

# Design and off-design analysis of a Tesla Turbine utilizing CO<sub>2</sub> as working fluid

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**Abstract.** The Tesla turbine is a bladeless expander; which principle of operation is based on the conversion of the viscous forces, developed by the flow while expanding through the rotor, in mechanical energy. It is especially suitable for small/micro size distributed energy systems (kW scale), mainly due to its very low cost, which results from the simple structure of the machine.

The Tesla turbine works well at relatively moderate expansion ratios. Therefore, it is fit for CO<sub>2</sub> power cycles applications that are characterised by small expansion ratio, despite the high pressure involved. In this work, the design and off-design analysis of a Tesla turbine for small/micro power application utilizing CO<sub>2</sub> cycles is proposed.

The optimized design was targeted for an inlet temperature of 150 °C and an inlet pressure of 220 bar. The final optimized geometry of the expander was defined, achieving a 23.4 W per channel power output with a 63% isentropic efficiency, when working with a 10.1 bar pressure drop at 2000 rpm. Furthermore, the turbine placement on the Baljè diagram was performed in order to understand the *direct competitors* of this machine.

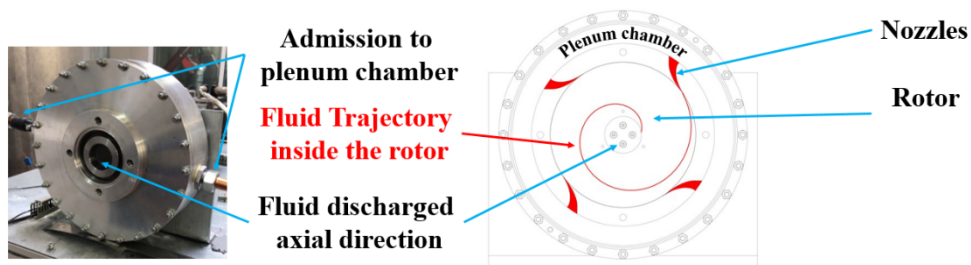
Finally, starting from the design configuration, the maps of efficiency at variable load and flow coefficients and that of reduced mass flowrate at variable pressure ratio were realized. Through the merging of these curves, the off-design maps of the Tesla turbine were obtained, highlighting a very limited sensitivity of the efficiency to variable working conditions, if rotational speed is adequately adjusted.

## 1 Introduction

The Tesla turbine is a peculiar expander, which converts the energy own by the fluid through the exchange of momentum due to shear forces. The turbine is thus also called viscous turbine, friction turbine or bladeless turbine. Indeed, its characteristic trait is the rotor, which, conversely to conventional turbomachine, is bladeless. This feature allows the turbine to be simple, affordable and reliable. The admission of the fluid to the rotor is realized through one or more tangential nozzles, which accelerate the fluid and direct it tangentially into the rotor. After entering the rotor from the outer radius, the fluid depicts a spiral path and exits axially from the inner radius. The most important feature of this machine is the very small gap between the disks, which is the geometric parameter that influences the most the possibility of converting the energy own by the fluid in mechanical energy (Fig.1).

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**Fig 1.** Schematic of Tesla turbine

The Tesla turbine was firstly introduced by Tesla in 1913 [1]; nonetheless, due to the run towards centralized power plant, his invention did not stir up much interest. It was only in the 1950s [2] and especially in the 1960s with the works of W. Rice, that the Tesla turbine started to be investigated more accurately. Indeed, W. Rice deeply studied the principle of operation of this machine, developing a 2D analytical model, as well as through the realization of six different Tesla turbine prototypes and testing them with air as working fluid. The maximum achieved experimental efficiency was of 25.8% [3]. Nevertheless, after W. Rice work, the Tesla turbine experienced another long period of indifference. Only in the last years, due to the increasing interest in micro power generation, the Tesla turbine research has flourished. Particularly, Guha and Sengupta intensively investigated the reasons for inefficiency of the turbine, through computational investigations, by the means of a 2D code and by experimental campaigns [4, 5]. Another fundamental contribution to the current state of the art of this technology was given by V.P. Carey and his research group, who took over the work developed by W. Rice and further polished the 2D code and extended the field of application of the Tesla turbine to Watt and sub-Watt scale [6, 7]. Only in the last few years the Tesla turbine has been deemed to be a possible alternative to micro expanders for organic Rankine cycles (ORCs). Particularly, Lampart et al. investigated the possibility of utilizing the Tesla turbine with Solkatherm (SES36) as working fluid [8], while Song et al. developed a 2D code, which allowed them to assess the thermodynamic efficiency, as well as the power produced by a Tesla turbine working with different organic fluids [9].

