

MONOPOLISTIC COMPETITION WHEN INCOME MATTERS*

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We analyse monopolistic competition when consumers have an indirect utility that is additively separable. This leads to markups depending on income (both in the short and long run) but not on the market size, which generates pricing to market, incomplete pass-through and pure gains from variety for countries that open up to trade. Firms' heterogeneity *à la* Melitz implies a Darwinian effect of consumers' spending on business creation and a Linderian effect on (endogenous) quality provision. We discuss extensions with an outside good and heterogeneous agents, and offer simple and tractable specifications (linear or log-linear) of the demand functions.

This article proposes an alternative model of monopolistic competition in the tradition of the studies of large markets (Chamberlin, 1933), where firms choose prices independently and entry is free. The model is based on a class of non-homothetic preferences, unexplored in the analysis of monopolistic competition, which satisfy indirect additivity and delivers convenient specifications for applied research, especially in trade and macroeconomics.

As is well known, the model introduced by Dixit and Stiglitz (D–S, 1977: Section I) and based on constant-elasticity-of-substitution (CES) preferences over differentiated goods has become a workhorse model in modern economics. It implies constant markups and an endogenous number of firms that is proportional to both the number of consumers and their per capita income. Moreover, under firm heterogeneity, CES preferences imply that changes in population or income do not affect the efficiency of the active firms. These features have key consequences, for instance on the structure of and the gains from trade (Krugman, 1980; Melitz, 2003) and on firms' behaviour over the business cycle (see Blanchard and Kiyotaki, 1987 and, more recently, Bilbiie *et al.*, 2012).

From an empirical point of view, however, the CES model has some drawbacks. Primarily, it cannot account for the variability of markups across countries and/or over the business cycle. There is indeed a consistent evidence in the trade literature that, over the long run, prices and markups tend to be higher in richer countries (Alessandria and Kaboski, 2011; Fieler, 2011); there is also some macroeconomic evidence that markups are variable over the business cycle (for instance, Nekarda and Ramey (2013) make a case for procyclical markup behaviour in reaction to demand shocks).¹ Moreover, although

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¹ There is also evidence on incomplete pass-through for changes of marginal costs and trade costs (De Loecker *et al.*, 2012).

the empirical analysis of the impact of market size on prices under monopolistic competition has rarely distinguished between income and population effects, Simonovska (2015) studies international pricing of traded goods (online sales of clothes shipped to foreign markets and in competition with many imperfect substitutes) and finds a positive elasticity of prices with respect to per capita income but no significant impact of population on prices.

To account for the variability of markups under monopolistic competition, one has to depart from CES preferences. But models based on quasilinear (Spence, 1976; Melitz and Ottaviano, 2008; Anderson *et al.*, 2012) or general homothetic preferences (Benassy, 1996) face similar limitations, since they remove any direct effect of income on demand (with quasilinearity) or on demand elasticity (with homotheticity). To obtain variable markups, D–S (1977: Section II) proposed non-homothetic preferences represented by additively separable direct utility functions (Krugman, 1979). As recently stressed, such ‘direct additivity’ generates equilibrium prices that can either decrease or increase in the number of consumers (Zhelobodko *et al.*, 2012), implying an ambiguous impact of market size on welfare and ambiguous selection effects under firm heterogeneity (Bertoletti and Epifani, 2014; Dhingra and Morrow, 2015). However, in spite of non-homotheticity, free entry neutralises the impact of income on markups and market structure: in the long-run equilibrium, when the number of firms adjusts to obtain zero expected profits, prices and firm selection cannot be affected by changes in consumers’ expenditure (Zhelobodko *et al.*, 2012; Bertoletti and Etro, 2014*a, b*). Notice that on the one hand monopolistic competition models *à la* D–S (1977) were developed to explain the number of varieties/competitors in the long run, on the other hand the empirical relation mentioned between markups and income emerging in the trade literature (Alexandria and Kaboski, 2011; Fieler, 2011; Simonovska, 2015) refers to a time span that explicitly requires a long-run theoretical foundation. We propose an alternative class of non-homothetic preferences and argue that it can easily account for the stylised facts outlined above and, in particular, for the relation between prices and income in the long run.²

Nevertheless, it is important to remark that, for a given number of firms, income affects markups also under direct additivity of preferences (Behrens and Murata, 2012*a*). Moreover, there exist other models based on non-homothetic preferences in which income heterogeneity between individuals or across countries ‘matter’ in the sense of affecting crucially the market results even in the long run. For instance, Simonovska (2015) studies a multi-country trade model where wages are endogenous and exert an impact on the equilibrium markups, while Behrens and Murata (2012*b*) show that differences in labour productivity affect the relative gains from trade. In another important work based on non-homothetic preferences, Murata (2009) exploits increasing returns due to specialisation arising out of differentiated intermediate goods. This implies that higher labour efficiency

² On the crucial role of non-homotheticity in trade models see also the recent works by Fajgelbaum *et al.* (2011), Markusen (2013) and Mrázová and Neary (2014).

affects prices (and the number and composition of varieties) even in the long run because it allows for more specialisation and lower marginal costs.³ Our complementary approach in this article is to present a simple model in the D–S tradition where income affects markups in the long run even in autarky, without heterogeneous firms/varieties or consumers and without returns from specialisation.

Our main assumption is that consumers' preferences can be represented by an additively separable indirect utility function. Such 'indirect additivity' amounts to assume that the relative demand of two goods does not depend on the price of other goods, while it depends in general on income. It is thus different from 'direct additivity', for which the marginal rate of substitution between any two goods does not depend on the consumption of other goods. In fact, duality theory (Hicks, 1969; Samuelson, 1969) tells us that direct and indirect additivity characterise two different classes of well-behaved preferences (remarkably, the homothetic case of CES preferences is the only common ground). A key implication of our assumption is that the number of goods provided in the market does not affect their substitutability and thus the demand elasticity, while income can affect both with crucial consequences. Remarkably, simple and common direct demand functions, such as linear demands and log-linear demands, emerge from our preferences (with 'addilog' and exponential subutilities) and generate simple closed form solutions for variable markups, which can be easily used in a variety of applied situations.

Monopolistic competition under indirect additivity produces two-sided results that can be useful for trade and macroeconomic applications. First, it generalises the neutrality of market size on the production structure which emerges in CES models, thereby yielding pure gains from varieties as in Krugman (1980). Second, it delivers markups that are variable in income/spending, generating pricing to market as a natural phenomenon: as long as demand is less elastic for richer consumers, markups are higher in markets with higher individual income. Moreover, markups do change when demand shocks affect individual spending or supply shocks affect marginal costs. We show that these results are robust to extensions in which an outside good represents the rest of the economy, there is consumer heterogeneity in both preferences and income, and also when firms differ in productivity *à la* Melitz (2003). At the very least, our results suggest that empirical works should carefully distinguish the effects due to per capita income from those associated to the population size.

The comparative statics for business creation is also of interest. Richer consumers with less elastic demand induce firms to increase their markups, which triggers a more than proportional entry of firms, while an increase in income inequality between consumers tends to exert the opposite effects. Instead, when firms are heterogeneous in productivity, indirect additivity establishes a Darwinian mechanism that is absent in the Melitz model: less productive firms enter in booms (when income increases) and exit during downturns (a sort of 'cleansing effect' of recessions). Finally, if firms can invest in the quality of their products, we find that more productive firms tend to react

³ The article by Murata (2009) is akin to our own in stressing the different role played by population and per capita income in affecting the elasticity of demand. It is different because it assumes directly additive preferences and asymmetric final products sold in competitive markets.

to an increase in consumers' income by offering products of higher quality sold at higher prices, which is consistent with the celebrated Linderian effect (Linder, 1961).

The work is organised as follows. In Section 1 we present our baseline model characterising the endogenous entry equilibrium and introducing convenient specifications to be used in applied research. We also compare our results with those of alternative models based on directly additive, homothetic and non-separable preferences. In Section 2 we extend the model in various directions and compare optimal and equilibrium market structures. In Section 3 we study a two-country model *à la* Krugman considering both costless trade between different countries and costly trade between identical countries. We conclude in Section 4. All the proofs are in the Appendix.

1. A Model of Monopolistic Competition

Consider a market populated by L identical agents with income $E > 0$ to be spent on a mass of n differentiated goods.⁴ We represent preferences through indirect utilities, which depend on the prices p_j of each variety j and on income or, exploiting homogeneity of degree zero of the indirect utility, on their ratios $s_j \equiv p_j/E$. Our key assumption is the adoption of the following symmetric and additively separable indirect utilities:

$$V = \int_0^n v\left(\frac{p_j}{E}\right) dj. \quad (1)$$

As we will clarify below, this assumption identifies a general class of well-behaved preferences that do not satisfy the D–S (1977) assumption of direct additivity, with the remarkable exception of the CES case. To satisfy sufficient conditions for (1) being a valid indirect utility function (while allowing for a possibly finite choke-off price and obtaining well-behaved demand functions), we assume that the indirect sub-utility $v(s)$ is at least thrice differentiable, with $v(s) > 0$, $v'(s) < 0$ and $v''(s) > 0$ for any $s < \bar{s}$, $v(s) = 0$ for $s \geq \bar{s}$, and $\lim_{s \rightarrow \bar{s}} v(s), v'(s) = 0$ for some $\bar{s} > 0$. These assumptions imply that demand and extra utility are zero for a good that is not consumed.

The Roy identity provides the following direct individual demand function for good i :

$$x_i(p_i, E, \mu) = \frac{v'\left(\frac{p_i}{E}\right)}{\mu}, \quad (2)$$

where

$$\mu = \int_0^n v'\left(\frac{p_j}{E}\right) \frac{p_j}{E} dj \quad (3)$$

depends on the marginal utility of income: namely, $\mu = -E(\partial V/\partial E) < 0$. The demand function of each variety depends on its price and on the same price aggregator, μ , which is unaffected by the price p_j of a single firm. Total market demand is $q_i = x_i(p_i, E, \mu)L$.

⁴ Using the wage as numeraire, E can be interpreted as the labour endowment of each agent (in efficiency units).

