



FLORE

Repository istituzionale dell'Università degli Studi di Firenze

Darrieus wind turbine blade unsteady aerodynamics: a threedimensional Navier-Stokes CFD assessment

Questa è la Versione finale referata (Post print/Accepted manuscript) della seguente pubblicazione:

Original Citation:

Darrieus wind turbine blade unsteady aerodynamics: a three-dimensional Navier-Stokes CFD assessment / Balduzzi, Francesco; Drofelnik, Jernej; Bianchini, Alessandro; Ferrara, Giovanni; Ferrari, Lorenzo; Campobasso, Michele Sergio. - In: ENERGY. - ISSN 0360-5442. - ELETTRONICO. - 128:(2017), pp. 550-563. [10.1016/j.energy.2017.04.017]

Availability:

This version is available at: 2158/1079611 since: 2021-03-30T12:02:33Z

Published version: DOI: 10.1016/j.energy.2017.04.017

Terms of use: Open Access

La pubblicazione è resa disponibile sotto le norme e i termini della licenza di deposito, secondo quanto stabilito dalla Policy per l'accesso aperto dell'Università degli Studi di Firenze (https://www.sba.unifi.it/upload/policy-oa-2016-1.pdf)

Publisher copyright claim:

(Article begins on next page)

Darrieus Wind Turbine Blade Unsteady Aerodynamics: a Three-Dimensional Navier-Stokes CFD assessment

3

Francesco Balduzzi¹, Jernej Drofelnik², Alessandro Bianchini¹, Giovanni Ferrara¹,
 Lorenzo Ferrari^{3*}, Michele Sergio Campobasso⁴

6

¹ Department of Industrial Engineering, University of Florence - Via di Santa Marta 3, 50139, Firenze, Italy Tel. +39 055 275 8773 - Fax +39 055 275 8755 - balduzzi@vega.de.unifi.it

⁹ ² School of Engineering, University of Glasgow - James Watt Building South, University Avenue, G12 8QQ
 ¹⁰ Glasgow, UK - Tel. +44 (0)141 330 2032 - j.drofelnik.1@research.gla.ac.uk

³ CNR-ICCOM, National Research Council of Italy - Via Madonna del Piano 10, 50019 Sesto Fiorentino, Italy Tel. +39 055 5225 218 - Fax +39 055 5225 203 - lorenzo.ferrari@iccom.cnr.it

⁴ Department of Engineering, Lancaster University - Gillow Avenue - LA1 4YW Lancaster, UK - Tel. +44
 (0)1524 594673 - Fax +44 (0)1524 381707 - m.s.campobasso@lancaster.ac.uk

- 15 * = *contact author*
- 16

17 Abstract

18 Thanks to the recent rapid progress in high-performance computing and the growing 19 availability of large computational resources, computational fluid dynamics now offers a 20 cost-effective, versatile and accurate means to improve the understanding of the unsteady 21 aerodynamics of Darrieus wind turbines, increase their efficiency and delivering more cost-22 effective structurally sound designs.

In this study, a Navier-Stokes CFD research code featuring a very high parallel 23 efficiency was used to thoroughly investigate the three-dimensional unsteady aerodynamics 24 of a one-blade Darrieus rotor. Highly spatially and temporally refined time-dependent 25 simulations were carried out using up to 16,000 processor cores per simulation on an IBM 26 BG/Q cluster. The study aims at providing a detailed description and quantification of the 27 main three-dimensional effects associated with the cyclical motion of this turbine type, 28 including tip losses, dynamic stall, vortex propagation and blade/wake interaction. On one 29 hand, the results corroborate the findings of several carefully designed two-dimensional 30 studies. On the other hand, they reveal that the three-dimensional flow effects affecting 31 Darrieus rotor blades are much more complex than assumed by the conventional lower-32 fidelity models often used for design applications, and strongly vary during the rotor 33 revolution. 34

35

36 Keywords

37

Darrieus wind turbine, unsteady Navier-Stokes simulations, CFD, tip-effects, 3D flows

38

39 Nomenclature

- 40 *Latin symbols*
- 41 AoA angle of attack

42	AR	aspect ratio	[-]
43	С	blade chord	[m]
44	C_m	moment coefficient	[-]
45	C_p	pressure coefficient on airfoils	[-]
46	BEM	Blade Element Momentum	
47	CFD	Computational Fluid Dynamics	
48	H	turbine height	[m]
49	k	turbulent kinetic energy	$[m^2/s^2]$
50	NS	Navier-Stokes	
51	р	static pressure	[Pa]
52	R	turbine radius	[m]
53	Т	torque per unit length	[Nm]
54	TKE	turbulent kinetic energy	
55	TSR	tip-speed ratio	[-]
56	U	wind speed	[m/s]
57	U_z	vertical velocity component	
58	VAWTs	Vertical-Axis Wind Turbines	
59	Wth	theoretic relative wind speed	[m/s]
60	<i>x, y, z</i>	reference axes	
61	\mathcal{Y}^+	dimensionless wall distance	[-]
62			
63	<u>Greek symbols</u>		
64	9	azimuthal angle	[deg]
65	μ_t	turbulent viscosity	[Kg/m/s]
66	ρ	air density	[kg/Nm ³]
67	Φ	computational domain diameter	[m]
68	Ψ	computational domain height	[m]
69	ω	specific turbulence dissipation rate	[1/s]
70	Ω	turbine revolution speed	[rad/s]
71			
72	<u>Subscripts</u>		
73	∞	value at infinity	
74	ave	averaged value	

76 **1. Introduction**

77 *1.1 Background*

After most research projects on vertical-axis wind turbines (VAWTs) came to a 78 standstill in the mid 90's [1], the Darrieus wind turbine [2] is receiving rapidly increasing 79 attention of both researchers and manufacturers [3-5]. For distributed wind power generation 80 in the built environment [6], inherent advantages of this turbine type, such as performance 81 independence on wind direction, generator often positioned on the ground, low noise 82 emissions [7], enhanced performance in skewed flows [8] may outweigh disadvantages such 83 as lower power coefficients and more difficult start-up with respect to typical horizontal axis 84 machines. Moreover, in densely populated areas VAWTs are often preferred to other turbine 85 types because they are perceived as aesthetically more pleasant and thus easier to integrate in 86 the landscape [9]. The applicability of Darrieus wind turbine for utility-scale power 87 generation making use of floating platforms also appears to present important benefits in 88 89 terms of overall dynamic stability [10].

