Effect of interactions on the localization of a Bose-Einstein condensate in a quasiperiodic lattice

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Effect of interactions on the localization of a Bose-Einstein condensate in a quasiperiodic lattice

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The transport properties of a Bose-Einstein condensate in a 1D incommensurate bichromatic lattice are investigated both theoretically and experimentally. We observe a blockage of the center of mass motion with low atom number, and a return of motion when the atom number is increased. Solutions of the Gross-Pitaevskii equation show how the localization due to the quasidisorder introduced by the incommensurate bichromatic lattice is affected by the interactions.

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The intrinsic perfection of lattices made from a standing wave of light surprisingly makes them an excellent candidate for the investigation of disorder in atomic systems. Free from uncontrollable defects, precise disorder can be added simply in the form of additional optical lattices [1–6] or with an optical speckle potential [6–9]. The combination of optical lattices with a Bose-Einstein condensate (BEC) offers the complexity of interactions in a setting unimpeded by large thermal fluctuations. The significant observation of the Mott-insulator phase was realized utilizing a strongly interacting BEC produced in a three-dimensional ordered crystal of light [10]. Extending this work to the strongly disordered regime with the inclusion of a bichromatic lattice, recently led to initial experimental evidence of a Bose-glass phase [11]. Another open question remains as to the effect of weak interactions on the Anderson localized phase [12].

To address this regime we have carried out investigations of the transport of a BEC in a quasiperiodic lattice. The complexity provided by the interplay of disorder and interactions that makes disordered BECs a stimulating topic, also can introduce instability inherent in nonlinear transport [13]. In fact we find that dynamical instability does occur for transport in a quasiperiodic lattice when the center of mass motion reaches a critical velocity, and has a nontrivial dependence on the interaction strength.

Anderson's seminal paper in 1958 showed that the wave function of a particle placed in a disordered lattice remains localized when the range of the on-site energies is sufficiently large compared to the hopping energy between neighboring sites [14]. Such a disordered potential can be approximated using a bichromatic lattice, obtained by superimposing a primary optical lattice with a weak secondary lattice of incommensurate wavelength. The secondary lattice breaks the discrete translational invariance of the system, thus allowing localization of the wave functions. However, the effect of the quasi*order* may be important depending on the exact parameters of the bichromatic lattice [3,5].

In our system, the 1D incommensurate bichromatic lattice is produced combining the primary optical lattice derived from a Titanium:Sapphire laser operating at a wavelength $\lambda_1=830.7(1)$ nm with a secondary lattice obtained from a diode laser emitting at $\lambda_2=1076.8(1)$ nm. Our ^{87}Rb BEC is produced in an Ioffe-Pritchard magnetic trap, elongated in the direction of the lattice. The trapping frequencies are $\omega_x=2\pi\times 8.7$ Hz axially and $\omega_\perp=2\pi\times 90$ Hz radially. The BEC can be produced in the range of $\simeq(1.5\times 10^4)-(2\times 10^5)$ atoms. The resulting potential along the lattice axis is

$$V(x)=s_1E_{R1}\sin^2(k_1x)+s_2E_{R2}\sin^2(k_2x)+\frac{m}{2}\omega_x^2x^2, \quad (1)$$

where s_1 and s_2 measure the height of the lattice potentials in units of the respective recoil energies $E_{R1}=h^2/(2m\lambda_1^2)\simeq h\times 3.33$ kHz and $E_{R2}=h^2/(2m\lambda_2^2)\simeq h\times 1.98$ kHz, k_1 and k_2 are the wave numbers of the two lasers, h is the Planck constant, and m is the mass of ^{87}Rb.

