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Frequency vs time domain identification of heritage structures

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Abstract

This paper discusses on the output only modal identification of historic structures. Modal identification has been carried out both in frequency and in time domain using the Frequency Domain Decomposition (FDD) and the Stochastic Subspace Iteration (SSI-data) respectively. To highlight the sensitivity of the two methods, two masonry towers under different environmental loads have been considered. In one case the dynamic excitation can be assumed as a very weak random white noise, in the other case, probably due the several external noises, the ambient noise is locally dominated by several harmonic forces. The paper highlights the challenges in the modal identification of heritage structures such as the very low weak operating response and the role of the harmonics originated by engines operating somewhere nearby the structures. Moreover, it is investigated the information trade-off between the frequency domain and the time domain identification for evaluating the modal properties of the structures in a straightforward manner.

Keywords: Modal Identification; Historic structures; Output only; FDD; SSI.

1. Introduction

From the early nineties the modal identification of structures under operative/environmental loads, the so-called OMA (Operational Modal Analysis), has been investigated by several authors. At the beginning, the common approach was the spectral analysis of the signals with simple techniques as the Peak Picking (PP) (Bendat and Piersol 2010) dealing with the peak identification in the Fourier transform of the recorded signals. Felber (1993)

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introduced the Averaged Normal Spectral Density (ANSPD) for identifying the natural frequencies and the mode shapes of bridges structures. The natural evolution of this technique is the Frequency Domain Decomposition (FDD) (Brincker et al. 2001) that allows to evaluate the natural frequencies of structures by the analysis of the Singular Values Decomposition (SVD) of the power spectral density matrix at each frequency. The basic hypotheses of these techniques are that the structures are lightly damped, the mode are well separated so that the response is dominated by a singular mode in a certain frequency bandwidth and the excitation is a Gaussian white noise. The second generation of OMA techniques in the frequency domain is the Enhanced Frequency Domain Decomposition (EFDD), which estimates the modal power spectral density of a modal coordinate with the information of the SVD around the chosen peak by considering all the spectral lines around the peak that have a sufficiently high modal assurance criterion (MAC) with the first mode shape estimation. Zhang (Zhang et al. 2010) introduced the modal filtering technique to isolate each modal coordinate avoiding the analysis of all the spectral lines around the peaks and the definition of a MAC threshold to merge the frequency points that define the modal spectral density of the analyzed coordinate. Once the power spectrum of the SDOF is available it is possible to take back in the time domain evaluating the autocorrelation function that can be viewed as the impulse response function (IRF) of the modal coordinate enabling the evaluation of the damping ratio and of the damped frequency.

The techniques in the time domain are based on the theory of the systems and control, using the state formulation of the dynamic problem to extract physical information from the signals. Among all the proposed techniques (Ibrahim time domain, Eigensystem realization, AR and ARMA models), the Stochastic Subspace Iteration (SSI) (Peeters et al. 1999) can be easily automated for identifying the modal property of the structures. The SSI-data procedure (Van Overschee et al. 1996) allows the modal identification from the recorded signals of a monitoring system, usually based on accelerometers data. The technology growth and the huge calculation capability of the modern computers support the use of this technique also for long-term monitoring purpose of civil structures (Magalhães et al. 2012; Zhang et al. 2017). One of the main issue is setting the model parameters and the extraction of the modal properties distinguishing between the spurious and the physical poles of the system. Furthermore, in the cultural heritage buildings the recorded response is very weak, and the level of signal-to-noise ratio can be very high. The quality of the signals and the presence of harmonics play key role on the modal identification of these structures.

In the last years there is a renewed attention in the OMA techniques applied to the cultural heritage for Structural Health Monitoring purpose, and several recent experiences are well described in literature (Ubertini et al. 2017; Gentile et al. 2016; Azzara et al. 2018). In this paper, some of the issues above reported are discussed examining two case studies selected with the aim to underlining the key role aspects of the modal identification of heritage buildings.

Nomenclature

$\mathbf{S_y}$	Power Spectral density matrix.
$\mathbf{s_n}$	Singular values matrix elements.
$\widetilde{\boldsymbol{\Phi}}$	Mode shape estimation matrix, each column represents the approximate mode shape.
$\mathbf{S_y}$	Power Spectral density matrix
$\mathbf{s_q}$	Auto-spectral density of the modal coordinate q.
$\mathbf{x(k)}$	State vector at the k*th* step.
$\mathbf{y(k)}$	Measurement vector at the k*th* step.
$\mathbf{A}$	System matrix.
$\mathbf{C}$	Output matrix.
$\mathbf{w(k)}$	External Gaussian white noise input.
$\mathbf{v(k)}$	Noise on the measurement modelled as Gaussian white noise.
$\mathbf{P}$	Projection matrix.
$\mathbf{H_i}$	Hankel block matrix.
$\boldsymbol{\Gamma_i}$	Observability matrix.
$\widehat{\mathbf{X}}_i$	Kalman filters matrix estimate.