Preferences represented by (1) are homothetic if and only if $v(s)$ is isoelastic, i.e., if $v(s) = s^{1-\theta}$ with $\theta > 1$. Indeed, in such a case they are of the CES type, with indirect utility $V = E(\int_j p_j^{1-\theta} dj)^{1/(\theta-1)}$, where θ is the elasticity of substitution. By an important result in duality theory (Hicks, 1969; Samuelson, 1969) the class of preferences which satisfy ‘direct additivity’, i.e., that can be represented by an additive direct utility $U = \int_0^n u(x_j) dj$, for some well-behaved subutility $u(\cdot)$, does not satisfy (1), with the only exception of CES preferences.⁵ Therefore, the indirect utility (1) encompasses an unexplored class of non-homothetic preferences whose corresponding direct utility functions are non-additive.

Suppose that each variety is sold by a firm producing with constant marginal cost $c > 0$ and fixed cost $F > 0$ (both in labour units): the profits of firm i can then be written as:

$$\pi(p_i, E, \mu) = \frac{(p_i - c)v'(\frac{p_i}{E})L}{\mu} - F. \tag{4}$$

The most relevant implication of (2) is that the elasticity of the direct demand corresponds to the (absolute value of the) elasticity of $v'(\cdot)$, which we define as:

$$\theta(s) \equiv -\frac{v''(s)s}{v'(s)} > 0.$$

This elasticity depends on the price as a fraction of income, p_i/E , but is independent from μ and L .⁶ Instead, in the case of direct additivity the elasticity of the inverse demand is uniquely determined by the consumption level.⁷

Any firm i maximises (4) with respect to p_i . The first order condition (FOC) is $v'(s_i) + (p_i - c)v''(s_i)/E = 0$, which requires that (locally) $\theta(s) > 1$. Moreover the second order condition (SOC) requires $2\theta(s) > \zeta(s)$, where $\zeta(s) \equiv -v'''(s)s/v''(s)$ is a measure of demand curvature. Notice that $\theta'(s)s/\theta(s) = \theta(s) + 1 - \zeta(s)$, therefore $\theta' > 0$ if and only if $\theta > \zeta - 1$, in which case demand becomes more elastic when price goes up or income goes down.⁸ The FOC for profit-maximising price can be rewritten as follows:⁹

$$\frac{p^e - c}{p^e} = \frac{1}{\theta(\frac{p^e}{E})}. \tag{5}$$

This pricing rule shows that under indirect additivity the profit maximising price is always independent from the mass of varieties supplied, because the latter does not

⁵ In addition, it can be shown that the assumptions of either direct or indirect additivity and homotheticity imply that preferences are CES: see Blackorby *et al.* (1978, section 4.5.3).

⁶ Notice that θ is a measure of the curvature of $v(\cdot)$: as such, it could be related to well-known risk aversion measures (Bertoletti, 2006; Behrens and Murata, 2007).

⁷ Under direct additivity, the (individual) inverse demand of variety i is given by $p_i(x_i, \lambda) = u'(x_i)/\lambda$, where $\lambda = \int_0^n u(x_j)x_j dj/E$ is the marginal utility of income. Notice that each inverse demand depends on its own quantity and on the same quantity aggregator, λ . Accordingly, both direct and indirect additivity satisfy the so-called ‘generalized additive separability’: see Pollak (1972).

⁸ If demand is (locally) concave ($v''' > 0$) the SOC is always satisfied and $\theta' > 0$. On the contrary, if demand is convex ($v''' < 0$) we may have $\theta' < 0$.

⁹ To guarantee the existence of a solution to (5) we assume $\bar{s}E > c$ (the consumer willingness to pay is large enough) and $\lim_{s \rightarrow \bar{s}} \theta(s) > \bar{s}E/(\bar{s}E - c)$. Notice that the SOC guarantees uniqueness.

affect the elasticity of demand. This property appears entirely consistent with the Chamberlinian treatment of the ‘large group equilibrium’ (Chamberlin, 1933, ch. V). It follows that an exogenous increase in the number of competitors would just proportionally reduce the level of individual consumption.¹⁰ At the same time, the optimal price grows with income if firms face a less elastic demand (and *vice versa*), which provides a demand-side rationale for markups that are variable across markets (or over the business cycle). Consider the realistic case of $\theta' > 0$: then, the model is consistent with typical forms of pricing-to-market, i.e., the same good should be sold at a higher price in richer (or booming) markets.¹¹ Similarly, under the same assumption a change in the marginal cost is transmitted (pass-through) to prices in a less than proportional way (undershifting). Summing up, we have:

PROPOSITION 1. *Under indirect additivity and monopolistic competition the profit-maximising prices are independent from the mass of active firms; they increase in the income of consumers and less than proportionally in the marginal cost, if and only if demand elasticity is increasing in the price.*

Since by symmetry the equilibrium profit is the same for all firms and it is decreasing in their mass, we can characterise the endogenous market structure through the zero profit condition $(p - c)EL/np = F$. This and the pricing rule (5) jointly deliver the free-entry equilibrium mass of firms and production size of each firm:

$$n^e = \frac{EL}{F\theta\left(\frac{p^e}{E}\right)}, \quad q^e = F \frac{\theta\left(\frac{p^e}{E}\right) - 1}{c}. \quad (6)$$

The following Proposition summarises the comparative statics for n^e :

PROPOSITION 2. *Under indirect additivity, in a monopolistic competition equilibrium with endogenous entry the mass of firms increases proportionally with the number of consumers; it increases more than proportionally with the income of consumers and decreases with the marginal cost if and only if the demand elasticity is increasing in the price.*

As a corollary, the equilibrium size of each firm q^e in (6) does not depend on the number of consumers and it decreases with individual income if and only if $\theta' > 0$. To understand these results and their applications, it is convenient to think of changes in L as changes in the scale of the economy, of changes in E as (due to) demand shocks on the disposable income of consumers and of changes in c as supply shocks to firms’ productivity. First of all, the impact of an increase in the scale of the economy is always the same as under CES preferences: a larger market size does not affect prices and production per firm but simply attracts more firms without inducing any other effect on

¹⁰ In the D–S case of direct additivity, an exogenous increase in the number of varieties n could either increase or decrease the price level, cutting more or less than proportionally the level of individual consumption.

¹¹ However, it is immediate to verify that p^e/E is always decreasing in income under our assumptions.

the market structure. This neutrality result and its key implications for the Krugman (1980) model of trade extend from CES preferences to the entire class described by (1).¹²

Second, an increase of the spendable income of consumers has more complex implications. Consider the realistic case where higher income makes demand more rigid ($\theta' > 0$). Then, a positive demand shock reduces the perceived demand elasticity and induces firms to increase their markups and reduce sales accordingly. For a given number of firms this leads to a large positive impact on gross profits which promotes business creation and increases more than proportionally the number of varieties provided in the market.

Third, consider an increase in firms' productivity associated with a reduction of the marginal cost (still assuming $\theta' > 0$): lower costs are translated less than proportionally to prices, which increases the markups and triggers additional entry (while the impact on firms size is ambiguous).¹³ Accordingly, and contrary to what happens with CES preferences, our more general model allows demand and supply shocks to generate additional processes of business creation/destruction. This would alter the dynamic path of macroeconomic models with endogenous entry (for instance Bilbiie *et al.*, 2012).

Under monopolistic competition and exogenous productivity, indirectly additive preferences exclude any impact of the number of goods on markups, which instead emerges under general additive or quadratic utilities (Melitz and Ottaviano, 2008; Zhelobodko *et al.*, 2012). However, it is important to stress that such an impact is not really due to changes in the 'intensity' of competition between firms (where there are no strategic interactions) but to changes in the elasticity of substitution between goods perceived by consumers. Only concentrated markets with a small number of firms competing *à la* Bertrand or *à la* Cournot produce a direct impact of the number of firm on markups and our model can be easily extended in this direction to generate competitive effects (see Bertolotti and Etro (2014*b*) for a discussion of these effects under imperfect competition).¹⁴ Alternatively, a link between market size and prices could be also introduced through endogenous productivity due to the adoption of specialised input, as in Murata (2009).

1.1. Examples

The results of our model can be illustrated with simple specifications that deliver closed-form solutions. Tractable cases arise if $v^{-1}(\cdot)$ is homogenous or logarithmic up to a linear transformation (see Behrens and Murata (2007) for a discussion under

¹² As an immediate consequence, increasing the population just induces pure gains from variety. This is a remarkable difference compared to the D-S model, where the existence of gains from trade can be guaranteed only when the equilibrium price is decreasing in the population (Zhelobodko *et al.*, 2012; Dhingra and Morrow, 2015).

¹³ One can compute $\partial \ln q^e / \partial \ln c = (\theta - 1)(\theta - \zeta) / (2\theta - \zeta)$, therefore q^e increases with the marginal cost if and only if $\theta > \zeta$: i.e., production increases with marginal cost if demand is log-concave. On the recent revival of the literature on incidence and pass-through, see for instance Fabinger and Weyl (2014). On empirical evidence on incomplete pass-through see De Loecker *et al.* (2012).