Historically, the aerodynamic performance analysis of these rotors has been carried out with low-fidelity methods, like the Blade Element Momentum (BEM) theory [1,11-13] or lifting line methods [14-15]. More recently, however, the intrinsic limitations of these models made clear that higher-fidelity tools are needed in order to understand in greater depth the complex physical phenomena taking place during the revolution of Darrieus rotors [16], like the interaction of the blades with macro vortices [17] or dynamic stall [18].

If experimental testing is often extremely difficult and expensive, Navier-Stokes (NS) 96 Computational Fluid Dynamics (CFD) can provide versatile and accurate means to improve 97 the understanding of VAWT unsteady aerodynamics and achieve higher-performance, 98 99 structurally sound and more cost-effective Darrius turbine designs. The use of NS CFD for simulating time-dependent Darrieus turbine aerodynamics is rapidly increasing due to both 100 the ongoing development and deployment of more powerful high-performance computing 101 102 hardware, such as large clusters of multi- and many-core processors [19], and also the development of computationally more efficient algorithms. 103

104

105

1.2 Previous CFD studies on Darrieus VAWTs

106 Early assessments of the Reynolds-Averaged Navier-Stokes (RANS) CFD technology for Darrieus rotor aerodynamics, aiming primarily at thoroughly investigating the complex 107 fluid mechanics of these machines, made use mainly of a two-dimensional (2D) approach 108 109 (e.g. [20-21]); an extensive literature review on these studies has been recently provided by Balduzzi et al. [22]. The use of a 2D approach was motivated by the need of maintaining the 110 computational cost of the simulations within acceptable limits, since the fully-unsteady 111 solution of the three-dimensional (3D) flow field past rotating Darrieus rotors requires very 112 large computational resources due to the very large temporal and spatial grid refinement 113 needed for obtaining reliable results [22-24]. Unfortunately, the use of 2D simulations results 114 in some important aerodynamic features being discarded (e.g. tip flow effects, secondary 115 flows, etc.). Notwithstanding this, recent work [25-28] showed that properly-set 2D approach 116 can provide estimations of the power curve of a rotor relatively close to the experimental 117 value and, thus, usable for preliminary design. 118

Since the early 2D studies of Darrieus rotor aerodynamics based 2D NS CFD, researchers have longed to perform 3D simulations of these machines in order to fully understand some phenomena that are presently modeled on the basis of a relatively small amount of data from existing turbines and assumptions based on overly simplistic analytical models.

In the past few years, thanks to the growth of available computational resources, 3D NS 124 CFD analyses have received increasing attention, and some preliminary studies have been 125 published. First, comparisons between two-dimensional simulations and three-dimensional 126 ones have been carried out (e.g. [29]). Purely 3D studies have been also carried out to 127 characterize the turbine wake [29], the flow field around the blades [30-31], the start-up of 128 small rotors [32], the effects of the finite aspect ratio [33-34] or the influence of supporting 129 arms [35] and different blade shapes [36]. Finally, further studies were also focused on the 130 performance variation in skewed flow conditions [37-38]. 131

Although many of the aforementioned analyses have indeed highlighted new important aerodynamic phenomena, in almost all cases limited computational resources imposed the use of fairly coarse spatial and temporal refinements, which often did not match the requirements indicated by proper sensitivity analyses. In particular, the common approach found in the literature was to progressively coarsen the meshes when moving to 3D analyses, in order to limit the total number of cells in the range between 1,000,000 and 10,000,000.

Most recent 2D parametric CFD analyses of Darrieus rotors (e.g. [22-24]) showed that the simulation reliability is tremendously affected by the quality of the meshing and time-

stepping strategies. In particular, it has been shown that the minimum temporal and spatial 140 refinement levels required to obtain grid-independent solutions is quite high, due to the 141 aerodynamic complexity of these unsteady rotor flows. Therefore, when using 3D unsteady 142 NS CFD for Darrieus rotor aerodynamics, the computational cost of the simulation becomes 143 very large due to necessity of maintaining high levels of time-refinement and a high level of 144 spatial refinement both in the grid planes normal to the rotor axis and the axialwise direction. 145 Refinement in the latter direction is mandatory to reliably resolve 3D flow features. Failing to 146 maintain suitably high grid refinement levels in all three direction results in losing the 147 potential of 3D simulations of improving the design of these machines by properly resolving 148 3D effects. As an example, one of the previous studies based on 2D RANS CFD showed that 149 temporal and spatial grid-independent solutions are obtained provided that grids with at least 150 400,000 elements are used [22]. To preserve the same accuracy level in a 3D RANS 151 simulation of the same turbine (modelling only half of the rotor making use of symmetry 152 boundary conditions on the plane at rotor midspan) the mesh would consist of about 153 90,000,000 cells, which is almost ten times the size of the finest meshes used in the 3D 154 RANS studies of Darrieus rotor flows published to date. 155

156 157

1.3 Study aim

In this study, a RANS CFD research code featuring a very high parallel efficiency is 158 used to investigate the detailed features of 3D flow effects of a rotating Darrieus rotor blade 159 and the impact of such effects on the power generation efficiency of the blade. To accomplish 160 this, a time-dependent 3D simulation using very high levels of spatial and temporal 161 refinements and yielding a fairly reliable assessment of the phenomena under investigation is 162 carried out using a large 98,304-core IBM BG/O cluster. The scope of the study is to analyze 163 the main 3D effects occurring during the cyclic motion of the considered one-blade rotor 164 configuration, including tip vortices, dynamic stall and downstream vortex propagation, and 165 to assess the impact of these phenomena on the overall performance of this rotor. 166

167 The paper is organized as follows: sections 2 and 3 summarize the main features of the 168 case study and the numerical framework, respectively. Section 4 presents the main results of 169 the 3D analysis and compares them to those of the 2D analysis of the same case study, in 170 order to highlight the impact of 3D effects by comparing the integral performance of a finite-171 length rotor and that of the infinite blade counterpart. A summary of the study and concluding 172 remarks are finally provided in section 5.

173

174 2. Case study

The case study selected for the 3D simulation is a one-blade H-Darrieus rotor using a 175 NACA 0021 airfoil. The chord (c=0.0858 m) and radius (R=0.515 m) of this virtual rotor 176 were taken equal to those used in the case-study of [21]; the blade was attached at midchord. 177 In order to reduce the computational cost of the simulation, the central symmetry of H-178 Darrieus rotors was exploited, allowing to simulate only one half of the blade rather than the 179 entire blade length of H=1.5 m. Thus, the aspect ratio (AR) of the simulated blade portion is 180 8.74 which is half that of the actual blade. The blade was contained in a cylindrical 181 computational domain (Fig. 1) having a radius $\Phi=240R$, a value chosen to guarantee a full 182 development of the wake [26]. The domain height was instead set to Ψ =2.53H, corresponding 183 to half the height (due to the central blade symmetry) of the wind tunnel where the original 184 model was tested [21,39]; data of these tests were used for the validation of the numerical 185 RANS CFD approach [26-28], that can be viewed as the 2D counterpart of the 3D 186 187 methodology used in the present study.



