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Dimensionless numbers for the assessment of mesh and timestep requirements in CFD simulations of Darrieus wind turbines

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Abstract

Computational Fluid Dynamics is thought to provide in the near future an essential contribution to the development of Vertical-Axis Wind Turbines.

The unsteady flow past rotating blades is, however, a challenging application for a numerical simulation and some critical issues have not been settled yet. In particular, if some studies in the literature report detailed analyses on the assessment of the computational model, there is still no adequate convergence on the requirements in terms of spatial and temporal discretizations.

In the present study, a multivariate sensitivity analysis was first carried out on a specific case study at different tip-speed ratios in order to define the optimal mesh and timestep sizes needed for an accurate simulation. Once full insensitivity had been reached, the spatial and temporal requirements needed to properly describe the flow phenomena were related to two dimensionless numbers, one for each domain, which can be used to assess the suitability of the selected settings for each specific simulation.

The simulations revealed that the spatial requirements must be selected in order to ensure an accurate description of velocity gradients in the near-blade region. To this purpose, a Grid-Reduced form of vorticity is proposed as the best indicator for the quality of the mesh refinement.

It is also shown that the temporal requirements are made stricter at low tip-speed ratios by the need of correctly describing the vortices detaching from the blades in the upwind region. To do so, proper thresholds for the Courant Number are highlighted in the study.

Keywords: CFD; Darrieus; VAWT; vorticity; Courant Number; mesh sensitivity;

1. Introduction

In the last decade, increasing interest among renewable energy sources has been paid to Vertical-Axis Wind Turbines (VAWTs) [1]-[5]. To study these machines, one-dimensional models based on the Blade Element Momentum (BEM) theory have been extensively used in the past, in particular to identify the first design solutions [6]-[10].

Due to the intrinsic limitations of one-dimensional models, however, more recently the attention of the scientific community has been focused on CFD simulations (e.g. [11]-[12]), which are thought to shortly

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42 enable a more in-depth understanding of the aerodynamic behavior of blades rotating around an axis
43 orthogonal to flow direction and of many connected issues, e.g. aero-acoustic noise [13]. The use of higher-
44 order methods to improve the understanding of wind energy phenomena is indeed one of the main challenges
45 for the research [14-15].

46 Since fully unsteady phenomena need to be captured, a proper numerical setup is needed for an accurate
47 numerical simulation. In particular, the low tip-speed ratios (TSRs) are characterized by a large variation of
48 the incidence angle during each revolution. In these conditions, non-stationary phenomena take place, e.g.
49 the onset of dynamic stall structures and vortex shedding. An accurate prediction of all these flow features is
50 therefore pivotal to correctly predict the machine performance in these functioning conditions. Since high
51 local gradients of the flow quantities are associated to the vortices generation, the computational method and
52 the mesh topology must be properly related to the requirements for resolving the physics of the problem.

53 While the suitability of the modeling strategy (turbulence, numerical schemes, algorithm, etc.) is not a
54 priori evaluable [16] the choices made in terms of spatial and temporal discretization lead to the generation
55 of numerical errors that can be theoretically evaluated and minimized. As a general reference, the mesh
56 resolution must be defined according to the gradients intensity in order to accurately compute the spatial
57 variation of the flow quantities and to limit the numerical diffusion, especially in the case of CFD approaches
58 using unstructured meshes. The greater are the gradients, the finer must be the mesh. The order of the
59 difference schemes can be preserved and the grid related error may be virtually eliminated. Verification is
60 mandatory to ensure that a CFD code can correctly produce a solution for the mathematical equations used in
61 the conceptual model, although it does not necessarily imply that the computational results actually represent
62 the physical phenomenon.

63 Some works have been presented in literature [17]-[28] reporting detailed analyses on the assessment of
64 the proper computational model but there is still no adequate comprehension or convergence on the CFD
65 requirements. In the majority of these works, the numerical model is investigated only at a superficial level,
66 with focus only on the sensitivity analyses to the grid refinement, timestep or turbulence model.

