The unusual shape of Palazzo Vecchio in Florence

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Abstract: Palazzo Vecchio is a well preserved Medieval building, but its plan and its volumes embarrass historians of architecture, who are not able to recognize the irregular form of its strange perimeter as the result of an intentional design. They ascribe it to external causes, like underground walls of pre-existent structures. A significant part of the monument logic (immaterial heritage in a material monument) has become an archaeological matter: we have lost the memory of intentions and meanings linked to its peculiar shape. CAD representation of the survey of the palace in twodimensional drawings leads to the discovery of unknown aspects of the plan. Through CAD drawings we can read the building's measures according to its metrical system and have a notion of the possible sense of its layout. If the length and square measures of a place are known, the peculiar shape of a plan can reveal an unforeseen sense. The whole metrical Florentine system of linear and surface measures enlightens the extraordinary design: the areas of its three parts (calculated by CAD) stress the fact that the goal of each portion's design was the rational expression of the area by means of special numbers. The geometrical process of the drawing comes out as an original creation. Cad drawings bring to light unknown relations between architecture and new scientific science in Tuscany at the end of thirteenth century (after Leonardo Pisano's work), as a specific immaterial ratio of the monument, impossible to be found without the simple, but not trivial, instruments offered by CAD.

Keywords: Palazzo Vecchio and Florentine Gothic Architecture; mathematics and architecture; Cad drawings from ancient buildings survey; metric systems and architecture

Fig. 1 – Western front of Palazzo Vecchio, ortho-photo

A monument difficult to understand

Palazzo Vecchio can by no means be likened to an archaeological monument (Fig. 1). It is in excellent state of repair and admirably performs its functions as municipal seat, masterpiece and stamp of identity. Yet this does not remove the ever-present suspicion of its bearing an incomprehensible design. Ever since Villani's *Chronicle*, which accused it of being *oblique* and *not foursquare*[1], historiographical judgements have looked upon its form with perplexity, justifying it on the basis of no longer recognisable contingent necessities and not architectural reasons (Fig. 2, A). In other words, it is deemed no longer legible.

The expectations of architectural design, as have evolved since the Renaissance (for example, evident design rules, symmetrical forms or weights, visible measurability), are not satisfied by the form of the Palazzo). As Vasari already put it, if the Grand Duke did not want to knock it down it was owing to its value as an emblem and not its architectural merit[2]. In recent times, the development of its parts has been judged lacking a single intentional design[3].

Its defects therefore were: it was not rectangular, it did not follow the lines of the city centre, there was no clear order to the layout of the rooms, it showed little unity in the development of its parts (Fig. 3).

CAD for a new knowledge from survey drawings

Having finished the survey, and formed a CAD (2001-2006) image, the design was examined, and the lengths, surfaces and corners measured with the digital programme's tools. Some surprising deductions emerge from the figures obtained. Still today the Palazzo continues to reveal the secrets of a sophisticated plan that would have been impossible were it not for skills, abilities and intentions in the Gothic architectural workshops that far surpass our expectations. Here I will show the operations to draw its layout, in the order that I consider they were probably performed. First, however, it is necessary to describe a fundamental guiding parameter behind the project. It derives from a paradigmatic fact of the scientific culture of the time and involves Leonardo Pisano, commonly known as Fibonacci, who gave it a definition.

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[1] Giovanni Villani, Villani's Chronicle, trans. Rose E. Selfe, Book IX, ch. XXVI, year 1298.

[2] Giorgio Vasari, Opere, Florence, Audin 1823, vol. VI, Ragionamenti, Ragionamento primo: speaks of *old walls built haphazardly by those first citizens who built them for their own comfort and not for pomp,* walls that needed to be *rebuilt according to a new order, … converted into a single body …*

[3] Franco Cardini*; Palazzo Vecchio, una storia tra Potere e Civiltà*, in Palazzo Vecchio a Firenze, ed. Maria C.Salemi, Florence, Nardini Ed. 2001 in p. 24: *In 1343 it was Gualtieri Duke of Athens who had a series of solid elements built to link the Via dei Gondi side of the Palazzo with the buildings in Via dei Leoni: in short, an attempt was made to rationalize a group of new buildings, older constructions and persisting ruins, the result of long vicissitudes from the Roman age to the levelling of the Uberti family houses in 1266,* own translation.