The literature review showed that there is an increasing interest in the investigation of Tesla turbine for micro power generation, especially in the field of ORCs; nonetheless, in literature there are no studies investigating this turbine with CO<sub>2</sub> as working fluid. Therefore, the main goals of this work are (i) to assess the possibility of designing a Tesla turbine for CO<sub>2</sub> and (ii) analysing its behaviour under off-design conditions.

## 2 Methodology

### 2D model

The assessment of Tesla turbine performance when working with CO<sub>2</sub> was carried out by the means of a home-made developed 2D code in Engineering Equation Solver (EES) environment [10]. The complete set of equations is reported in [11, 12]. The model was obtained through the reduction of Navier-Stokes equations in cylindrical coordinates, under the assumptions of viscous, compressible steady flow and neglecting body forces influence. The fluid was modelled as real, calculating its properties from local thermodynamic data. The model assumed the fluid flow regime as fully developed laminar, imposing a standard parabolic axial velocity distribution. This assumption is based on the knowledge that the most common operating regime of the Tesla turbine is laminar. Furthermore, in order to take into account the transitional conditions occurring in the entry region, a control coefficient was introduced [13]. Pumping and windage losses were not taken into account, assuming that the no leakages are present.

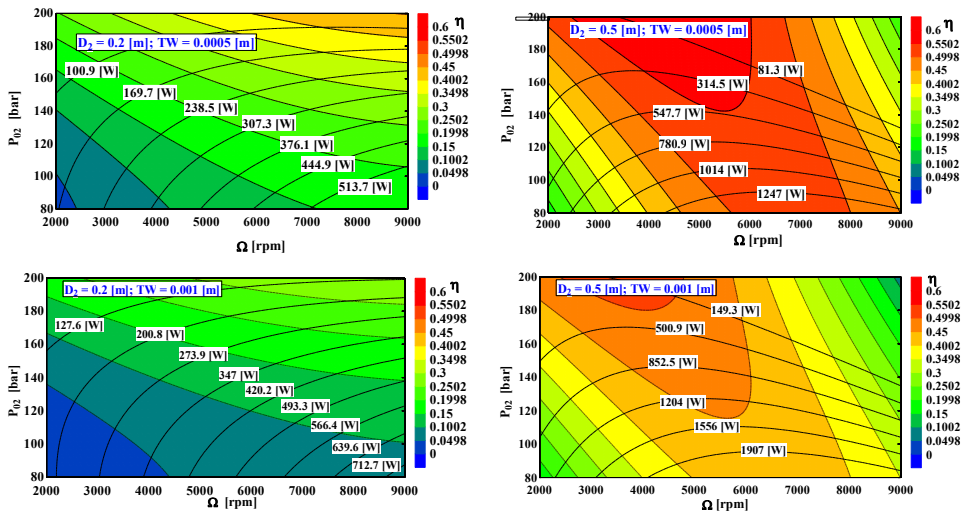
### Boundary conditions

The Design analysis started from a sensitivity analysis of the main geometric parameters of the Tesla turbine. The main investigated dimensions were the ratio between inlet and outlet stator radii, the external diameter of the rotor, the width of stator throat and the thickness of the rotor disks. The stator inlet/outlet ratio was assumed at 1.25, as suggested by Glassman [14]. Seven different rotor diameter sizes (0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5 m), two different throat widths (0.5 and 1 mm) and two different disk thicknesses (0.8 and 1 mm) were investigated. These values were selected taking into account both thermodynamic and mechanical constraints of the machine. Alongside the geometric optimization, the operational range of the turbine was defined. Particularly, the field of low temperature applications (100-150°C) was selected, assuming also maximum total inlet pressure of 200-250 bar. These pressure values were assumed with the objective of achieving good performance of the CO<sub>2</sub> supercritical cycle, as suggested in [15]. Once the main geometric and thermodynamic boundary conditions were selected, a parametric analysis was performed, including the variation of rotational speed, rotor channel width and mass flow rates.