67 The information provided are mostly case dependent and, therefore, of scarce practical use. An example
68 of a higher-level study was proposed by Trivellato [29], aimed at assessing the size of the grid elements at
69 the rotating interface and the angular marching step by exploiting the Courant-Friedrichs-Lewy (CFL)
70 criterion. In particular, it was found that angular timesteps in the order of $1/15^\circ$ or $1/30^\circ$ are advisable to
71 minimize numerical errors, due to the effects of the CFL criterion on relevant local properties (such as the
72 torque coefficient as a function of the blade azimuthal position).

73 In order to achieve a mesh independent solution, Almohammadi [21] pointed out by means of an
74 extensive literature review that, although extensive research has been carried out to obtain reasonable
75 agreement between CFD results and experimental data, no single study exists in the literature that adequately
76 covers a grid independency analysis. To this purpose, Almohammadi suggested the use of advanced methods
77 for the investigation of the mesh independency. In particular, he made use of extrapolation-based error
78 estimators as the General Richardson Extrapolation (GRE) and the Grid Convergence Index (GCI), which
79 are largely recommended for verification studies in computational fluid dynamics [30]-[33].

80 In previous works ([11] and [34]), the authors showed the necessity of defining very heavy meshes,
81 particularly due to the high number of nodes needed on the airfoils' surface. Moreover, the need of both an
82 increase of the mesh resolution and a reduction of the angular timestep were noticed in case of low
83 revolution speeds of the rotor. These operating conditions are more critical due to the wider range of
84 incidence angles that enhances the stall phenomenon.

85 In the present study, the results of a deep and systematic sensitivity analysis were analyzed in order to
86 identify the correlation between the requirements in terms of spatial and temporal resolution and the physics
87 to be solved. In the first part of the activity, the CFD requirements throughout the whole operating range of
88 the machine were identified. This goal was achieved by analyzing a large set of operating points which were
89 simulated considering different meshes, angular timesteps and rotating speeds. In the second part, the authors
90 identified the relationship between the physical properties of the flow and the assessed discretization
91 properties of the numerical model by making use of newly proposed dimensionless numbers. The main aim
92 of the study was then to define some guidelines of generalizable validity for the CFD simulation of Darrieus
93 VAWTs.

2. Numerical Setup

2.1 Simulation Settings

In two previous works ([11] and [34]), the authors developed and successfully validated a two-dimensional approach to the simulation of Darrieus rotors. The aforementioned references extensively report and discuss the assessment of the main simulation settings, which are however also briefly reported here to provide a clear overview of the work to the reader.

The commercial code ANSYS Fluent [35] was used for the 2D simulations, which made use of a time-dependent unsteady Reynolds-averaged Navier-Stokes (U-RANS) approach, in the pressure based formulation. The Coupled algorithm was employed to handle the pressure-velocity coupling. The second order upwind scheme was used for the spatial discretization of the whole set of RANS and turbulence equations, as well as the bounded second order for time differencing to obtain a good resolution [11]. Air was modeled as an ideal compressible gas with standard ambient conditions, i.e. a pressure of 1.01×10^5 Pa and a temperature of 300 K.

The global convergence of each simulation was monitored by considering the difference between the mean values of the torque coefficient over two subsequent revolutions normalized by the mean value over the second period of the pair. The periodicity error threshold was set to 0.1% [11].

Exploiting the sliding-mesh model of the solver, the simulation domain was divided into two subdomains in order to allow the rotation of the turbine, as proposed by Maître et al. [18] and Raciti Castelli et al. [19].

Concerning the turbulence closure problem, Balduzzi et al. [36], showed the effectiveness of Menter's shear stress transport (SST) [37] model in performance simulations involving unsteady aerodynamics for VAWTs, as also confirmed by the wide use in recent literature [37]. The same model was then used in the present study.

The presented CFD approach was validated against several experimental data [11]. In particular, extensive comparisons were made with experiments collected on a wind tunnel [34] by the authors.

The tested turbine was a real full-scale model of an industrial rotor with three blades and cambered airfoils, obtained by a conformal transformation of the NACA0018 section by the turbine's radius to compensate the flow curvature effects [11],[39]-[40]. The geometric features of the rotor considered in the former studies are summarized in Table 1.