Figure 1 - Computational domain.

The 3D mesh (details are reported in Fig. 2) was obtained by first generating a 2D mesh past the airfoil using the optimal mesh settings identified in [26-27], and then extruding this mesh in the spanwise (z) direction and filling up with grid cells the volume between the blade tip and the circular farfield boundary.

196

189 190

191



197 198

199

Figure 2 - Some details of the computational mesh.

The 3D grid is structured multi-block. Its 2D section normal to the z-axis and in the z-200 interval occupied by the blade (Fig. 2(a)) consisted of 4.3×10^5 quadrilateral cells. The airfoil 201 was discretized with 580 nodes and the first element height was set to $5.8 \times 10^{-5} c$ to guarantee 202 a dimensionless wall distance y^+ lower than 1 throughout the revolution. As recommended in 203 [22], a proper refinement of both leading and the trailing edge regions was adopted (Fig. 204 2(b)), as well as a globally high refinement in the region around the airfoil within one chord 205 206 from the walls in order to properly resolve the detached flow regions at high angle of attack (AoA) [40]. After extrusion in the z direction, 80 layers in the half-blade span were formed 207 (Fig. 2(c)), with progressive grid clustering from midspan to the tip in order to ensure an 208 accurate description of tip flows. Moreover, a high grid refinement level was provided in the 209 210 whole tip region above the blade in order to properly capture the flow separation and the tip vortices. The final mesh was made of 64×10^6 hexahedral cells and able to fulfill the 211 numerical requirements prescribed in [24]. 212

Due to the large burden associated with running the 3D time-dependent simulation, only a single operating condition was simulated, corresponding to a tip-speed ration (*TSR*) of 3.3. The free stream wind speed was U=9 m/s. The turbulence farfield boundary conditions were a turbulent kinetic energy (k) based on 5% turbulence intensity and a characteristic length of 0.07 m (according to [27], production limiters were used in the simulations on k and ω with a setting ratio of 10).

219

221

220 **3. Computational framework**

All RANS simulations were carried out with the COSA research code.

COSA is a compressible density-based structured multi-block finite volume RANS 222 code featuring a steady flow solver, a time-domain (TD) solver for the solution of general 223 unsteady problems [41-42], and a harmonic balance solver for the rapid solution of periodic 224 flows [43-45]. The RANS equations are obtained by time-averaging the Navier-Stokes 225 equations on the characteristic turbulent time-scales of the problem at hand. The RANS 226 equations are formally similar to the Navier-Stokes equations, and differ from those for the 227 fact that all thermodynamic and kinematic variables are mean rather than instantaneous 228 229 values, and for the presence of an additional term, the Reynolds stress tensor accounting in a mean fashion for the effects of turbulence. Making use of Boussinesq's approximation, the 230 Reynolds stress tensor is given by the product of an eddy viscosity μ_t and the strain rate 231 tensor of the mean velocity field. COSA determines μ_t with Menter's k- ω shear stress 232 transport turbulence model [46]. The second-order space discretization of the convective 233 fluxes of both the RANS and the SST equations uses an upwind scheme based on Van Leer's 234 235 MUSCL extrapolations and Roe's flux difference splitting. The second order discretization of all diffusive fluxes is instead based on central finite-differencing. The space-discretized 236 RANS and SST equations are integrated in a fully-coupled fashion with an explicit solution 237 238 strategy based on full approximation scheme multigrid featuring a four stage Runge-Kutta smoother. Convergence acceleration is achieved by means of local time-stepping and implicit 239 residual smoothing. For general time-dependent problems, the TD equations are integrated 240 using a second order dual time-stepping approach. 241

Comprehensive validation analyses of COSA are reported in [43,45] and other 242 references cited therein. For unsteady problems involving oscillating wings and cross-flow 243 open rotors such as the Darrieus rotor configuration investigated in this paper, COSA solves 244 the governing equations in the absolute frame of reference using an arbitrary Lagrangian-245 Eulerian formulation and body-fitted grids. In the case of Darrieus rotors this implies that the 246 entire computational grid rotates about the rotational axis of the turbine. The suitability of 247 248 COSA for the simulation of Darrieus wind turbines has been recently assessed through comparative analyses with both commercial research codes and experimental data [26-27]. 249

Present simulations were run on an IBM BG/Q cluster [47], featuring 8,144 16-core 250 nodes with a total of 98,304 cores. Thanks to the excellent parallel efficiency of the COSA 251 code, the simulations yielding the results presented in this paper could be performed using 252 about 16,000 cores. This required partitioning the grid into 16384 blocks, making use of in-253 house utilities. Using a time-discretization of 720 steps per revolution, the simulation needed 254 12 revolutions to achieve a fully periodic state. The flow field over the consider period was 255 256 assumed to be periodic once the difference between the mean torque of the last two revolutions was smaller than 0.1% of the mean torque in the revolution before the last. The 257 wall-clock time required for the complete simulation was about 653 hours (27.2 days). 258 259

260 **4. Results**

Figure 3(a) reports the periodic profile of the torque coefficient per unit blade length at different span lengths along the blade (0% and 100% correspond to midspan and tip,

respectively). The angular position $9=0^{\circ}$ corresponds to the blade leading edge facing the 263 oncoming wind and entering the upwind half of its revolution. 264

The instantaneous torque coefficient per unit length C_{mz} is defined by Eq. (1). Here T_z 265 denotes the instantaneous torque per unit blade length at the considered z position, U_{∞} and ρ_{∞} 266 denote the wind speed and the air density, respectively, and c denotes the blade chord. 267

(1)



9 [deg] 270 271 272

273

269

268

Figure 3 - Moment coefficient vs azimuthal angle: (a) variation at different span lengths; (b) 2D simulations compared to the 3D profile at midspan and average 3D profile.