Fig. 2 – Palazzo Vecchio, ground floor plan, author survey

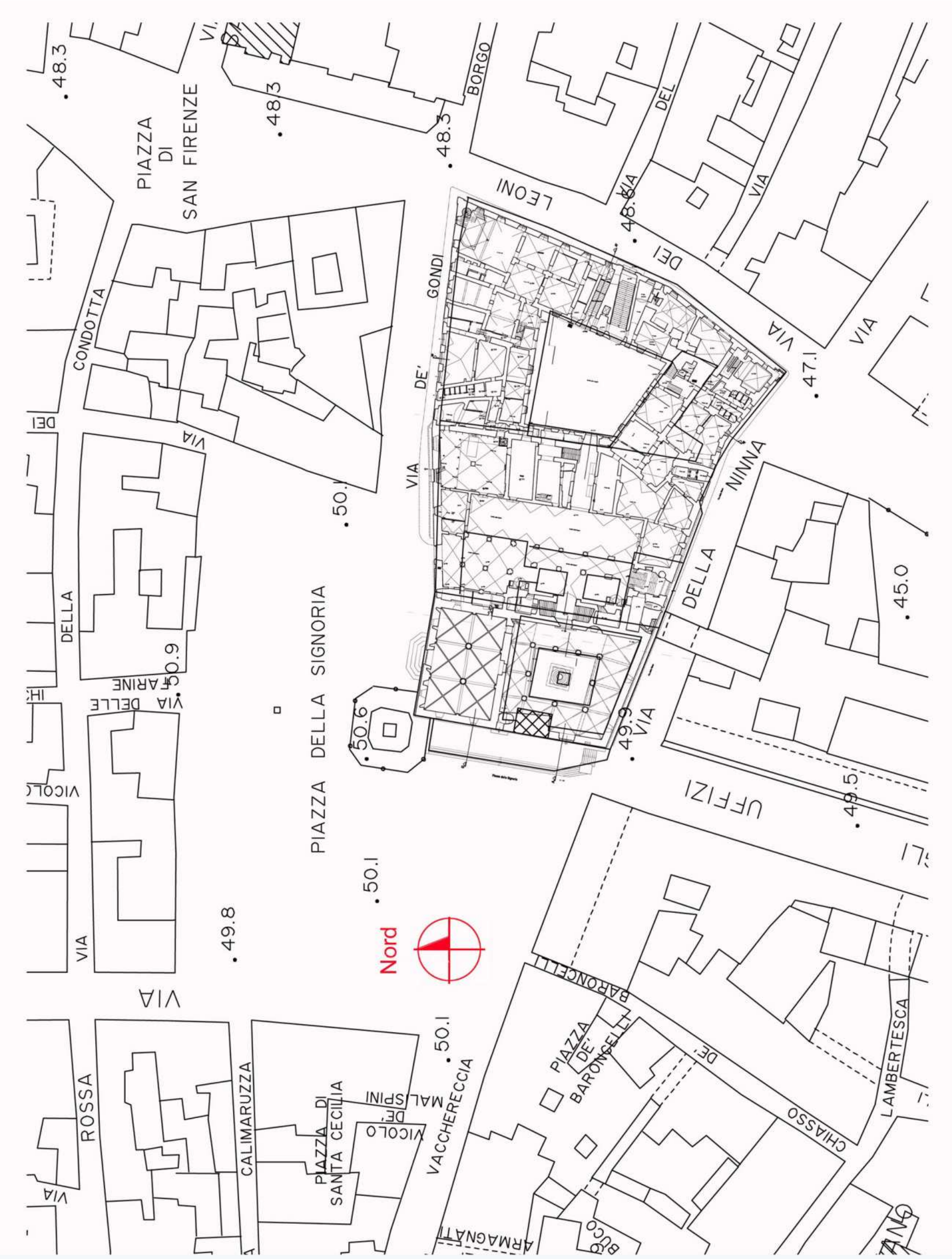

Fig. 3 – Palazzo Vecchio and Signoria square

Two paradigms from Fibonacci

The Fibonacci's *Liber Abaci* (Book of Calculation – 1200) and *Practica Geometriae* (Compendium of surveying Techniques – 1220) describe the Pisan system of measuring lengths and surfaces which would go on be used in Florence: they define the **4-arm Pisan cane** (in Florence an arm or *braccio* was 58.36 cm) and the **6-foot land-survey perch or *pertica agrimensoria*** (equivalent to **5 arms**), with their multiples and sub-

multiples; the **36-square-foot surface perch or *pertica superficialis*** (equivalent to **25 square arms**), and the **bushel of 66 *perticae superficialis*** = **1650 square arms = 561 m2** (Fig. 4a). The layout of the Gothic city, in the Pisan and Florentine area, is commensurate to these sizes, on the basis of the axiom that every urban organism designed must have an express extension of either a small and whole number of bushels, or a number of square arms that is a multiple of 1,000 or 100. The figures with which this result is achieved are not only rectangles, but also less obvious forms, comprising triangles or trapeziums, so long as they have known geometrical characteristics. They have to be figures whose measurements must all be possible to deduct on the basis of a single metric datum, or with linear equations, or square root extraction.
With this premise, the design of Palazzo Vecchio is associated with a particular fundamental triangle (Fig. 4b), the isosceles triangle that halves the surface of 10 bushels (= 16,500 sq. arms, a rectangle with sides of 82,5 and 200 arms, Fig.4a), therefore a triangle with a base of 82.5 arms and height of 200 arms (the same triangle was used to generate the oldest part of Piazza S. Maria Novella, in the same period as Palazzo Vecchio). Why this became part of the architectural design I would not know, but I may note that the top corner measures 23°26'43", almost the same value as the ecliptic angle, and in this period there was probably very intense research on astronomy and topography in the Dominicans' Studio in Santa Maria Novella. Inside the triangle of 5 bushels that I have described, the trapezium of 2 bushels resting on its base (Fig. 5, A and B), with an area of 2/5 x 8,250 = 3,300 sq. arms, was the starting point for the plan for a communal palace. Its depth needs to be found. It is obtained by difference from the height of the triangle with a surface of 3 bushels (4,950 sq. arms): **4,950 : X^2= 8,250:200^2; X = 155; 200-155 = 45 height of the trapezium; 3,300:45 = 73,333 half-sum of the bases. Minor base = 64.16 arms**. This is the sequence that calculates the measurements of the trapezium. Of the omitted triangle, base 64 arms and height 155 arms, there remains a trace in the main aspect of the palazzo. With the ball of the wooden spire, under the gilded copper lion, it reaches the same height, reflecting the path generating the design on the ground (Fig. 6).

