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Review text:

This paper presents a system of propositional logic equipped with suitable modal operators to speak of rationality of players in finite games in normal form, and tries to characterize the set of its theorems. The language this logical system is built upon, is made in such a way that, given a game with a finite set N of players, for each of which there is a equally finite set S_i of strategies (these sets S_i being mutually disjoint), it contains atomic formulas s_i^j, for $1 \leq i \leq |N|$ and $1 \leq j \leq |S_i|$, where $|N|, |S_i|$ indicate the size of N and S_i respectively, with the intended meaning: "player i plays strategy s^j of hers", and formulas $r_i(S)$ for: "strategy $r_i(S)$ is rational to player i if the opponents choose strategies in S". Operators $r_i(x)$ are the axiomatic counterpart of functions, say r_i by abuse of notation, whose domain is the set of all product sets $\mathcal{T}_{-i} := \mathcal{T}_{i_1} \times \mathcal{T}_{i_2} \times \ldots \times \mathcal{T}_{i_m}$, where $\{i_1, \ldots, i_m\} = N \setminus \{i\}$ and each $\mathcal{T}_{i_j}$ is a subset of S_{i_j}, and whose co-domain is the set of all subsets of S_i (hence, r_i returns a finite set of strategies of player i, those that are rational for a given combination of sets of strategies of i's opponents).

Complex formulas of the language are then built by means of a modal operator for logical inference $\vdash_i$, in which case they take the form $\vdash_i \varphi$ for "player i infers φ", by means of the usual logical connectives, and by combinations thereof. Special attention is devoted to *simple statements*, i.e. formulas $T_i := (s_i^1 \vee s_i^2 \vee \ldots \vee s_i^k)$ with $k \leq |S_i|$, which intuitively say that action by player i is determined in the sense that is chosen among strategies in the list $s_i^1, s_i^2, \ldots, s_i^k$. Equally important are combinations of simple statements $T_I := T_{i_1} \wedge T_{i_2} \wedge \ldots \wedge T_{i_k}$, where $I = \{i_1, \ldots, i_k\} \subseteq N$.

Axioms of the system include all classical propositional tautologies, the axiom asserting that at least one of all possible actions combinations in the game will occur, and all axioms of the form $(\vdash_i T_{-i}) \rightarrow r_i(T_{-i})$, where $-i := N \setminus \{i\}$, stating that players play rationally once they have made inferences about the opponents' choices. Admitted rules of inference are *modus ponens* and necessitation for $\vdash_i$.

One notable aspect is the lack of assumptions regarding both the monotonicity of the rationality operator and the monotonicity of the inference operator (in the sense that from $\vdash_i \varphi$ and $\vdash_i \varphi \rightarrow \psi$, it follows $\vdash_i \psi$). This makes the formalism flexible and prone to a number of applications, as the author discusses. Monotonicity, however, does play an important role in the main theorem of the paper. As a matter of fact, call the *lower monotone envelope* of a function the largest monotone function that approximates it from below with respect to set inclusion $\subseteq$. Let R_i be the lower monotone envelope of r_i and define $R(T_I) = R_{i_1}(T_{-i_1}) \times R_{i_2}(T_{-i_2}) \times \ldots \times R_{i_k}(T_{-i_k})$ for $I = \{i_1, \ldots, i_k\}$. Then, it is proved that statements T_N are theorems of this logical system if and only if all strategy combinations belonging to the largest fixpoint of R occur in T_N.

The article also presents a way to avoid the use of the modal operators $\vdash_i$, and contains a discussion of examples and extensions that help clarifying the scope and interest of the results.