of the SFAS game is maintained in the SCAS game when c does not exceed the value for the receiver to acquire the information on the sender's current type, which is the same for all separating strategies of the sender; however, when c exceeds such value, all separating equilibria cease to exist in this SCAS game. Finally, from Proposition 1 we can also conclude that separating equilibria where are more informative will disappear at higher cost levels than those which are less informative (think, for instance, of a fully separating equilibrium compared to a partially separating equilibrium). More precisely, a signaling strategy for the Sender can be seen as an information-revealing experiment, and according to the Blackwell's theorem (Blackwell et al., 1951, 1953), the more informative an experiment the higher its value for any expected-utility maximizing decision-maker.

5.2. Refinements

Given the multiplicity of equilibria in a signaling games (a SCAS game is not an exception to this), some notion of equilibrium refinement is often applied to get rid of implausible equilibria. So, even if the set of equilibria of a SCAS game differs from that of a SFAS game, an equivalence might be restored through refinements. In the following we argue that this is not the case.

An important class of refinements applied to signaling games follow the idea that out-of-equilibrium beliefs should not be totally free, but need to satisfy some criterion of reasonableness.⁹ With the aim of extending such class of refinements to SCAS games, we define the notion of robustness to restrictions on beliefs at out-of-equilibrium signals, and in point (a) of Proposition 2 we show that in a SCAS game the equilibrium where all sender's types pool on the minimum signal satisfies such robustness criterion. Indeed, refinements acting through restrictions on out-of-equilibrium beliefs rely on the possibility that a deviation by the sender triggers a path of play along which the receiver gets some piece of information that is unexpected along the equilibrium path; but in a pooling equilibrium of a SCAS game the receiver does not acquire the signal, so that this possibility does not exist. This is why refinements acting through restrictions on beliefs at out-of-equilibrium signals do not have a bite in such case. We now formalize, for SCAS and SFAS games, the concept of an equilibrium profile that is robust to restrictions on beliefs at out-of-equilibrium signals. Given a sender's strategy μ , we denote with $X^e(\mu)$ the set of signals that are chosen by at least one sender's type, i.e., $X^e(\mu) = \{x \in X : \exists t \in T, \mu(t) = x\}$. Similarly, we denote with $X^o(\mu)$ the set of signals that are not chosen by any sender's type, i.e., $X^o(\mu) = X \setminus X^e$. Moreover, given a sender's strategy μ , a restriction on beliefs at out-of-equilibrium signals is a collection of sets $\{\mathcal{B}(x, s_2)\}_{x \in X^o(\mu)}$, with $\mathcal{B}(x, s) \subseteq \Delta T$, $\mathcal{B}(x, s) \neq \emptyset$.¹⁰ An equilibrium is said to be robust to a restriction on beliefs at out-of-equilibrium signals if it is compatible with the restricted set of admissible beliefs.¹¹

We are ready for the following proposition on the robustness of equilibria to restrictions on beliefs at out-of-equilibrium signals.

⁹ A non-exhaustive list of refinements acting through restrictions on out-of-equilibrium beliefs include: the Intuitive Criterion (Cho and Kreps, 1987), Divinity (Banks and Sobel, 1987), D1 (Cho and Kreps, 1987), D2 (Cho and Kreps, 1987), the Undeclared Equilibrium (Mailath et al., 1993).

¹⁰ We observe that even a signal $x \in X^e(\mu)$ triggers an out-of-equilibrium belief if the receiver deviates from s_1 to s_2 . However, such beliefs are already restricted by the definition of Perfect Bayes Nash equilibrium given in Appendix A (see the E3.2 requirement).

¹¹ More formally, it must satisfy the definition of a perfect Bayes-Nash equilibrium with a strengthening of the E3.3 requirement (see Appendix A), that is $\beta(x, s_2) \in \mathcal{B}(x, s_2)$, for all $x \in X^o(\mu)$.

Proposition 2. Consider a SCAS game and its associated SFAS game.

- (a) In the SCAS game, given μ such that $\mu(t) = 0$ for all $t \in T$, for any restriction on beliefs at out-of-equilibrium signals there exists (s, α) such that $(\mu, (s, \alpha))$ is a pooling equilibrium robust to the restriction.
- (b) If $(\mu, (s, \alpha))$ is a separating equilibrium that is robust to a given restriction on beliefs at out-of-equilibrium signals in the SCAS game, then it is also a separating equilibrium that is robust to the same restriction in the SFAS game; if $(\mu, (s_2, \alpha))$ is a separating equilibrium of the SFAS game that is robust to a given restriction on beliefs at out-of-equilibrium signals, then it is a separating equilibrium of the SCAS game that is robust to the same restriction if and only if $c \leq v(\mu)$.