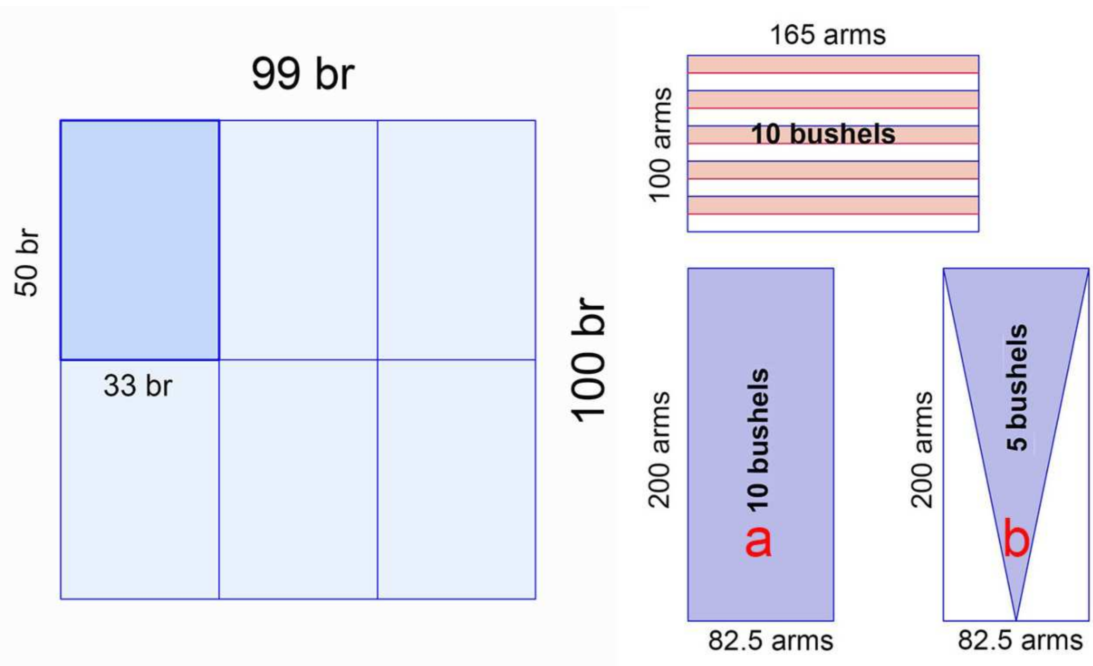

Fig. 4 – The bushel area unity in the Florentine metric system

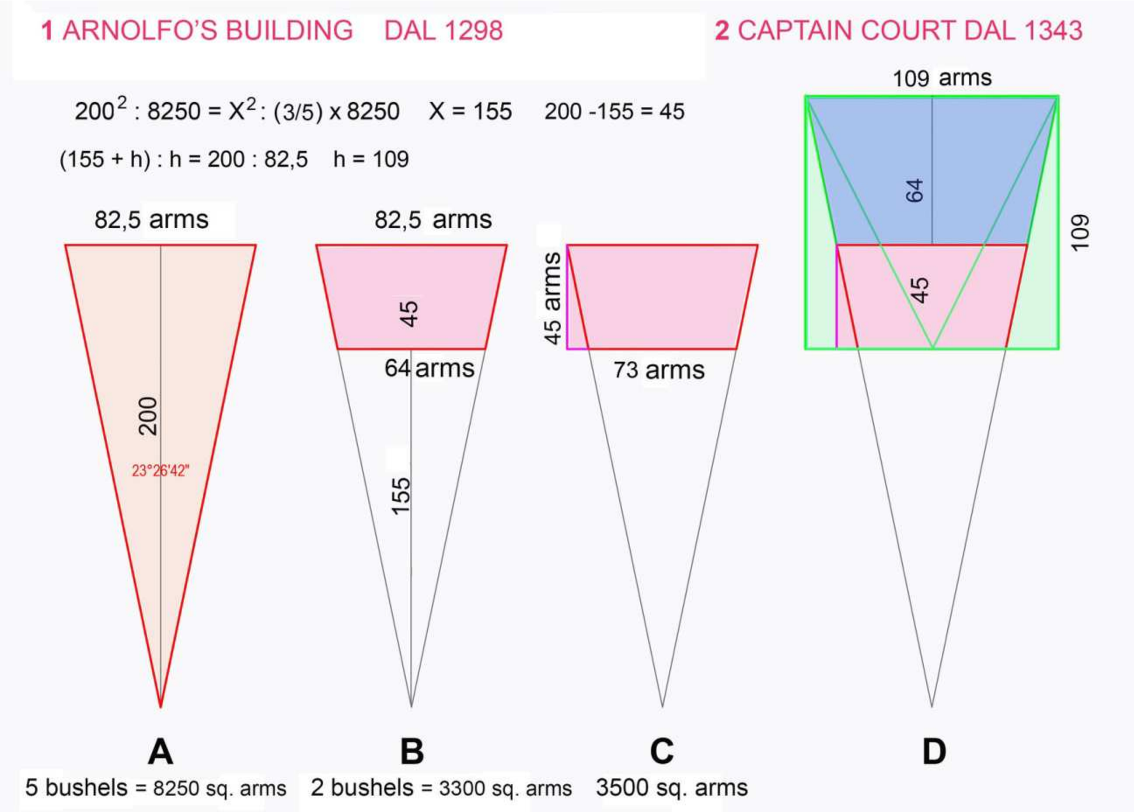

Fig. 5 – The triangle of 5 bushels, the trapezium of 2 bushels, the rectangular trapezium and the Captain Court

A second outline is associated with the Palazzo:[4] a rectangular room and a square courtyard that together form a new rectangle with the proportions of the first. It reflects the numbers in Fibonacci's series (the sequence is: 1,2,3,5,8,13,21,34,55,89,144). In Palazzo Vecchio the numbers 3,5,21,34 are involved (Fig.7), but the isosceles trapezium of 2 bushels (45 x 73) transforms into a rectangular trapezium, absorbing the 73 x 45 rectangle (good approximation of a golden rectangle: 73/45 = 1,622), and it also enters the geometrical process that defines the design, taking the area from 3,300 to 3,500 sq. arms.

Therefore, the design includes a trapezium with a height of 45, major base (back) of 82.5 and minor base of 73 (front). Inside this there must be a (golden) rectangular room and a courtyard, whose form derives from a square with one side that opens, giving rise to a trapezium. The sides of the fundamental triangle will continue behind the palace, with the goal of forming another trapezoidal courtyard, which needs to be defined. Let us think of the realisation of this design on the ground.

[4] And this comes from another convent, Santa Croce (Fig. 8), where the hall of the first Gothic convent that lies underneath the present sacristy (ancient chapter?) has the form, measurements and static layout of the Sala d'Arme in Palazzo Vecchio and was preceded by a square courtyard.

The layout of the ground plan

We are in the empty area set to host the palace and the square and we have to trace the lines of its perimeter (Fig. 8). The topographer sets up in a chosen area, to the west of the future palace, at a convenient distance. Having drawn the cross of the cardinal directions, he fixes the east-west direction (that of churches) as the starting line. He aligns the first of the levels located on the extremes of a square prepared with the desired angle of 23°26’43” on this and places a series of pegs in a sequence, generating a line (exact East-West astronomical direction); then he lines up other pegs at the same distances as the first on the second sight. He builds the bisecting line, that is, the height of the triangle. He traces a 200-arm measure from this to then get the 82.5 arm-long back base. He traces the orthogonal lines at the top of the triangle and looks for the intersections with the lines of the catheti (Fig. 9).

