Some Observations On Role Of Uncertainty In Forensic Geotechnical Engineering

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ABSTRACT

Forensic geotechnical engineering addresses the posterior assessment of geotechnical design with the goal of ascertaining responsibility. An important factor in such an assessment lies in the appropriate recognition of the existence and magnitude of uncertainty in the design phase. In the case of deterministic design by means of the factor of safety, uncertainties are included in design implicitly only. Thus, in the posterior assessment of deterministic design, statistical and probability theory can only be applied by formulating hypotheses regarding expected performance and presumed level of uncertainty. This paper illustrates a statistical method for the back-assessment of responsibility of design in geotechnical failures, and provides useful guidelines for the indirect quantitative estimation of geotechnical uncertainty where the latter is not available.

INTRODUCTION

One key aspect that distinguishes geotechnical engineering from structural engineering is the large variability (natural/intrinsic variability, testing errors, and transformation uncertainties introduced when measured parameters are converted to engineering parameters) related to naturally occurring geomaterials. Engineering decisions are less straightforward in the presence of such variabilities. Forensic geotechnical engineering is also inevitably complicated by these variabilities.

Phoon and Kulhawy (1999a) identified three primary sources of geotechnical uncertainties, namely: (a) inherent variability, (b) measurement error, and (c) transformation uncertainty. Inherent variability results primarily from the “real” heterogeneity which is inherent to geomaterials. Such heterogeneity stems from natural geologic and physical/chemical/biological processes that produced and continually modify the soil/rock mass in-situ. Uzielli et al. (2006) provided a state-of-the-art report on the estimation of inherent variability of soft soils, along with a comprehensive database of statistics related to such source of uncertainty for a large number of geotechnical parameters. Kulhawy et al. (2000) provided similar statistics for rock. Measurement error arises from equipment, procedural/ operator, and random testing effects. Phoon and Kulhawy (1999a) and Uzielli (2008) provided quantitative statistics for measurement error for laboratory and in-situ tests. The third component of uncertainty is introduced when field or laboratory measurements are transformed into design parameters using empirical or other correlation models (e.g., correlating the standard penetration test N value with the undrained shear strength). Robust model statistics can only be evaluated using: (1) realistically large scale prototype tests, (2) a sufficiently large and representative database, and (3) reasonably high quality testing where extraneous uncertainties are well controlled. A systematic statistical compilation of geotechnical transformation uncertainty is currently lacking. Phoon & Kulhawy (2005a) characterized transformation uncertainty for laterally loaded rigid drilled shafts. Phoon & Kulhawy (2005b) investigated the transformation uncertainty for drilled shafts under undrained axial loading. Phoon et al. (2006) modeled load-displacement uncertainty for augered cast-in-place piles under axial compression. Uzielli & Mayne (2010) illustrated the statistical characterization and the probabilistic modeling of load-displacement uncertainty for vertically loaded shallow footings on sand from a large load testing database. The key observation in these studies is that geotechnical predictive models are usually conservative (correctly so), but they are *not always conservative* because of model uncertainties.

For each combination of soil type, measurement technique, and correlation model, the uncertainty in the design soil property is evaluated systematically by combining the appropriate component uncertainties such as in the simple second-moment probabilistic approach proposed by Phoon & Kuhawy (1999b).

The large magnitude of geotechnical uncertainty is not surprising given that the volume of geo-materials investigated by direct or indirect means is extremely small in comparison to the volume of interest. Chiles and Delfiner (1999) cited volume fractions investigated at Brent field, North sea, to be 10^{-9} for cores and cuttings and 10^{-6} for logging.

In the occurrence of failure, one naïve interpretation is that “failure” is always possible and geotechnical variability is responsible, rather than human errors. Sowers (1993) noted that the majority of foundation failures were due to human shortcomings. This naïve interpretation essentially misses the key principle of forensic geotechnical engineering, which is to assess if - and to which degree – the existence and magnitude of uncertainty were adequately accounted for in design.