The possibility of localization with our bichromatic lattice in the absence of interactions is demonstrated by numerical diagonalization of the stationary 1D Schrödinger equation using the potential defined by Eq. (1). A strong primary lattice is chosen, $s_1=10$, $s_1E_{R1}/h=33$ kHz, and a perturbing secondary lattice of maximum height $s_2=2$, $s_2E_{R2}/h=4$ kHz. The ground state resulting from the bichromatic lattice is shown in Fig. 1(a) and is contrasted with the ground state of a pure random case in Fig. 1(b). The random potential is simulated using only the primary lattice, $s_1=10$, with additional random on-site energies in a box distribution in the range $[0,\ \Delta]$, where $\Delta/h\leqslant 4$ kHz. The amount of the disorder is given by either the height of the secondary lattice in (a) or by the maximum on-site energy in the random case (b), and is denoted by Δ in both (a) and (b). We define J as the tunneling in the primary lattice. The thin line is the ground state with only the primary lattice, showing the total length of the system in the harmonic trap. The last graph in each column is an enlargement of the actual potentials. The quasiperiodic system mimics true disorder to a certain extent, showing localized states characterized by an exponential decay in the envelope of the density moving away from the localization center, $|\Psi(x)|^2\propto\exp(-|x-x_0|/l)$, where l is the localization length. The exponential localization occurs only above a threshold level at approximately $\Delta/J\simeq 6$ in the bichromatic case. This is in clear contrast with random disorder where in a 1D infinite system the localization persists for any infinitesimal amount of disorder.

In the presence of weak interactions one expects localization effects to persist only to a certain extent. In fact, repulsive interactions tend to delocalize the atoms competing against the disorder [12,15].

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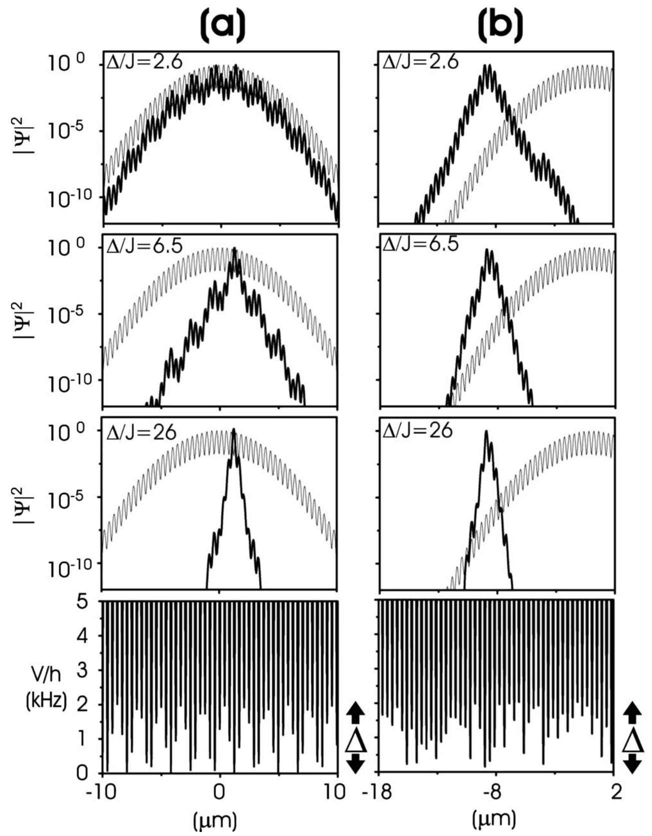

FIG. 1. Density profiles in log scale of the noninteracting ground state with increasing disorder in (a) a bichromatic lattice and (b) a lattice with random on-site energies. The thin line represents the ground state with only the primary lattice. The last graph in each column shows the on-site energies in the respective potentials with the amount of disorder shown by $\Delta/J \simeq 26$. In all cases the height of the primary lattice is $s_1=10$ giving a tunneling energy of $J/h=75$ Hz.