¹⁴ Competitive effects due to strategic interactions are widely studied in industrial organisation. For trade implications see Bertolotti and Epifani (2014) and Etro (2015). For macroeconomic implications, see Etro and Colciago (2010).

direct additivity). This is clearly the case of the isoelastic function but also of the 'addilog' function $v(s) = (a - s)^{1+\gamma}/(1 + \gamma)$ with $a > 0$, which is the choke-off price, and $\gamma > 0$, and of the exponential function $v(s) = e^{-bs}$ with $b > 0$.¹⁵

Let us consider the example based on the addilog preferences, which delivers a classic set of demand functions $q_i = \text{const} \times (a - p_i/E)^\gamma$ whose elasticity depends on the parameter γ : in particular, when $\gamma = 1$ the demand is linear, when $\gamma \rightarrow 0$ the demand is perfectly rigid and when $\gamma \rightarrow \infty$ it is perfectly elastic. The addilog model leads to the following closed form solutions:

$$p^e = \frac{\gamma c + aE}{1 + \gamma}, \quad n^e = \frac{(aE - c)EL}{F(aE + \gamma c)}, \quad q^e = \frac{F(1 + \gamma)}{aE - c}. \quad (7)$$

The second example, based on exponential preferences, generates another widely used demand function with a log-linear specification, $\log q_i = \text{const} - bp_i/E$, where b affects the semi-elasticity of demand. The associated free-entry equilibrium can be easily derived as:¹⁶

$$p^e = c + \frac{E}{b}, \quad n^e = \frac{E^2 L}{F(bc + E)}, \quad q^e = \frac{Fb}{E}. \quad (8)$$

As expected, population is neutral on prices and firm' size. Moreover, both examples satisfy $\theta' > 0$, therefore higher income makes demand more rigid, which leads firms to increase their prices and reduce their production, with a more than proportional increase in the number of firms. In addition, a marginal cost reduction is not fully translated on prices, which attracts more business creation and has a limited impact on firm size (none with log-linear demand).

At this point, one may wonder what kind of direct utility functions are associated with indirect additivity and, in particular, with the above examples.¹⁷ We can answer this question by solving for the inverse demand functions and then substituting them into (1) to recover the direct utility. The Roy identity (2) provides $p_i(x_i, E, \mu) = E v^{-1}(\mu x_i)$ for each variety i . Employing the budget constraint, we obtain that μ is implicitly defined by $1 = \int_j v^{-1}(\mu x_j) x_j dj$. In the addilog example, we have:

$$p_i(x_i, E, \mu) = E \left[a - (x_i |\mu|)^{\frac{1}{\gamma}} \right] \text{ where } \mu = - \left(\frac{a \int_0^n x_j dj - 1}{\int_0^n x_j^{\frac{1+\gamma}{\gamma}} dj} \right)^\gamma.$$

and for the log-linear demand we have:

$$p_i(x_i, E, \mu) = \frac{E}{b} \left(\ln \frac{b}{|\mu|} - \ln x_i \right) \text{ where } \mu = -b \exp \left(- \frac{b + \int_0^n x_j \ln x_j dj}{\int_0^n x_j dj} \right),$$

¹⁵ Other simple examples of $v(s)$ are generalisations of the isoelastic function such as $v(s) = (s + g)^{1-\theta}$, with $\theta > 1$, or 'mixtures' such as $v(s) = s^{1-\alpha} + s^{1-\beta}$ with $\theta \neq \alpha > 1$.

¹⁶ It is worth noticing that this case differs from the case of Logit demands (Anderson *et al.*, 2012), which derives from quasilinear preferences and exhibits no income effects.

¹⁷ Standard results ensure that, under our assumptions, preferences represented by (1) can be also represented by a well-behaved direct utility: see Blackorby *et al.* (1978, section 2.2.1).

We can derive the direct utility for the general case as follows:

$$U = \int_0^n v[v'^{-1}(\mu x_j)]dj \equiv \int_0^n u(|\mu x_j|)dj \text{ with } 1 = \int_0^n v'^{-1}(\mu x_j)x_jdj, \tag{9}$$

where the ‘subutility’ u for each good is increasing in its consumption level. As expected, preferences are not directly separable: (9) shows that the marginal rate of substitution between two varieties is affected by the consumption of all the others through μ . In our two examples we obtain:

$$U = \frac{(a \int_0^n x_jdj - 1)^{1+\gamma}}{(1 + \gamma) \left(\int_0^n x_j^{\frac{1+\gamma}{\gamma}} dj \right)^\gamma} \text{ and } U = \int_0^n x_jdj \exp\left(-\frac{b + \int_0^n x_j \ln x_jdj}{\int_0^n x_jdj}\right),$$

respectively for the addilog and log-linear case, where utility depends on total consumption and on other aggregators of the consumption levels.¹⁸

1.2. *Comparison with Other Models*

To clarify the role of the assumptions on preferences in monopolistic competition, it is important to understand that demand elasticity in a symmetric equilibrium is ultimately determined by the elasticity of substitution.¹⁹ In particular, indirect additivity amounts to assume that the optimal consumption ratio of any two goods does not depend on the price of any other good. This implies that in case of a common price $p_i = p_j$ the elasticity of substitution between varieties i and j does not depend on the number of goods, while it might depend on income. Instead, under direct additivity of preferences (the D–S model), it is the marginal rate of substitution between any two goods $u'(x_i)/u'(x_j)$ which is independent from the consumption of the other goods, leading to the property that their inverse price ratio, p_i/p_j , is independent from the quantities of the other goods consumed. As an implication, the elasticity of substitution between varieties i and j in the case of a common consumption level $x_i = x_j$ depends only on this consumption level. Finally, monopolistic competition under homothetic preferences (a case studied in Benassy, 1996) implies that the elasticity of substitution cannot depend on the level of income nor, in a symmetric equilibrium, on the price level. However, it may still depend on the number of varieties (as in the Translog example of Feenstra, 2003).

The different implications of these three classes of preferences are a consequence of their differences in the relevant demand elasticities and in the entry process. In case of direct additivity (D–S, 1977),²⁰ income affects prices in the short run but not in the

¹⁸ Another example generating closed form solutions arises if $v(s) = (s + g)^{1-\theta}$, with $\theta > 1$. Here the equilibrium price is $p^e = (\theta c + gE)/(\theta - 1)$ and the direct utility can be written as $U = [\int_j x_j^{(\theta-1)/\theta} dj]^\theta (1 + g \int_j x_jdj)^{1-\theta}$. Notice that $\theta' \geq 0$ if $g \geq 0$.

¹⁹ The elasticity of substitution between goods i and j is a logarithmic derivative of x_i/x_j with respect to p_i/p_j ; see Blackorby and Russell (1989) for a formal discussion of the concept and Bertoletti and Etro (2014b) for its crucial role in models of imperfect competition and product differentiation. See Mrázová and Neary (2013) for a article that investigates how the assumptions on demand determine the relevant comparative statics properties of market equilibria.

²⁰ Under direct additivity the indirect utility function depends on a price aggregator given by the marginal utility of income λ : namely, $V = \int_0^n u[u^{-1}(\lambda p_j)]dj$ with λ implicitly defined by $E = \int_0^n u^{-1}(\lambda p_j)p_jdj$.

long run (Behrens and Murata, 2012*a*; Bertolotti and Etro, 2014*a*). To see this, notice that the reciprocal of the inverse demand elasticity, $\varepsilon(x_i) = -u'(x_i)/u''(x_i)x_i$, depends on the consumption level only; the free-entry equilibrium can be summarised as follows:

$$\frac{p^e - c}{p^e} = \frac{1}{\varepsilon(q^e/L)}, \quad n^e = \frac{EL}{F\varepsilon(q^e/L)} \quad \text{and} \quad q^e = \frac{F[\varepsilon(q^e/L) - 1]}{c}, \quad (10)$$

where the equilibrium price does depend on the population L , which in turn affects non-linearly the number and the size of firms: the exact impact depends on whether ε is increasing or decreasing in consumption (Zhelobodko *et al.*, 2012; Bertolotti and Epifani, 2014). However, the price and the production of each firm are determined independently from income E : in spite of non-homotheticity free entry neutralises the impact of income and markups cannot be affected by changes in consumer spending over the business cycle if the number of firms adjusts to ensure the zero profit condition.²¹

With homothetic preferences (Benassy, 1996), the demand elasticity in the case of symmetric consumption $\varepsilon(n)$ can only depend on the number of varieties offered,²² and the free-entry equilibrium can be summarised as follows:

$$\frac{p^e - c}{p^e} = \frac{1}{\varepsilon(n^e)}, \quad n^e = \frac{EL}{F\varepsilon(n^e)} \quad \text{and} \quad q^e = \frac{F[\varepsilon(n^e) - 1]}{c}. \quad (11)$$

In this case, the number of firms increases more or less than proportionally with total income EL depending on whether $\varepsilon(n)$ is increasing or decreasing in the number of firms, but is independent from the marginal cost, whose changes are (inversely) proportional to the firm size. Accordingly, markup is now neutral to productivity shocks (complete pass-through).

Summing up, under free entry, each one of the previous three classes of preferences is characterised by a different form of 'neutrality' and CES preferences inherit all of them:

REMARK 1. *Under endogenous entry, with indirect additivity the population is neutral on markups but income and productivity affect them; with direct additivity income is neutral on markups but population and productivity affect them; with homotheticity productivity is neutral on markups but population and income affect them.*

1.3. Non-separable Indirect Utilities

As the reader might expect, it is possible to extend the setting of monopolistic competition to the general class of non-separable symmetric preferences: for a general treatment and new examples see Bertolotti and Etro (2014*b*).²³ The relevant indirect

²¹ The reason of the different free-entry results of direct and indirect additivity is rooted in the market adjustment process, which takes place through shifts of demand due to changes in the mass of firms affecting the marginal utility of income. Since the profit expression with direct additivity is $\pi = [u'(x)/\lambda - c]Lx - F$, there is a unique (symmetric) equilibrium (zero-profit) value of $\lambda = [nu'(x)x]/E$. On the contrary, under indirect additivity, there is a unique equilibrium value of $L/\mu = LE/[nu'(p/E)\beta]$.

²² Homotheticity implies that the indirect utility function can be written $V = E\lambda$, where the marginal utility of income λ is a homogenous of degree -1 function of all prices.

²³ In the case of a finite number of goods, Bertolotti and Etro (2014*b*) compare equilibria under monopolistic, Cournot and Bertrand competition.

utility functions will in general depend on the number of the consumed varieties (as in Feenstra, 2003). The Roy identity provides always the relevant demand of each firm i as a function of its price p_i and of some symmetric price aggregators whose values are given for each firm (so that strategic considerations are neglected). In a symmetric equilibrium demand elasticity can be shown to be equal to the Morishima Elasticity of Substitution (defined in Blackorby and Russell, 1989), which depends either on the common price-income ratio p/E or on the common consumption value x , and on the number of varieties n (all related by the budget condition $xpn = E$). Accordingly, the equilibrium markup can be affected by all the exogenous parameters E , L/F and c . From this perspective, the three types of preferences discussed in the previous subsection represent the polar cases in which the equilibrium elasticity depends just either on p/E , x or n .