Phoon et al. (2009) opined that Leonards’ (1982) definition of “failure” - *unacceptable difference between expected and observed performance* – cannot be evaluated in a meaningful way using factor of safety approach based on deterministic methods. To elaborate, “expected performance” must vary given the backdrop of potentially significant geotechnical variability. It is realistic and perhaps more credible to quantify “unacceptable difference” in a statistical sense. Hence, an objective statistical measure of “unacceptable difference” (specifically, a difference not explainable by underlying variability) should provide useful additional information in the formulation of such an opinion.

Geotechnical engineering analysis and design have traditionally been performed in a deterministic perspective, in which performance and safety are most frequently indexed by the factor of safety (FS). The factor of safety (FS) is computed as a single deterministic number based on some cautious estimates of the design parameters and a conservative model of the physical response. Thus, albeit formally deterministic, FS implicitly accounts for uncertainty.

The utilization of the factor of safety is a rather reasonable approach at the design stage, but it is questionable at the forensic stage when an actual failure has occurred. In principle, if the factor of safety is much larger than 1, it is not possible to reconcile with an observed failure. In forensic engineering, one is thus led to the conclusion that some human errors are involved. This is the other extreme in the spectrum of reasons postulated for observed failures. The first extreme as mentioned above is that Nature is at fault (natural variability). The second extreme is that humans are at fault. While human errors are indeed responsible for most failures as noted by CIRIA (1977), Sowers (1993) and many others, it is possible that the factor of safety is not sufficient because parametric variabilities are too large and/or the predictive model is less conservative (this is related to model uncertainties as discussed above).

Phoon et al (2009) presented a statistical framework for acceptance criteria linking factor of safety, reliability index and coefficient of variation of factor of safety. In their approach, "expected performance" is described by a target reliability index, and conventional hypothesis testing was employed to ascertain if the target reliability level had been achieved in the original design based on the mean of a sample of size n from a population of "observed" factors of safety, FS^*. This paper illustrates a more general criterion for the assessment of responsibility in geotechnical failures, which is applicable to any target reliability level and statistical significance level. The paper also provides guidelines for the indirect estimation of the uncertainty associated with the factor of safety, which serves as a basic input to the aforementioned criterion.

FORENSIC ASSESSMENT OF RESPONSIBILITY IN GEOTECHNICAL DESIGN

In the geotechnical reliability literature, the factor of safety (FS) is typically modeled as a lognormal random variable. Say FS follows a lognormal random variable with mean μ, standard deviation σ and coefficient of variation $\sigma/\mu=\theta$. Then ln(FS) is normally distributed with mean (λ) and variance (ξ^2) given by, respectively:

$$\lambda = \ln(\mu) - 0.5\xi^2 \tag{1}$$

$$\xi^2 = \ln(1+\theta^2) \tag{2}$$

It follows that the standard deviation ξ of ln(FS) is approximately equal to θ=COV(FS) up to θ of about 0.5. If FS follows a log-normal population, the reliability index is given by:

$$\beta = \frac{E[\ln(FS)]}{\sigma[\ln(FS)]} = \frac{\lambda}{\xi} \tag{3}$$

Based on the definition of the reliability index and setting a target reliability index β_T, the following null and alternate hypotheses on the population mean [actually, the mean of ln(FS)] can be formulated as (Phoon et al. 2009):

H0: $\lambda = \beta_T \cdot \xi$
H1: $\lambda < \beta_T \cdot \xi$

Assuming that ξ is known and a sample size of n is available, the null hypothesis is rejected at a level of significance α if:

$$\frac{\ln\left(FS^*\right) - \lambda}{\left(\xi / \sqrt{n}\right)} = \frac{\ln\left(FS^*\right) - \beta_T \cdot \xi}{\left(\xi / \sqrt{n}\right)} < \Phi^{-1}(\alpha) \tag{4}$$

or

$$FS^* < \exp\left\{\left[\beta_T + \Phi^{-1}(\alpha)/\sqrt{n}\right]\sqrt{\ln\left(1+\theta^2\right)}\right\} \tag{5}$$

where Φ^{-1} is the inverse standard normal cumulative distribution and FS* is the average factor of safety from a sample of size n. The rejection criterion in Eq. (5) provides a simple numerical yardstick to evaluate "unacceptable difference between expected and observed performance" in the presence of potentially significant geotechnical variability. A rejection means that the observed average factor of safety (FS*) does not support the claim that the target reliability level β_T has been achieved.