Such behavior can be seen in Fig. 2, showing the ground states in our bichromatic lattice with interactions calculated by means of a 1D effective equation, namely the nonpolynomial Schrödinger equation (NPSE) [16]. This model includes an effective radial-to-axial coupling thus providing a more realistic description than a strictly 1D GPE, but still avoiding the complication of the full 3D theory (see [13] and references therein). In the presence of interactions the strongly localized ground state of (a) transforms into a state with multiple peaks with partially overlapping tails (b). Upon increasing the interactions, the overlap between these peaks increases until the state eventually becomes extended (c). These results show similar behavior to previous simulations of a BEC in a three-color lattice [6]. Even with the increased interactions, a state with multiple peaks can be recovered by increasing disorder (d)–(f). The crossover behavior between an extended superfluid state and a localized state is very difficult to quantify in this picture. We take a pragmatic approach and characterize the system by investigating its transport properties.

In the experiment we excite dipole oscillations by abruptly shifting the center of the magnetic trap. With a single color lattice, the superfluid BEC oscillates freely at a frequency modified by the effective mass [17,18] differently from what has been observed for 1D atomic gases in [19] or for a 3D BEC in much deeper optical lattices in [20]. Adding an incommensurate bichromatic lattice, we expect the oscillations to be blocked by localization effects. Figures 3(a)–3(d) show the center of mass motion after a trap shift of 6 μm, with a fixed number of atoms $N=1.5\times10^4$, a fixed height of the primary lattice $s_1=10$, and a variable height of the secondary lattice. At $s_2=0.1$ ($\Delta/J=2.6$), the BEC oscillates with some damping. Increasing s_2 the motion is strongly damped, until the BEC stops moving at $s_2=0.5$ ($\Delta/J=13$). The height of the secondary lattice at which the BEC becomes localized is greater than the noninteracting threshold for localization shown in Fig. 1. This could be due to the screening effect of interactions shown in Fig. 2.

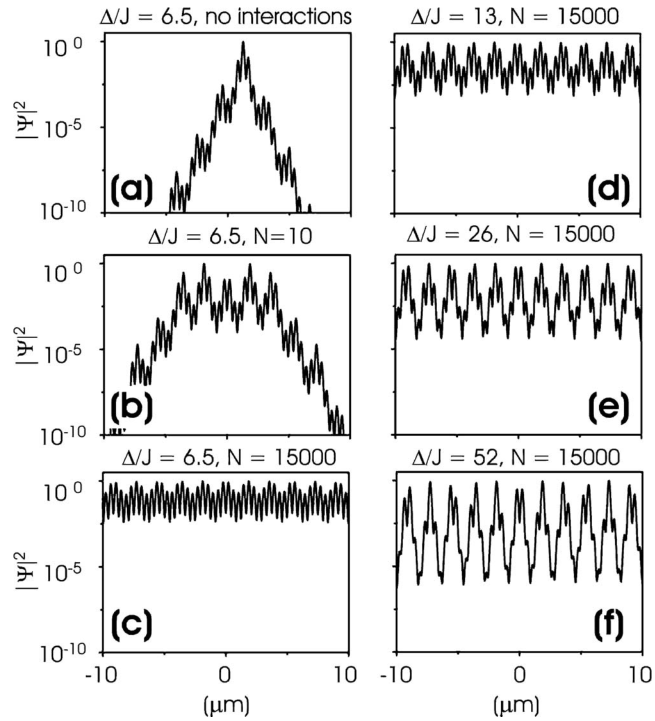

FIG. 2. Density profile in log scale of the ground state with interactions for (a)–(c) increasing atom number and fixed $\Delta/J=6.5$, and for (d)–(f) increasing Δ/J and fixed atom number $N=1.5\times10^4$. In all cases the height of the primary lattice is $s_1=10$.