We remark that, in point (b) of Proposition 2, refinements acting through restrictions on beliefs at out-of-equilibrium signals, even if ineffective against pooling equilibria on the minimum signal, allow to get rid of many separating equilibria of the SCAS game, in a manner similar to what they do in a signaling game without costly acquisition of signals. Indeed, in any separating equilibrium R must be playing s_2 , so that a deviation by S allows to reach an out-of-equilibrium information set of R , where the freedom in the choice of beliefs is now restricted by the refinement under consideration, possibly inducing R to play in a way that is favorable to S .

There is another class of refinements which considers out-of-equilibrium play not as intentional deviations, but as the result of mistakes that players make when choosing equilibrium strategies.¹² These refinements require players' strategies to be optimal against completely mixed strategy profiles that approach the equilibrium profile. Proposition 3 establishes, in point (a), the robustness of the pooling equilibrium on the minimum signal with respect to any kind of trembles. The intuition is as follows. Starting from the pooling equilibrium on the minimum signal, if R is forced to choose s_2 with some small probability due to trembles, then S may have some incentive to opt for a signal that is not the minimum; however, since signaling is a costly activity, if the tremble is sufficiently small then the benefit from signaling falls short of the cost. An analogous reasoning applies if we consider R 's optimal behavior when S 's choice is trembling, due to the fact that s_2 is more costly than s_1 .

To formalize the idea of robustness to any kind of trembles, we make use of the notion of truly perfect equilibrium (Kohlberg, 1981).¹³ To present it formally, we have first to modify our setting by considering a finite set of signals \hat{X} , with $0 \in \hat{X}$, and a finite set of actions \hat{Y} , so that the set of strategies is also finite for both S and R , which we denote with \hat{M} and $\{\hat{s}_1, \hat{s}_2\} \times \hat{A}$.¹⁴ Moreover, we have to introduce mixed strategies to model players' trembles in the choice of strategies. We denote by σ_S a mixed strategy for S , with $\sigma_S(\mu)$ indicating the probability attached to pure strategy μ . Similarly, we denote by σ_R a mixed strategy for R , with $\sigma_R(s, \alpha)$ indicating the probability attached to pure strategy (s, α) . We note that in a SFAS game $\sigma_R(s_1, \alpha) = 0$ for all $\alpha \in \hat{A}$, since s_1 cannot be chosen by R either by mistake. Given a profile (σ_S, σ_R) , we refer to a tremble as a sequence of completely mixed strategy profiles $(\sigma_S^k, \sigma_R^k)_{k=1, \dots, \infty}$ such that $(\sigma_S^k, \sigma_R^k) \rightarrow (\sigma_S, \sigma_R)$ as $k \rightarrow \infty$. Given a degenerate mixed strategy profile (σ_S, σ_R) where $\sigma_S(\mu) = 1$ and $\sigma_R(s, \alpha) = 1$, and a tremble $(\sigma_S^k, \sigma_R^k)_{k=1, \dots, \infty}$, we say that $(\mu, (s, \alpha))$ is robust to the tremble if there exists \hat{k} such that, for all $k \geq \hat{k}$, μ is best reply against σ_R^k and (s, α) is best reply against σ_S^k ; furthermore, we say that $(\mu, (s, \alpha))$ is a truly perfect equilibrium if $(\mu, (s, \alpha))$ is robust to every tremble. Finally, to simplify the statement of Proposition 3, we introduce the following definition: given a degenerate mixed strategy profile, a tremble in the SCAS game $(\sigma_S^k, \sigma_R^k)_{k=1, \dots, \infty}$, and a tremble in the SFAS game $(\hat{\sigma}_S^k, \hat{\sigma}_R^k)_{k=1, \dots, \infty}$, we say that these two trembles are equivalent if (i) for every $\mu \in \hat{M}$, the expected utility for S of playing μ is the same against σ_R^k and $\hat{\sigma}_R^k$, for all k , and (ii) for every $(s, \alpha) \in \{\hat{s}_1, \hat{s}_2\} \times \hat{A}$, the expected utility for R of playing (s, α) is the same against σ_S^k and $\hat{\sigma}_S^k$, for all k .