With reference to this rotor, in [11] an extended sensitivity analysis was carried out on the main simulation settings. Comparative analyses assessed the influence of each numerical parameter both on the solution stability and on the accuracy with respect to purposefully collected experimental data on the study turbine. Figure 1 reports the comparison between simulated data and experiments in terms of torque coefficient of the whole turbine (three blades) to assess the capability of correctly simulating VAWT flow physics. Very good agreement is readily noticeable almost in every point of the functioning curve of the turbine. Such an impressive match between the two data sets was probably favored by the fact that the experimental data were purged from the tare torque and the blades of the rotor were long enough ($AR > 10$) to reduce the influence of the tip-losses.

Moreover, in [11] the authors also demonstrated that the proposed numerical approach suitably predicted the azimuthal distribution of blade torque over a revolution of an additional literature test case.

Based on the experience of past studies, the following settings (Table 2) were proposed for this type of simulations and also endorsed for the present study:

2.2 Study Case

Since the work was focused on the assessment of the meshing and time-stepping strategies in order to minimize the numerical error related to the spatial and temporal discretization, the choice of the turbine to be investigated was not necessarily imposed by the availability of experimental data. A mesh-independent solution should indeed be achieved independently from experimental results [21], which can be compared only afterwards for validating purposes.

Therefore, the authors decided to perform the analysis considering a reference case represented by a hypothetical single-bladed rotor (having all other geometric features equal to those of the tested rotor presented in Table 1), where only the aerodynamic behavior of an undisturbed blade has to be solved. In this way, the interaction between different blades was not accounted for since it was thought not to be part of the main focus of the analysis. Specific attention was indeed put on a proper resolution of the flow around the

146 rotating airfoil in a curved flow-path. The refinement levels identified in the present study are anyhow fully
147 compatible with those selected for the three-blade full rotor [34].

148 Figure 2 shows the simulation domain, where all the boundary distances are given as a function of the
149 rotor diameter ($D=2*R$). The final dimensions of both the stationary and the rotating domains were defined
150 according to the sensitivity analysis reported in [11] in order to allow a full development of the turbine wake.
151 The same dimensions have indeed been used successfully by the authors in similar analyses (e.g. [40]).

152 The velocity-inlet boundary condition is supplied by the imposition of a uniform wind profile
153 considering an undisturbed speed of 8 m/s. The ambient pressure condition is instead imposed at the outlet
154 boundary. A symmetry condition was finally assigned to the lateral boundaries. The symmetry condition for
155 lateral boundaries is indeed the most common solution for this type of simulations; the authors have anyhow
156 demonstrated in [11] that the selected width of the domain is largely sufficient not to induce any influence of
157 the boundaries on the flow field around the turbine.

158 Figure 3 shows the torque characteristic prediction, obtained from the 2D CFD simulations at the
159 conclusion of the sensitivity analysis reported in the next section. The results are here anticipated to allow
160 the reader to understand the choices made by the authors in selecting the operating points to be analyzed.

161 The maximum torque output ($c_P=0.127$) is produced at a TSR=3.3. Positive torque outputs, although in
162 unstable conditions, are produced by the blade starting from TSR=1.7.

163 Based on previous studies establishing the necessity of defining different settings to be adopted
164 depending on the considered operating condition [11], four different tip-speed ratios were here selected for
165 the sensitivity analysis. In detail, two unstable conditions were considered, i.e. TSR=1.7 and TSR=2.2, in
166 addition to the peak point (TSR=3.3) and a stable operating point (TSR=4.4), corresponding to the 75% of
167 the maximum power output.

168 2.3 Design Points

169 To more clearly understand the points' selection, some details on the working conditions and flow
170 properties are given in this section, with the goal of identifying the presence of criticalities in the physical
171 functioning.

172 The instantaneous torque coefficient versus the blade angular position over a revolution (ϑ) is shown in
173 Figure 4. Starting from the angular position of zero incidence (blade aligned with the absolute wind flow), all
174 the four cases exhibit an increase of torque due to the increase of the blade lift, having approximately the
175 same slope. After reaching the peak values, two different working conditions are readily distinguishable:

- 176 • The stable points (TSR=3.3 and TSR=4.4) show a uniform decrease in the second quadrant (from
177 90° to 180°), followed by a constant and almost null torque extraction in the downwind section of the
178 rotor;
- 179 • The unstable points (TSR=1.7 and TSR=2.2) show a sudden drop before 90° , leading to negative
180 torque, which is caused by the decrease of the lift associated to the development of the stall.