We extended the investigations to a variable interaction energy effectuated by changing the BEC number of atoms. Figure 3(d) shows the measured center of mass motion of the atoms for two different atom numbers $N=1.5\times10^4$ and $N=2\times10^5$, after an initial trap shift of 6 μm, $s_1=10$ and $s_2=0.5$ ($\Delta/J=13$). Transport is stopped only for the low number of atoms, although some damping is seen with the higher atom number. To improve the signal to noise ratio, the experimental data were taken after abruptly switching off all the confining potentials allowing 20 ms of time of flight. The dipole oscillations were simulated using the time-dependent NPSE followed by a free expansion and are shown by the dashed and dotted lines in Fig. 3. The experimental behavior is nicely described by the solutions of the NPSE. We note

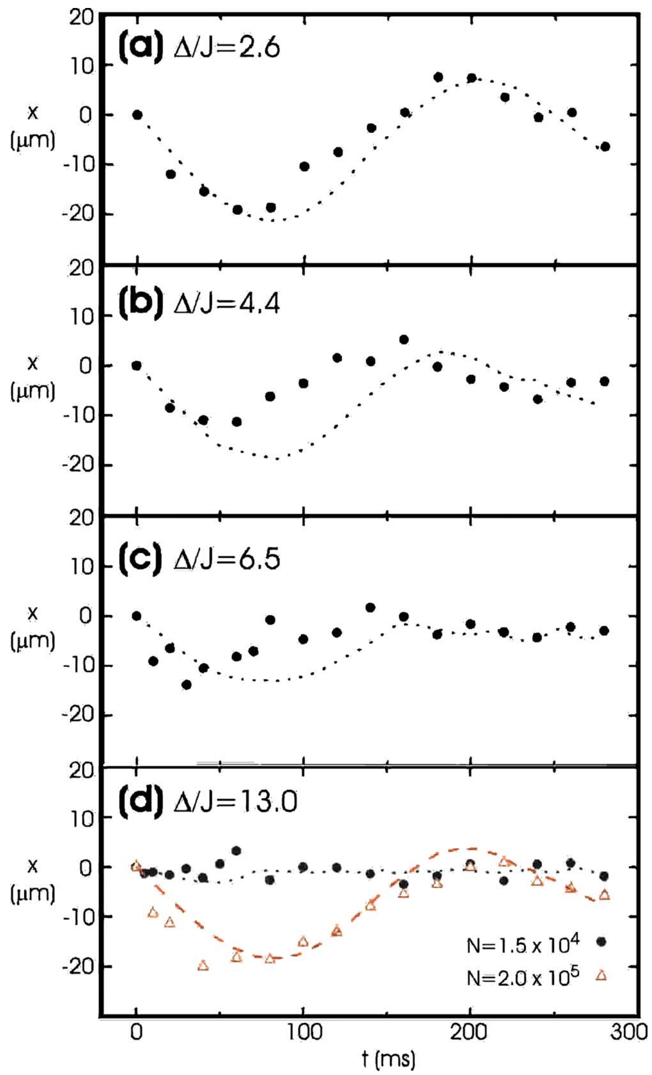

FIG. 3. (Color online) (a)–(c) Measured dipole oscillations with increasing intensity of the secondary lattice. For $s_1=10$ and $N=1.5\times10^4$. (d) Measured oscillations with $s_1=10$, $\Delta/J=13$ and two atom number, $N=1.5\times10^4$ and $N=2\times10^5$. The motion of the atoms was measured after 20 ms of expansion, taking the position of the central peak. The origin of the x axis is set to the initial position after expansion. The dotted and dashed lines show NPSE simulations.

that for intermediate values of Δ/J the dynamics are critically sensitive to the actual experimental parameters hampering a quantitative comparison with the simulations. Furthermore, in this regime dynamical instability strongly modifies the dynamics, the details depending on the initial populations of the unstable modes. This could explain the larger discrepancy between experimental data and simulations in this regime [Figs. 3(b) and 3(c)].