Here we limit our discussion to emphasise that the results obtained under indirect additivity extend also to other interesting non-separable preferences. Consider, for example, preferences that can be represented by the following ‘quadratic’ (non-homothetic) indirect utility function:

$$V^n = \frac{1}{2} \int_0^n \left(a - \frac{p_j}{E} \right)^2 dj - \frac{1}{2n} \left(\int_0^n \frac{p_j}{E} dj \right)^2. \quad (12)$$

By the Roy identity we obtain the demand function:

$$x_i = \frac{aE - (p_i - \bar{p})}{|\mu|E}, \quad (13)$$

where $\mu = -E(\partial V^n / \partial E) < 0$, the additional price aggregator $\bar{p} = \int_0^n p_j dj / n$ is the average price.²⁴ Accordingly, a symmetric equilibrium yields the demand elasticity $\varepsilon = p^e / aE$ and the equilibrium price:

$$p^e = c + aE, \quad (14)$$

which is increasing in income but again independent from the number of consumers, with firm size $q^e = F/aE$ and number of firms $n^e = aE^2 L/F(c + aE)$.

2. Extensions

In this Section we extend our baseline model in a number of directions to emphasise its tractability and to investigate new issues.

2.1. *Outside Good and Optimum Product Diversity*

It can be useful to introduce an outside good representing the rest of the economy, as in many general equilibrium models.²⁵ Let us consider a second sector producing a homogenous good under perfect competition and constant returns to scale. We follow

²⁴ The demand presented in (13) is similar to that used to introduce monopolistic competition in the textbook of Krugman *et al.* (2012, ch. 8). Income is irrelevant there since its microfoundation is inspired to the quasilinear model of Melitz and Ottaviano (2008).

²⁵ A relevant part of the literature, starting with Spence (1976), has focused on quasilinear preferences, thus eliminating income effects: see in particular Melitz and Ottaviano (2008) and Anderson *et al.* (2012).

D–S (1977, Section II) and adopt an indirect utility that has an intersectoral Cobb–Douglas form:²⁶

$$V = \left(\frac{E}{p^0}\right)^\omega \left[\int_0^n v\left(\frac{p_j}{E}\right) dj\right]^{1-\omega}, \quad (15)$$

where p^0 is the price of the outside good and $\omega \in [0,1]$: clearly (15) collapses to (1) for $\omega = 0$.²⁷ In the Appendix, we show that the pricing rule for the differentiated goods remains the same as in (5), but the equilibrium mass of firms also depends on the elasticity of the indirect sub-utility v , defined as $\eta(s) \equiv -v'(s)s/v(s) > 0$, which reflects the relative importance of the differentiated sector:

PROPOSITION 3. *In a Cobb–Douglas two-sector economy with indirect additivity and monopolistic competition with endogenous entry in the differentiated sector, an increase in the number of consumers is neutral on prices and increases linearly the mass of firms but higher income increases prices if and only if the demand elasticity is increasing in the price.*

It is interesting to evaluate the welfare properties of this generalised setting. As is well known, firms do not fully internalise the welfare impact of their entry decision, which may lead to too many or too few firms.²⁸ The constrained optimal allocation (controlling prices and number of varieties under a zero profit constraint) is derived in the Appendix and provides a simple comparison with the decentralised equilibrium for any $\omega \in [0,1]$:

PROPOSITION 4. *In a Cobb–Douglas two-sector economy with indirect additivity, monopolistic competition with endogenous entry generates excess entry (insufficient entry) with too little (too much) production by each firm if the elasticity of the indirect sub-utility is everywhere increasing (decreasing) in the price.*

Paralleling D–S, an intuition for this result can be obtained by noticing that $\eta(s)$ approximates the ratio between the revenue of each firm and the additional utility generated by its variety. If $\eta' > (<)0$ they diverge and at the margin each firm finds it more profitable to price higher (lower), i.e., to produce less (more), than what would be socially desirable. This, in turn, attracts too many (too few) firms. One may find more reasonable the case in which the elasticity of the sub-utility decreases when income gets higher, which requires $\eta' > 0$. This is the case for the exponential and addilog cases: accordingly, they both imply excess entry.

²⁶ A general specification of intersectoral preferences would not change the pricing rule, but the equilibrium number of firms would not necessarily be linear in the number of consumers.

²⁷ We can offer two interpretation of this setting. First, it corresponds to a typical two-sector trade model in general equilibrium. Second it can be re-interpreted in terms of a two-period model where young agents have income E to be spent in the homogenous good or saved to consume the differentiated goods when old (with discount factor $1/\omega - 1$ and a zero interest rate). Such a set-up can be easily introduced in an overlapping generations (OLG) model.

²⁸ See the original D–S (1977) paper, Kuhn and Vives (1999) and Dhingra and Morrow (2015) for key references on this issue. Notice that the first-best allocation would require marginal cost pricing and subsidies to the firms.

2.2. *Heterogeneous Consumers and Income Distribution*

In this subsection, we generalise our model to the case of consumers with different preferences and income. The model remains tractable and allows one to draw implications on the impact of income distribution on the market structure (which is not neutral as in the CES case). We assume that there is a mass L of consumers of different ‘types’. Types are distributed across the population according to the cumulative distribution function $C(h)$ with support $[0,1]$.²⁹ The consumer of type h has income E_h and indirect utility function given by:

$$V_h = \int_0^n v_h \left(\frac{p_j}{E_h} \right) dj. \quad (16)$$

As we prove in the Appendix, in the symmetric equilibrium, each firm adopts a simple extension of the pricing rule (5) for homogenous consumers:

$$\frac{p^e - c}{p^e} = \frac{1}{\bar{\theta}(p^e, C)} \text{ with } \bar{\theta}(p, C) \equiv \int_0^1 \theta_h \left(\frac{p}{E_h} \right) \frac{E_h}{\bar{E}} dC(h), \quad (17)$$

where $\bar{\theta}$ is a weighted average of the individual demand elasticities θ_h and the weight is the consumer of type h 's ‘fraction’ of average income \bar{E} . Under free entry, the mass L of consumer is again neutral but the distribution of types is not.³⁰ Moreover, we can prove additional results on the impact of income distribution on the market structure:

PROPOSITION 5. *Under indirect additivity with heterogeneous consumers and monopolistic competition with endogenous entry, an increase in the mass of consumers is neutral on prices and increases linearly the number of firms. With identical preferences:*

- (i) *a change in income distribution according to the likelihood-ratio dominance raises (decreases) prices and increases the mass of firms more (less) than proportionally to average income if the demand elasticity is increasing (decreasing); and*
- (ii) *a mean preserving spread decreases prices and the mass of firms if and only if the demand elasticity is convex.*

The long-run impact of a change in the income distribution is in line with the spirit of the baseline model and breaks the neutrality emerging in models based on CES and exponential direct utilities or other equivalent microfoundations (Behrens and Murata, 2012*b*; Tarasov, 2014). However, the impact of a change in inequality is ambiguous. To fix ideas, let us consider the case of identical preferences with a demand elasticity increasing and convex with respect to the price, as in our addilog example: in such a case a mean preserving spread of the income distribution increases the average demand elasticity that is expected by the firms, which in turn reduces prices and induces business destruction.

²⁹ We arrange consumer types in such a way that $h > k$ implies $E_h > E_k$, exclude any form of price discrimination (i.e., there is no market segmentation) and focus on the symmetric equilibrium. For an analysis of heterogeneity in income (not in preferences) under direct additivity see Foellmi and Zweimüller (2004).

³⁰ A special case arises if preferences are of the exponential type, i.e., $v_h = e^{-b_h p/E_h}$. In such a case $\theta_h(p/E_h) = b_h p/E_h$ and therefore $\bar{\theta} = p\bar{b}/\bar{E}$, where $\bar{b} = \int_h b_h dC(h)$: the market structure depends only on \bar{b} and average income \bar{E} .

2.3. *Heterogeneous Firms and Endogenous Quality*

Melitz (2003) has shown that under heterogeneous productivity of the firms and CES preferences there are no selection effects on the set of active firms when markets expand, for instance in a boom or when the country opens up to costless trade. However, under direct additivity this neutrality holds for changes in income but not in the population, whose increase can give rise to ambiguous effects (depending on the shape of the elasticity of substitution). In particular, when prices are increasing with the size of consumption, an expansion of the market scale induces a selection effect, forcing the exit of the least productive firms, while less productive firms are able to survive during a contraction of the market (Zhelobodko *et al.*, 2012; Bertoletti and Epifani, 2014). In this subsection we show that under indirect additivity and firms heterogeneity the market size remains neutral but income growth matters, exerting a sort of Darwinian effect as long as $\theta' > 0$: income expansions allow less productive firms to survive but downturns lead to the exit of the least productive firms.

Following Melitz (2003), we assume that, upon paying a fixed entry cost F_e , each firm draws its marginal cost $c \in [\underline{c}, \infty)$ from a continuous cumulative distribution $G(c)$ with $\underline{c} \geq 0$. In the Appendix, we show that the equilibrium price function $p(c)$ of an active c -firm satisfies the pricing rule (5), that high-productivity firms produce more, get larger revenues and are more profitable, as in Melitz (2003), but they also charge higher markups if and only if $\theta' > 0$. Firms are active if their variable profits π_v cover the fixed cost F , that is if they have a marginal cost below the cut-off \hat{c} satisfying:³¹

$$\pi_v(\hat{c}) = \frac{[p(\hat{c}) - \hat{c}]v'[p(\hat{c})/E]L}{\mu} = F. \quad (18)$$

Moreover, the equilibrium must satisfy the endogenous entry condition:

$$\int_{\underline{c}}^{\hat{c}} [\pi_v(c) - F]dG(c) = F_e, \quad (19)$$

i.e., firms must expect zero profit from entering in the market. The two equations determine \hat{c} and μ as a function of L , F , F_e and E but in the Appendix we show that a change in L produces no selection effects: an increase of market size is completely neutral on all the prices and on the productivity cut-off beyond which firms are active, even when preferences are not CES. Instead, changes in income induce novel effects on the structure of production:

PROPOSITION 6. *Under indirect additivity, monopolistic competition with endogenous entry and cost heterogeneity between firms, an increase in population is neutral on prices and on the productivities of the active firms (it increases proportionally their mass) but higher income increases prices of all firms and makes less productive firms able to survive if and only if the demand elasticity is increasing in the price.*

³¹ Notice that even when the fixed costs of production are null ($F = 0$) but there is a finite choke-off price \bar{s} (as for instance in the case of addilog preferences) the marginal cost cut-off $\hat{c} = \bar{s}E$ is increasing in income and independent from population.