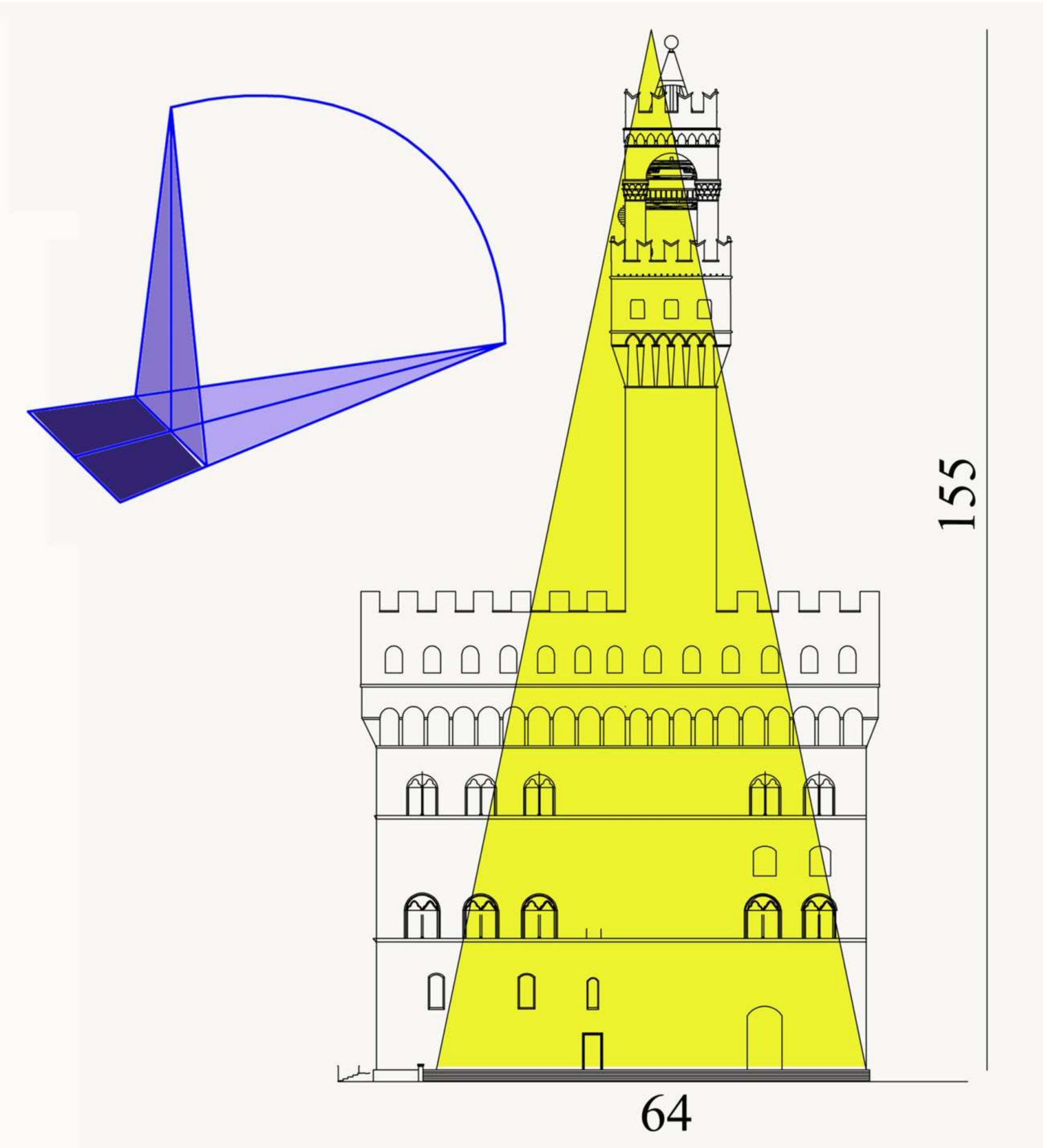

Fig. 6 – The triangle of three bushels on the Western front

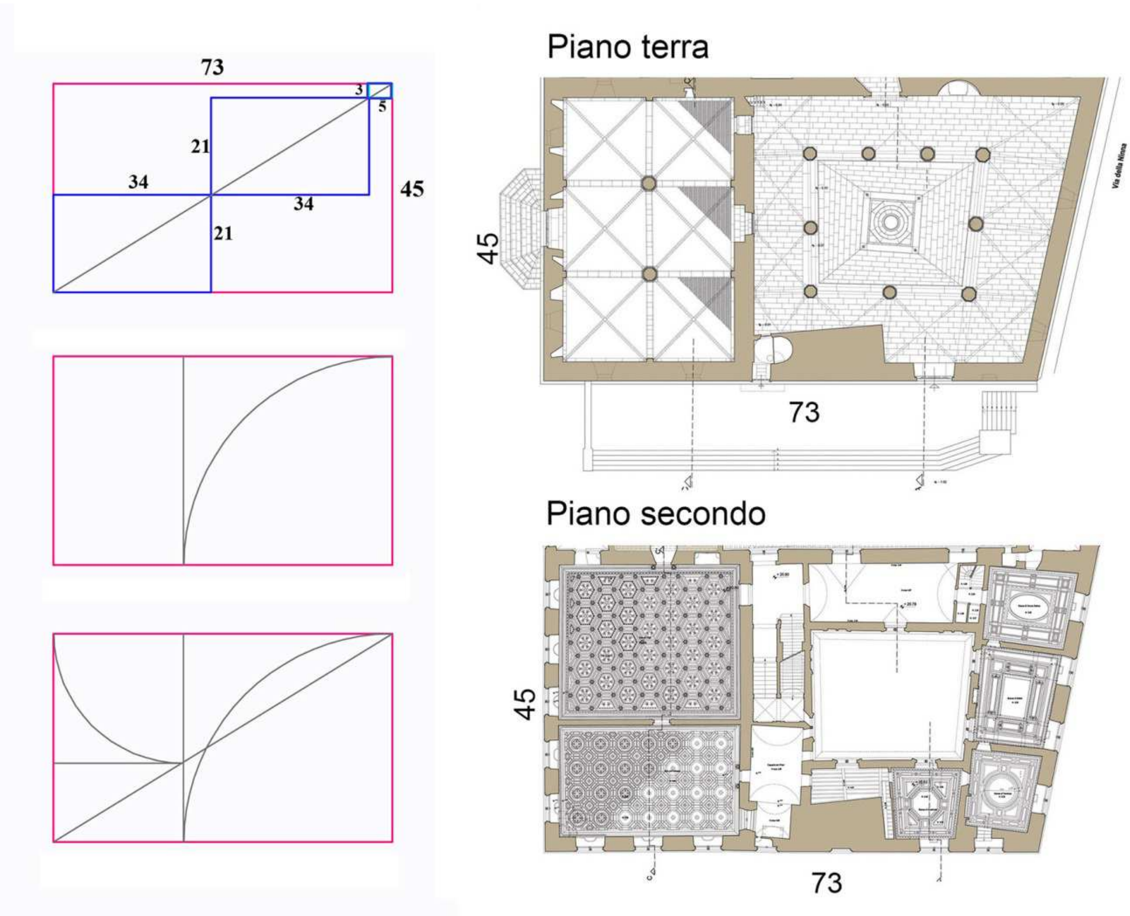

Fig. 7 – The Fibonacci Sequence in the Palace design

To the left (northern face of the Palazzo), the side of the trapezium will be cancelled by the rectangular Sala d'Arme. It is necessary to make a definite tracing of the back portion of how the face is due to continue before it is hidden from the topographer. The scarp holding up the wall that we see beyond the access door to the Corte del Capitano (Figs.2, 9) was the provision made to fix the extreme of the joining segment with the corner of the Sala. The north-east direction of the fundamental triangle converges on it. It was singled out on the ground.[5] What distance was the end of the north face placed at? The final point was put at the convergence of two optical alignments (Fig.10): one was the north-east direction observed from the vertex of the fundamental triangle; the other was that of a second topographer who, standing on the median point of the minor base of the isosceles trapezium, looked through the sights of a Jacob's staff formed by a square with the levels lined up on the extremes of the opposite side to that of the sight set on the median point (this instrument was described by the medieval philosopher Levy ben Gerson in its treatise on astronomy).

