

*This is a review submitted to Mathematical Reviews/MathSciNet.*

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**Title:** The fundamental problem of general proof theory.

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**Primary classification:**

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**Review text:**

Among the original aims of David Hilbert's famous program known as *Beweis-theorie*, which was later on translated as "proof theory", there was the attempt of making the very concept of mathematical proof the object of a rigorous investigation. This part of Hilbert's goal survives in the sense of the paper under review as the objective of *general proof theory* (as distinguished from *reductive proof theory*, which is intended as the realization of the other part of Hilbert's view related to the attempt of proving the consistency of mathematical theories by finitary means). The contribution achieved here in that direction of study is to present a new notion of "validity" for proofs, called *analytical validity*, that stems from a critical reconsideration of a previous proposal made by the author of the paper (in particular, those in D. Prawitz [Logic Methodology and Phil. of Science IV, 225-250, North Holland, Amsterdam, 1973; MR0465811], and D. Prawitz [Natural Deduction. A Proof-Theoretical Study, Almqvist and Wiskell, Stockholm, 1965; MR0193005]).

The whole issue is connected to the attempt of isolating the epistemic character that makes arguments capable of providing the *ground* for asserting their own conclusions, i.e. capable of letting someone *know* that these conclusions hold, as distinguished from arguments intended as constructions which make them legitimate assertions only owing to a concept of validity as preservation of truth.

The previous proposal the author of this paper had made in the said respect took inspiration from Gentzen's early work and ideas about the justification of

inferences from [Math. Zeitschrift, 39:176-210, 1935; MR1545497], that lead to introduce the notion of *canonical form* of a proof as a natural deduction derivation that ends in a introduction rule, and to consider the main relation between introduction and elimination rules in that context to be guided by an *inversion principle* guaranteeing that every non canonical proof can be turned into a canonical one. This proposal is stated here to be critical under several respects, and the new notion of *analytical valid argument* is offered as a substitute. It stems from making precise the idea of *containment* of an analytical valid argument in canonical form into a non-canonical argument (something that also the previous notion was referring to, albeit only “metaphorically” as the author itself acknowledges). The newly defined notion is then briefly discussed with respect to some selected case studies.

The last part of the paper offers an analysis of the notion of proof in the intuitionistic tradition that goes from Heyting to Martin-Löf, within which a quite articulated view of proofs as constructions carrying, or not carrying epistemic value is argued to be held. Besides contributing to a better understanding of the general aim of the paper, the related remarks are also used to hint at future refinements on the notion of validity attained at here so to achieve some further, natural desiderata.