This result rationalises a Darwinian effect of recessions: these induce the exit of low-productivity firms, while on the contrary expansionary shocks associated with higher spending make low-productivity firms able to survive. Notice that such a cyclical process cannot be reproduced in the baseline Melitz model or in its extension to directly additive preferences.³²

The heterogeneous costs model can be easily extended to take into account endogenous quality choices. This possibility has been recently explored to account for positive correlations of productivity with both quality and prices (see for instance Fajgelbaum *et al.* (2011) and Kugler and Verhoogen (2012)), whereby non-homothetic preferences are essential to explain the positive association of income with both quality and prices associated with the Linder hypothesis (Linder, 1961). Let us suppose that for a variety j with price p_j and quality $k_j \geq 0$ the sub-utility is given by $v_j = v(p_j/E)\varphi(k_j)$, where $\varphi, \varphi' > 0$ (higher quality increases both utility and demand without affecting demand elasticity) and $\lim_{k \rightarrow 0} \varphi(k) = 0$ to avoid corner solutions. For simplicity, let us assume that a c -firm can produce goods of quality k at the marginal cost ck , obtaining variable profits $\pi_v = (p - ck)v'(p/E)\varphi(k)L/\mu$. Under some regularity conditions,³³ the equilibrium choices $p(c)$ and $k(c)$ satisfy the FOCs:

$$\frac{p - ck}{p} = \frac{1}{\theta(p/E)} \text{ and } \theta\left(\frac{p}{E}\right) = 1 + \epsilon(k), \tag{20}$$

where $\epsilon(k) \equiv \varphi'(k)k/\varphi(k)$ is the elasticity of demand with respect to quality. Price and quality are again independent from L ³⁴ but their relation with productivity and consumers' income is more complex and can be derived through total differentiation as follows:

$$\begin{aligned} \text{sign}\left\{\frac{\partial p}{\partial c}\right\} &= -\text{sign}\{\epsilon'\} & \text{and} & & -\text{sign}\left\{\frac{\partial p}{\partial E}\right\} &= \text{sign}\{\theta'\}, \\ \text{sign}\left\{\frac{\partial k}{\partial c}\right\} &= -\text{sign}\{\theta'\} & \text{and} & & -\text{sign}\left\{\frac{\partial k}{\partial E}\right\} &= \text{sign}\{\theta'\}. \end{aligned}$$

First, notice that under CES preferences ($\theta' = 0$) quality is (endogenously) independent from productivity and consumers' income, while when demand is isoelastic in quality ($\epsilon' = 0$) the price is the same for all firms and more productive firms invest more in quality. Under our standard assumption $\theta' > 0$, more productive firms produce goods of higher quality. Moreover, they can even invest so much to sell them at higher prices compared to low productivity firms: this happens when demand becomes more sensitive to quality for products of higher quality ($\epsilon' > 0$).³⁵ Finally, again in line with the Linder hypothesis, higher income induces specialisation in high quality, high price goods. Notice that, given price and quality choices, variable profits are still increasing in productivity and income and the free-entry mechanism operates as above.

³² See Ottaviano (2012) for a related result in a model with a 'quadratic' direct utility and income effects.

³³ The SOCs require $2\theta > \zeta$, $\xi \equiv k\varphi''/\varphi' < 2(\theta - 1)$ and $\zeta(\zeta - 2\theta) > (\theta - 1)(2\zeta - 3\theta)$. Accordingly, it must be the case that $\theta' > 0$ if $\epsilon' \geq 0$.

³⁴ The neutrality of market size on quality is robust to more general specifications of the indirect sub-utility as $v_j = v(s_j, k_j)$ but, once again, it breaks down with direct additivity.

³⁵ For empirical evidence in this direction see for instance Kugler and Verhoogen (2012).

3. A Two-country Story

One of the main limits of the trade models based on monopolistic competition with CES preferences (Krugman, 1980; Melitz, 2003) is their inability in providing simple reasons why firms change markups when selling in different countries or facing different trade costs. It is well known that pricing to market is a pervasive phenomenon: identical products tend to be sold at different markups in different countries and, in particular, prices appear to be positively correlated with per capita income (Alessandria and Kaboski, 2011) but hardly with country population (Handbury and Weinstein, 2014; Simonovska, 2015). We have also evidence of incomplete pass-through of cost reductions due to trade liberalisation (De Loecker *et al.*, 2012). In this Section, we generalise the basic Krugman (1980) model to indirectly additive preferences and emphasise its implications for the structure of trade.³⁶

We consider trade between two countries sharing the same preferences (1) and technology, as embedded into the costs c and F , but possibly with different numbers of consumers L and L^* and different productivity (i.e., labour endowment in efficiency units). In particular, we assume that the labour endowments of consumers in the home and foreign countries are respectively e and e^* , so that income levels are $E = we$ and $E^* = w^*e^*$. Accordingly, the marginal and fixed costs in the domestic and foreign countries are respectively wc and w^*c and wF and w^*F .³⁷ Let us assume that to export each firm bears an ‘iceberg’ cost $\tau \geq 1$ and, as standard, let us rule out the possibility of parallel imports aimed at arbitraging away price differentials (i.e., international markets are segmented). Consider the profit of a firm i , based in the home country, which can choose two different prices for domestic sales p_i and exports p_{zi} :

$$\pi_i = \frac{(p_i - wc)v'\left(\frac{p_i}{E}\right)L}{\mu} + \frac{(p_{zi} - \tau wc)v'\left(\frac{p_{zi}}{E^*}\right)L^*}{\mu^*} - wF, \quad (21)$$

where μ and μ^* are the home and foreign values of (3). A symmetric expression holds for a foreign firm j , choosing prices p_j^* and p_{zj}^* .

The optimal price rules for the home firms are:

$$\frac{p - wc}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \quad \frac{p_z - \tau wc}{p_z} = \frac{1}{\theta\left(\frac{p_z}{E^*}\right)}, \quad (22)$$

and the price rules for the Foreign firms are similarly obtained. Therefore, four different prices emerge in the symmetric equilibrium, with $\mu = nv'(p/E)p/E + n^*v'(p_z^*/E)p_z^*/E$

³⁶ Several recent articles have studied trade in multi-country models with non-homothetic preferences. Behrens and Murata (2012a) and Simonovska (2015) use specific types of directly additive preferences: the former article assumes that markets are not segmented while the latter deals with the case of international price discrimination. Mrázová and Neary (2014), Bertolotti and Epifani (2014) and Bertolotti and Eto (2014a) assume direct additivity: the first two articles consider identical countries (with the former focusing on welfare with symmetric firms and the latter dealing with the case of heterogeneous firms), while the third one investigates the case of different countries and market segmentation. Fajgelbaum *et al.* (2011) consider products of different qualities within a Logit demand system.

³⁷ Identical results would emerge assuming different productivities (affecting proportionally both marginal and fixed cost) and equal labour endowments. This would be reflected on the equilibrium wages and through this on incomes.

and $\mu^* = nv'(p_z/E^*)p_z/E^* + n^*v'(p^*/E^*)p^*/E^*$. The endogenous entry condition for the firms of the home country reads as:

$$\frac{(p - wc)v'\left(\frac{p}{E}\right)L}{\mu} + \frac{(p_z - \tau wc)v'\left(\frac{p_z}{E^*}\right)L^*}{\mu^*} = wF, \tag{23}$$

and a corresponding one holds for the firms of the foreign country. We can normalise the home wage to unity, $w = 1$, and close the model with the domestic resource constraint (or, equivalently, the labour market clearing condition):

$$eL = n(cxL + \tau cx_2L^* + F), \tag{24}$$

where $x = v'(p/E)/\mu$ and $x_2 = v'(p_z/E^*)/\mu^*$. This provides a system of seven equations in seven unknowns ($p, p_z, p^*, p_z^*, w^*, n$ and n^*). With non-homothetic preferences, population and productivity of a country have a distinct impact on the relative wages, with complex implications for price differentials and the structure of trade. However, we can get the main insights focusing on the two cases traditionally analysed in the literature: costless trade between different countries, and costly trade between identical countries. The former case is obtained setting $\tau = 1$ and is characterised as follows:

PROPOSITION 7. Under indirect additivity, monopolistic competition with endogenous entry and costless trade, firms adopt a higher price in the country with higher per capita income; opening up to trade reduces (increases) the number of firms in the country with higher (lower) per capita income if and only if the demand elasticity is increasing, and generates pure gains from variety.

Since costless trade induces wage equalisation (wages must be the same in both countries otherwise the zero-profit conditions could not be simultaneously satisfied; also see Behrens and Murata, 2012a), the price rules show immediately the emergence of pricing to market: under the standard assumption $\theta^i > 0$, prices of identical goods are higher in the country with the higher per capita income because demand is more rigid compared to the other country and these prices are independent of the population sizes.³⁸ Consumers enjoy new varieties produced abroad and bought at the same price of the domestic goods. Nevertheless, opening up to trade induces a redistribution of firms and production across countries which is absent in the Krugman (1980) model. Firms exporting to the country with poorer consumers sell there at a lower mark up and face entry of foreign firms in the domestic market: accordingly, they obtain lower variable profits, which leads to business destruction at home. Therefore, the richer country is characterised by a process of concentration in fewer and larger firms. On the contrary, business creation takes place in the poorer country, where firms start selling abroad at higher mark ups and reduce their size. Finally, prices and the total number of firms across countries remain the same as in autarky, therefore the gains from trade are always pure gains from variety as in Krugman (1980).³⁹

³⁸ Notice that pricing to market can arise in a multi-country setting also under direct additivity (Markusen, 2013; Bertolotti and Etro, 2014a; Simonovska, 2015): a larger income implies a larger individual consumption for each variety, which in turn affects markups. However, exactly for this reason, direct additivity also preserves the (ambiguous) impact of the number of consumers on markups.