## 3 Results

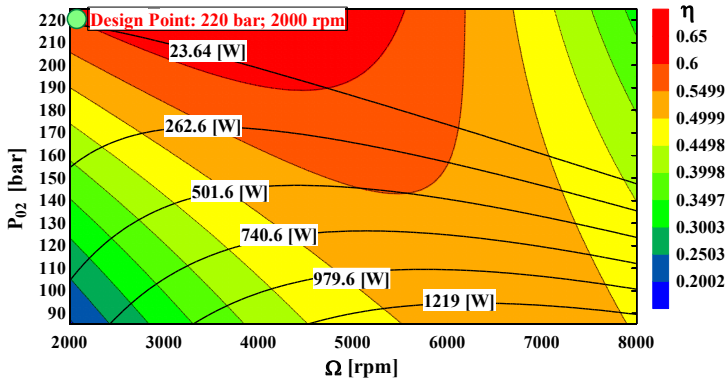
### Design optimization

Figure 2 displays the efficiency  $\eta = W/\Delta h_{0s}$  (coloured area) and the power output of the Tesla turbine (black iso-curves) at variable rotational speed and total outlet pressure, for a total inlet pressure of 220 bar and a total inlet temperature of 150 °C. The assumed rotor disk thickness is of 0.8 mm, which guarantees the highest performance of the machine for the same thermodynamic conditions compared to disks with higher thickness. This is due to the lower mass flow rate, which is caused by the reduction of inlet area (continuity equation). Considering the two borderline cases, therefore diameters of 0.2 and 0.5 m, it can be observed that, considering the same throat section, a higher performance was achieved for larger turbines. Furthermore, when considering the same external diameter, higher efficiencies were achieved with smaller throat sections; conversely, the power output was lower due to the lower mass flow rate (reduction of throat area keeping same thermodynamic conditions).



**Fig. 2.** Performance maps at various rotational speed and total outlet pressure, for an external rotor diameter of 0.2 (a, c) and 0.5 (b, d) for a 220 bar total inlet pressure and 150°C total inlet temperature

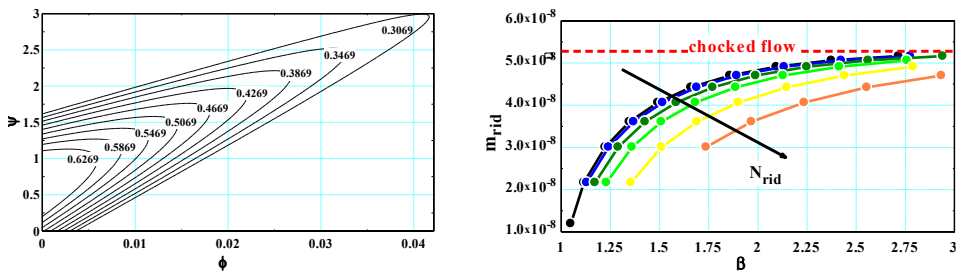
Following an extensive parametric analysis with all the above described parameters variable, the final design was reached and resumed in Figure 3. Particularly, the suggested configuration considers a very large rotor diameter (0.5 m), as higher efficiencies are obtained at higher rotor diameters. This allows the expander to work with reduced rotational speed (2000 rpm). Indeed, for a Tesla turbine, the rotor diameter and rotational speed are inversely proportional. In order to have high efficiencies at low rotational speeds, the rotor size should be larger; while for smaller diameters, the rotational speed should drastically increase. In order to have the highest efficiency, the total pressure drop of the turbine was found to be really small (10 bar). Under these conditions, the turbine achieves 62.9% efficiency and the total power produced per channel is of 23.4 W. The mass flow rate in each channel is 0.013 kg/s. The advantage of this turbine is that multiple channels may be added in the axial direction. Therefore, if 50 channels are considered, the turbine can produce up to 1.17 [kW] at 62.9% efficiency.



**Fig. 3.** CO<sub>2</sub> Tesla turbine Design point ( $D_2 = 0.5$  [m];  $P_{00} = 220$  bar,  $\Omega = 2000$  [rpm])

**Performance maps**

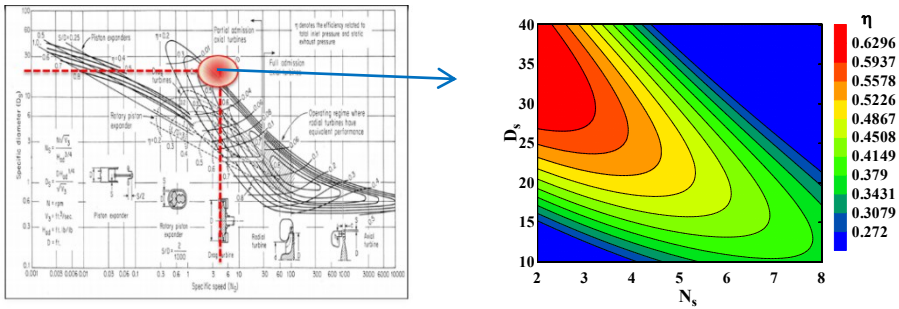
Once the design of the CO<sub>2</sub> Tesla turbine was defined, the related performance maps were built in order to determine its behaviour under design and off-design conditions. Figure 4a shows the efficiency of the turbine on a flow ( $\Phi = v_{r2}/U_2$ ) - load ( $\Psi = W/U_2^2$ ) coefficient chart. It is important to remark that the behaviour of efficiency is similar to that of standard bladed turbines. It should also be remarked that Tesla turbines work well at low mass flow rates (therefore, at low values of flow coefficient) and relatively low load coefficient. Therefore, the suggested operative range would be  $0.05 < \Phi < 0.08$  and  $0.5 < \Psi < 1$ . Figure 4b shows the expansion ratio ( $\beta = P_0/P_3$ ) -reduced mass flow rate ( $m_{rid} = P_0/P_3$ ) curve at different reduced speeds. Also in this case, the shape of the curves is similar to that of bladed turbines, with an expansion ratio between 1 and 3 and a reduced mass flow rate per channel between  $1 \cdot E^{-8}$  and  $5 \cdot E^{-8}$ .



