An Enhanced Algorithm for 2D Indoor Localization on Single Anchor RSSI-based Positioning Systems

Marco Passafiume, Stefano Maddio, Giovanni Collodi and Alessandro Cidronali
Department of Information Engineering
University of Florence, Italy
Email: marco.passafiume@unifi.it, stefano.maddio@unifi.it, giovanni.collodi@unifi.it, alessandro.cidronali@unifi.it

***Abstract*—In this paper a cost-effective RSSI-based Range and Direction of Arrival (DoA) estimator using a single anchor node is proposed. Exploiting the Space Division Multiple Access enabled by a anchor node equipped with a switched beam front-end, the DoA of a nomadic node is estimated on the basis of maximum likelyhood approach driven by the expected power distribution. The distance is subsequently estimated on the basis of a robust inversion of the power-range relationship. Compared to conventional estimators, the proposed approach offers a precise DoA estimation as well as a adequate range estimation despite the simple implementation. The combination of the two information enables also the estimation of the absolute position with a very low-cost solution, suitable for Wireless Sensor Networks and for 802.11n/ac IEEE compliant networks.**

Experimental validations in an anechoic chamber confirm that the proposed method can provide a complete 2D localization with a mean error of 95cm and a sub-metric localization area coverage of about 60% within a typical home room area size ($28m^2$), without the needing of a multi-anchor system setup.

I. INTRODUCTION

The topic of indoor positioning has gained a lot of interest in the last years. The applications of the position awareness is going to dominate the scene of pervasive computing and wireless connectivity in GPS-denied scenarios. In this context, the improved capability of typical WiFi mobile devices are the driving force of the topic of indoor localization.

Among the various positioning strategies, the one based on the RSSI(Receive Signal Strengh Indicator), a packet-related received power estimation embedded in any 802.11/802.15.4 communication flow, is the most promising despite its limits [1] thanks to its ease of deployment. Some localization solutions based on 802.11/802.15.4 standard and operating on RSSI-data are already operating, but they need a constellation of nodes to achieve a localization estimation [1], [2].

More conservative approaches for a home environment have been investigated, based only on a single smart node capable to monitor an entire area. Recently, the capability of 802.11n PHY link operating on both 2.45GHz and 5.2GHz bands has been exploited to improve DoA localization accuracy even in presence of strong multipath impairment [3].

In this paper this approach is extended to estimate the range of a nomadic node with the same hardware used to estimate the DoA. This enables an absolute positioning information based on distance and angle. Despite the coarse accuracy of the range, the proposed solution is an improvement over the DoA estimation only.

II. DESCRIPTION OF THE SYSTEM

A RSSI-based DoA estimator is a system capable of radio signal Direction of Arrival identification exploiting only RSSI estimation values, using a Space Division Multiple Access enabled by a Switched Beam Antenna (SBA) [4], [5].

The vector of received RSSI by a set of available antenna beams is the key for the DoA estimation. The angle estimation is obtained on the basis of maximum likelihood criteria driven by the comparison of the actual RSSI with expected signal distribution. Since all the beams are affected by the same propagation impairment, the noise due to radio channel effects is minimized due to the evaluation of the relations between the variables obtained in the same range [1], [6].