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[5] Perhaps the internal scarp erected on it was then to be demolished to make a new wall with the same characteristics as the others; instead the new one was made (probably by Brunelleschi) inside the scarp.

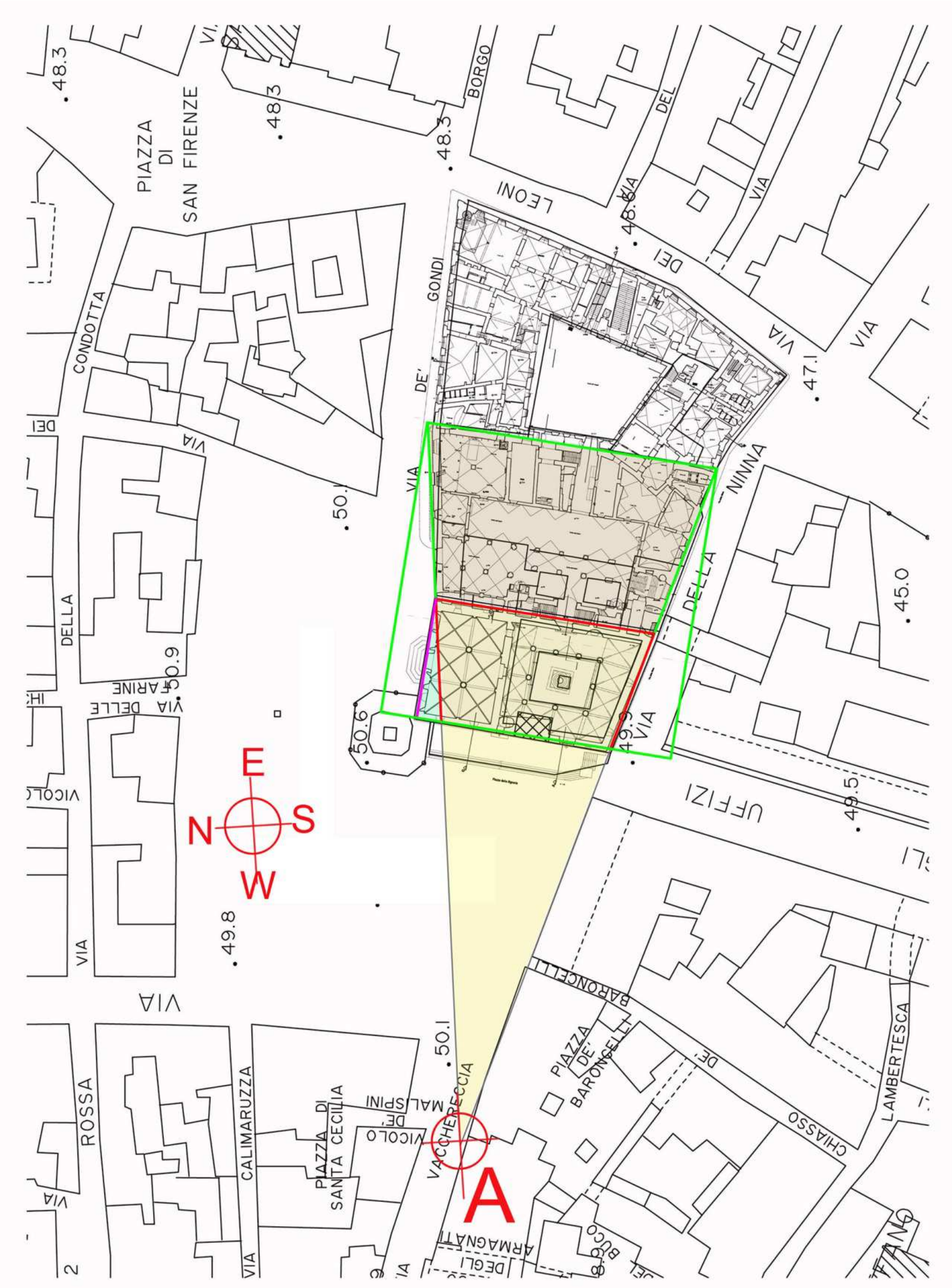

Fig. 8 – The generative layout in the square

Fig.a 9 – The Northern front with the scarp wall on the left of the Captain Court entrance

Having traced the alignment of the sides of the 82.5/200 triangle, the second topographer would have seen the sights of the directions come to rest on them in a ratio of 1:2 and would thus have calculated the extremes of the back wall. The calculation definitely first found the distance of that wall from the front of the palazzo, which would have confirmed the result of the operation:

64 : 155X = X : (155+X). From which X=109.

The area of this part of the building would have measured as follows: **(109 - 45) x (109 + 82.5)/2 = 6,128 sq. arms = 3.71 bushels.**

The whole area is 3,500 + 6,128=5.83 bushels. The number is not whole, but the entire method followed parametric equations.

Inside the isosceles trapezium of the Corte del Capitano, the transversal structures are distributed in such a way as to divide a determined area into strips: 1.5 bushels, 1 bushel, another 1 bushel; around ¼ of a bushel remains. Now we know how it was possible to achieve this result, as every polygon was entirely described by mathematical functions (Fig.11).

The back end of the palazzo was formed following another method: the mathematicians that flanked the architects in the design workshop had become very good and humored themselves by finding triangles that resolved any layout needs, each time managing to obtain surfaces with the desired requirements. A surface of exactly five bushels is achieved with a special trick, by assembling triangles of particular proportions, obtained with parametric equations, starting from the area that each of them needed. Without the CAD it would have been impossible to discover the intent of this most singular design (Fig.12). Without an expert mathematician or topographer, the project could not have been drawn. Without familiarity with their skills, it could not have been devised (Fig.13).