Localization due to disorder is not the only physical effect which can block the motion of the atoms. In the case of an incommensurate bichromatic lattice the Bloch theorem cannot be applied, nevertheless the energy spectrum still shows energy bands. The band structure for the single lattice is complicated by the emergence of "minigaps" opening up almost everywhere across the spectrum [5]. However, the dominant modification to the first Brillouin zone of the primary lattice, for a weak addition of the secondary lattice, are the extra energy gaps at k_b and k_1-k_b, where $k_1=2\pi/\lambda_1$ gives the boundary of the first Brillouin zone of the primary lattice, and $k_b=2\pi/\lambda_b$ corresponds to the quasiperiodicity introduced on the larger length scale $\lambda_b=\lambda_1\lambda_2/(\lambda_2-\lambda_1)=4.38\lambda_1$ from the beating between the two colors. This simplification to the energy spectrum is particularly true when interactions are introduced, since they can effectively screen the potential varying on length scales larger than the healing length, washing away the energy gaps at smaller k. As a consequence, in the presence of interactions one should carefully investigate the possible contribution of dynamical instability that in a single lattice has been observed to block dipole oscillations [13,18] when the quasimomentum becomes greater than $\simeq 0.5k_1$. In the bichromatic lattice dynamical instability may occur at a small quasimomentum $\gtrsim 0.5k_b$ corresponding to the beat periodicity, much smaller than the onset of instability with only the primary lattice.

To understand the contribution of the various effects instigating localization, the center of mass motion can be compared to the momentum spectrum taken from the NPSE. Figure 4(a) shows the center of mass motion using the same parameters as the measurements taken in Fig. 3(d); an initial magnetic trap shift of 6 μm, $s_1=10$, $s_2=0.5$, and for both $N=1.5\times10^4$ and $N=2.0\times10^5$. Note that the simulations are the same as shown in Fig. 3(d); however, Fig. 4(a) shows the motion in the trap without expansion for the first 100 ms.

From the momentum spectrum it is possible to distinguish when dynamical instability is present. Figure 4(b) shows the progression of the momentum spectra with $N=1.5\times10^4$, and Fig. 4(c) shows the momentum spectra with $N=2\times10^5$. At $t=25$ ms, the main components of the momentum spectra correspond to the peaks of the primary lattice at integer multiples of $\pm2\hbar k_1$, and peaks of the beat periodicity at integer multiples of $\pm2\hbar k_b$ around the primary peaks. For both high and low atom number, at $t=50$ ms extra momentum components begin to rapidly grow signifying the onset of dynamical instability [13]. We observe that the instability occurs when the quasimomentum becomes greater than $0.5k_b$. By 75 ms there is a marked difference between the cases of high and low atom number. With $N=1.5\times10^4$ atoms, the initial spectrum has been obscured by the additional momentum components. In contrast, with $N=2\times10^5$ atoms, the spectrum retains much of its original structure. Interestingly, the increased nonlinearities inhibit the growth of the instability.

Augmenting the interactions reduces the healing length of the condensate. In previous experiments, with a single lattice of spacing 0.4 μm and typically a healing length of $\simeq 0.3$ μm, the lattice spacing is not significantly greater than the healing length, and therefore measurements are largely indifferent to the atom number [13,18]. However, in the case of the bichromatic lattice the large spacing of the beating (1.8 μm) allows interactions to effectively smooth over the large scale beat periodicity. The growth of dynamical instability in our bichromatic lattice is governed by the competition between augmentation by increased nonlinearity and diminution by screening of the beat periodicity [21].