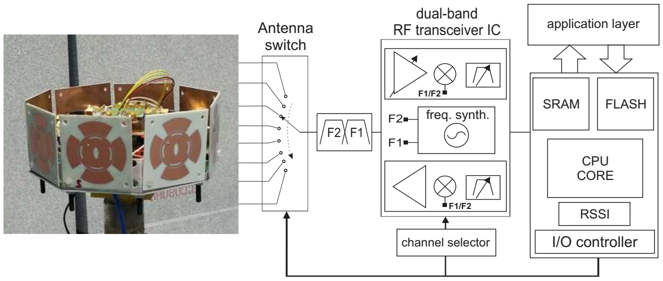

Fig. 1. Pictorial of SBA multi-band anchor system

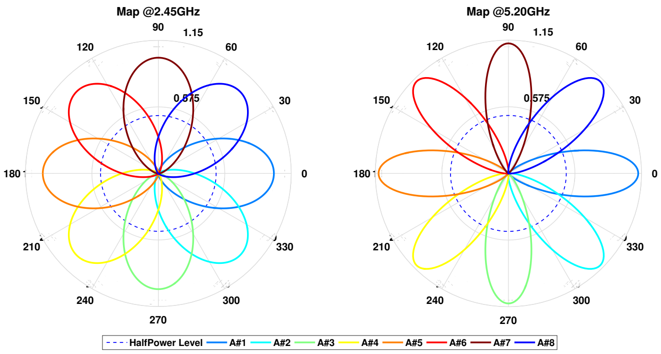

Fig. 2. SBA 1-D azimuthal ideal antenna gains (linear)

The proposed SBA multiband system (fig.1) operates with a set of 8 dual-frequency Circularly Polarized directive antennas (fig.1-2) [7], designed for this purpose. The antennas provide good-enough directivity for azimuthal DoA estimation at both frequencies, following the design criteria as shown in [1]. Having a set of antennas defined as in fig.2 and collecting RSSI for both 2.45GHz and 5.2GHz packet streams, DoA estimation is done using a doubled RSSI set improving estimation accuracy [6], [8].

DoA estimation is achieved applying a Maximum-Likelihood algorithm between the vector of observed RSSI values S and each vector defined within a “reference map” $\mathcal{M}$ [3]. The steering vector S is defined as

$$S = \left\langle \overbrace{\underbrace{\mathrm{RSSI}_{1@f_A}, \ldots, \mathrm{RSSI}_{8@f_A}}_{\text{RSSI values}@f_A = 2.45\text{GHz}}, \underbrace{\mathrm{RSSI}_{1@f_B}, \ldots, \mathrm{RSSI}_{8@f_B}}_{\text{RSSI values}@f_B = 5.20\text{GHz}}}^{2\times N \text{ elements}} \right\rangle^{H} \tag{1}$$

making use of multi-frequency RSSI observations as becoming from a doubled antenna set, thus dealing with an equivalent array having $2N = 8 \times 2 = 16$ different elements.

Following eq.1 definition the “base reference map” $\mathcal{M}_0$ is defined overall the azimuthal DoA domain φ as

$$\begin{aligned} \mathcal{M}_0(\varphi) &= S_{expected}(\varphi) - P_{inc} = \\ &= \big\langle G_{1@f_A}(\varphi), \ldots, G_{N@f_A}(\varphi), \\ &\qquad G_{1@f_B}(\varphi), \ldots, G_{N@f_B}(\varphi) \big\rangle^{H} \end{aligned} \tag{2}$$

Defining the standard LSE cost function as in [3]

$$\mathrm{C_{LSE}}(\varphi) = ||\,\boldsymbol{R}(\varphi)\,|| = \sqrt{\sum_{n=1}^{2N} (S_n - \mathcal{M}_n(\varphi))^2} \tag{3}$$

the final DoA estimation is achieved simply resolving

$$\hat{\varphi} = \underset{\varphi}{\operatorname{argmin}}\, \mathrm{C_{LSE}}(\varphi) \tag{4}$$

exploiting multi-frequency diversity as an equivalent antenna array elements increase, improving estimation accuracy as established by [6], [8].

Note that fig.2 shows pure angular pattern definition without dealing with different frequency propagation properties. Considering standard Friis propagation model [3] effective incident power to overall SBA equals to

$$P_{inc}(D, \varphi_{tx}) = \underbrace{P_T + G_T(\varphi_{tx})}_{\text{transmitted power } P_{tx}} + 10\eta \log\left(\frac{\mathrm{c}_0}{4\pi f D}\right) \tag{5}$$

considering φ_{tx} as the SBA direction respect transmitter, while η parameter models scenario specific propagation properties. The i-th antenna RSSI evaluation at frequency f_A equals to

$$\mathrm{raw\,RSSI}_{i@f_A} = P_{inc@f_A}(D, \varphi_{tx}) + G_{i@f_A}(\varphi) \tag{6}$$

Note that DoA estimation relies only on reciprocal RSSI differences observed by the SBA transceiver (within the RSSI vector), thus a constant term variation spread overall steering vector values will not affect DoA estimation [2], [6].