181 The differences between the two conditions can be explained by comparing the vorticity field (ω) for all
182 the revolution speeds. The authors consider vorticity as the most representative quantity for the
183 determination of the level of complexity of flow structures. Indeed, high vorticity is produced when the
184 velocity gradients are large, i.e. when the flow quantities suffer from abrupt spatial variations. To capture
185 these structures, the size of the mesh elements must be reduced as the gradients increase.

186 For example, the angular position of $\vartheta=140^\circ$ was analyzed in Figure 5, nearly corresponding to the
187 location of the negative peak for TSR=1.7 and TSR=2.2. In both cases, the separation bubble gives rise to the
188 creation of a large vortex from the leading edge. Just after this vortex has detached, a further bubble starts
189 growing at the trailing edge. Conversely, the contours at TSR=3.3 and TSR=4.4 reveal the presence of a
190 thickened but stable wake, indicating that the flow is still attached to the blade surface.

191 The vorticity level was then evaluated over the entire revolution of the blade. A dimensionless form of
192 the vorticity ($\tilde{\omega}$), divided by the rotating speed Ω , was essential in order to be comparable throughout
193 different operating regimes (Eq. 1):

$$194 \quad \tilde{\omega} = \frac{\omega}{\Omega} \quad (1)$$

195 First, the maximum value of the dimensionless vorticity $\tilde{\omega}_{max}$ reached in the overall domain was
196 extracted, independently from the location. Figure 6 shows the trends of $\tilde{\omega}_{max}$ as a function of the blade
197 azimuthal position for the four TSRs. The highest intensities can be observed for the angular positions of
198 maximum torque output for each considered case. The maximum vorticity is therefore generated in condition

199 of attached flow, when the lift is maximized due to the highest accelerations of the flow following the
200 curvature of the blade profile. Thereafter, the reduction of $\tilde{\omega}_{max}$ follows the torque decrease, with minimum
201 values in case of vortices generation for lower revolution speeds.

202 At a first glance, these results may appear controversial and can lead to a misinterpretation of the
203 phenomenon, since $\tilde{\omega}_{max}$ decreases with the vortices onset. Actually, this outcome is due to the fact that $\tilde{\omega}_{max}$
204 is a local value of a single element, but is not representative of the overall flow structures, as the extent of
205 high-vorticity zones can be substantially altered in different operating conditions.

206 To overcome this aspect, the vorticity level was evaluated by means of an aggregate point of view
207 through the calculation of the extent of the regions at high $\tilde{\omega}$. Conventionally, it was assumed that the
208 vorticity is “high” when it is greater than the revolution speed by an order of magnitude, i.e. when $\tilde{\omega} > 10$. The
209 area covered by the fluid regions at high vorticity was computed along the whole blade revolution and it is
210 reported in Figure 7 in a dimensionless form $A_{\tilde{\omega}}$ (i.e. divided by the rotor area). The trends now agree with
211 expectations, since $A_{\tilde{\omega}}$ increases as the torque decreases due to stall phenomena, with the highest values
212 occurring across the 2nd and 3rd quadrants. It can be observed that the high-vorticity area grows for lower
213 angular velocities. In particular, a marked distinction between stable and unstable operating points is clearly
214 noticeable, with values that are more than quadrupled for the latter.

215 This introductory analysis was necessary to point out the modifications of the flow features throughout
216 the operating range of the studied rotor, in order to provide some preliminary considerations for an easier
217 comprehension of the outcomes shown in the following sections. The flow fields at low-TSR points show
218 indeed a notably higher degree of complexity, with an oscillating torque output caused by the alternate
219 separation of bubbles from the leading and trailing edges. It will be shown later that in these regimes the
220 discretization requirements are stricter in order to prevent the increase of the discretization errors.

221 2.4 Meshing and time-stepping strategies

222 It is well known that a CFD code solves the turbulent-flow by the discretization of the continuous space
223 and time into finite intervals. The continuous solution of the differential equations is replaced with discrete
224 values of the variables, which are computed at only a finite number of grid points. The introduced error must
225 be verified and minimized by systematically refining the grid size and time step.