Considering the center of mass motion shown in Fig. 4(a) together with the momentum spectrum we note that the con-

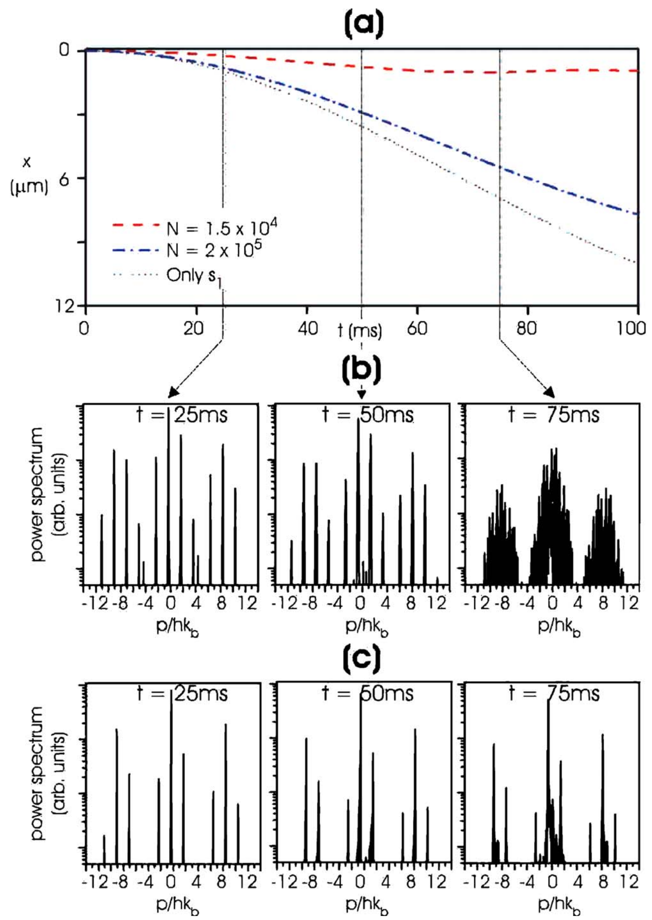

FIG. 4. (Color online) (a) The center of mass motion from NPSE simulations for $s_1=10$, $\Delta/J=13$, and two different N. The dotted line shows the expected superfluid oscillation in a one color lattice with $s_1=10$. (b) The corresponding momentum spectra in log scale at different times for $N=1.5\times10^4$. The vertical arrows show the time at which each momentum spectrum is taken. (c) The momentum spectra at different times for $N=2\times10^5$.

taminated momenta at 75 ms for low atom number, shown in Fig. 4(b), is reflected in the complete blockage of the center of mass motion. However, most importantly, in the absence of dynamical instability, which is macroscopically observed only after 50 ms, the movement of the atoms is strongly damped with respect to the superfluid case [see dotted line in Fig. 4(a)], suggesting that strong damping of the oscillations is not only due to dynamical instability originating from the beat periodicity of the bichromatic lattice. The presence of interactions and the screening of the minigaps at small k from the incommensurate lattice renders interpretation of the blocked motion at short times difficult. As already pointed out, with sufficiently strong interactions the energy spectrum could be dominated by the periodicity at λ_b, and correspondingly the strongly damped motion at short times could be described by a highly effective mass. A further increase of the interactions produces a screening up to larger k, thus reducing the effective mass down to that of the primary lattice [21,22]. Similarly to the screening behavior seen in Fig. 2, the contribution of the disorder will depend on the relative strength of interactions and the intensity of the disordering potential.

In conclusion, maintaining a minimal number of atoms we have observed a transition from oscillations to blocked motion with increasing intensity of the incommensurate lattice. Simulations for our experimental parameters using the NPSE show that the quasiorder inherent in the bichromatic lattice leads to dynamical instability that contributes to the blocked motion only after a critical time. Screening of both localization due to the disorder and dynamical instability due to the beat periodicity was observed with strengthened interactions in the simulations. Increasing the number of atoms in the experiment, we observed a return of oscillating motion.

This work shows that the exact choice of parameters is crucial to separate and isolate the effect of disorder-induced localization, or the nontrivial onset of dynamical instability in a bichromatic lattice. In the noninteracting limit, the bichromatic lattice is also a very promising tool to investigate Anderson-like localization that could be accessed utilizing fermions or Feshbach resonances, which also provide the important possibility of tuning the interactions.

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