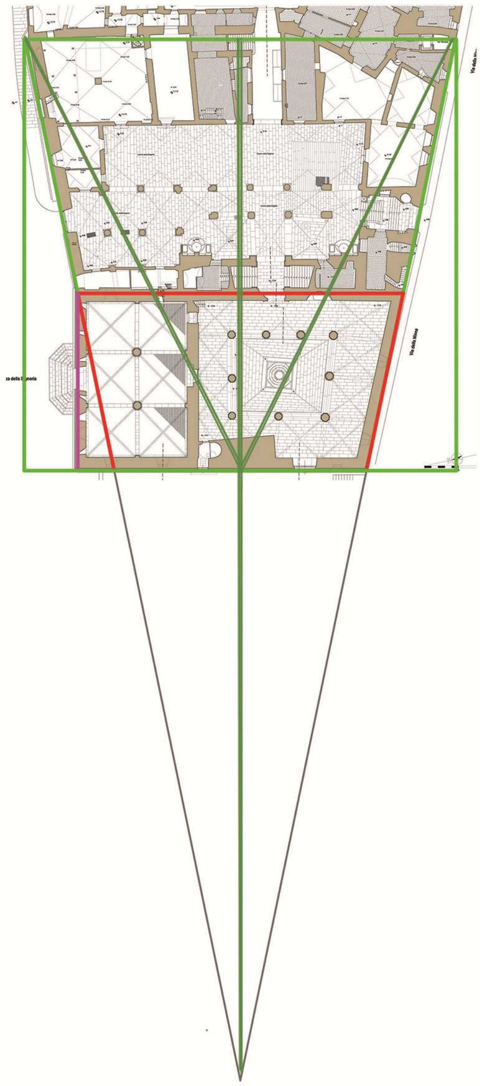

Fig. 10 – The two topographic operations

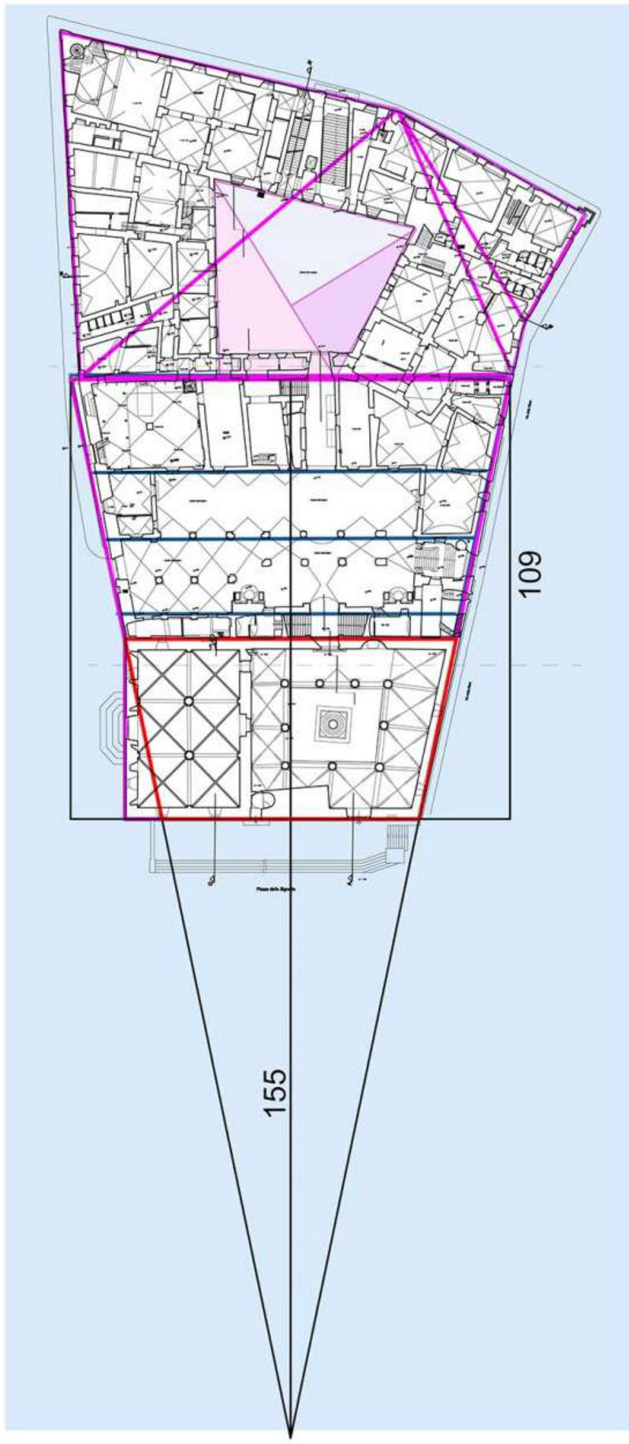

Fig. – 13 Whole layout of mathematical requirements in the Palace design

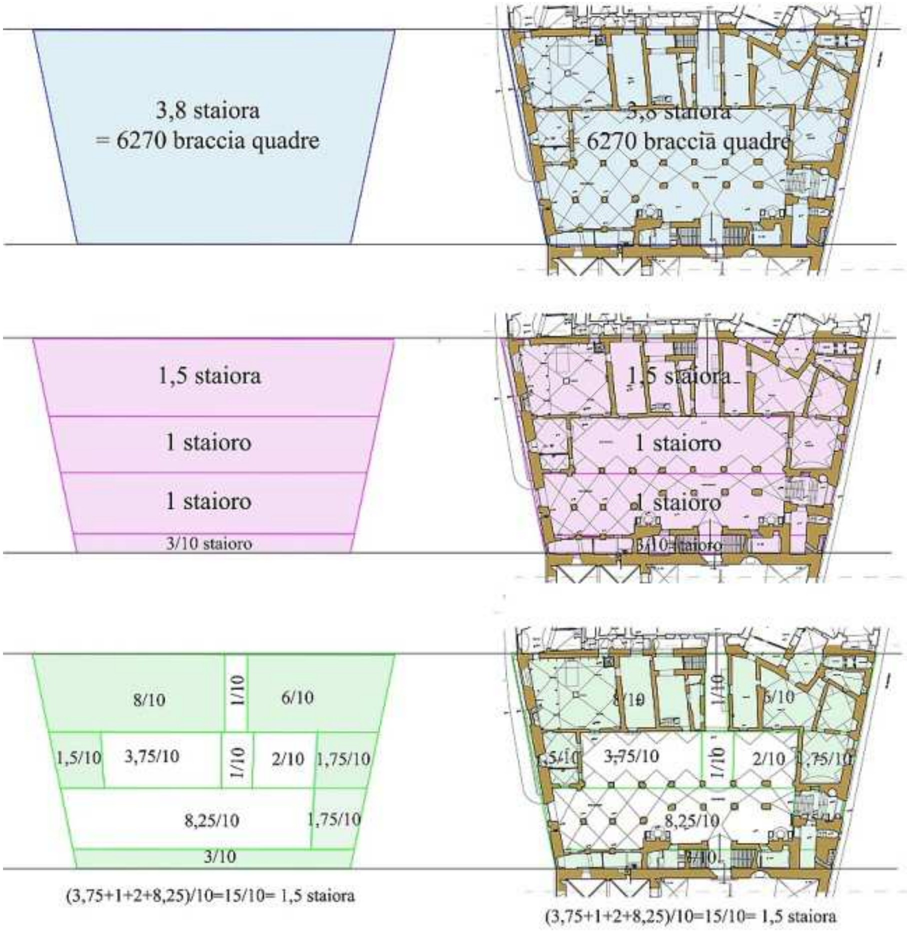

Fig. 11 – Area paradigms for the Captain Court plan

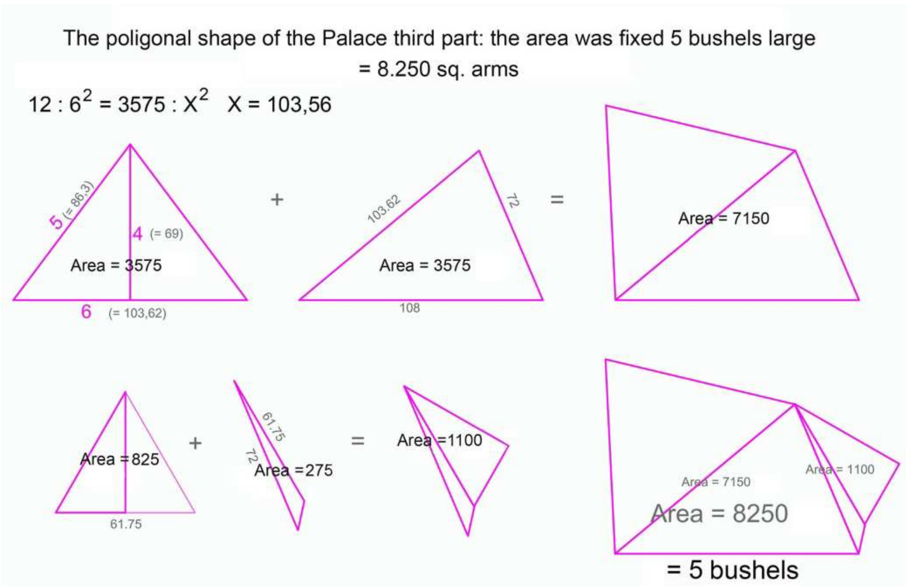

Fig. – 12 Area paradigms for the back block

Two strange things

Of Arnolfo's edifice, as well as not being at a right angle, two things strike the surveyor: 1) the different thickness of the walls in the rectangular room (c. 1.70 m, 2.92 arms) and the courtyard (c. 1.57 m, 2.7 arms), measurements without a simple relationship to the arm; 2) the strange direction of the sides of the tower inside the courtyard. The doubts arising in the preserver and historian's mind are: 1) do the two walls belong to different times and should we deem the Sala to be a first autonomous project? 2) does the tower follow the directions of an existent structure, or does it reflect a first different intention?

1) The external measurements of the Sala d'Arme (45 x 31.10 arms) are such as to produce a total area of exactly 1,400 sq. arms. The area of the void is 990 sq. arms. The walls therefore occupy 410 sq. arms. The outside perimeter of the courtyard is such as to produce an area of exactly 2,100 sq. arms (the total area of the palazzo is 3,500 sq. arms). The internal area, also excluding the space occupied by the tower, measures 1,670 sq. arms. The walls therefore, including the tower, occupy 430 sq. arms. The quadrilateral surface of the tower (with an almost 90° angle, but crooked with regard to the courtyard walls) measures 70 sq. arms and forms a continual homogeneous body with the courtyard wall.
   Let us observe: 3,500, the area of the palazzo, is a multiple of 70. So are 1,400 (surface area of the room) and 2,100 (surface area of the courtyard); the base of the tower outside the wall measures 70. In all, the wall surfaces total 840, also a multiple of 70, but the division between the room and courtyard is not the same, albeit the difference slight (Fig.14). Why? To me it seems that the goal of this not banal calculation must have been important and that if the tower is the single module of reference, this cannot but be in relation with the stability of the building, with its extraordinarily high tower. Just before the plans for Palazzo Vecchio, the Leaning Tower was built in Pisa