1.1. Background on Frequency Domain Decomposition (FDD)

The main idea of the FDD is based on the SVD decomposition of the power spectral density matrix (PSD) that is a positive definite Hermitian matrix. The decomposition lead to the diagonal singular value matrix $\mathbf{S}$, left multiplied for a matrix $\mathbf{U}$ and right multiplied for the transpose complex conjugate matrix of $\mathbf{U}$:

$$\mathbf{S_y} = \mathbf{U S U^H} \tag{1}$$

This decomposition can be interpreted as the multiplication of the mode shape matrix for the auto-spectral density of the modal coordinates:

$$\mathbf{S_y(f)} = \widetilde{\boldsymbol{\Phi}} \mathbf{s_n(f)} \widetilde{\boldsymbol{\Phi}}^{\mathbf{H}} \tag{2}$$

The FDD is a biased technique because the decomposition (2) is not the exact decomposition of the PSD matrix of a structure excited with a white noise. If the system is light damped and the modes are well separated the decomposition (2) is a good approximation of the modal properties of the system.

Usually the hypothesis of white noise input and low values of damping are satisfied, but the separation between the modes is very often violated. For this reason, the PSD matrix on a certain frequency band can be written as the superposition of the modal auto spectral density of each modal coordinate:

$$\mathbf{S_y(f)} = \mathbf{s_1(f)} \widetilde{\boldsymbol{\Phi}}_1 \widetilde{\boldsymbol{\Phi}}_1^{\mathbf{H}} + \mathbf{s_2(f)} \widetilde{\boldsymbol{\Phi}}_2 \widetilde{\boldsymbol{\Phi}}_2^{\mathbf{H}} + \ldots \tag{3}$$

Defining a set of orthogonal vectors $\mathbf{V} = [\mathbf{v_1}, \mathbf{v_2}, \ldots]$ such that:

$$\mathbf{V^H} \widetilde{\boldsymbol{\Phi}} = \mathbf{I} \tag{4}$$

it is now possible to isolate the spectral density of a single modal coordinate by projecting the PSD matrix in the new reference system $\mathbf{V}$, in that way the auto-spectral density is available for each modal coordinate:

$$\mathbf{s_q(f)} = \mathbf{V^H S_y(f) V} \tag{5}$$

With the full EFDD procedure is now possible taking back the auto-spectral density in the time domain estimating the modal damping ratio as the logarithmic decrement of the autocorrelation function that can be interpreted as free decay (this is beyond the aim of the paper, because the identification procedure uses also the SSI technique that furnishes directly the poles of the system).

1.2. Background on Stochastic Subspace Iteration (SSI)

The SSI is a parametric technique developed in the time domain based on the discrete time state space form of a linear time invariant system under unknown excitation:

$$\begin{cases} \mathbf{x(k+1)} = \mathbf{A x(k)} + \mathbf{w(k)} \\ \quad \mathbf{y(k)} = \mathbf{C x(k)} + \mathbf{v(k)} \end{cases} \tag{6}$$

The data driven approach based on the analysis of the recorded data directly follows these operation: the construction of the Hankel block matrix $\mathbf{H_i}$ that con be interpreted as the two-block matrix of the observation of the future $\mathbf{Y_f}$ and the observation of the past $\mathbf{Y_p}$. The number of column j is equal to the number of samples in the time history and the number i of rows is a user defined property of the system.

$$\mathbf{H_i} = \begin{bmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_2 & \cdots & y_j \\ y_{i-1} & y_i & \cdots & y_{i+j-2} \\ \hline y_i & y_{i+1} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+j} \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+j-2} \end{bmatrix} = \begin{bmatrix} \mathbf{Y_p} \\ \mathbf{Y_f} \end{bmatrix} = \begin{bmatrix} \text{"past"} \\ \text{"future"} \end{bmatrix} \tag{7}$$

The orthogonal projection of the row space of the observation of the past onto the row space of the observation of the future, can be calculated yielding the projection matrix **P**:

$$\mathbf{P} = \mathbf{\Gamma}_i \hat{\mathbf{X}}_i \tag{8}$$

Following the decomposition (7) the projection matrix can be expressed as the multiplication of the observability matrix $\mathbf{\Gamma}_i$ and the Kalman filters state estimate $\hat{\mathbf{X}}_i$. The observability matrix can be estimated by the SVD decomposition of the projection matrix. The number of the singular values considered in the estimate of the observability matrix is the order of the model. Usually the projection matrix **P** is pre and post multiplied for some convenient weight matrices $\mathbf{W}_1$ and $\mathbf{W}_1$. Based on the choice of the weighted matrices we have different algorithm as the Principal Components (PC), Unweighted Principal Components (UPC) and the Canonic Variate Algorithm.