By this, dealing with single frequency and single anchor DoA estimations effective P_{inc} value does not affect DoA estimation [4], [6]. Dealing with single anchor but multi-frequency RSSI steering vectors, RSSI for different frequencies will differ both for the antenna gain as well as for the incident power term. While $G_{i@f_A}(\varphi)$ gain term is included within reference map as defined in eq.2, the incident power term introduces a not constant term offset due to straight frequency dependency. Such term can be rewritten as

$$P_{inc}(D, f) = P_{tx}(f) + \underbrace{10\eta \log\left(\frac{\mathrm{c}_0}{4\pi f}\right)}_{\text{freq. related term}} - 10\eta \log D \tag{7}$$

separating frequency related terms from scenario specific ones.

To correctly equalize terms within each reference map vector the frequency related attenuation term must be added. Because the reference map is intended to be defined once during localization system setup, [3] shown that a typical value of $\eta = 2$ is good enough to obtain a good DoA estimation accuracy.

Equalized reference map vectors equal to

$$\mathcal{M}(\varphi) = \begin{pmatrix} G_{1@f_A}(\varphi) + 20\log\left(\frac{\mathrm{c}_0}{4\pi f_B}\right) \\ \vdots \\ G_{N@f_A}(\varphi) + 20\log\left(\frac{\mathrm{c}_0}{4\pi f_A}\right) \\ G_{1@f_B}(\varphi) + 20\log\left(\frac{\mathrm{c}_0}{4\pi f_B}\right) \\ \vdots \\ G_{N@f_B}(\varphi) + 20\log\left(\frac{\mathrm{c}_0}{4\pi f_B}\right) \end{pmatrix} \tag{8}$$

Equalized map vectors as in eq.8 do not include a correction factor for the $P_{tx}(f)$ term. Note that a full defined 802.11n/ac radio-link defines a known transmission power level for each different sub-channel transmission, so such parameter can be considered known for each packet transfer so each different i-th antenna obtained RSSI value can be corrected simply placing

$$\mathrm{RSSI}_i = \underbrace{\mathrm{raw\,RSSI}_i}_{\text{obtained value}} - P_{tx}(\text{mode\_id}) \tag{9}$$

where the $P_{tx}(\text{mode\_id})$ function is totally defined by 802.11n/ac standard while the “mode_id” identifier is given within each collected data packet [9].

III. RANGE ESTIMATION ALGORITHM

Previous paragraph summarized multi-frequency DoA estimation algorithm: DoA estimation accuracy improvement is deeply investigated in [3].

In [2], [6] is clearly described how actual proposed system is thought to work within a network of anchors: such solution achieves a sub-metrical localization within typical office environments, but it can appear to be oversized for smaller scenarios. Dealing with a single anchor DoA estimation, each (x, y) position on the anchor plane can be defined as

$$\hat{\overline{\mathbf{x}}} = \hat{D}\begin{pmatrix} \cos\hat{\varphi} \\ \sin\hat{\varphi} \end{pmatrix} + \overline{\mathbf{x}}_0 \qquad \begin{aligned} \text{with } \overline{\mathbf{x}}_0 &= \text{anchor position} \\ \hat{\varphi} &= \text{estimated DoA} \\ \hat{D} &= \text{estimated range} \end{aligned} \tag{10}$$

thus while $\overline{\mathrm{x}}_0$ and $\hat{\varphi}$ can be considered known, the distance $\hat{D}$ between the mobile node and the SBA is still unknown.