We are ready for the following proposition on the robustness of equilibria to players' trembles.

Proposition 3. Consider a SCAS game with finite sets of signals and actions, and its associated SFAS game.

- (a) In the SCAS game, any pooling equilibrium on the minimum signal is a truly perfect equilibrium.
- (b) If $(\mu, (s, \alpha))$ is a separating equilibrium that is robust to a given tremble in the SCAS game, then it is also a separating equilibrium that is robust to an equivalent tremble in the SFAS game; if $(\mu, (s_2, \alpha))$ is a separating equilibrium that is robust to a given tremble in the SFAS game, then it is a separating equilibrium that is robust to an equivalent tremble in the SCAS game if $c < v(\mu)$.

We remark that, in point (b) of Proposition 3, similarly to what happens for restrictions on beliefs at out-of-equilibrium signals, the set of separating equilibria that are robust to a given tremble is the same in a SCAS game and in the associated SFAS game, provided that acquisition costs are small enough.¹⁵ On the whole, we can reinforce an observation already made after Proposition 1: it is not only the set of separating equilibria of a SCAS game, but the set of the separating equilibria

¹² Another possibility is to consider perturbations in payoffs, rather than in the choice of strategies. A refinement based on this idea is the essential equilibrium (Wen-Tsun and Jia-He, 1962).

¹³ Indeed, a truly perfect equilibrium is perfect (Selten, 1975), proper (Myerson, 1978), strictly perfect (Okada, 1981), strictly proper (Van Damme, 1991). Moreover, a truly perfect equilibrium can be seen as a one-point set that is Kohlberg-Mertens stable (Kohlberg and Mertens, 1986).

¹⁴ The results in Proposition 1 hold in this setting with finite strategy sets as well.

¹⁵ Point (b) of Proposition 3 is weaker than point (b) of proposition 2 because when $c = v(\mu)$ the result depends on the given tremble of the SFAS game.

that are robust to refinements (belief-based or tremble-based), which converges to the analogous set of the associated SFAS game as c tends to 0.

One further remark that may be worth doing concerns the inability of tremble-based refinements to restrict R 's behavior at information sets where a non-minimum signal has been chosen, and R has already chosen s_2 . We observe that such information sets cannot be reached by means of S 's mistakes only; indeed, a mistake by R when choosing between s_1 and s_2 is also required. In order to recover some refining power for trembles at the above-mentioned information sets, we may resort to the agent-normal form game and apply trembles at the agent level, with the result that optimal behavior would be required at such information sets as well. In this case a statement similar to the one in Proposition 2 applies: for any sequence of trembles there exists a pooling equilibrium on the minimum signal that is comprised of strategies that are best replies against each sufficiently-large-indexed element of the sequence.

One last remark is about the role of the assumption that signal acquisition is a deliberate and costly choice for the results in Propositions 2 and 3. Having a choice for the receiver between s_1 and s_2 is what really matters to obtain that the pooling outcome on the minimum signal is robust to restrictions on beliefs at out-of-equilibrium signals; indeed, if R chooses s_1 then he switches off her listening ability, and hence any sender's deviation is not observable. The robustness to refinements based on players' trembles, instead, depends not only on R choosing to listen (s_2) or not to listen (s_1), but also on the fact that s_2 is more costly than s_1 , which makes s_1 strictly better than s_2 when S is choosing a pooling strategy and trembles are small enough.

6. Discussion

In this section we discuss the robustness of our results if we assume a smooth process of information acquisition by the receiver (Section 6.1), if the sender has the possibility to invite the receiver to acquire the signal (Section 6.2), if the signal is not purely costly to the sender (Section 6.3), and if the receiver can commit to acquire the signal (Section 6.4). The discussion is kept at an informal level.

6.1. Smooth acquisition costs

In this paper we have considered a model that is reminiscent of Reis (2006): acquisition of information is always all-or-nothing, and signal acquisition is binary, i.e., R either pays the acquisition cost and acquires x with certainty, or pays nothing and acquires nothing. One can instead think of the process of signal acquisition as a smooth or uncertain one: the greater the cost incurred to acquire the signal, the greater the acquisition of the signal content or the likelihood that acquisition is successful. In this respect, a natural question to ask is whether separation of sender's types becomes more likely under such a smooth or uncertain process. In general, the literature has shown that the information acquisition technology does matter for the selection of equilibria (see Yang, 2015 and Denti, 2016 for recent contributions on the issue). What we argue here is that the pooling outcome retains its prominence also when signal acquisition is not all-or-nothing or is not binary.