³⁹ Augmenting the model with strategic interactions generates a genuine competitive effect of trade on markups leading also to gains from competition (Etro, 2015).

The second case we consider, the one of costly trade, is obtained by setting $\tau > 1$ with $L = L^*$ and $e = e^*$ and is characterised as follows:

PROPOSITION 8. *Under indirect additivity, monopolistic competition with endogenous entry and costly trade between identical countries, opening up to trade reduces the markup on the exported goods and the mass of firms in each country relative to autarky if and only if the demand elasticity is increasing.*

Because also trade costs are symmetric, wages and prices are equalised in both countries. However, the markup applied to goods sold at home and abroad is not the same when preferences are not homothetic. In particular, the markup (on the marginal cost $c\tau$) is lower for the exported goods if $\theta' > 0$, because firms undershift transport costs on prices (De Loecker *et al.* (2012) provide convincing evidence for such incomplete pass-through of the cost reductions on prices during trade liberalisations). This shows a different form of pricing to market, which has the additional consequence of affecting the entry process compared to the neutrality of the Krugman (1980) model: as long as the average markup diminishes because of undershifting of the transport costs on export prices, opening up generates a process of business destruction in both countries. Welfare gains from liberalisation, therefore, cannot be guaranteed, since the changes in the price of the imported goods also affects the gains from variety. In particular, trade liberalisation (a reduction of τ) reduces the price of imports and increases their consumption but also affects the number of firms through changes in the average markup, which leads to an ambiguous impact on welfare. We prove the following in the Appendix:⁴⁰

REMARK 2. *Trade liberalisation has a non-monotonic welfare impact when $\theta' > 0$, since it is negative around autarky (assuming the existence of a finite choke-off price) and it must be positive in the case of an opening to free trade.*

Finally, under some additional conditions, we can show that richer countries trade relatively more between themselves than poorer countries, which is also in line with the evidence (for instance see Fielser, 2011). The export share on GDP, say $\varpi \equiv x_z p_z / (x_z p_z + x p)$, can be derived as:

$$\varpi = \frac{v'(p_z/E)p_z}{v'(p_z/E)p_z + v'(p/E)p} \quad (25)$$

where prices satisfy (22). Under CES preferences this ratio is $1/(1 + \tau^{\theta-1})$, therefore the export share is independent from income. Under non-homotheticity, weak additional conditions satisfied with linear and log-linear demand ($\zeta' > 0$ is sufficient) guarantee that the export share increases with income because the relative demand for imported goods becomes more elastic. This is in line again with the Linder hypothesis that richer countries trade more between themselves.

In conclusion, we note that the assumption of indirect additivity of preferences may prove useful also to disentangle the impacts of income and population on other

⁴⁰ An analogous result has been derived under direct additivity by Bykadorov *et al.* (2015).

aspects of trade such as the volume of trade between countries, the emergence of multinationals and multiproduct firms and the role of trade policy.

4. Conclusion

The contribution of this work is to propose a new tractable model of monopolistic competition. We study the equilibria of large markets under indirect additivity of consumers’ preferences, an alternative to the classic assumptions of direct additivity or quasilinearity. Under reasonable conditions (namely more rigid demand for higher income), indirect additivity generates simple predictions that are in contrast with the traditional approach and await for empirical tests: higher income (and productivity) in a market should increase markups and more than proportionally the number of firms, while the number of consumers should be neutral. Our framework encompasses a number of convenient cases such as those with addilog and exponential preferences generating linear and log-linear demand functions. Therefore it could be applied to analyse complex issues usually considered exclusive territory for CES modelling: in particular, we hope that indirect additivity will prove useful in building new multi-country trade models and dynamic macroeconomic models with monopolistic competition and heterogeneous firms and consumers.

Appendix. Proofs

Proof of Proposition 1. By using $\theta'p/\theta E = \theta + 1 - \zeta \geq 0$ if and only if $\theta + 1 \geq \zeta$, the result follows from the total differentiation of (5):

$$\frac{\partial \ln p^e}{\partial \ln n} = 0, \quad \frac{\partial \ln p^e}{\partial \ln E} = \frac{\theta + 1 - \zeta}{2\theta - \zeta} \quad \text{and} \quad \frac{\partial \ln p^e}{\partial \ln c} = 1 - \frac{\theta + 1 - \zeta}{2\theta - \zeta}, \tag{A.1}$$

after noticing that $2\theta - \zeta > 0$ from the SOC.

Proof of Proposition 2. Using the comparative statics in (A.1) and differentiating (6) we obtain:

$$\frac{\partial \ln n^e}{\partial \ln L} = 1, \quad \frac{\partial \ln n^e}{\partial \ln E} = 1 + \frac{(\theta + 1 - \zeta)(\theta - 1)}{2\theta - \zeta} \quad \text{and} \quad \frac{\partial \ln n^e}{\partial \ln c} = -\frac{(\theta + 1 - \zeta)(\theta - 1)}{2\theta - \zeta}.$$

Proof of Proposition 3. By the Roy identity, the demand of each differentiated good is given by:

$$x_i = \frac{v'(p_i/E)}{\int_0^n \left[v'(p_j/E) \frac{p_j}{E} - \frac{\omega}{1 - \omega} v(p_j/E) \right] dj},$$

and the profits of each firm i are given by:

$$\pi_i = \frac{v'(p_i/E)(p_i - c)L}{\int_0^n \left[v'(p_j/E) p_j/E - \frac{\omega}{1 - \omega} v(p_j/E) \right] dj} - F, \tag{A.2}$$

where the denominator is unaffected by a single price. It is immediate to verify that, independently from the value of ω , each firm adopts the same pricing rule as in (5) and the comparative static properties of the profit-maximising price p^e are then the same as in Proposition 1. The number of goods produced in the free-entry equilibrium can be derived as:

$$n^e = \frac{EL(1-\omega)\eta(p^e/E)}{F\theta(p^e/E)[(1-\omega)\eta(p^e/E)+\omega]}, \quad (\text{A.3})$$

which now depends on ω and both the elasticities θ and η : changes in income affect prices and the allocation of expenditure between the differentiated goods and the outside one. Nevertheless, the number of firms is always proportional to L .

Proof of Proposition 4. We compare the market equilibrium with a constrained optimal allocation which maximises utility under a zero-profit condition for the firms. The problem boils down to:

$$\max_{n,p} nv(p/E) \quad \text{s.t.} \quad p = c + \frac{F}{x(p, n, E)L},$$

where

$$x(p, n, E) = \frac{v'(p/E)}{n \left[v'(p/E) \frac{p}{E} - \frac{\omega}{1-\omega} v(p/E) \right]}$$

is the symmetric demand of a variety. Notice that the zero-profit constraint implicitly defines n as a function of p that is continuous on $[c, \bar{s}E]$, with $n(c) = 0$ and $\lim_{p \rightarrow \bar{s}E} n(p)$ which is a finite number. Accordingly, the objective function is null for $p = c$ and $p \rightarrow \bar{s}E$, and positive for at least some intermediate price due to our assumptions. Therefore, an internal optimum satisfying the FOCs must exist:

$$\begin{aligned} v(p/E) &= -\rho \frac{F}{Lx^2} \frac{\partial x}{\partial n}, \\ nv'(p/E)/E &= -\rho \left(1 + \frac{F}{Lx^2} \frac{\partial x}{\partial p} \right), \end{aligned}$$

where ρ is the relevant Lagrange multiplier. The FOCs imply:

$$\eta(p/E) = -\frac{\frac{xpL}{F} + \frac{\partial \ln x(p, n, E)}{\partial \ln p}}{\frac{\partial \ln x(p, n, E)}{\partial \ln n}} = \frac{p}{p-c} + \frac{\partial \ln x(p, n, E)}{\partial \ln p},$$

and it is easily computed that:

$$\eta' = \frac{\eta[1-\theta+\eta]}{p/E}. \quad (\text{A.4})$$

Since

$$\frac{\partial \ln x(p, n, E)}{\partial \ln p} = \frac{\omega \frac{d \ln \eta(p/E)}{d \ln p/E}}{\omega + (1-\omega)\eta} - 1 = \frac{(2\omega-1)\eta - \omega\theta}{\omega + (1-\omega)\eta},$$

we obtain the optimal markup:

$$\frac{p^* - c}{p^*} = \frac{\omega + (1-\omega)\eta}{(1-\omega)\eta(1+\eta) + \omega\theta}. \quad (\text{A.5})$$

Finally, the optimal mass of firms is:

$$n^* = \frac{(1 - \omega)\eta LE}{F[(1 - \omega)\eta(1 + \eta) + \omega\theta]}. \tag{A.6}$$

Comparing (A.5) with (5), it follows that the RHS of (A.5) is larger (smaller) than $1/\theta$ if (everywhere) $\theta \geq 1 + \eta$. Since from (A.4) it follows that $\eta' \leq 0$ is equivalent to $1/(1 + \eta) \geq 1/\theta$, then (everywhere) $\eta' \leq 0$ is equivalent to $p^e \leq p^*$, which in turn implies $x^* \leq x^e$ by the zero-profit constraint. Using the fact that the RHS of (A.6) is larger (smaller) than the RHS of (A.3) if η' is smaller (larger) than zero we obtain $n^e \leq n^*$ if $\eta' \leq 0$, which completes the proof.