From the Kalman filters state estimate it is possible to evaluate the system matrix **A** governing the problem (6), with the state space formulation for a linear dynamic system it is possible to evaluate the mode shape and the poles of the system. The main issue is the choice of the parameters that characterize the model, and the choice of the physical meaningful modes among all the spurious modes. Because the modes identified by the procedure are equal to the order of the model chosen by the user. To overcome this problem, it is possible to use the stabilization chart or different type of clustering of the data (Reynders et al. 2012; Magalhães et al. 2011), performing different SSI analysis with different parameters. In the following applications we used a two steps post processing technique proposed by Ubertini et al. (2012) based first on the identification of the stables modes imposing soft criterions on the identified modal parameters and a hard criterion on the maximum value of the damping ratio (set at the 5%). The number of elements that satisfy both conditions is the criteria for identifying a stable mode. The second stage is based on the assembly of the clusters with the distance between the elements lower than a user-defined value. Moreover, it is imposed a hard condition on the minimum number of elements for considering representative a certain cluster. The mean value of the cluster is considered as the representative statistical value for all the modal parameters estimated.

2. Illustrative examples

The techniques summarized above are here used for the automated modal identification of two masonry towers characterized by different level of excitation and noise. It is discussed how the two techniques can be integrated for the validation of the modal identification, that is a crucial point for the automated procedures employed in the long-term monitoring systems.
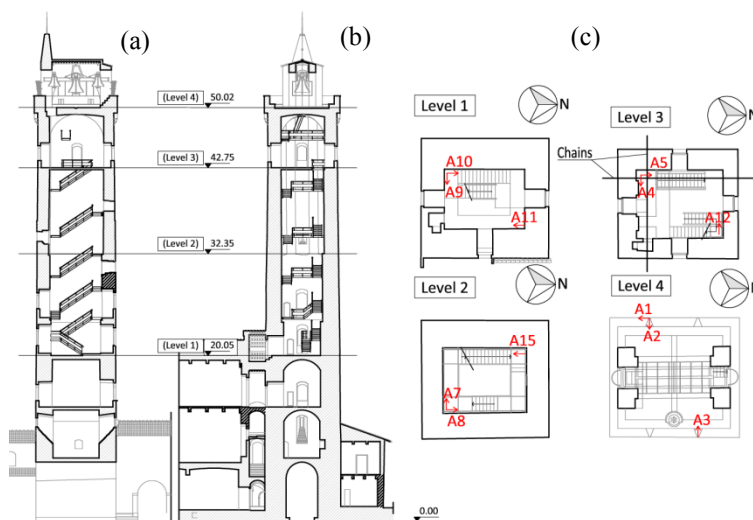

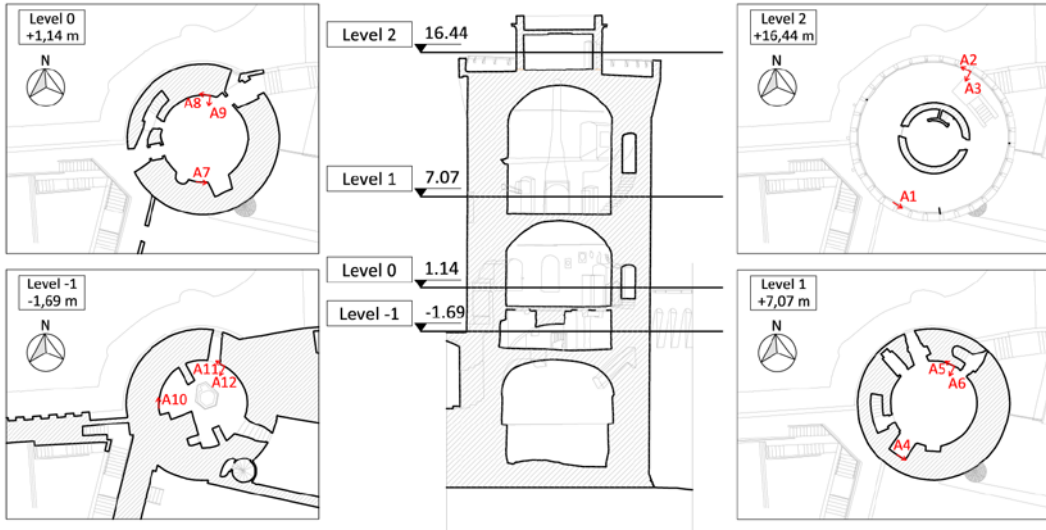Fig. 1. Tower A: (a) N-S cross-section (b) E-W cross-section (c) Accelerometers positions.