It is possible for a rejection to arise because the underlying "true" geotechnical variability was grossly over-estimated. A "do not reject" scenario means that the observed average factor of safety is not unreasonably "low" and failure may be caused by geologic "surprises", limitations in the existing factor of safety, critical failure mechanism not identified, gross human errors, etc. Again, it is possible for "do not reject" to arise because we have grossly under-estimated the underlying geotechnical variability. The rejection criterion depends significantly on expected performance indexed here by β_T. Target reliability levels are not well established in geotechnical engineering. Numerous reliability calibration studies (e.g., Phoon et al. 1995) have shown that existing foundations are typically designed to achieve a target reliability index of about 3. For β_T=3 and θ=0.3, for instance, the rejection criterion for β_T takes the form:

$$FS^* < \exp\left(\beta_T\xi - 1.645\,\xi/\sqrt{n}\right) = \exp\left[\sqrt{\ln(1+\theta^2)}\ \left(\beta_T - 1.645/\sqrt{n}\right)\right] \tag{6}$$

The rejection curves for β=3.0 corresponding to various sample sizes (n) of FS are shown in Fig. 1.

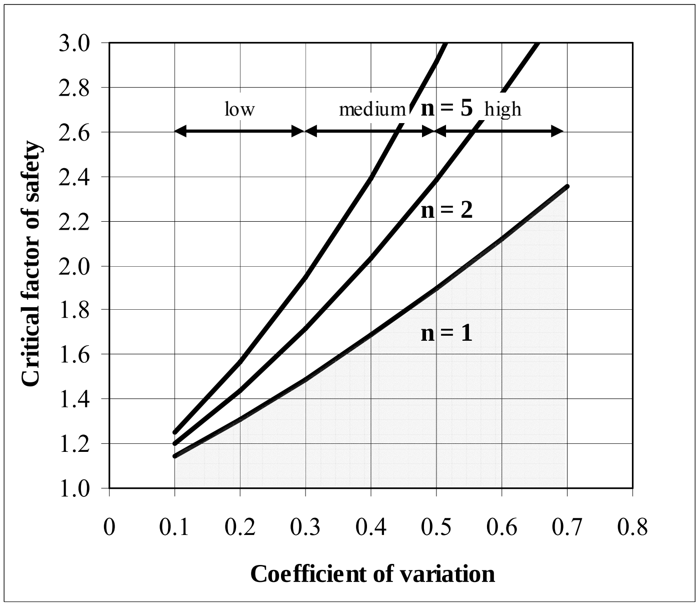

Fig. 1 Rejection criteria for different sample sizes of observed factors of safety based on a target reliability level β_T=3.

Fig. 2 presents the critical values of factor of safety below which the difference between expected and observed performance is "unacceptable" for a single observed FS* (n=1). For example, if the expected performance corresponds to $\beta_T=3$ and the underlying variability corresponds to $\theta=0.3$, an observed factor of safety of 1.4 is too "low" and not explainable as a random outcome from a population of FS with $\beta_T=3.0$ and $\theta=0.3$.

INDIRECT ESTIMATION OF UNCERTAINTY

The explicit presence of θ=COV(FS) in Eq. (5) attests for the importance of investigating the variability in the factor of safety. Such variability stems from the uncertainties in the estimation of demand and capacity of a geotechnical design.

In principle, geotechnical uncertainty related to a specific design should be estimated at the basic component level (e.g. Phoon & Kulhawy 1999b). The uncertainty associated with FS is then evaluated through one or more available methods, including Monte Carlo simulation, First-Order Second-Moment, Point Estimation and others (see e.g. Baecher & Christian 2003). However, since deterministic design does not entail explicit characterization of uncertainties, it is likely that an a posteriori assessment of design must rely on indirect estimation of uncertainty itself.