Proof of Proposition 5. The demand of a consumer h for good i can be written as $x_{hi}(p_i, E_h, \mu_h) = v'_h(p_i/E_h)/\mu_h$, where $\mu_h = \int_j v'_h(p_j/E_h)(p_j/E_h)dj$. Since types are distributed according to $C(h)$ with support $[0,1]$, profits of firm i are:

$$\pi(p_i) = (p_i - c)L \int_0^1 x_{hi}(p_i, E_h, \mu_h)dC(h) - F,$$

which implies that the profit-maximising price p_i satisfies the FOC:

$$(p_i - c) \int_0^1 \frac{v''_h\left(\frac{p_i}{E_h}\right)}{\mu_h E_h} dC(h) + \int_0^1 \frac{v'_h\left(\frac{p_i}{E_h}\right)}{\mu_h} dC(h) = 0.$$

Symmetric pricing implies $\mu_h = nv'_h(p^e/E_h)(p^e/E_h)$ and thus:

$$\frac{p^e - c}{p^e} = \frac{- \int_0^1 \frac{E_h}{np^e} dC(h)}{\int_0^1 \frac{v''_h\left(\frac{p^e}{E_h}\right)}{nv'_h(p^e/E_h)} dC(h)} = \frac{1}{\int_0^1 \theta_h\left(\frac{p^e}{E_h}\right) \frac{E_h}{E} dC(h)} \equiv \frac{1}{\tilde{\theta}(p^e, C)},$$

where $\theta_h \equiv -v''_h(p/E_h)p/[v'_h(p/E_h)E_h]$ and $\bar{E} = \int_h E_h dC(h)$. The price rule is thus independent from L . Endogenous entry implies the following mass of firms:

$$n^e = \frac{\bar{E}L}{F\tilde{\theta}(p^e, C)},$$

which proves the first part of the proposition since L affects linearly n^e .

To prove the second part, suppose that all consumers share the same preferences. It is then convenient to rewrite $\tilde{\theta}$ directly as $\tilde{\theta}(p, I) = \int_{E_0}^{E_1} \theta(p/E)(E/\bar{E})dI(E)$, where $I(\cdot)$ is the income distribution function implied by $C(\cdot)$. To prove (1), consider a change in I according to likelihood ratio dominance: i.e., a change from I^0 to I^1 such that $i^1(E)/i^1(T) \geq i^0(E)/i^0(T)$ for all $E > T$, $E, T \in [E_0, E_1]$, where $i(\cdot) = I'(\cdot)$ is the relevant density function. This implies that I^1 also (first-order) stochastically dominates I^0 : thus, by a well-known result, this raises the average income (i.e., $\bar{E}^1 > \bar{E}^0$). We can write:

$$\tilde{\theta}(p^e, I) = \int_{E_0}^{E_1} \theta\left(\frac{p^e}{E}\right)d\Phi(E; I) \text{ with } \Phi(E; I) = \int_{E_0}^E \frac{T}{\bar{E}} dI(T),$$

where the cumulative distribution function Φ has density Φ' . Notice that:

$$\frac{\Phi'(E; I^1)}{\Phi'(T; I^1)} = \frac{Ei^1(E)}{Ti^1(T)} \geq \frac{Ei^0(E)}{Ti^0(T)} = \frac{\Phi'(E; I^0)}{\Phi'(T; I^0)} \text{ for all } E > T, E, T \in [E_0, E_1].$$

Accordingly, $\Phi(E; I^1)$ dominates in terms of the likelihood ratio $\Phi(E; I^0)$ and it must then be the case that $\Phi(E; I^1) \leq \Phi(E; I^0)$ for all $E \in [E_0, E_1]$ (i.e., the former distribution first-order stochastically dominates the latter). It follows that when $\theta(p/E)$ is a decreasing (increasing) function of E an improvement of income distribution according to likelihood-ratio dominance implies $\tilde{\theta}(p, I^1) \leq (\geq) \tilde{\theta}(p, I^0)$ for all p , which in turn decreases (increases) the equilibrium value of θ and thus raises (decreases) the equilibrium price level and the mass of active firms more (less) than proportionally to the rise of average income.

To prove (2), suppose that I^1 is a mean-preserving spread of I^0 . Then $\bar{E}^1 = \bar{E}^0 = \bar{E}$. The function $\theta(p/E)E/\bar{E}$ in the definition of $\tilde{\theta}(p, I)$ is a concave (convex) function with respect to E if and only if $\theta' < (>) 0$. By a standard result, it must then be the case that $\tilde{\theta}(p, I^0) > (<) \tilde{\theta}(p, I^1)$ when $\theta' < (>) 0$. It follows that a mean-preserving spread decreases (increases) the equilibrium value of θ , and then raises (decreases) prices and the mass of firms when θ is a concave (convex) function of the price.

Proof of Proposition 6. The variable profits of an active c -firm are given by $\pi_v = (p - c)v'(p/E)L/\mu$, where μ is independent of its price choice. Therefore, the pricing rule (5) applies to all firms. We denote with $p = p(c)$ the profit-maximising price of a c -firm, with $x(c) = v'[p(c)/E]/\mu$ the individual consumption of its product, and with:

$$\pi_v(c) = [p(c) - c]v'[p(c)/E]L/\mu$$

its variable profit for given μ defined as:

$$\mu = n \int_{\underline{c}}^{\hat{c}} v'[p(c)/E][p(c)/E] \frac{dG(c)}{G(\hat{c})} < 0.$$

Note that the optimal price of firm c does not depend upon L and μ and follows the same comparative statics as in Proposition 1, with $\partial \ln p(c)/\partial \ln c \leq 1$ and $\partial \ln p(c)/\partial \ln E \leq 0$ if and only if $\theta' \geq 0$. Moreover, the FOCs and SOC for profit maximisation imply the following elasticities with respect to the marginal cost:

$$\frac{\partial \ln x(c)}{\partial \ln c} = -\theta[p(c)/E] \frac{\partial \ln p(c)}{\partial \ln c} < 0, \quad (\text{A.7})$$

$$\frac{\partial \ln p(c)x(c)}{\partial \ln c} = \frac{-\{\theta[p(c)/E] - 1\}^2}{2\theta[p(c)/E] - \zeta[p(c)/E]} < 0, \quad (\text{A.8})$$

$$\frac{\partial \ln \pi_v(c)}{\partial \ln c} = 1 - \theta[p(c)/E] < 0. \quad (\text{A.9})$$

Accordingly, high-productivity (low- c) firms are larger, make more revenues, are more profitable and they charge higher (lower) markups if θ is increasing (decreasing). In addition, for the given μ , we have the following elasticities with respect to income:

$$\frac{\partial \ln x(c)}{\partial \ln E} = \frac{\theta[p(c)/E]^2 - \theta[p(c)/E]}{2\theta[p(c)/E] - \zeta[p(c)/E]} > 0, \quad (\text{A.10})$$

$$\frac{\partial \ln p(c)x(c)}{\partial \ln E} = \frac{\theta[p(c)/E]^2 + 1 - \zeta[p(c)/E]}{2\theta[p(c)/E] - \zeta[p(c)/E]} > 1, \quad (\text{A.11})$$

$$\frac{\partial \ln \pi_v(c)}{\partial \ln E} = \theta[p(c)/E] > 1. \quad (\text{A.12})$$

The size of each firm increases with E (for given μ), and revenues and profits increase more than proportionally. However, each price increases with respect to income only when $\theta' > 0$ and decreases otherwise. The set of active firms is the set of firms productive enough to obtain

positive profits. Denote by \hat{c} the marginal cost cutoff, namely the value of c satisfying the zero cutoff profit condition $\pi_v(\hat{c}) = F$, or:

$$[p(\hat{c}) - \hat{c}]v'[p(\hat{c})/E]L = \mu F. \tag{A.13}$$

The relation (A.13) implicitly defines $\hat{c} = \hat{c}(E, |\mu|F/L)$. Differentiating it yields:

$$\frac{\partial \ln \hat{c}}{\partial \ln E} = \frac{\theta[p(\hat{c})/E]}{\theta[p(\hat{c})/E] - 1} > 0, \tag{A.14}$$

$$\frac{\partial \ln \hat{c}}{\partial \ln |\mu|} = \frac{\partial \ln \hat{c}}{\partial \ln F} = -\frac{\partial \ln \hat{c}}{\partial \ln L} = \frac{1}{1 - \theta[p(\hat{c})/E]} < 0. \tag{A.15}$$

Endogenous entry of firms in the market implies that expected profits:

$$\mathbf{E}\{\pi\} = \int_{\underline{c}}^{\hat{c}} [\pi_v(c) - F]dG(c), \tag{A.16}$$

must be equal to the sunk entry cost F_e . The expected profits decrease when the absolute value of μ increases, that is $\partial \mathbf{E}\{\pi\} / \partial |\mu| < 0$. Accordingly, the condition $\mathbf{E}\{\pi\} = F_e$ pins down uniquely the equilibrium value of μ as a function $\mu(E, L, F, F_e)$. Moreover, using (A.13) the free entry condition can be written as:

$$\int_{\underline{c}}^{\hat{c}} \left\{ \frac{[p(c) - c] v'[p(c)/E]}{[p(\hat{c}) - \hat{c}] v'[p(\hat{c})/E]} - 1 \right\} dG(c) = \frac{F_e}{F}. \tag{A.17}$$