A simple way to model a smooth process of signal acquisition that is not binary is to consider a stochastic acquisition where the probability of acquiring x is an increasing function of the cost paid. Suppose R has the possibility to choose a level of acquisition effort $e \in [0, 1]$, which replaces the choice of $s \in \{s_1, s_2\}$; also, with probability $1 - e$ no signal is acquired, while with probability e the signal is acquired.

A simple way to model a smooth process of signal acquisition that is not all-or-nothing is to have the signal x always acquired but with some blurring noise whose impact negatively depends on the acquisition effort, so that the signal observed is equal to the signal sent plus a noisy term. We denote with $\pi \in [0, +\infty)$ the precision (i.e., the inverse of the variance) of the noisy term, which is chosen by the receiver.

We observe that the pooling equilibrium on the minimum signal is still supported by a form of strategic complementarity in both models sketched above: if S chooses $x=0$ for all $t \in T$, then R has no reason to spend in signal acquisition, therefore R 's optimal choice is $e=0$ (the signal is never acquired) or $\pi=0$ (the signal is completely uninformative); if R chooses $e=0$, or $\pi=0$, then S has no reason to spend in costly signaling, therefore S 's optimal choice is $x=0$ for all $t \in T$. Moreover, note that when $e=0$ no out-of-equilibrium information set of R can be reached by means of a single deviation by S , and when $\pi=0$ no out-of-equilibrium information set exists, in that zero precision of the signal does not constrain the set of possible signals. In both cases intentional deviations by S are ineffective at triggering a different path of play by R ; this intuitively leads to the robustness of the pooling outcome on the minimum signal to refinements acting through restrictions on out-of-equilibrium beliefs. Finally, note that no benefit can be obtained by unilateral deviations to $e' > 0$, or $\pi' > 0$, and to $x' > 0$, while such deviations would entail a strictly larger cost; this allows us to conclude that the pooling outcome on the minimum signal is also robust to refinements based on players' trembles, in a version of these games with finite strategy sets.

6.2. Inviting to acquire the signal through further signaling

It seems natural to ask whether the prominence of separation is restored if S has the possibility to communicate to R that he is actually sending an informative signal – i.e., a signal that separates (at least partly) types – and that therefore the signal is worth acquisition.

One can think of many situations where indeed the sender can send, together with the main signal x , an accompanying costly signal, say z , that acts as an invitation for the receiver to engage in the costly acquisition of x . We argue here that, in fact, not much can be restored by the use of z .

We note that in this setting information on types can be transmitted, i.e., separation can occur, either on x or on z , but not on both. This is so because, if separation is attained on x (or on z) and the receiver acquires x (or z), then all types would strictly prefer to save on costs and pool on a null z (or x), and similarly the receiver would strictly prefer not to incur the cost of acquiring z (or x), since its acquisition would add no useful information. So, suppose that separation is effectively attained on z only. In order for this kind of separation to be more robust than a pooling equilibrium, it is *necessary* that the sender's utility function satisfies an equivalent of the single-crossing property on types and z , which is not guaranteed in general. Even if such a necessary condition holds, to restore the prominence of separation the receiver must acquire z automatically. In fact, if the acquisition of z requires a deliberate choice and is costly to R , then the kind of strategic complementarity between signaling and acquiring the signal that we have illustrated for the SCAS game is at work here as well; moreover, out-of-equilibrium information sets are not reachable through deviations by S only when starting from the pooling outcome on the minimum signal, and also unilateral deviations from it would be strictly costly for both S and R . Therefore, results similar to those for the SCAS game can be intuitively obtained in this setup as well.

6.3. Signal not purely costly to the sender

The SCAS game studied in this paper accommodates cases where the signal x is purely dissipative – it is always a net cost for S and of no intrinsic utility (or some disutility) for R – as well as cases where the signal x is of some intrinsic value to the receiver. However, the model does not accommodate the case where x is not a pure net cost for the sender.