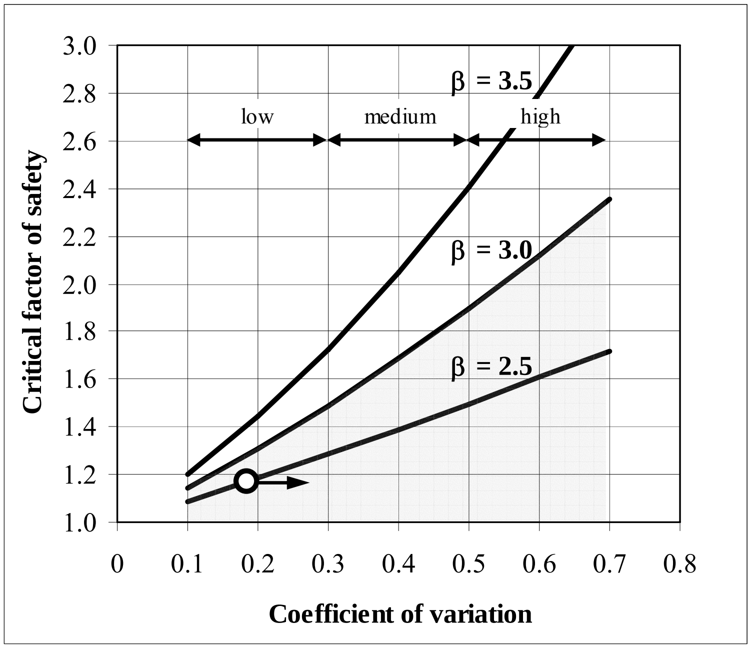

Fig. 2 Rejection criteria for different target reliability levels for one observed factor of safety

It is possible to infer the underlying coefficients of variation of FS from the empirical evidence presented in Silva et al. (2008), who proposed several relationships between the annual probability of failure and the factor of safety based on 75 projects (zoned and homogeneous earth dams, tailings dams, natural and cut slopes, and some earth retaining structures) and expert judgment. The same authors defined 4 categories of earth structures (see Fig. 3):

Category I—facilities designed, built, and operated with state-of-the-practice engineering. Generally these facilities have high failure consequences;

Category II—facilities designed, built, and operated using standard engineering practice. Many ordinary facilities fall into this category;

Category III—facilities without site-specific design and substandard construction or operation. Temporary facilities and those with low failure consequences often fall into this category;

Category IV—facilities with little or no engineering.

Silva et al. (2008) compared the empirical data in Fig. 3 with some theoretical curves, but did not provide any mathematical details. Their theoretical curves can be easily reproduced using the following procedure:

1) Assume that FS is lognormally distributed with parameters λ and ξ.
2) The horizontal axis of Fig. 5 is the mean factor of safety, μ.
3) If θ=COV(FS) is sufficiently small, $\lambda \approx \ln(\mu)$ and $\xi \approx \theta$.
4) The vertical axis of Fig. 5 is the probability of failure, given by:

$$p_f = \Pr\left(FS < 1\right) = \Pr\left[\ln(FS) < 0\right] = \Phi\left(\frac{0-\lambda}{\xi}\right) = \Phi\left[\frac{-\ln\left(FS\right)}{\theta}\right] \tag{6}$$