Fig. 2. Tower B: cross-section and accelerometer positions.

The two cases study are the "Torre Grossa" (Tower A, Fig. 1) and the "Mastio di Matilde" (Tower B, Fig. 2). The cases are interesting since the level of ambient excitation is completely different and the presence of noise can lead to false identified modal properties of the structures. Moreover, these two old masonry towers are slightly different, both for the slenderness and for the boundary conditions, seeming good cases of study for testing the modal identification techniques under ambient loads to capture the critical issues of the analyzed techniques.

2.1. Tower A ("Torre Grossa")

Tower A is the tallest of the survived towers in the city centre of San Gimignano (Italy). The tower was built in the thirteenth century with a square cross section of about 9.5×9.5 m with an overall height of about 55 m. The sustaining walls, with variable thickness from 2.6 m at the base until 1.6 m at the top, are built with the multilayer technique: two external layers (travertine stones and brick masonry) and a cohesive internal filling (Fig. 1).
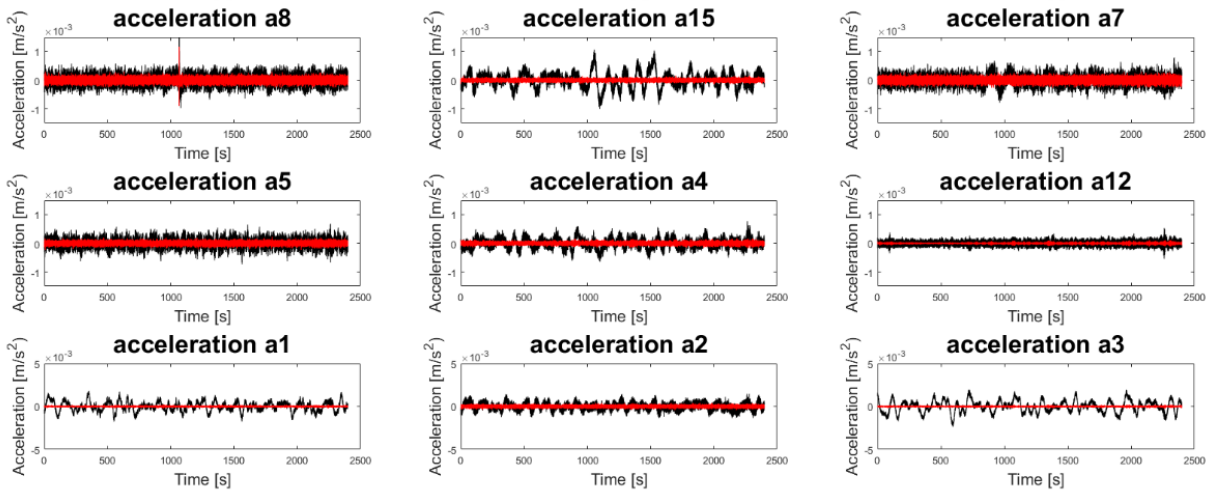

Fig. 3. Tower A: Recorded accelerations of the analyzed time history (raw data in black, filtered and resampled data in red).

Up to 20 meters the tower is confined in the East direction by the "Palazzo Comunale" (Town Hall), and on the West direction by a masonry building until the height of 10 meters. The slabs of the first two levels are built by masonry vaults, while in the last level there is a concrete slab. Internally, the levels of the tower are connected by a steel stair that arrive until the top level where is located a big bell. The tower was repaired in the past with two steel chains in the SW corner that was collapsed during a thunder storm.

The experimental tests were developed between the 29th-30th of March 2017 by the DICEA-UNIFI Lab, employing 12 accelerometers. The sampling frequency was of 400 Hz and the length of all the time series was about 2400 s. All the signals are filtered with a bandpass filter between 0.3-10 Hz, and the signal is resampled at 25 Hz to have a good frequency resolution and a limited quantity of data points (Fig. 3). The position of the accelerometers is shown in Fig. 1(c). Each level has three accelerometers for identifying the torsional mode shape. The N-S direction is the X axis and the E-W direction (directed to the Town Hall) is the Y axis.