and a debate may have been underway in the Valdarno regarding the provisions needed to make a tower safe. The science of building at the time to a large extent followed statics or the science of gravity. The distribution of the weights in the perimeter wall resulting on the ground floor can be considered uniform throughout the height of the palazzo. The tower, whose height above the gallery is around twice that of the portion underneath (48 arms high), is a shaft emerging from the homogeneous and continual solid which provides a wide base and counterweight. From this, the tower, around 144 arms high, protrudes by just over 96 arms (56 m, the same as the height of the tower of Pisa). The geometry of the distribution of the masses is such that the position of the barycentre, before the tower comes out from the garret, is located precisely at the intersection of the diagonals traced by the internal corners. A better 'mechanic' than me perhaps can see the purpose of this calculation.
2) The strange quadrilateral form of the base of the tower inside the courtyard, on the ground floor, only becomes a regular rectangle at the height of the gallery. The unusual directions of the wall lines have been imputed to the existent Torre della Vacca: and yet the walls are coeval to the external wall of the palazzo. If an existing tower has been clad, why not follow the directions of the walls it was part of? The surface of 70 sq. arms next to the walls makes one wonder if the surface against the walls at ground level could not exceed that measure, the protrusion of the tower at the top instead had to be much bigger: a 34 : 21 Fibonacci rectangle, with sides of 13. 6 arms, and 8.4 arms, in metres 7. 93 x 4.90, with an area of 114. 24 sq. arms (which would grow further). On the ground, having removed the 2.7 x 7.94 =21.57 strip of wall, it would have been 92.67 sq. arms, quite a lot more than 70. Therefore, the tower was thinned down to 70, but cut down so that the positions of the final lines, to be found using parallels to the external walls from the furthest points of its perimeter, were already fixed at the vertices of its internal perimeter (Fig.15). These are the real reasons for the strange ground plan of the tower. I would

define this as generative architecture, because it produces the final measurements of the form from its inside.

Thanks to CAD, the reasons for its design have been explained, in part at least.