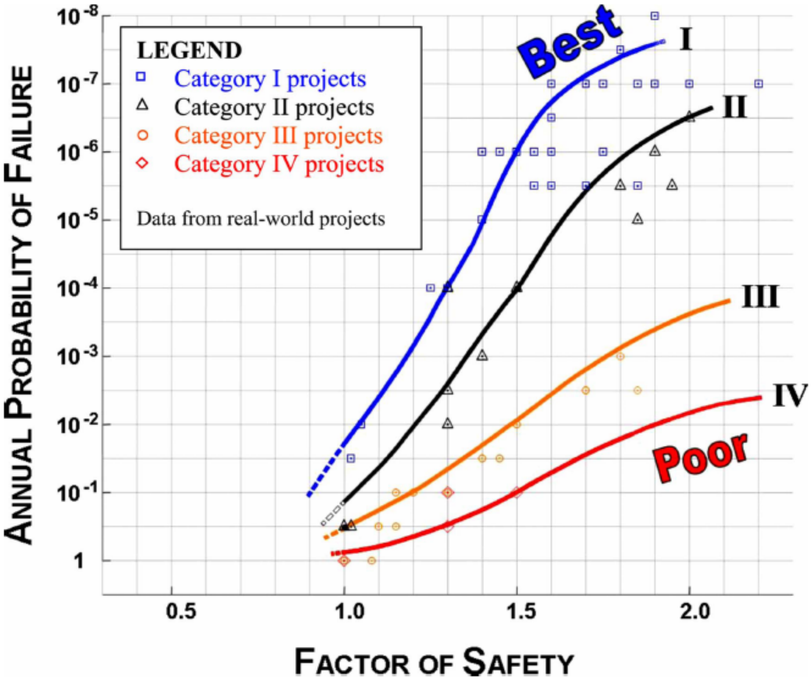

Fig. 3 Annual probability of failure versus factor of safety for earth structures (Silva et al. 2008)

The theoretical lognormal curves from Silva et al. (2008) are reproduced in Fig. 4 using the above procedure with θ = 0.072 (Category I), 0.109 (Category II), 0.174 (Category III), 0.316 (Category IV). Lognormal probability curves for a complete and more systematic range of COVs are shown in Fig. 5. It is important to note that the horizontal axis in Fig. 3 "Factor of Safety" refers to the mean factor of safety, which is not the same as the critical factor of safety shown along the y-axis in Figs. 1 and 2. The mean factor of safety is denoted by μ in Eq. (1).

EXAMPLE APPLICATION

An example of the use of Fig. 2 can be illustrated with reference to the results presented in Duncan (2000) wherein a case study of underwater slope failure was reported. The failure took place entirely within San Francisco Bay mud, a normally consolidated, slightly organic clayey silt or silty clay of marine origin. Previous experience in the area indicated that 1(H):1(V) slopes with a factor of safety of 1.25 were satisfactory. To reduce the quantity of excavation, slopes of 0.875(H): 1(V) having a factor of safety of 1.17 were excavated which failed subsequently. A risk-based back-analysis indicated a probability of failure of 18% which is unacceptable though the factor of safety is 1.17. Duncan (2000) mentioned that the coefficient of variation of shear strength parameters was high and hence the probability of failure was high. This case study can be plotted in Fig. 2 as an open circle. It is clear that an observed FS^*=1.17 cannot support the claim that β_T=2.5 for any COV larger than about 0.2. Following the same argument, we can conclude that β_T=3 is not supported for any COV larger than about 0.1. Because COV of 0.1 is the lower bound for most geotechnical problems, it is reasonable to say that β_T=3 was not achieved in the original design with a fair degree of confidence.

Alternately, one can attempt to interpret this example using the results presented by Silva et al. (2008) and Fig. 2. First, it is reasonable to classify the slope as a Category II to III project. In other words, the COV of FS should lie between 0.109 and 0.174. Next, based on Fig. 2 and the observation that a factor of safety of 1.25 is satisfactory, it may be deduced that precedents are constructed with $\beta_T > 3$. To achieve $\beta_T > 3$ for a factor of safety of 1.17, it is clear from Fig. 2 that the COV of FS must be *at most about 0.1*, which is quite unlikely