Figure 3(b) reports three torque profiles. One, that labelled 2D, refers to the results of a 274 2D simulation of the same rotor, and corresponds to the "ideal" torque of a blade with infinite 275 span, i.e. without any secondary effects at the blade tip. This 2D simulation was carried out 276 using a mesh equal to the midspan section of the 3D mesh and the same numerical parameters 277 of the 3D simulations. The torque profile labelled 0% is the torque per unit blade length at the 278 midspan position of the finite-length rotor, whereas the torque profile labelled 3D is the 279 overall torque coefficient C_m of the 3D rotor defined as: 280

$$C_m = \frac{2}{H} \int_0^{\frac{1}{2}} C_{mz} dz$$
(2)

Н

Examination of these profiles reveals several important facts. Firstly, the ideal 2D 282 torque and the 3D torque profiles are characterized by similar patterns, including the 283 occurrence of two relative maxima, one in the upwind the other in the downwind regions, and 284 also similar blade azimuthal positions of both maxima: the maximum torque in the upwind 285 portion of the revolution is located at $9 \approx 88.5^{\circ}$ and the maximum torque in the downwind 286

portion of the revolution is located at $\vartheta \approx 257^{\circ}$ in both cases. An almost perfect match is visible between the 2D curve and the curve at midspan of the 3D rotor, highlighting that 3D flow effects due to tip flows do not reach this position.

Examination of all profiles of Fig. 3(b) shows that the effects of blade finite-length 290 effects are very small when the blade loading is low, i.e. when the angle of attack on the 291 airfoil is low (azimuthal positions between $0^{\circ} < 9 < 40^{\circ}$, $130^{\circ} < 9 < 210^{\circ}$): in these portions of the 292 revolution, the 2D and both 3D curves (i.e. the local profile at 0% span and the average one) 293 are indeed almost superimposed. When the incidence increases, the blade load also increases. 294 Consequently, torque reduction due to tip effects also increases both in the upwind and the 295 downwind zones. This is because the strength of tip vortex flow increases with the flow 296 incidence. By further looking at the comparison between the 2D curve and the mean 3D 297 curve, one can notice that these effects are strongest in the upwind region of the period, 298 299 where a maximum difference of 9.7% between the torque peaks occurs.

Focusing now on the torque profiles at different semispan lengths reported in Fig. 3(a),some additional observations can be made, more specifically:

- The torque profiles of the blade sections at 20%, 40% and 50% of the semispan are almost identical, indicating that at least half of the blade is characterized by a predominantly 2D flow with negligible impact of tip flow effects;
- The torque profiles of the blade sections at 60%, 70% and 80% start showing a progressive reduction of the torque peak, down to -14% with respect to the midspan section. The remainder of the torque curve is less affected, especially in the downwind zone;
- The torque profiles of the blade sections at 90%, 95% and 97.5% show that at these distances from the blade tip the 3D effects are strong throughout the whole revolution.
 In particular, in the regions of positive torque production, the efficiency is remarkably reduced;
- In proximity of the blade tip (99%) almost no positive contribution to the torque output
 is given, due to the large load reduction;
- In general, the azimuthal position of the torque peak occurs later in the cycle as one moves towards the tip, with a 5° shift between the 0% and 97.5% sections. This can be explained with a reduction of the incidence angle (downwash), as will be shown in continuation of the study.

To quantify the predicted impact of tip effects in comparison to existing knowledge, Fig. 4 reports the comparison of the 2D and mean 3D torque profiles obtained with the CFD analyses and the corresponding estimates obtained with the VARDAR research code, a stateof-the-art BEM code developed at the University of Florence [6,13-14] using the ubiquitous Leicester-Prandtl model for the finite-wing correction [48].

The two BEM profiles of Fig. 4 differ in that one includes tip flow corrections and the other does not.



327 328 329

Figure 4 - Moment coefficient profiles: 2D and 3D CFD data vs. BEM simulations either including or neglecting the finite-wind effects.

Examination of these profiles highlights, that the extent of the upwind moment peak 331 reduction predicted by the CFD analyses is in good agreement with that estimated with the 332 simplified tip flow model included in the BEM theory. On the other hand, the lumped 333 parameter model appears not to be able to properly capture the moment coefficient reduction 334 in the downwind portion of the revolution. Conversely to the CFD results, the BEM model in 335 fact predicts a slight moment increase downwind, since the lower energy extraction predicted 336 in the upwind zone leads to a lower induction factor and then to a higher attended wind speed 337 in the downwind flow. This comparative analysis highlights the potential of using CFD for 338 339 further improving the analysis, and ultimately the design, of Darrieus rotor aerodynamics 340 since it may provide a new insight for the calibration of proper corrections in lower-order models. 341

To provide a different quantitative perspective of the impact of tip losses, Fig. 5 reports the profile of mean moment coefficient per unit length. The mean value for each blade height is obtained by averaging the profiles of Fig. 3(a) over one revolution. The figure also reports the constant mean torque values of the 2D and 3D simulations for both the CFD and BEM models. All curves are normalized with respect to the mean 2D moment coefficient.

347



One sees that the average blade performance is almost unaffected by tip-effects up to approximately 70% of its semispan. Expressing these results with reference to the blade chord and aspect ratio, it is found that tip flow effects adversely affect the performance of the blade for a span length of approximately 2.6c (yellow zone in Fig. 5). In terms of aggregate data, the tip effects yield a reduction of the rotor torque of 8.6% with respect to the theoretical 2D calculation with virtually infinite span. This can be globally seen as an equivalent reduction of the actual blade's height by 0.75c (red colored zone in Fig. 5) and

such a correction factor needs to be accounted for when estimating the turbine's performanceby means of 2D CFD simulations.

To further investigate the phenomena that lead to the efficiency reduction, the Mach contours and streamlines at the angular position of maximum separation (i.e. ϑ =120°) are reported in Fig. 6(a). Different spanwise sections are reported to analyze the flow pattern alterations as one moves towards the blade tip.

364



Figure 6 – Downwash effect at 9=120°: (a) Mach contours and streamlines at different semispan
 locations; (b) flow streamlines in the tip region, skin friction lines and z velocity component on the blade
 surface.

In the central portion of the blade (i.e. from midspan to approximately 70% of the semispan) the flow streamlines lay on a 2D plane and a region of extensively detached flow is present along the entire blade. When moving closer to the tip (90% of the semispan), the discussed effect of the downwash field is apparent: the effective AoA on the airfoil is reduced with respect to that at midspan. As a consequence, the extension of the stall region is also reduced due to the decrease of the incidence angle.

