# Review of S. Mackereth "Fixed-point posets in theories of truth", Journal of Philosophical logic, vol. 48, 189-203, 2019

*This is a review submitted to Mathematical Reviews/MathSciNet.*

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**Title:** Fixed-point posets in theories of truth.

**MR Number:** MR3918699

**Primary classification:**

**Secondary classification(s):**

**Review text:**

Fixed-point theories of truth built over partial (three-valued) models have been notoriously presented as a way to solve issues related to the existence of paradoxes like the Liar, which arise in connection with formalized languages that contain their own truth predicate. Theories of that partial sort the paper under review is about, are those proposed by S. Kripke [Journal of Philosophy, 72(19): 690-716, 1975], R. Martin & P. Woodruff [Philosophia, 5(3): 213-217, 1975], and B. van Frassen [Journal of Philosophy, 63(7): 481495, 1966]. One way to roughly describe in general terms the common idea behind all of these proposals is as follows.

Let a ground first-order language $\mathcal{L}$ be given, let $M$ be a three-valued model of it (with domain $D_M$, and set of semantic values $\{\mathbf{t}, \mathbf{n}, \mathbf{f}\}$), and let $\mathcal{L}^+$ be the language obtained from $\mathcal{L}$ by adding a distinctive predicate symbol $T$ for "truth in $\mathcal{L}^+$"; let $I_M$ be an interpretation of $\mathcal{L}$ in $M$ which assigns a value in $D_M$ to individual constants and variables of $\mathcal{L}$, assigns a $n$-ary function $I_M(f) \in D_M^{D_M^n}$ to each and every $n$-place function symbol $f$ of $\mathcal{L}$, assigns distinct elements of $\{\mathbf{t}, \mathbf{n}, \mathbf{f}\}^{D_M^n}$ to each and every $n$-place predicate symbol of $\mathcal{L}$, and assigns to each formula $\varphi$ of $\mathcal{L}$ a value in $\{\mathbf{t}, \mathbf{n}, \mathbf{f}\}$ according to a three-valued scheme $l$ that tells how the value of complex formulas of $\mathcal{L}$ must be calculated in terms of the values of their subformulas. Let also $\tau$ be an interpretation for the truth predicate $T$, i.e., $\tau \in \{\mathbf{t}, \mathbf{n}, \mathbf{f}\}^{Sent_{L^+}}$, where $Sent_{L^+}$ is a subset of $D_M$ containing names $\#\varphi$ for each and every sentence $\varphi$ of $\mathcal{L}^+$, and is such that $\tau(d) = \mathbf{f}$ for every $d \in D_M \setminus Sent_{L^+}$. Finally, let $j_M^l$ be the *jump operation* that turns $\tau$ into an interpretation of the full language $\mathcal{L}^+$ (i.e., such that $j^l_M(\tau)(T(\#\varphi)) = \tau(\#\varphi)$ and is coherent with $I_M$ and $l$ in assignign values to the other formulas of it).

This paper concerns with cases in which $l$ is either taken to coincide with the strong Kleene three-valued scheme $k^+$, or the weak Kleene scheme $k^-$, or the supervaluation scheme $s$ by van Fraasen. The corresponding jump operations $j^{k^+}_M$, $j^{k^-}_M$, and $j^s_M$ can all be proved to have fixed points among elements of $\{\mathbf{t}, \mathbf{n}, \mathbf{f}\}^{D_M}$. Let $F^l_M$ be the set of the fixed points of $j^l_M$. This set can be equipped with a very natural relation $\preceq$ of partial ordering, given by

$$f \preceq g \Leftrightarrow \forall d \in D_M(f(d) \in \{\mathbf{t}, \mathbf{f}\} \Rightarrow f(d) = g(d))$$

for every $f, g \in \{\mathbf{t}, \mathbf{n}, \mathbf{f}\}^{D_M}$. About the structure of these fixed-point posets $(F^l_M, \preceq)$ for $l \in \{k^+, k^-, s\}$, A. Visser [Handbook of Phil. Logic, 2nd edn., vol. 11: 149-240, Springer, 2004] proved that: (i) each of them (actually, each structure $(F^m_M, \preceq)$ where $F^m_M$ is the set of fixed points of a jump operator $j^m_M$ that is *monotonic*), is a coherent complete partial order (ccpo for short, henceforth); (ii) for any finite ccpo $(X, \preceq_X)$, i.e., where $X$ is *any* finite set, there exists (a first-order language and) a ground model $M$ such that the fixed-point poset $(F^{k^+}_M, \preceq)$ is isomorphic to it.

This paper presents two main results, the first one of which is an extension of Visser's result (ii): Theorem 1 of the paper shows indeed that for every ccpo $(X, \preceq_X)$, i.e., not necessarily a finite one, there exists a ground model $M$ such that the fixed-point poset $(F^l_M, \preceq)$ is isomorphic to $(X, \preceq_X)$, where $l$ is either $k^+$ or $s$. Theorem 2 is instead a negative result, as it shows that the same claim holds not in case $l$ is $k^-$ since the property fails even in the finite case: a finite ccpo is exhibited for which there is no model $M$ whose fixed-point poset $(F^{k^-}_M, \preceq)$ is isomorphic to it.