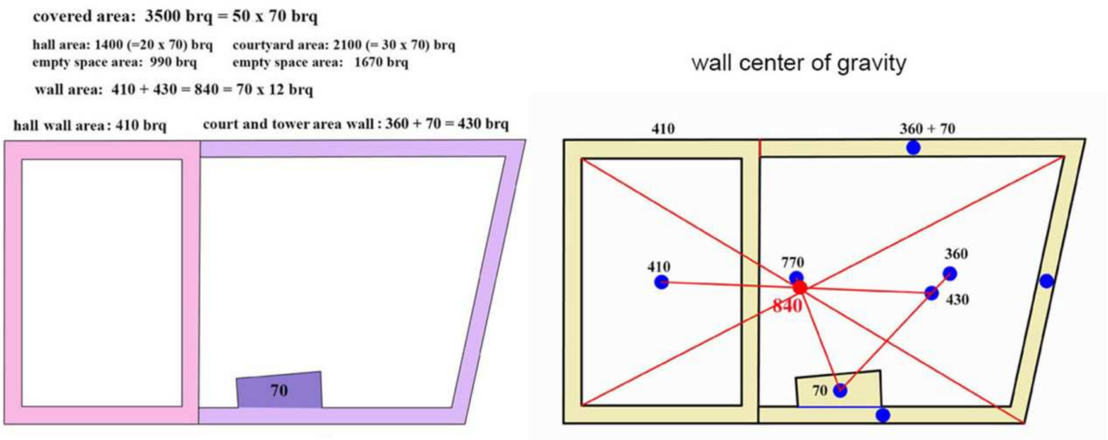

Fig. 14 – Walls thicknesses around the hall and around the court

Conclusions

Analysing the strange plan of the Palace from the inner perspective of the geometry of its shape, we pointed out different kinds of ratios: the paradigm of the five staiora triangle, some topographical techniques of the period, the statics of heavy bodies, the idea of correlating different plan directions at different levels. Current historiography attaches those unusual shapes to outer necessary conditions, not easily recognizable, hidden in former foundations, pre-existent to the Palace building. The described ratios were brought to light by objective data, whose compelling inner logic opens to a new scenario, diverging from the historiographic narrative we are used to. They are well grounded on computer science, which puts them at the disposal of those who have the idea of looking for them. Two kinds of Autocad-processed data that we are not inured to look for, come afloat: angles and surface measures. Current historiography makes us think that medieval culture could not mind of them, as it was not scientific enough: such a prejudice draws a veil over the documents and is misleading. When Palazzo Vecchio was devised, scientific research was alive and cultivated by many scholars of the Florentine Studia; by means of its achievements the architecture of the periods could reach the quality level that paved the way to the coming Renaissance in the following century.

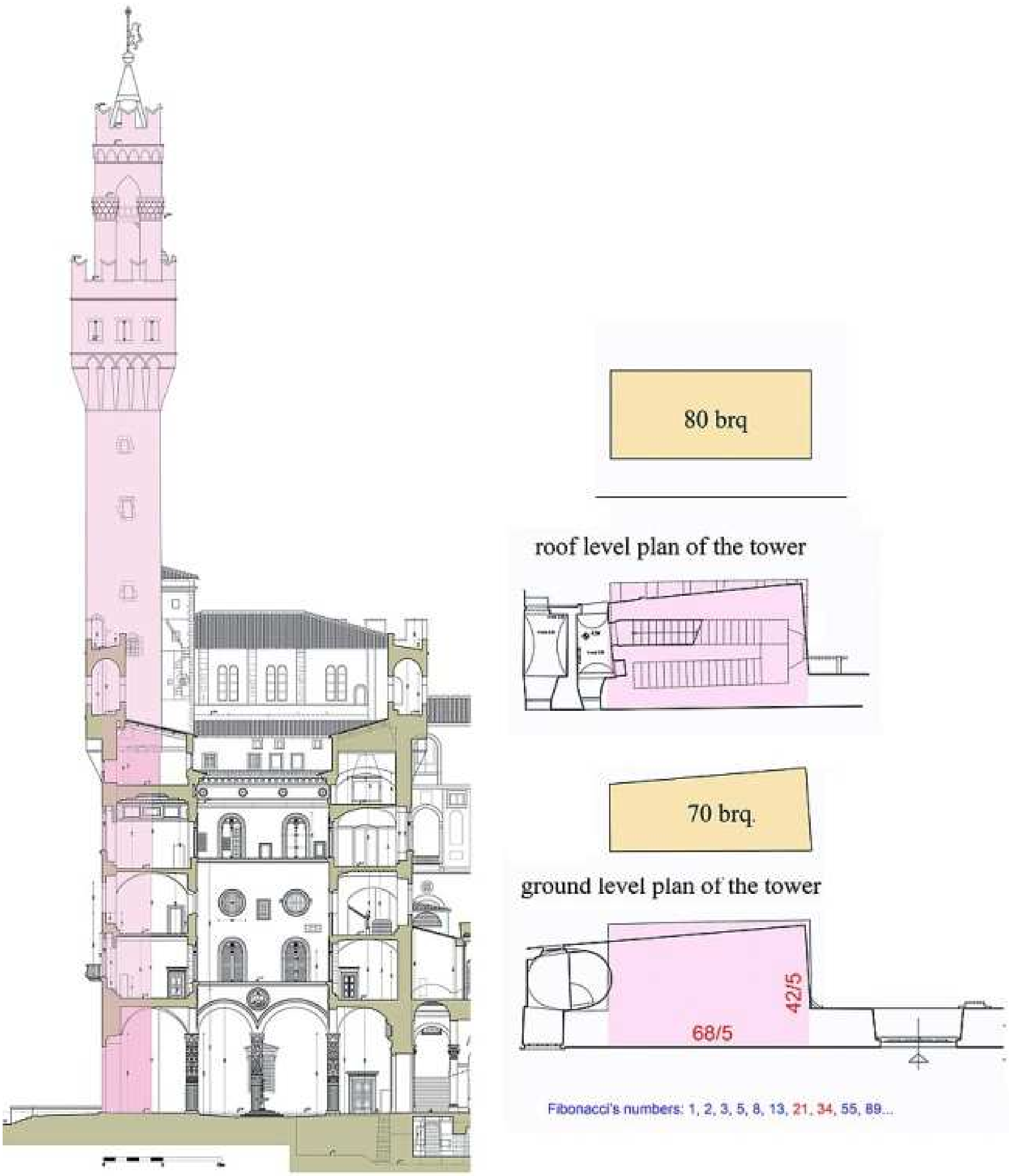

Fig.15 – The different quadrilateral poligons of the tower at ground floor and in the attic

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Imprint:

Proceedings of the 19th International Conference on Cultural Heritage and New Technologies 2014 (CHNT 19, 2014)

Vienna 2015

http://www.chnt.at/proceedings-chnt-19/

ISBN 978-3-200-04167-7

Editor/Publisher**:** Museen der Stadt Wien – Stadtarchäologie

Editorial Team: Wolfgang Börner, Susanne Uhlirz

The editor's office is not responsible for the linguistic correctness of the manuscripts.