The skin friction lines and contours of the z velocity component on the blade surface 378 reported in Fig. 6(b) show the extension of the region affected by downwash on the suction 379 side of the blade. Near the tip, the flow on the pressure side is no longer able to follow the 380 blade profile, and travels over the tip due to the pressure difference between the pressure side 381 and the suction side. The tip vortex flow is responsible for the downwash velocity component 382 and therefore for the incidence variation along the span, in accordance with the theory of 383 384 finite wings [48]. It is noted that the finite wing effects occurring in Darrieus rotors are more complex than those encountered in fixed finite wings. This is primarily because of the flow 385 curvature associated with the circular trajectory of the blade, and also the flow nonlinearities 386 due to dynamic stall. To quantify the impact of these effects, relevant outcomes can be 387 388 obtained by examining the torque coefficient trends per unit length produced by two relevant slices in Fig. 7, e.g. midspan and 80%. The percentage difference between the two curves (i.e. 389 the moment coefficient difference between the curves at each azimuthal angle divided by the 390 391 average moment coefficient at midspan) is also reported to quantify the dependence of the torque variation on the azimuthal position. 392

393

394 395

396



Figure 7 - Comparison of moment coefficient curves at 0% and 80% of the semispan.

397 It is observed that a notable torque reduction occurs between $40^{\circ} \le 9 \le 130^{\circ}$. In addition, large and sudden torque reduction is obtained at the end of the revolution, i.e. when the 398 incidence angle is progressively reducing towards values lower than the stall one. On the 399 other hand, an inversion in the expected trend was noticed close to $9=150^{\circ}$, while the torque 400 output is positive. Even if not apparent from the figure, a higher torque production (relative 401 difference of +11% between 80% and midspan) was noticed when getting closer to the tip. 402 According to the finite wing theory, the lift should be in fact always reduced in proximity of 403 the tip and the reduction is assumed to be proportional to the lift itself. Therefore, the 404 inversion at $9=150^{\circ}$ cannot be justified with this theory only, as well as the significant torque 405 406 reduction at the end of the revolution, since the reduced load on the blades should not generate such a large downwash effect. Detailed flow analyses will be shown in the 407 continuation of the study to comprehend this phenomenon. 408

First, the reasons of the sudden torque reduction at the blade tip between $315^{\circ} < 9 < 340^{\circ}$ were investigated. To this purpose, Fig. 8 shows the evolution of the turbulence kinetic energy (TKE) field at selected azimuthal positions based on TKE iso-surfaces. The color scale is based on the intensity of the velocity component along the z-axis (U_z) . Three azimuthal positions of the blade are considered in Fig. 8, i.e. $\vartheta=60^{\circ}$ (upper right), 180° (left) and 315° (lower right).

415





Figure 8 - Iso-surfaces of TKE colored with the contour of U_z .

419 During the upwind half of the revolution (θ =60°) the tip vortex is strong, since the vertical component of velocity is very intense. A high turbulence region is then generated in 420 the tip's wake. For $\mathcal{P}=180^\circ$, U_z is reduced around the blade tip, but the region of the blade 421 wake characterized by high TKE is increased in size and length, and is still associated with 422 large values of U_z . This generates a strong vortex, which is then detached from the blade, 423 after being convected by the wind, is re-encountered by the blade itself at $9=315^{\circ}$. The 424 interaction with this macro-vortex induces a more pronounced modification of the torque 425 curve with respect to the 2D case, where this effect is totally absent. 426

Focusing now on the azimuthal position where the torque inversion takes place, i.e. $\mathcal{G}=150^{\circ}$, Fig. 9 reports a top view of the flow streamlines at two azimuthal positions. The comparison is carried out with $\mathcal{G}=48^{\circ}$, which is the other position during the upwind half of the revolution experiencing the same AoA of $\mathcal{G}=150^{\circ}$. Streamlines on both the pressure and suction sides of the blade are visualized at four different span locations. 432



Figure 9 - Streamlines at different span lengths: 9=48° and 9=150°.

For $\mathcal{P}=48^{\circ}$ the "conventional" downwash effect is clearly visible: moving from midspan to the tip, the incidence of the oncoming flow is progressively reduced and the velocity past the trailing edge is shifted towards the pressure side. This phenomenon is not very pronounced due to the low load on the blade at this angular position and the deflection occurs only in proximity of the tip.

For $9=150^{\circ}$, moving from midspan to the tip, the incidence of the oncoming flow is 441 progressively reduced similarly to what is seen at $9=48^{\circ}$ and no evident difference with 442 respect to the $9=48^{\circ}$ degrees case can be observed. Conversely, the flow condition on the 443 suction side is completely different: at midspan, even if the AoA is similar to that obtained at 444 $9=48^{\circ}$, a large separation region is evident at $9=150^{\circ}$ due to dynamic stall. As a result of the 445 finite wing effect, a strong modification of the flow is then observed when moving across the 446 447 blade span towards the tip: starting from 70% semispan, the flow re-starts to be deflected towards the pressure side. At 90% semispan, the flow direction is completely changed with 448 respect to midspan: the separation region is indeed almost absent, making the flow become 449 very similar to that measured at $9=48^{\circ}$. 450

451 This can be explained by a combined effect of downwash and dynamic stall. During the advancing 90-degree sector of the blade trajectory, only the sections at the tip do not reach 452 stall conditions since they experience a lower AoA. When the incidence decreases in the 453 454 retreating 90-degree sector, the central portion of the blade suffers from dynamic stall, with a marked reduction of lift production. Conversely, the flow remains attached for the sections at 455 the tip, which are still able to produce the theoretical lift. It can be summarized that, even if 456 457 the incidence is lower close to the tip, the absence of dynamic stall guarantees higher lift and therefore higher torque production than at midspan in this specific angular position. The 458 explanation is straightforward: when the incidence is maximum, the sections at the tip do not 459 460 reach stall conditions since they experience lower AoAs and when the incidence decreases they are still able to produce the theoretical lift. It can be summarized that, even if the 461 incidence is lower close to the tip, the absence of dynamic stall guarantees, in this specific 462 condition, higher lift and therefore higher torque production. 463

Fig. 10 shows an analogous comparison for the angular positions of $9=210^{\circ}$ and 464 $9=300^{\circ}$, which again are characterized approximately by the same AoA value. It can be 465 noticed that the behavior at $9=210^{\circ}$ is the same of that at $9=48^{\circ}$, being an angular position in 466 the downwind half of the rotation where the incidence is increasing (in magnitude). At 467 $9=300^{\circ}$ the incidence (decreasing) is again the same of $9=210^{\circ}$ but in this case no evident 468 modification of the streamlines can be observed from midspan to the tip. Indeed, the suction 469 470 side is characterized by just a slight separation and the downwash effect is very similar to that at $\theta = 210^{\circ}$. 471

472



Figure 10 - Streamlines at different span lengths: 9=210° and 9=300°.