**Fig. 4.** CO<sub>2</sub> Tesla turbine a) Smith diagram; b)  $\beta$ -reduced mass flow rate diagram

### Baljè diagram

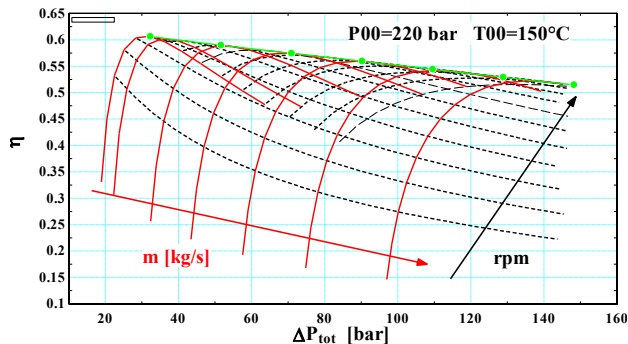
The Tesla turbine belongs to the family of viscous turbines, which are characterised by an opposite behaviour compared to the bladed ones. Conventional turbines usually hold high specific speeds and low specific diameters, while the Tesla turbine is characterised by relatively high specific diameters and moderate specific speeds. Indeed, the Tesla turbine has closer traits to drag turbines. Indeed, its location on the Baljè diagram (Fig. 5) is the one occupied by drag turbines and very close to volumetric expanders, e.g. low specific speed and relatively high specific diameter ( $10 < N_s < 40$  and  $2 < D_s < 8$  respectively [16]). According to the calculations performed with the developed model, the thermodynamic efficiency of the CO<sub>2</sub> Tesla turbine seems to be potentially above that of traditional drag turbines and in line (or slightly lower) with that of volumetric expanders (above 50% when designed for optimal super or trans critical CO<sub>2</sub> cycle efficiency).



**Fig. 5.** CO<sub>2</sub> Tesla turbine Baljè diagram

### Off-design maps

Finally, Fig. 6 presents the off-design maps of the CO<sub>2</sub> Tesla turbine. The red lines were obtained fixing the mass flow rate of the turbine while varying the rotational speed, whereas the black dotted lines were assessed varying the mass flow rate at fixed rotational speed. Coupling the two series of analyses allows drawing and interpolation curve of the maximum efficiency off-design loci (the green curve), where high efficiency values are achieved for a wide range of off – design conditions by suitably tuning the three parameters. In this way, it is possible to keep the Tesla turbine off-design curve modestly sensitive to off-design conditions by adjusting the rotational speed accordingly with the increase of mass flow rate. Low pressure drop implies low mass flow rates, which permits running the turbine at relatively moderate rotational speeds (2000 rpm). While, at higher pressure drop, higher mass flow rates are obtained and therefore also the rotational speed needs to be increased (at about 7000 rpm for the investigated turbine with  $D_2 = 0.5$  m).



**Fig. 6.** Off-design performance map

## 4 Conclusions

The design of a Tesla turbine for supercritical and trans-critical CO<sub>2</sub> cycles was carried out in this study. Several parametric analyses, varying both geometric and thermodynamic conditions were performed, defining the most effective geometric design of the turbine. An expander with an external 0.5 m diameter rotor was selected to work with 220 bar total inlet pressure and 150 °C total inlet temperature. Under these conditions, the power produced per channel is 23.4 W, which, with a 50 channel configuration, allows an overall power output of 1.17 kW with a mass flow rate of 0.65 kg/s and 62.9% efficiency. The turbine performance maps were also devised. When reported on the Baljè diagram, in addition to demonstrating a positioning appropriate to its physical functioning, the turbine results as a possible competitor of drag and volumetric expanders. Finally, an off-design chart was built: it showed that the CO<sub>2</sub> Tesla turbine may achieve a relatively flat maximum efficiency curve by adjusting the rotational speed when the pressure drop is changed from the design value.

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