The system {(A.13), (A.17)} can actually be seen as determining \hat{c} and μ in function of L, F, F_e and E . The second equation, (A.17), fixes \hat{c} and is independent from L , while the first one, (A.13), determines μ as linear with respect to L . The cut-off \hat{c} is therefore independent of market scale, because μ proportionally adjusts in such a way to keep constant the ratio L/μ and thus the variable profit of the cut-off firm. As a consequence, as in Melitz (2003), a change in L produces no selection effect, even when preferences are not CES. However, (A.17) shows that changes in the fixed costs are not neutral on the set of active firms: an increase in the fixed cost of production F or a reduction in the fixed cost of entry F_e require a reduction in the equilibrium value of the cut-off \hat{c} , which correspond to the typical selection effect *à la* Melitz (2003). The impact of income E is instead more complex. Since by (A.12) and (A.14) an increase of E raises $\mathbf{E}\{\pi\}$, it must increase the equilibrium value of $|\mu|$ to satisfy zero expected profits. In particular, we have:

$$\frac{\partial \ln |\mu|}{\partial \ln E} = -\frac{\frac{\partial \mathbf{E}\{\pi\}}{\partial E} E}{\frac{\partial \mathbf{E}\{\pi\}}{\partial |\mu|} |\mu|} = \bar{\theta}(\hat{c}) > 0, \text{ where} \tag{A.18}$$

$$\bar{\theta}(\hat{c}) = \left\{ \int_{\underline{c}}^{\hat{c}} \frac{1}{\theta[p(c)/E]} \frac{p(c)x(c)}{\int_{\underline{c}}^{\hat{c}} p(c)x(c)dG(c)} dG(c) \right\}^{-1}$$

is the harmonic mean of the θ values according to $G(\cdot)$ and \hat{c} . Computing the total derivative of \hat{c} with respect to E using (A.14), (A.15) and (A.18), we obtain:

$$\frac{d \ln \hat{c}}{d \ln E} = \left(\frac{\partial \hat{c}}{\partial E} + \frac{\partial \hat{c}}{\partial |\mu|} \frac{\partial |\mu|}{\partial E} \right) \frac{E}{\hat{c}} = \frac{\theta[p(\hat{c})/E] - \bar{\theta}(\hat{c})}{\theta[p(\hat{c})/E] - 1}, \tag{A.19}$$

which is positive if and only if (everywhere) $\theta' > 0$: that is, in addition to increasing (decreasing) the mark up of the inframarginal firms, a rise of E creates an anti-selection (selection) effect if θ increases (decreases) with respect to the price. To close the model, the expected mass of active firms

n is determined by the budget constraint, requiring average expenditure to equal E/n , and thus:

$$n = \frac{E}{\int_{\underline{c}}^{\hat{c}} p(c)x(c) \frac{dG(c)}{G(\hat{c})}}. \tag{A.20}$$

Since an increase of the mass of consumers L affects proportionally μ and, thus, proportionally reduces individual consumption $x(c)$, it follows from (A.20) that it also proportionally increases the mass of varieties.

Proof of Proposition 7. Let us assume $\tau = 1$. In such a case each firm faces the same demand functions, independently from the country in which it is based. However, the firms based in the home country have a cost advantage (disadvantage) with respect to firms from the foreign country if $w < (>) w^*$. Since a necessary condition for a monopolistic equilibrium with endogenous entry in both countries is $\pi = \pi^* = 0$, it follows that it must be $w/w^* = 1$. Accordingly, we can normalise the common wage to $w = w^* = 1$ (which restores the notation of the baseline model with $E = e$ and $E^* = e^*$), and conclude that in a symmetric equilibrium $p = p_z^*$ and $p^* = p_z$. This means that all firms adopt the same price in the same country, with:

$$\frac{p - c}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \quad \frac{p^* - c}{p^*} = \frac{1}{\theta\left(\frac{p^*}{E^*}\right)}, \tag{A.21}$$

where $p > p^*$ when $E > E^*$ if and only if $\theta' > 0$ (everywhere). The opening of costless trade has no impact on prices and mark ups relative to autarky, extending this property of the Krugman (1980) model to our entire class of indirectly additive preferences. From the symmetry, we infer that all firms have the same profit and using the price rules in (A.21) the zero-profit constraint provides the total mass of firms as follows:

$$n + n^* = \frac{EL}{F\theta\left(\frac{p}{E}\right)} + \frac{E^*L^*}{F\theta\left(\frac{p^*}{E^*}\right)}. \tag{A.22}$$

This is the sum of the masses of firms emerging under the autarky equilibrium in each separate country, say $n^a = EL/F\theta(p/E)$ and $n^{a*} = E^*L^*/F\theta(p^*/E^*)$, therefore the total mass of firms remains the same. This implies that after opening up to trade welfare unambiguously increases because of the increase in the number of consumed varieties. However, the mass of firms active in each country is not the same as in autarky. In fact, by using the resource constraints one can obtain:

$$\frac{n}{n^*} = \frac{EL}{E^*L^*} \begin{cases} \leq & \frac{n^a}{n^{a*}} \\ & = \\ \geq & \frac{n^a}{n^{a*}} \end{cases} = \frac{EL\theta(p^*/E^*)}{E^*L^*\theta(p/E)} \text{ if } E > E^* \text{ and } \theta' \begin{cases} \geq \\ \leq \end{cases} 0, \tag{A.23}$$

where we used the fact that $p/E \begin{cases} \leq \\ \geq \end{cases} p^*/E^*$ if $E > E^*$ and $\theta' \begin{cases} \geq \\ \leq \end{cases} 0$. Since the total number of firms is constant, the number of firms in the rich (poor) country must decrease (increase) if $\theta' > (<) 0$. Finally, the resource constraint of each contry implies that whenever the domestic mass of firms increases (decreases) their size must decrease (increase).

Proof of Proposition 8. Let us assume $\tau > 1$ but $L = L^*$ and $e = e^*$. In such a case, all the equilibrium variables must be the same across countries by symmetry. Therefore, we can again normalise $w = w^* = 1$, which implies that $E = E^*$. The internal prices and the prices of exports must be the same in both countries, i.e., $p = p^*$ and $p_z = p_z^*$. These prices satisfy:

$$\frac{p - c}{p} = \frac{1}{\theta\left(\frac{p}{E}\right)}, \quad \frac{p_z - \tau c}{p_z} = \frac{1}{\theta\left(\frac{p_z}{E}\right)}.$$

By Proposition 1 we know that $p_z > p$ and $(p - c)/p \gtrless (p_z - \tau c)/p_z$ if and only if $\theta' \gtrless 0$. By symmetry the number of firms in each country is the same, say n , but this does not need to be the same as in autarky, $n^a = EL/F\theta(p/E)$. To find the number of firms in each country after opening up to trade, let us combine the free entry condition (23) with the price rules (22) to obtain:

$$\frac{pxL}{\theta\left(\frac{p}{E}\right)} + \frac{p_z x_z L}{\theta\left(\frac{p_z}{E}\right)} = F.$$

By using $E = n(px + p_z x_z)$ the number of firms can be derived as follows:

$$n = \frac{EL}{F\bar{\theta}} \text{ with } \bar{\theta} \equiv \left[\varpi \theta\left(\frac{p_z}{E}\right)^{-1} + (1 - \varpi) \theta\left(\frac{p}{E}\right)^{-1} \right]^{-1}. \tag{A.24}$$

Notice that $\bar{\theta}$ is the harmonic average of $\theta(s)$ and $\theta(s_z)$, where $s = p/E$ and $s_z = p_z/E$, with weights corresponding to the share values of total sales (ϖ is given in (25)). Under CES preferences this is a constant: the number of firms is the same as in autarky (independent from the transport costs). Otherwise, since $s_z > s$, we have $n \lesseqgtr n^a$ if and only if $\theta' \gtrless 0$.

Proof of Remark 2. Let us consider welfare in the model with costly trade between two symmetric countries. Since we can write:

$$V = n[v(s) + v(s_z)],$$

the impact of trade liberalisation (a reduction of trade iceberg cost τ) can be accounted by:

$$d \ln V = d \ln n + d \ln[v(s) + v(s_z)].$$

Notice that the second term on the right-hand side of last expression always has a sign opposite to that of the associated change in trade costs (for $s < \bar{s}$), since:

$$d \ln[v(s) + v(s_z)] = \frac{v'(s_z) s_z d \ln s_z}{v(s) + v(s_z)}$$

and $d \ln s_z > 0$ if and only if τ increases. Also notice that (A.24) implies:

$$d \ln n = \bar{\theta} \left[\frac{d\varpi - \varpi d \ln \theta(s_z)}{\theta(s_z)} - \frac{d\varpi}{\theta(s)} \right],$$

where ϖ is the GDP export share. With CES preferences $d \ln n = 0$, and trade liberalisation is always welfare enhancing. Beyond this case, we can sign the welfare impact of trade liberalisation starting from autarky when there exists a finite choke-off price, and around free trade. First, let us consider autarky, where $\varpi^a = 0$ and $s_z^a = \bar{s}$ so that $d \ln[v(s^a) + v(\bar{s})] = 0$. Trade liberalisation implies:

$$d \ln V^a = d \ln n^a = \left[\frac{\theta(s^a)}{\theta(\bar{s})} - 1 \right] d\varpi^a,$$

since the welfare contribution of imported goods is negligible. As $d\varpi^a > 0$, it follows that $d \ln V^a \lesseqgtr 0$ whenever $\theta' \gtrless 0$ everywhere. Therefore, trade liberalisation from autarky reduces welfare if and only if $\theta' > 0$, due to a reduction in the number of goods associated with a reduction in the average markup; an analogous result has been derived under direct additivity (and an elasticity of substitution decreasing with respect to the size of consumption) by Bykadorov *et al.* (2015). Notice that, since moving from autarky to free trade is equivalent to doubling of the market size, which implies a welfare improvement from Proposition 7, the effects of trade liberalisation on welfare are necessarily non-monotonic when $\theta' > 0$ everywhere. In fact, we can evaluate the impact of an increase in trade costs starting from free trade (denoted by ' ft '), where $\varpi^{ft} = 1/2$, $s^{ft} = s_z^{ft} = s^a$ and $d \ln s_z^{ft} > 0$, so that:

$$d \ln V^{ft} = \left[-\frac{\theta'(s^a)s^a}{2\theta(s^a)} + \frac{v'(s^a)s^a}{2v(s^a)} \right] d \ln s_z^{ft},$$

which is certainly negative if $\theta'(s^a) \geq 0$: in such a case trade liberalisation is welfare enhancing in the neighbourhood of free trade because it increases the number of consumed goods due to a business creation effect caused by the larger average markup, and reduces prices as well.⁴¹

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⁴¹ It is easy to prove that an increase of trade costs starting from free trade must reduce welfare also when $\eta' < 0$ everywhere, since in such a case it would possibly raise the number of consumed varieties only when it is already excessive (see subsection 2.1).

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