In addition, Fig. 11 shows the streamlines at $\vartheta=315^{\circ}$ to see the effect of the turbulent vortex on the flow field that was described in Fig. 8: it is apparent that a sudden distortion of the incoming flow in the tip region takes place due to the blade-vortex interaction, with a large variation of the incidence angle, which is now not related to the downwash effect.







494

Figure 11 - Streamlines at different span lengths: 9=315°.

484 All aforementioned results can be more quantitatively described by directly evaluating 485 the pressure coefficient (C_p) distributions and the vorticity contours along the blade span.

Fig. 12 reports the pressure coefficient distributions at different span lengths for three 486 of the previously discussed critical angular positions: maximum load (9=80°), inversion of 487 load between midspan and tip ($9=150^{\circ}$) and maximum load in the downwind half of the 488 rotation ($9=240^{\circ}$). The pressure coefficient used in this study is defined by Eq. (3), where p 489 denotes the static pressure at the airfoil surface. It has to be noted that, due to the complexity 490 of properly defining the actual relative wind speed on the airfoil, the pressure profiles were 491 here normalized by the theoretic relative wind speed (w_{th}) among a null induction factor (i.e. 492 the vectorial sum between the wind speed at infinity U_{∞} and the peripheral speed ΩR). 493

$$C_{p} = \frac{p - p_{\infty}}{\frac{1}{2}\rho_{\infty}w_{th}^{2}}$$
(3)





Figure 12 - Pressure coefficient profiles at different span lengths: 9=80°, 9=150° and 9=240°.

In conditions of high load (high AoA and high relative speed, like at $9=80^{\circ}$), it is 498 confirmed that the 3D effects can reach up to 40% of the semispan from the tip, since moving 499 from midspan to the tip, the C_p profile at 60% already shows a slight reduction of the load 500 with respect to midspan. Close to the tip, the suction side of the blade is characterized by an 501 almost constant pressure, indicating that this section does generate a very small lift force. As 502 a result, a negative torque production is noticed at this location. When the AoA is high but 503 the relative speed magnitude is lower ($9=240^\circ$), the 3D effects extend only to the last 20% of 504 the semispan (i.e. from 80% semispan to tip): significant differences in the C_p distribution can 505 be appreciated, however, only above 90% of the semispan. Close to the tip, only a reduction 506 of the load can be observed, without any additional effect. The position $9=150^{\circ}$ is instead 507 characterized by stalled flow, as shown by the large separation in the pressure distributions at 508 0% and 60%. In this case, the effect of the tip vortex is to slightly reduce the load, inducing 509 the flow to remain attached to the blade, with a lower separation and even a small 510 511 improvement of produced lift.

512 The evolution of the vorticity contours at different span locations along the blade is 513 finally presented in Fig. 13.





529

Figure 13 - Vorticity contours at different span lengths during the revolution.

Focusing on the upwind half of the rotation, the two extreme positions of $9=40^{\circ}$ and 518 $9=140^{\circ}$, are in fact of particular interest. It can be noticed that, although (as expected) the 519 behavior in terms of torque distribution along the span is comparable, with analogous values 520 at each span location (see Fig. 3(a) and Fig. 3(b)), the 3D flow field in these two positions is 521 remarkably different. Indeed, for $9=40^{\circ}$ the vorticity contours are very similar, moving from 522 midspan to the tip, while for $\vartheta = 140^{\circ}$ the large separation region due to stall is clearly present 523 along the largest portion of the blade but tends to disappear when getting close to the tip due 524 to tip effects. Moving to the downwind half of the revolution, it is apparent that the vorticity 525 contours are now very similar, moving from midspan to the tip, at all azimuthal positions. 526 This behavior is in agreement with previous analyses of streamlines and pressure coefficient 527 528 profiles.

530 **5. Concluding remarks**

In the study, a 3D, time accurate CFD study of a rotating Darrieus wind turbine blade was carried out. Particular attention was given to the description of 3D flow effects and to the modifications induced by them in comparison to the "infinite-wing" ideal case, which is the one generally optimized by both 2D numerical calculations and low-fidelity simulation models. The main outcomes of the analysis can be summarized as follows:

a) 3D effects do not modify the general trend of torque extraction over the revolution
(with the only exception of the near-tip region): indeed, maximum and minimum relative
values of torque take place at analogous blade azimuthal positions.

b) The impact of tip effects is strongly dependent on the azimuthal position of the blade, according to oncoming relative speed in terms of incidence angle and modulus. In this view, the accuracy of a low-fidelity model like the BEM theory could be remarkably increased by re-correcting the aerodynamic coefficients for finite-wing effects at each azimuthal position (or streamtube). c) In the present study case, the average torque reduction due to finite-blade effects corresponded to a reduction of the effective overall blade length of 1.5c (0.75c for each half blade).

547 d) A strong interaction between the tip-vortex released in the upwind portion of the 548 blade trajectory and the blade moving in the downwind region was noticed in correspondence 549 to an azimuthal position of $\vartheta=315^{\circ}$.

550 Future work will include the investigation of 3D flow effects at different tip-speed 551 ratios, particularly the lower ones, at which the impact of dynamic stall is thought to be more 552 relevant, and should also extend the analysis to a three-blade turbine, in order to describe in 553 detail all aspects of wake/blade interactions.

554

555 Acknowledgements

We acknowledge use of Hartree Centre resources in this work. The STFC Hartree 556 Centre is a research collaboratory in association with IBM providing High Performance 557 Computing platforms funded by the UK's investment in e-Infrastructure. The Centre aims to 558 develop and demonstrate next generation software, optimised to take advantage of the move 559 towards exa-scale computing. Part of the reported simulations were also performed on two 560 other clusters. One is POLARIS, part of the N8 HPC facilities provided and funded by the N8 561 consortium and EPSRC (Grant No.EP/K000225/1). The Centre is co-ordinated by the 562 Universities of Leeds and Manchester. The other resource is the HEC cluster of Lancaster 563 University, which is also kindly acknowledged. Finally, thanks are due to Prof. Ennio 564 Antonio Carnevale of the Università degli Studi di Firenze for supporting this research. 565 566