Taking into account that the city center of San Gimignano, where the tower is, is restricted to the ordinary vehicular traffic and that the wind was completely absent during the investigation, the recorded acceleration at each point were very weak (of the order of 10^{-4} m/s^2). With that low energy signals, the modal identification become challenging and almost a low level of noise can compromise the results of the modal identification. This make the case study interesting.
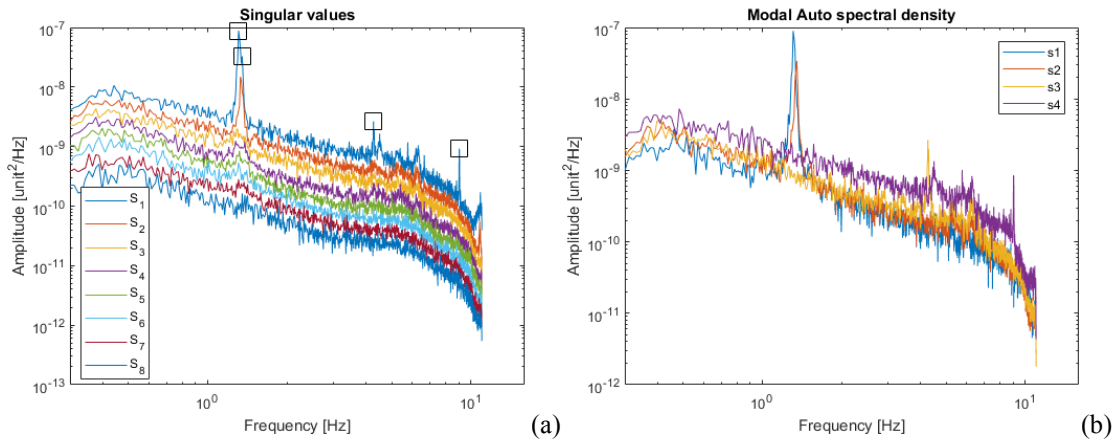

Fig. 4. Tower A: (a) Singular values of the PSD matrix; (b) Auto-spectral density of the modal coordinates.
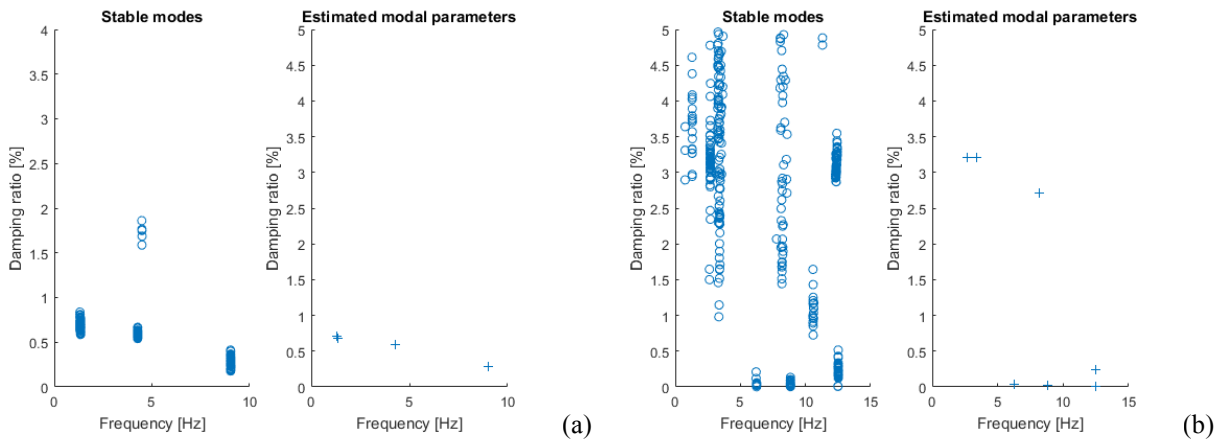