In our model, all sender's types strictly prefer, other things being equal, to set $x=0$:

$$x^*(t, y) \equiv \arg \max_{x \in X} U(t, x, y) = 0, \text{ for all } t \in T, y \in Y. \quad (2)$$

It turns out that the kind of strategic complementarity that supports the pooling outcome in a SCAS game may not exist if (2) does not hold. However, we stress that what is crucial to our results is not that 0 is the common best signal for all sender's types in the absence of a signaling value – an assumption which, in fact, can easily be substituted with a common optimal $x^* > 0$ for all $t \in T$; what really matters for the existence of the needed strategic complementarity is that *a common best signal exists for all types*. To see why, consider the extreme case where $x^*(t, y)$ is one-to-one in t for any given y . This implies that, in the case that R chooses $s=s1$, each sender's type finds it optimal to choose a distinct x . If the information about the sender's type is sufficiently valuable to R , it becomes impossible for a profile with no signal acquisition to be an equilibrium because types separate independently of R 's behavior, and therefore R always find it optimal to acquire the signal. Therefore, in the absence of a common set signal for all types, the pooling outcome on the minimum signal would even fail to be a Perfect Bayes-Nash equilibrium.

Let us conclude with a few remarks that, in our opinion, indicate that acquisition costs – and in general the analysis conducted in this paper – might be relevant even when (2) does not hold.

One remark regards the refinement potential of arbitrarily small acquisition costs in a standard signaling game. Note that if $x^*(t, y)$ is one-to-one in t for any given y , then the incentive for S 's types to separate does not come from the fact that R acquires the signal, but from the fact that each type has its own preferred x . This rules out all pooling equilibria in a SCAS game, but it does not so in the associated SFAS game (i.e., in a standard signaling game). In fact, in a SFAS game R always chooses $s=s2$, and in particular it does so also when all S 's types pool on the same \bar{x} ; this allows for beliefs at out-of-equilibrium signals on the part of R that harshly punish types who deviate from \bar{x} , sustaining the pooling equilibrium. In a SCAS game, instead, R would switch from $s=s2$ to $s=s1$, leaving each type $t \in T$ free to switch to his preferred $x^*(t, y)$. Perhaps interestingly, this argument shows that an arbitrarily small acquisition cost rules out all pooling equilibria in signaling games where types strictly prefer different signal levels.

Another remark regards the potential backfiring of mandatory disclosure policies. Consider a SCAS game where x represents costly disclosure of some characteristic on the part of the sender, and suppose that a public authority wants to keep x above a certain threshold. If $x^*(t, y)$ is one-to-one in t for the relevant range of y , then some disclosure will certainly happen as no pooling can be sustained in equilibrium. However, if the public authority imposes a minimum \bar{x} , then it can happen that separation collapses and a pooling equilibrium on \bar{x} with no signal acquisition emerges. In particular, this can happen whenever $\bar{x} \geq \max_{t \in T} x^*(t, \bar{y})$, where \bar{y} is the best action for R under $s1$ when all types pool on \bar{x} (for smaller values of \bar{x} a partial pooling can emerge, instead). This may lead to a loss in terms of information transmission that more than offsets the targeted benefits of a high x .

6.4. Publicly committing to acquire the signal

In the SCAS game studied in Sections 4 and 5, as well as in the variants discussed in Sections 6.1, 6.2, and 6.3, there is no possibility for the sender, prior to choosing the signal, to observe the choice of the receiver between acquiring and not acquiring the signal. This neglected case is strategically equivalent to a situation where the receiver R must publicly commit to choose $s1$ or $s2$, so that S learns if R will acquire the signal *before* choosing what signal to send. It is not too

complicated to see that if R can publicly commit to a given $s \in \{s1, s2\}$, and information is sufficiently valuable to her, then the prominence of separation is restored. Consider a variant of the SCAS game where R must¹⁶ initially commit herself to choose $\bar{s} \in \{s1, s2\}$ and suppose also that such commitment is observed by S before he chooses what signal to send. This configures an additional initial stage of the game where R announces \bar{s} , followed by a second stage of the game where S chooses the signal x , and then a third stage where R plays \bar{s} and chooses an action y . In this setup, S can condition the choice of the signal on \bar{s} , so that his strategy is now represented by function $\bar{\mu} : T \times \{s1, s2\} \rightarrow X$. Note that this setup configures two distinct subgames: one signaling (sub)game in which R has committed to $s1$ and S knows that his signal will never be acquired by R (basically, signalling is impossible), and another signaling (sub)game where R has committed to $s2$ and S knows that his signal will always be acquired (basically, the associated SFAS game). Such dynamic structure of the game naturally calls for an equilibrium concept that entails backward reasoning. A simple way to do so is to look for subgame perfection after having refined the equilibria of each signaling subgame using standard refinements acting through out-of-equilibrium beliefs.