567 **References**

- Faraschivoiu I. Wind turbine design with emphasis on Darrieus concept. Polytechnic
 International Press: Montreal (Canada), 2002.
- 570 [2] Darrieus GJM. Turbine having its rotating shaft transverse to the flow of the current.
 571 US Patent No.01835018, 1931.
- 572 [3] Tjiu W, Marnoto T, Mat S, Ruslan MH, Sopian K. Darrieus vertical axis wind turbine
 573 for power generation I: Assessment of Darrieus VAWT configurations. Renewable
 574 Energy 2015; 75(March 2015): 50-67. DOI: 10.1016/j.renene.2014.09.038
- Tjiu W, Marnoto T, Mat S, Ruslan MH, Sopian K. Darrieus vertical axis wind turbine
 for power generation II: Challenges in HAWT and the opportunity of multi-megawatt
 Darrieus VAWT development. Renewable Energy 2015; 75(March 2015):560-571.
 DOI: 10.1016/j.renene.2014.10.039
- 579 [5] Bianchini A, Ferrara G, Ferrari L. Design guidelines for H-Darrieus wind turbines:
 580 Optimization of the annual energy yield. Energy Conversion and Management
 581 2015;89:690-707. DOI: 10.1016/j.enconman.2014.10.038
- 582 [6] Balduzzi F, Bianchini A, Carnevale EA, Ferrari L, Magnani S. Feasibility analysis of a
 583 Darrieus vertical-axis wind turbine installation in the rooftop of a building. Applied
 584 Energy 2012; 97: 921–929. DOI: 10.1016/j.apenergy.2011.12.008
- 585 [7] Mohamed MH. Aero-acoustics noise evaluation of H-rotor Darrieus wind turbines.
 586 Energy 2014; 65(1): 596-604. DOI: 10.1016/j.energy.2013.11.031.
- 587 [8] Bianchini A, Ferrara G, Ferrari L, Magnani S. An improved model for the performance
 588 estimation of an H-Darrieus wind turbine in skewed flow. Wind Engineering 2012;
 589 36(6): 667-686. DOI: 10.1260/0309-524X.36.6.667

- 590 [9] Mertens S. Wind Energy in the Built Environment. Multi-Science: Brentwood (UK),
 591 2006.
- 592 [10] Borg M, Collu M, Brennan FP. Offshore floating vertical axis wind turbines:
 593 advantages, disadvantages, and dynamics modelling state of the art. Marine & Offshore
 594 Renewable Energy Congress, London (UK), 26-27 September, 2012.
- [11] Brahimi M, Allet A, Paraschivoiu I. Aerodynamic analysis models for vertical-axis
 wind turbines. International Journal of Rotating Machinery 1995; 2(1): 15-21. DOI:
 10.1155/S1023621X95000169
- 598 [12] Paraschivoiu I, Delclaux F. Double Multiple Streamtube Model with Recent
 599 Improvements. Journal of Energy 1983, 7(3), pp. 250-255.
- [13] Bianchini A, Ferrari L, Carnevale EA. A model to account for the Virtual Camber
 Effect in the Performance Prediction of an H-Darrieus VAWT Using the Momentum
 Models. Wind Engineering 2011; 35(4): 465-482. DOI: 10.1260/0309-524X.35.4.465
- [14] Marten D, Bianchini A, Pechlivanoglou G, Balduzzi F, Nayeri CN, Ferrara G,
 Paschereit CO, Ferrari L. Effects of airfoil's polar data in the stall region on the
 estimation of Darrieus wind turbines performance. Proc. of the ASME Turbo Expo
 2016, Seoul, South Korea, June 13-17, 2016.
- [15] Marten D, Lennie M, Pechlivanoglou G, Nayeri CD, Paschereit CO. Implementation,
 Optimization and Validation of a Nonlinear Lifting Line Free Vortex Wake Module
 within the Wind Turbine Simulation Code QBlade. Proc. of the ASME Turbo Expo
 2015, Montréal, Canada, June 15-19, 2015.
- [16] Deglaire P. Analytical Aerodynamic Simulation Tools for Vertical Axis Wind
 Turbines. Digital Comprehensive Summaries of Uppsala Dissertations from the Faculty
 of Science and Technology 2010, 704, ISSN 1651-6214.
- 614 [17] Amet E, Maitre T, Pellone C, Achard JL. 2D Numerical Simulations of Blade-Vortex
 615 Interaction in a Darrieus Turbine. Journal of Fluids Engineering 2009; 131: 111103.1–
 616 111103.15. DOI: 10.1115/1.4000258
- 617 [18] Simao-Ferreira C, van Zuijlen A, Bijl H, van Bussel G, van Kuik G. Simulating
 618 dynamic stall on a two-dimensional vertical-axis wind turbine: verification and
 619 validation with particle image velocimetry data. Wind Energy 2010; 13: 1-17. DOI:
 620 10.1002/we.330
- [19] Salvadore F, Bernardini M, Botti M. GPU accelerated flow solver for direct numerical simulation of turbulent flows. Journal of Computational Physics 2013; 235: 129-142.
 DOI: 10.1016/j.jcp.2012.10.012
- [20] Howell R, Qin N, Edwards J, Durrani N. Wind tunnel and numerical study of a small
 vertical axis wind turbine. Renewable Energy 2010; 35: 412-422. DOI:
 10.1016/j.renene.2009.07.025
- [21] Raciti Castelli M, Englaro A, Benini E. The Darrieus wind turbine: Proposal for a new performance prediction model based on CFD. Energy 2011; 36: 4919-4934. DOI: 10.1016/j.energy.2011.05.036
- [22] Balduzzi F, Bianchini A, Maleci R, Ferrara G, Ferrari L. Critical issues in the CFD simulation of Darrieus wind turbines. Renewable Energy 2016; 85(01): 419-435. DOI: 10.1016/j.renene.2015.06.048

