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Review of "A Dynamic Solution to the Problem of Logical Omniscience" by Jens Christian Bjerring and Mattias Skipper

The problem of logical omniscience notoriously affects possible-worlds models of belief, i.e., triples $M = \langle W, V, f \rangle$, owing to the fact that (i) an agent believes a proposition p at world w (or, the sentence $Bp \in V(w)$, where B is the belief operator of the chosen object language), if and only if p is true at each and every world that is possible for the agent at w (i.e., $p \in V(w')$ for every $w' \in W$ such that $w' \in f(w)$), and the fact that (ii) the set of truths at each world in the model is a deductively closed and consistent set of sentences. It follows that, if an agent believes p at w and q is a logical consequence of p, then q is true at each world w' that is possible for the agent at w owing to (ii), hence the agent believes q as well owing to (i).

The usual routine set to avoid logical omniscience is based upon introducing "impossible worlds" in the possible-world model, i.e. worlds which violate the laws of classical logic somehow (see R. Fagin et al. [*Reasoning about knowledge*, MIT Press, 1995; MR1345612] for a survey). This move turns the above models into quadruples $M = \langle W^P, W^I, V, f \rangle$, where W^P is the set of possible worlds, while W^I is the collection of impossible worlds instead. However, this approach is inadequate if one wants to avoid logical omniscience without abandoning *logical competence* of agents, i.e., informally speaking, the ability of agents to engage in performing deductive reasoning starting from their own beliefs. It is actually shown in the paper under review that no given model that features impossible worlds to avoid logical omniscience is capable of equally grant logical competence with respect to any chosen set of logical rules $\mathcal{R}$. As a matter of fact, if one assumes that impossible worlds in the model are closed under derivations in $\mathcal{R}$ to preserve logical competence even in a restricted form, for instance up to derivations of a fixed length n for some $n \geq 1$ (thereby causing any world w in the model to verify q if w verifies p, and if q is derivable in $\mathcal{R}$ from p in n-many steps, or $p \vdash^n_{\mathcal{R}} q$ for short), then it turns out that every world w that verifies all sentences in a given set of sentences Γ, also verifies q whenever $\Gamma \vdash^s_{\mathcal{R}} q$ holds for every s. This brings logical omniscience back in the model which was set to avoid it.

The paper introduces a new variety of models that make use of features from dynamic logic. The object language, beside the usual belief operator B, includes countably many dynamical operators $\langle n \rangle, [n]$ for $n \in \mathbb{N}$, to form sentences $\langle n \rangle p$ and $[n]p$ with the intended meanings: p holds after some n steps of logical reasoning", and p holds after any n steps of logical reasoning" respectively. Formulas involving the dynamical operators are equipped with a semantics that captures these meanings. This is done by considering *n-expansions* of worlds, that is worlds w' that expand the set of sentences $V(w)$ that are verified at w by sentences which are derivable from them in $\mathcal{R}$ in n-many steps (i.e., worlds w' such that $V(w) \vdash^n_{\mathcal{R}} V(w')$). This notion is further used to define a relation between pointed models, i.e. pairs (M, w) where M is an impossible-world model and w a distinguished world of it, which holds between (M, w) and (M', w) whenever M' is M except for the fact that the set $f'(w)$ of possible worlds at w contains only n-expansions of w, hence worlds which only verify sentences derivable in $\mathcal{R}$ from $V(w)$ in n-many steps.

The semantics for sentences of the object language is then modified so to make use of this relation between pointed models to express the intended meaning of formulas involving dynamical operators through a suitably defined validity relation $\models$. In particular, a formula of the form $\langle n \rangle Bp$ turns out to be verified in (M, w) if and only if p is verified at all $w' \in f'(w)$, therefore just in case it is derivable from $V(w)$ through a chain of n applications of the rules in $\mathcal{R}$.

The main result that is achieved, is a theorem stating that if $\{p_1, \ldots, p_k\} \vdash^n_{\mathcal{R}} q$, then $\{\langle m_1 \rangle Bp_1, \ldots, \langle m_k \rangle Bp_k\} \models \langle m_1 + \ldots + m_k + n \rangle Bq$. That is: if q follows from $p_1, \ldots, p_k$ after some n-step derivation in $\mathcal{R}$, then an agent that comes to believe each p_i after some m_i-step derivation will end up believing q after some $m_1 + \ldots + m_k + n$-step one. The theorem has two notable corollaries as special cases, namely, under the same hypothesis, (i) that an agent who believes all the premises $p_1, \ldots, p_k$, will believe q after performing some n-step derivation in $\mathcal{R}$; and (ii) that if q follows from the empty set in $\mathcal{R}$ in n-many steps, then an agent believes it after some n-step derivation.

The significance of this result is further discussed in order to argue that the original goal for setting up these models is achieved, and the issue is analyzed even further to argue that the goal is better achieved by the means presented here than by those available through the approach fostered by M. Jago in [*Erkenntnis*, 79(6), 2014, pp. 1151-1168; MR3261925] and [*The impossible: an essay on hyperintensionality*, Oxford Univ. Press, 2014].