In the signaling subgame where R has committed to $s1$, there is just one Perfect Bayes-Nash equilibrium: all types of S pool on $x=0$, since $x>0$ is costly and S is certain that R will never observe x . Denote S 's strategy in this subgame with μ^P , with $\mu^P(t)=0$ for all t . In the signaling subgame where R has committed to $s2$, there are many Perfect Bayes-Nash equilibria, both pooling and separating. Since the cost c of choosing $s2$ is sunk, in this subgame there are exactly the same equilibria of the associated SFAS game, i.e., they are the same of a standard signaling game.

We consider here the typical case where the best fully separating equilibrium stands up as most prominent, i.e., we focus on the equilibrium where all sender's types separate and each type spends on signal x the minimum required for separation (Riley, 2001). Denote S 's strategy in this equilibrium with μ^S . So, S 's strategy in the full game can be written as $\bar{\mu} = (\mu^P, \mu^S)$.

Using backward reasoning, R anticipates that by committing to $s1$ she will end up in a pooling equilibrium where she plays the optimal y against the beliefs associated with no signal acquisition, while by committing to $s2$ she will end up in a fully separating equilibrium where she plays, for each t , $y(t) \in \arg\max_{y \in Y} u(t, \mu^S(t), y)$. If the information conveyed by μ^S is valuable to R in the SFAS game, then it is straightforward to see that for a positive acquisition cost c which is small enough, R strictly prefers to end up in the fully separating equilibrium, and therefore she will commit to $s2$. In such a case, the prominence of the outcome of full separation is restored.

Let us end this discussion with a more general point regarding the actions that the receiver can make to facilitate communication. To the extent that information transmission is valuable to the receiver, it is reasonable to expect that the receiver acts (and even incurs costs) in order to facilitate the emergence of a fully separating equilibrium. If public commitment to the acquisition of the signal is possible or if the acquisition can be made before the signal is sent, then separation is actually the most prominent outcome. But how likely it is that this is the case? For the signal to be effectively acquired *before* the sender sends it, we must be in a situation where the sender and the receiver communicate through a channel that the receiver can "switch on" at her will and whose on/off status is easily observable (i.e., at no cost) by the sender. In addition, as the cost of keeping the channel switched on reasonably depends on how long it is left in such a state, another crucial requirement is that the receiver must know approximately *when* the sender is going to send the signal, otherwise another coordination problem arises. While this setup certainly fits some real cases of signaling, it hardly fits most of them. When signal acquisition takes the form of buying the necessary "hearing" tools, or when it takes the form of moving to the correct "listening" location, then public commitment sounds reasonable. However, if signal acquisition is a matter of cognitive effort or attention, then the possibility of public commitment seems far less likely, especially for what concerns letting the sender know about the commitment.

7. Conclusions

In Proposition 1 we have shown that in a SCAS game the presence of costs to acquire the signal restricts the feasibility of pooling equilibria to those where all sender's types pool on the minimum signal, while in the associated SFAS game there can be pooling equilibria on non-minimum signals; furthermore, any separating equilibrium of the SFAS game exists in the SCAS game as well if and only if the acquisition cost is not larger than the value of acquiring information on the signal for the receiver. Furthermore, there is no refinement acting through restrictions on beliefs at out-of-equilibrium signals, or relying on players' trembles when choosing strategies, that can get rid of the pooling equilibrium on the minimum signal, as it is shown in Propositions 2 and 3, respectively. These findings suggest that in a SCAS game a pooling outcome on the minimum signal is much more prominent than it is in the associated SFAS game, and such prominence grows in the acquisition costs (at the extreme, no separating equilibrium exists for sufficiently high acquisition costs).

Is this good or bad news? From a welfare perspective, a pooling equilibrium on the minimum signal can lead to either a better or worse outcome with respect to an alternative separating equilibrium. Of course, this is true for standard signaling games as well, since the transmission of information which occurs in a separating equilibrium – while surely beneficial for the receiver – may well be detrimental for some types of the sender. On top of this, we stress that in the presence of costs to acquire the signal, a pooling equilibrium where the receiver exerts no effort in signal acquisition allows to save the costs

¹⁶ We stress that if R is not forced to commit, but has just the option to do so, the main thrust of the argument still applies. The reason is that, as long as information is valuable to R , the option to commit to $s2$ will always be exercised.

of information acquisition. Without further assumptions on the relationship between sender's types and receiver's actions, it remains undecided whether suppressing information but saving in information acquisition is desirable or not.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.jebo.2017.10.022>.

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