A typical method to estimate D is applying Friis formula inversion. A classical formulation for range estimation given an RSSI observation equals to

$$\begin{aligned} 20\log\hat{D} &= \left[\text{raw}\,\text{RSSI}_i - P_{tx}\left(\text{mode\_id}\right)\right] - \\ &- \left[G_{rx}\left(\varphi\right) + 20\log\left(\frac{\mathrm{c}_0}{4\pi f}\right)\right] = \boldsymbol{R}_i \end{aligned} \tag{11}$$

defining $\boldsymbol{R}_i$ as the steering vector term "residual" (cfr. eq.3), or rather the effective term which should be represent the free space path length dependency. Note that dealing with RSSI effective components (eq.6-9) given a steering vector the LSE estimator is expected to be minimal only when a perfect match between reference map vector gains and steering vector gain terms happens [6].

Dealing with a non-ideal RSSI observation or with a not perfect DoA estimation an added measure noise term $\Delta\,\boldsymbol{R}_i \geq \epsilon_{\text{RSSI}}$ is introduced, so final range estimation becomes

$$\hat{D} = 10^{\frac{\boldsymbol{R}_i + \Delta\,\boldsymbol{R}_i}{10\eta}} = 10^{\frac{\boldsymbol{R}_i}{10\eta}} \cdot 10^{\frac{\Delta\,\boldsymbol{R}_i}{20}} = D \cdot 10^{\frac{\Delta\,\boldsymbol{R}_i}{10\eta}} \tag{12}$$

thus estimated range uncertainty will be equal to

$$\Delta\hat{D} = \left(10^{\frac{\Delta\,\boldsymbol{R}_i}{10\eta}} - 10^{-\frac{\Delta\,\boldsymbol{R}_i}{10\eta}}\right) D \tag{13}$$

Eq.13 clearly shows how the RSSI estimation error leads to an uncertainty straight proportional to the effective real range. The single residual term $\boldsymbol{R}_i$ appears to include a too unpredictable noise term (due to both straight RSSI estimation and DoA estimation algorithm approximations): since each $\Delta\,\boldsymbol{R}_i$ noise term is uncorrelated to each other, it can be placed

$$\begin{gathered} \mathrm{E}\left\{\boldsymbol{R}_i\right\} = \mathrm{E}_i\left\{\boldsymbol{R} + \Delta\,\boldsymbol{R}_i\right\} = 20\log D + \mathrm{E}\left\{\Delta\,\boldsymbol{R}_i\right\} \\ \text{with } \left|\mathrm{E}\left\{\Delta\,\boldsymbol{R}_i\right\}\right| < \left|\Delta\,\boldsymbol{R}_i\right|, \forall i = 1 \ldots 2N \end{gathered} \tag{14}$$

so a better range estimation is given by

$$\hat{D} = 10^{\frac{\mathrm{E}\left\{\boldsymbol{R}_i\right\}}{10\eta}} = D \cdot 10^{\frac{\mathrm{E}\left\{\Delta\,\boldsymbol{R}_i\right\}}{10\eta}} \tag{15}$$

Applying eq.15 the multi-frequency range estimation could be given as the mean of range estimations done using different frequencies steering vectors residuals. Applying the $\eta = 2$ approximation in eq.15 leads to a negligible error on DoA estimation [3], but on range estimation any real η parameter fluctuation can heavily affect final results (fig.4b-c) [10].

Reference map for DoA estimation as calculated in eq.8 implicitly impose $\eta = 2$. Because we are trying to estimate η, trying to extract it from residuals obtained through eq.11 will lead to serious methodological errors. Corrected RSSI values (as in eq.9) from each i-th antenna for each frequency are equal to

$$\begin{cases} RSSI_{i@f_A} = G_{i@f_A}\left(\varphi\right) + 10\eta\log\left(\frac{\mathrm{c}_0}{4\pi f_A}\right) - 10\eta\log D \\ RSSI_{i@f_B} = G_{i@f_B}\left(\varphi\right) + 10\eta\log\left(\frac{\mathrm{c}_0}{4\pi f_B}\right) - 10\eta\log D \end{cases} \tag{16}$$