Fig. 5. Stabilization chart and mean value of frequency and damping for each cluster: (a) Tower A; (b) Tower B.
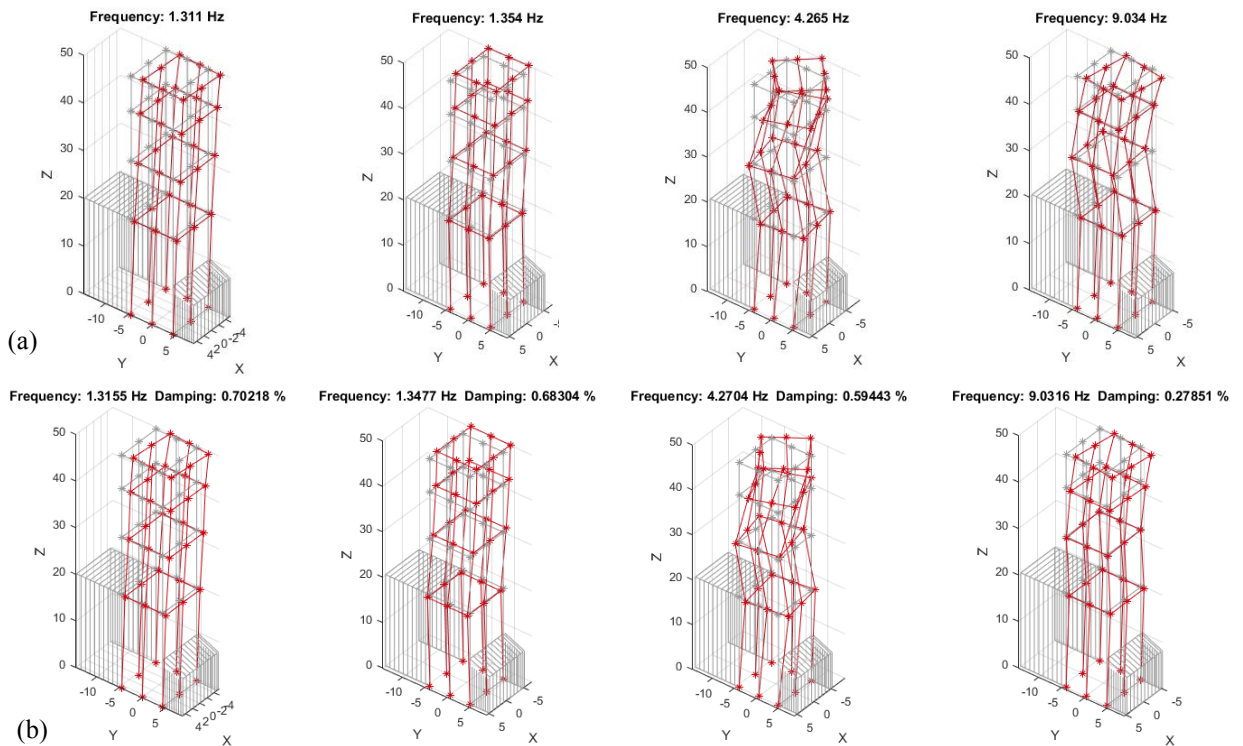
Fig. 6. Tower A: (a) The first four modes identified by the FDD (b) The first four modes identified by the SSI.

Table 1: Tower A: comparison of the results.

Identified mode	Frequency (Hz)		Damping ratio (%)		MAC
	FDD	SSI	FDD	SSI	
Mode 1 (translational *Y-Y*)	1.311	1.315	-	0.70	0.998
Mode 2 (translational *X-X*)	1.354	1.347	-	0.68	0.962
Mode 3 (torsional)	4.265	4.270	-	0.59	0.986
Mode 4 (translational *Y-Y*)	9.034	9.031	-	0.27	0.794

Despite the low level of excitation, confirmed by the spectral analysis (Fig. 4), it is possible to clearly distinguish the first two translational modes (with very close frequency). Both the SSI and the EFDD (Fig. 6)catch the two close modes that agree with the results obtained by a previous dynamic testing campaign carried out by performing a forced vibration test (Bartoli et al. 2013). Furthermore, the difference between the identified modal parameters with the two methods is less than the 1% in terms of frequency, more than 95% in terms of MAC except for the last mode were decreases until the 80 % (Table 1).

2.2. Tower B ("Mastio di Matilde")

The Tower B is a massive circular tower located in the "Fortezza Vecchia" (old fortress) in front of the Livorno's harbor (Italy). The external diameter of this masonry structure is about 12 and a total height of about 29 m. The masonry walls have a thickness of 2.5 m and an helicoidal stair to reach the different levels is embedded in the walls (Fig. 2). The slabs are built with masonry vaults except the last level that is made of concrete. The tower built in the thirteenth century, as a stand-alone structure, is now surrounded by masonry walls. In the West side the corner of a

little fortress, called "Quadratura dei Pisani", confined the tower up a height of thirteen meters; on the North side there is another masonry wall of the same height. On the South-East side stands the ruins of the Cosimo's dei Medici palace until a height of about ten meters.

For the mode shapes identification, the *X* axis is along the E-W direction (parallel to the first wall of the "Quadratura dei Pisani") and the *Y* axis is along the N-S direction (harbor direction). From the radial and the tangential direction, signals are calculated the components in the desired reference system considering rigid plane section displacements.