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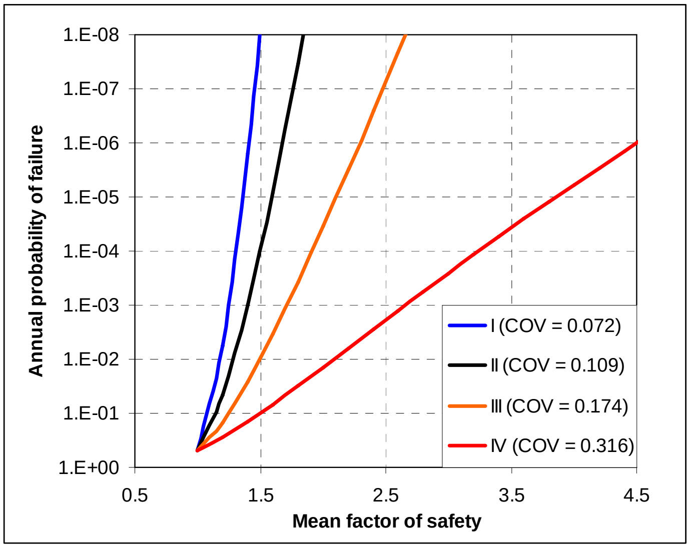

Fig. 4 Lognormal probability curves back-calculated from Silva et al. (2008)

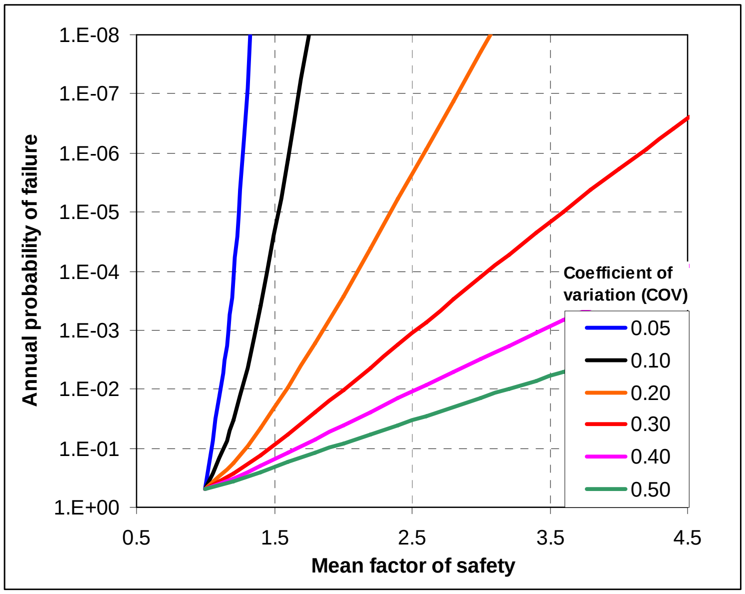

Fig. 5 Lognormal probability curves for higher COVs of FS

CONCLUSIONS

This paper illustrates a statistical method for the objective assessment of responsibility in geotechnical design from a forensic perspective. In particular, for a given expected level of reliability based on past experience and a first-order estimate of the likely COV of the factor of safety based on forensic evidence, it is possible to establish if the factor of safety is "sufficient". If the factor of safety is deemed sufficient, then the observed failure is not likely to be explainable by underlying geotechnical variabilities and attention could be productively focused on other causes such as geologic surprises, gross human errors, etc. If the factor of safety is deemed insufficient, then failure may simply be due to unexpectedly large geotechnical variabilities or insufficient appreciation of their magnitudes.

The framework is by no means perfect and comprehensive, given the diversity and complexities of actual failures. In this paper, the parametric and transformation uncertainties described previously are lumped into a single parameter. More refined estimates of the variability of the factor of safety could be obtained through the direct estimation of geotechnical uncertainty related to a specific design. From a statistical viewpoint, more sophisticated rejection criteria can be developed based on a sample estimate of ξ (rather than the population version used in the above equations). It suffices to note here that they do not follow the t-distribution in standard statistical texts.

A comprehensive evaluation based on more case studies is needed to validate the usefulness of the proposed method in forensic geotechnical engineering, and to fine-tune it if necessary. Despite the aforementioned limitations and the considerable room for refinement, the proposed framework allows practitioners to perform an initial objective evaluation of the observed factor of safety, particularly to eliminate the more obvious claim that it is an "unfortunate" realization caused by geotechnical variability.

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