Range dependency can be removed evaluating

$$\begin{aligned} &RSSI_{i@f_B} - RSSI_{i@f_A} = \\ &= \underbrace{\left[G_{i@f_B}\left(\varphi\right) - G_{i@f_A}\left(\varphi\right)\right]}_{\text{(A)}} + \underbrace{10\eta\log\left(\frac{f_A}{f_B}\right)}_{\text{(B)}} \end{aligned} \tag{17}$$

Eq.17 value is known because it is calculated from obtained corrected RSSIs (removing P_{tx} known term as shown in eq.9).

Note that the (A) term is known, and for each antenna it simply equals the straight difference between i-th antenna gains for different frequencies. The argument $\varphi \approx \hat{\varphi}$ can be considered known because range estimation is done after DoA estimation, so it can be written

$$\text{(A)} = \mathcal{M}_{0(N+i)}\left(\hat{\varphi}\right) - \mathcal{M}_{0i}\left(\hat{\varphi}\right) \tag{18}$$

By this, each i-th antenna can give an η estimation

$$\underbrace{\hat{\eta}_i = \frac{\left(RSSI_{i@f_B} - RSSI_{i@f_A}\right) - \text{(A)}}{10\left(\log f_A - \log f_B\right)}}_{\hat{\eta} = \mathrm{E}\left\{\hat{\eta}_i\right\}} \tag{19}$$

thus a more refined η approximation can be averaged from each different estimation, thereafter improving ranging and localization accuracy.

Applying method described above, thanks to P_{tx} term knowledge given from 802.11n/ac protocol level and exploiting multi-frequency antenna patterns predictability, a self-consistent single anchor bi-dimensional (x, y) coarse localization system can be implemented.

IV. EXPERIMENTAL RESULTS

Effective DoA estimation improvement is deeply analyzed in [3]: a brief idea of multi-frequency improvement on DoA accuracy is shown in fig.4, given the experimental setup shown in fig.3. The SBA router (the "anchor node") is rotated throughout the overall azimuth domain while an example user device (implemented using a signal generator to emulate the 802.11n PHY layer) is placed in front of it, at various distances from 1mt to 3mt with a step of 0.5mt, covering an equivalent area of $\pi 3^2 m^2 \approx 28m^2$. The signal generator is set to generate a 0dB CW signal to automatically remove P_{tx} terms from RSSI collected values, simplifying step described in eq.9.

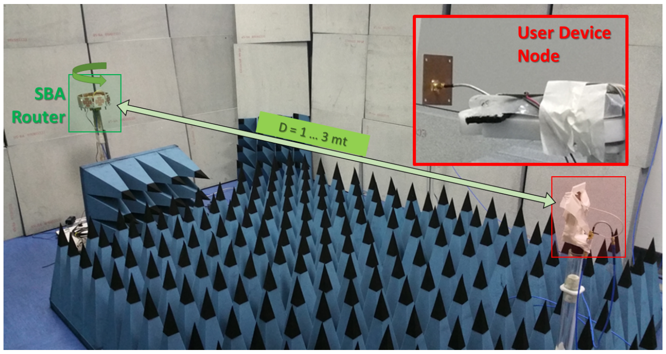

Fig. 3. Experimental setup

Overall DoA estimation error results are shown in fig.4: the solid red line shows CDF for DoA estimation error applying estimation algorithm presented in par.II and in [3] while red dashed one shows results applying a standard a-posteriori average on single frequencies DoA estimations.