- [23] Almohammadi KM, Ingham DB, Ma L, Pourkashan M. Computational fluid dynamics
 (CFD) mesh independency techniques for a straight blade vertical axis wind turbine.
 Energy 2013; 58(1 September 2013): 483-493. DOI: 10.1016/j.energy.2013.06.012
- [24] Balduzzi F, Bianchini A, Ferrara G, Ferrari L. Dimensionless numbers for the
 assessment of mesh and timestep requirements in CFD simulations of Darrieus wind
 turbines. Energy 2016; 97(15 February 2016): 246-261. DOI:
 10.1016/j.energy.2015.12.111
- [25] Daróczy L, Janiga G, Petrasch K, Webner M, Thévenin D. Comparative analysis of
 turbulence models for the aerodynamic simulation of H-Darrieus rotors. Energy 2015;
 90(1 October 2015): 680-690. DOI: 10.1016/j.energy.2015.07.102
- [26] Balduzzi F, Bianchini A, Gigante FA, Ferrara G, Campobasso MS, Ferrari L.
 Parametric and Comparative Assessment of Navier-Stokes CFD Methodologies for
 Darrieus Wind Turbine Performance Analysis. Proc. of the ASME Turbo Expo 2015,
 Montreal, Canada, June 15-19, 2015. DOI: 10.1115/GT2015-42663
- 647 [27] Gigante FA, Balduzzi F, Bianchini A, Yan M, Ferrara G, Ferrari L, Campobasso MS.
 648 On the Computational Fluid Dynamics Analysis of Darrieus Wind Turbines Using the
 649 Reynolds-Averaged Navier-Stokes Equations and the Shear Stress Transport
 650 Turbulence Model. Paper submitted for publication to: Energy, 2016.
- [28] Bianchini A, Balduzzi F, Ferrara G, Ferrari L. Influence of the blade-spoke connection
 point on the aerodynamic performance of Darrieus wind turbines. Proc. of the ASME
 Turbo Expo 2016, June 13-17, Seoul (South Korea), 2016.
- [29] Lam HF, Peng HY. Study of wake characteristics of a vertical axis wind turbine by
 two- and three-dimensional computational fluid dynamics simulations. Renewable
 Energy 2016; 90(May 2016): 386-398. DOI: 10.1016/j.renene.2016.01.011
- [30] Ghosh A, Biswas A, Sharma KK, Gupta R. Computational analysis of flow physics of a combined three bladed Darrieus Savonius wind rotor. Journal of the Energy Institute 2015; 88(4): 425-437. DOI: 10.1016/j.joei.2014.11.001
- [31] Raciti Castelli M, Pavesi G, Battisti L, Benini E, Ardizzon G. Modeling strategy and 660 numerical validation for a Darrieus vertical axis micro-wind turbine. Proc. of the 661 ASME 2010 International Mechanical Engineering Congress & Exposition (IMECE), 662 Canada, Vancouver, British Columbia, November 12-18, 2010. DOI: 663 10.1115/IMECE2010-39548 664
- [32] Untaroiu A, Wood HG, Allaire PE, Ribando RJ. Investigation of Self-Starting
 Capability of Vertical Axis Wind Turbines Using a Computational Fluid Dynamics
 Approach. ASME Journal of Solar Energy Engineering 2011; 133(November 2011):
 041010-1-8. DOI: 10.1115/1.4004705
- [33] Gosselin R, Dumas G, Boudreau M. Parametric study of H-Darrieus vertical-axis
 turbines using uRANS simulations. Proc. of the 21st Annual Conference of the CFD
 Society of Canada, May 6-9, Sherbrooke (Canada), 2013
- [34] Alaimo A, Esposito A, Messineo A, Orlando C, Tumino D. 3D CFD Analysis of a
 Vertical Axis Wind Turbine. Energies 2015; 8: 3013-3033. DOI: 10.3390/en8043013
- [35] De Marco A, Coiro DP, Cucco D, Nicolosi F. A Numerical Study on a Vertical-Axis
 Wind Turbine with Inclined Arms. International Journal of Aerospace Engineering
 2014; 2014: 1-14. DOI: 10.1155/2014/180498

- [36] Raciti Castelli M, Benini E. Effect of Blade Inclination Angle on a Darrieus Wind
 Turbine. ASME Journal of Turbomachinery 2012; 134(May 2012): 031016-1-10. DOI:
 10.1115/1.4003212
- [37] Orlandi A, Collu M, Zanforlin S, Shires A. 3D URANS analysis of a vertical axis wind
 turbine in skewed flows. Journal of Wind Engineering and Industrial Aerodynamics
 2015; 147(December 2015): 77-84. DOI: 10.1016/j.jweia.2015.09.010
- [38] Bedon G, De Betta S, Benini E. A computational assessment of the aerodynamic
 performance of a tilted Darrieus wind turbine. Journal of Wind Engineering and
 Industrial Aerodynamics 2015; 145(October 2015): 263-269. DOI:
 10.1016/j.jweia.2015.07.005
- [39] Dossena V, Persico G, Paradiso B, Battisti L, Dell'Anna S, Brighenti A, Benini E. An
 Experimental Study of the Aerodynamics and Performance of a Vertical Axis Wind
 Turbine in a Confined and Non-Confined Environment. Proc. of the ASME Turbo
 Expo 2015, Montreal, Canada, June 15-19, 2015
- [40] Rainbird J, Bianchini A, Balduzzi F, Peiro J, Graham JMR, Ferrara G, Ferrari L. On the
 Influence of Virtual Camber Effect on Airfoil Polars for Use in Simulations of Darrieus
 Wind Turbines. Energy Conversion and Management 2015;106:373-384. DOI:
 10.1016/j.enconman.2015.09.053
- [41] Campobasso MS, Piskopakis A, Drofelnik J, Jackson A. Turbulent Navier-Stokes
 Analysis of an Oscillating Wing in a Power Extraction Regime Using the Shear Stress
 Transport Turbulence Model. Computers and Fluids 2013; 88: 136-155. DOI:
 10.1016/j.compfluid.2013.08.016
- [42] Drofelnik J and Campobasso MS, Comparative Turbulent Three-Dimensional Navier Stokes Hydrodynamic Analysis and Performance Assessment of Oscillating Wings for
 Renewable Energy Applications, International Journal of Marine Energy, Vol. 16,
 2016, pp. 100-115
- [43] Campobasso MS, Gigante F, Drofelnik J. Turbulent Unsteady Flow Analysis of Horizontal Axis Wind Turbine Airfoil Aerodynamics Based on the Harmonic Balance Reynolds-Averaged Navier-Stokes Equations. ASME paper GT2014-25559, Proc. of the ASME Turbo Expo 2014, Düsseldorf (Germany), 2014. DOI:10.1115/GT2014-25559.
- [44] Campobasso MS, Drofelnik J, Gigante F, Comparative Assessment of the Harmonic
 Balance Navier-Stokes Technology for Horizontal and Vertical Axis Wind Turbine
 Aerodynamics, Computers and Fluids, Vol. 136, 2016, pp. 354-370.
- [45] Campobasso MS, Baba-Ahmadi MH. Analysis of Unsteady Flows Past Horizontal Axis
 Wind Turbine Airfoils Based on Harmonic Balance Compressible Navier-Stokes
 Equations with Low-Speed Preconditioning. ASME Journal of Turbomachinery 2012;
 134(6): 061020-1-13. DOI: 10.1115/1.4006293.
- [46] Menter FR. Two-equation Turbulence-models for Engineering Applications. AIAA
 Journal 1994; 32(8): 1598-1605. DOI: 10.2514/3.12149
- 717 [47] http://community.hartree.stfc.ac.uk/wiki/site/admin/resources.html, last accessed
 718 10/05/2016.
- [48] Abbott IH, Von Doenhoff AE. Theory of Wing Sections. New York, USA: DoverPublications Inc.; 1959.