The experimental tests were developed between the 23th of January 2017 by the DICEA-UNIFI Lab, with 12 accelerometers. The sampling frequency was of 400 Hz and the length of all the time series was about 2400 s. All the signals are filtered with a bandpass filter between 0.3-15 Hz and the signal is resampled at 40 Hz (Fig. 7).
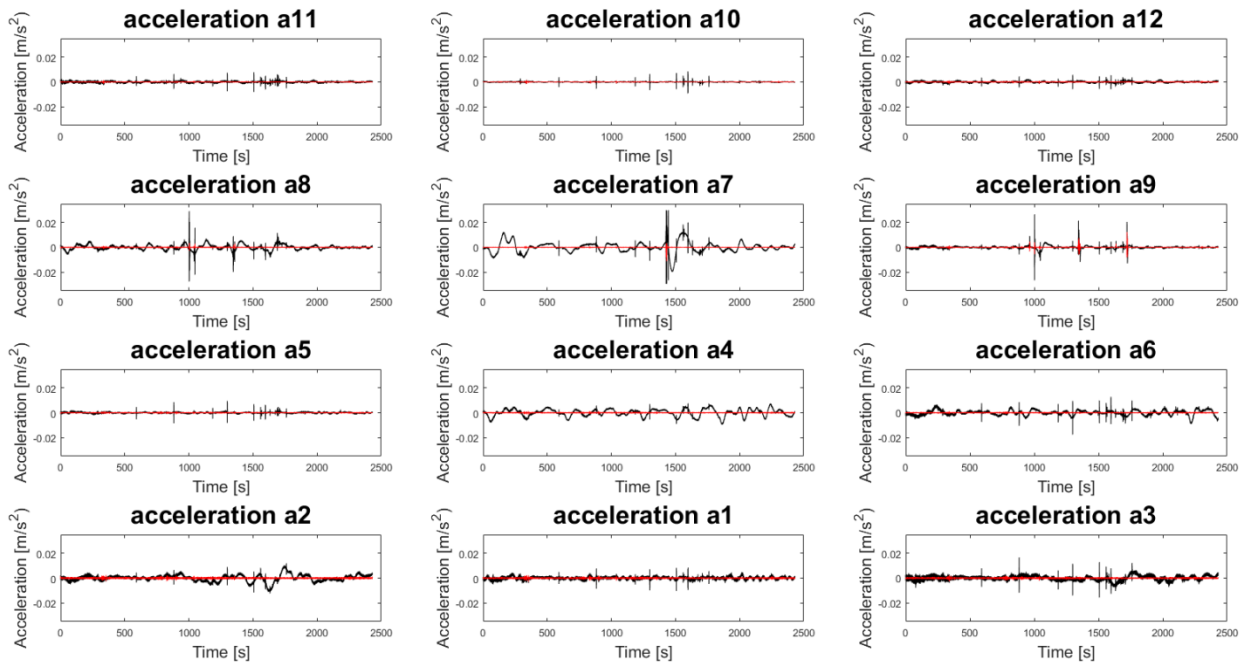

Fig. 7. Tower B: Recorded accelerations of the analyzed time history (raw data in black, filtered and resampled data in red).
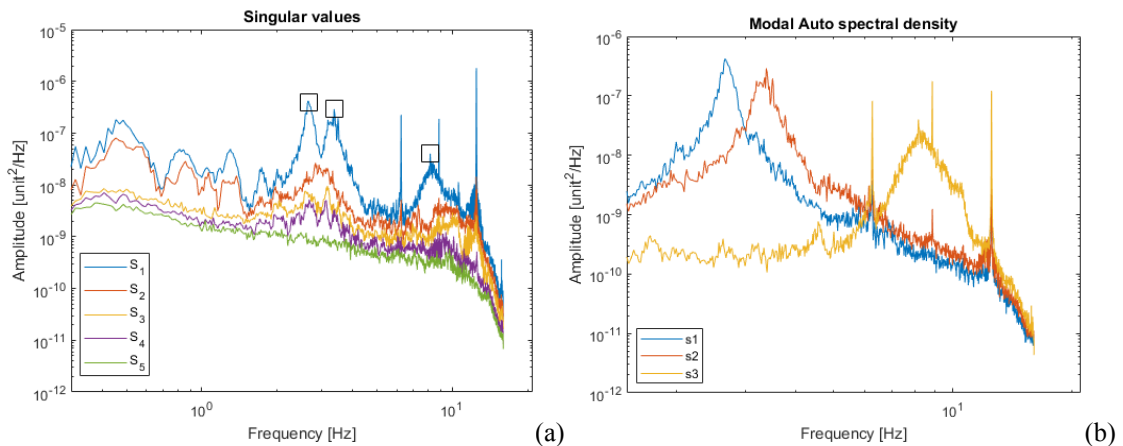

Fig. 8. Tower B: (a) Singular values of the PSD matrix; (b) Auto-spectral density of the modal coordinates.