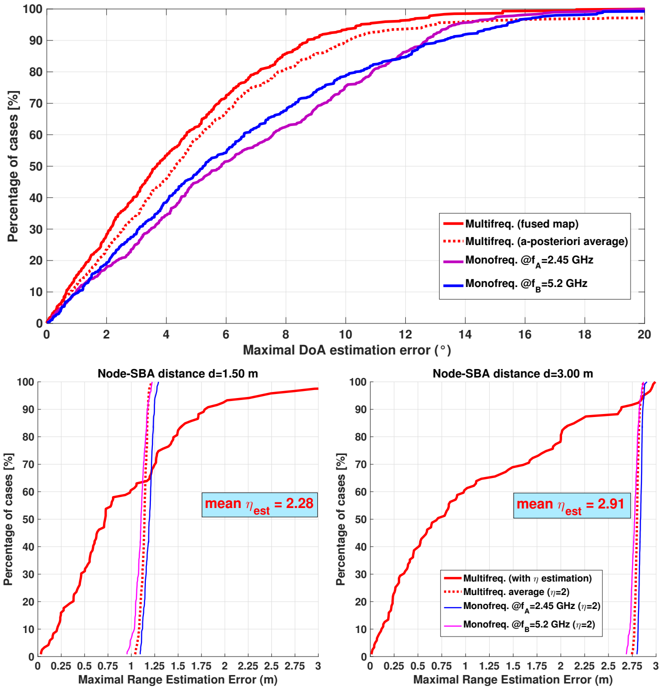

Fig. 4. Overall CDF for DoA estimation error (a) and CDFs for Range Estimation error at d=1.50m (b) and d=3.00m (c)

Proposed method improvements are clear observing range estimation error CDFs. Fig.4 depicts range error CDFs for two node-SBA distances (d=1.50m, d=3.00m): the effect of a wrong η estimation applied to a Friis formula inversion as in [8] leads to an high error in range estimation (eq.13) [10].

The reference map should be corrected using a better η estimation (eq.7-8), but such method requires a system calibration phase. Proposed method estimates the η parameter for each localization, so it becomes mandatory using a single anchor calibration-free 2D localization system setup.

Fig.5 CDFs depicts a particular staircase-function trend for typical methods. Such trend is strictly related to experimental points distribution, collected on circular paths for increasing ranges: because η wrong estimation causes an huge range estimation offset error which is proportional to range (eq.13), experimental point batches for each range will give their provision of huge offset error on overall estimations.

Table I summarizes overall 2D localization results (applying eq.10): localizations are done within an equivalent area of about $28m^2$, thorough comparable to a typical home room size. Note that equivalent area coverage belonging to a sub-metric localization error is about the 60% of total area (showing a 3x increase), so the system is suitable for 802.11n/ac nodes localization within highly constrained scenarios.

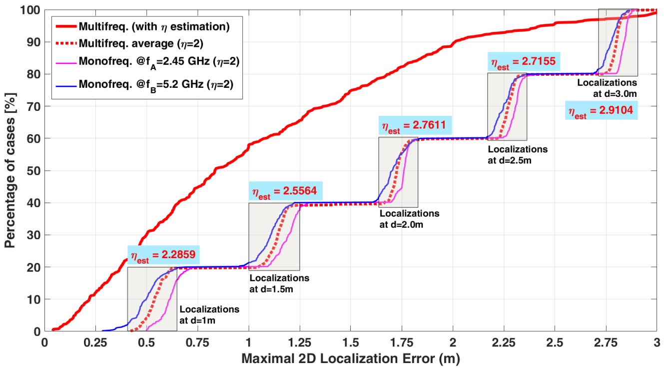

Fig. 5. Localization error CDF on overall $28m^2$ area

TABLE I
MAXIMAL 2D LOCALIZATION ERROR SUMMARY

Method	DoA Mean Error	Localization Mean Error	Cov.Area MLE<1m
@2.45 GHz (η=2) std.DoA [8]	6.58 deg	1.74m	20.00 %
@5.2 GHz (η=2) std.DoA [8]	6.35 deg	1.68m	21.01 %
Multifreq. (η=2) fused DoA [3]	**4.42 deg**	1.70m	20.00 %
This Work: Multifreq. fused DoA [3] + η est.		**0.95m**	**60.87%**

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