The decomposition of the PSD matrix by the SVD shows the first four singular values contain physical information about the system. The decomposition via modal filtering of the first three singular values lead to the auto-spectral density of each modal coordinate. The harmonics are clearly visible with narrow band peaks in all the singular values causing a local increase of the PSD matrix rank (Fig. 8).
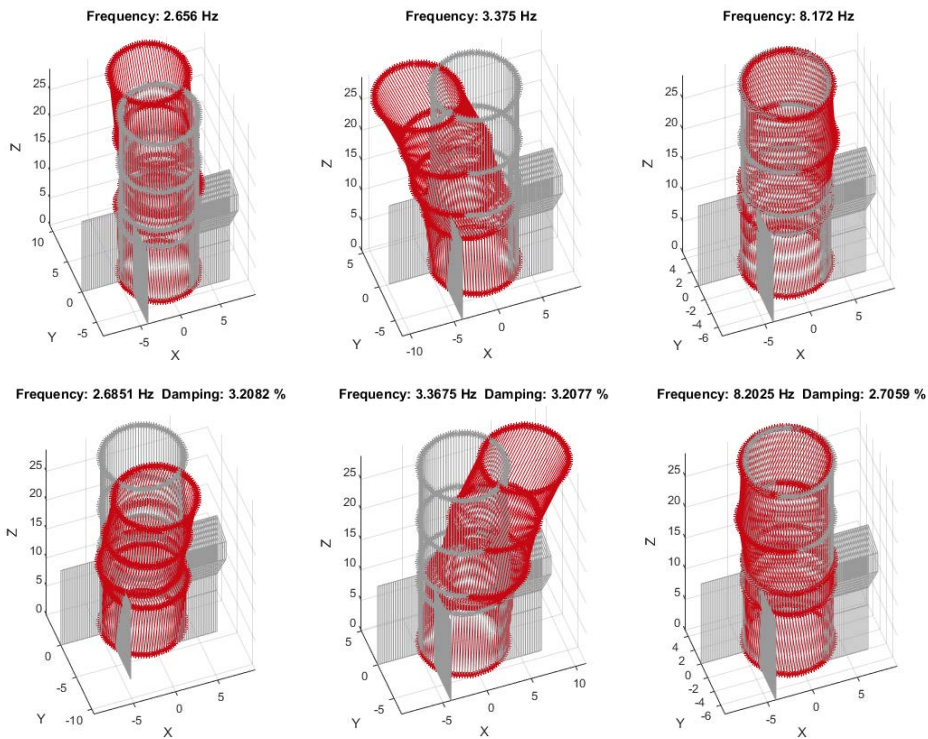

Fig. 9.Tower B (a) First three mode identification with FDD; (b) First three mode identification with SSI.

Table 2. Tower B: comparison of the results

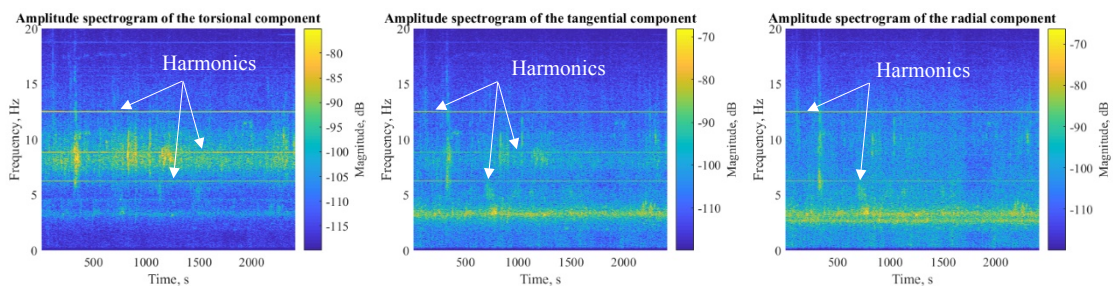| Identified mode	Frequency (Hz)		Damping ratio (%)		MAC
	FDD	SSI	FDD	SSI	
Mode 1 (translational y-y)	2,656	2,685	-	3,20	0,998
Mode 2 (translational x-x)	3,375	3,367	-	3,20	0.990
Mode 3 (torsional)	8,172	8,202	-	2,70	0,973

Fig. 10. Tower B: Short Fourier Transform of the displacements components at the last level.

The short Fourier Transform confirms the presence of harmonic components persistent during the whole length of the signal at 6 Hz, 9Hz and 12 Hz (Fig. 10). Moreover, the identified modes are confirmed with the first two bands representing the translational modes in the horizontal direction and the third torsional mode is clearly visible in the frequency band of 8 Hz.

3. Conclusions

The OMA techniques are widely used nowadays for extracting from the collected signals the modal properties of the structures. This paper discussed the efficiency of the most effective techniques in the frequency and in the time domain. The FDD have the great advantage that is not a parametric technique and the peak on the spectral density can be easily determined. Furthermore, with the EFDD is also possible to detect very close modes distinguishing them in the modal auto-spectral density.

Instead the SSI need to be tuned to obtain reliable modal parameters (Fig. 5) but is clear that the procedure can be easily automated enhancing the analysis of a huge amount of data from the long-term monitoring systems. These techniques are consolidated for the analysis of great civil structures which usually present higher level of recorded signals. For heritage buildings the recorded signals are very weak with a higher signal to noise ratio. The paper underlines how it is possible to overcome these issues by merging together the two techniques and paying attention to the signal analysis.

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