226 The verification step is not equivalent to the validation step, as stated by Roache [42]. Verification means
227 “solving the equations right” while validation means “solving the right equations”. Theoretically speaking,
228 when the grid size and time step approach zero, the discretization error becomes negligible ensuring a correct
229 solution of the discretized equations. Therefore, the verification consists in reducing the error to an
230 acceptable level for the considered application. On the other hand, the suitability of the solved equations in
231 representing the physical problem of interest is the subject of validation. The results may not be accurate
232 because the selected models do not accurately represent the physical reality.

233 Once the main simulation settings have been assessed, the verification of both meshing and time-
234 stepping strategies then becomes the key point for a successful simulation.

235 It is worth remarking that, in a Darrieus VAWT simulation, the sudden variation of the flow conditions
236 on the airfoil during the revolution is responsible for a strong mutual influence between the temporal and
237 spatial characteristic scales. To correctly reproduce a flow structure, e.g. a stall vortex, both a fine mesh (to
238 capture the gradients) and a very small advance of the rotating frame (to avoid any undesired discontinuity of
239 the variables between two instants) are needed. As a consequence, it might be not sufficient to perform CFD
240 computations on a single fixed grid. A multivariate sensitivity analysis has to be carried out accounting for
241 the mesh features and the timestep. The difference in grid size and time step between two cases should be
242 finally sufficiently large to identify the differences in CFD results.

243 In the present application, the CFD domain discretization was obtained using an unstructured triangular
244 mesh, except for the use of a structured O-grid of quadrilateral cells in the boundary layer region to improve
245 the near wall accuracy [11]. The first cell height was imposed such as to guarantee that the y^+ values from
246 the flow solutions did not exceed the limit of the SST turbulence model, i.e. $y^+ \sim 1$. To ensure a high quality
247 of the mesh near the blade, the total height of the O-grid was set to 8 mm, i.e. equal to 3% of the chord. This
248 solution was considered adequate for the application since a boundary layer thickness of about 1.9 mm at
249 TSR=1.7 was estimated based on the blade Reynolds number. This is indeed the most precautionary
250 condition, since the thickness is furthermore reduced for higher revolution speeds.

251 Figure 8 shows the main details on the spatial discretization for the baseline and coarsest mesh (named
252 *MI*). In the stationary domain (Figure 8a) the grid density is coarsened from the rotor to the boundaries. A

253 size function was set in the wake region downwind the rotor to guarantee an appropriate grid refinement. The
 254 size of the elements at the sliding interface between rotating and stationary domain is equal to $0.1c$ on both
 255 size, corresponding to 540 nodes on the circumference (Figure 8b). Almost 550 nodes were placed on the
 256 blade surface for the $M1$ mesh (Figure 8c), adopting a clustering in the leading and trailing edges to provide
 257 the required refinement in regions characterized by higher curvature (Figure 8d and Figure 8e). The
 258 boundary layer is discretized with an extrusion of 40 layers of quadrilateral elements, having a growth rate of
 259 1.1. The number of nodes in which the airfoil is discretized (N_N) is pivotal for the determination of both the
 260 attack angle of the incoming flow on the blade and the boundary layer evolution from the leading edge to the
 261 trailing edge. Moreover, the discretization level adopted in the near-blade region also controls the total
 262 number of mesh elements (N_E), since the growth of the cell's size has to be accurately controlled. Starting
 263 from the $M1$ setup, a node density study was performed. Four additional meshes, as illustrated in Table 3,
 264 were created in order to examine the mesh independent solution for the studied VAWT, ranging from $1.3 \cdot 10^5$
 265 to $8.2 \cdot 10^5$ cells. The main parameters used to control the final mesh size were the resolution of the airfoil
 266 profile, by varying N_N and the resolution of the boundary layer, by varying progressively the rows' number
 267 of quadrilateral elements (N_{BL}). The growth ratio of the quad layers was reduced progressively in order to
 268 keep a constant total height of about 8 mm. Figure 9 displays a detail of the boundary layer discretization at
 269 the leading edge for meshes $M2$ to $M5$. The increase in the cell density is evident, since the average sizing of
 270 the elements on the blade profile (Δ_B) is dropped to one-fifth from the coarsest to the finest grid.

271 A sufficient temporal resolution is necessary to ensure an accurate unsteady simulation of the turbine.
 272 Different timestep sizes Δt were tested that are equivalent to specific rotational displacements along the
 273 azimuth $\Delta\vartheta$. The shortest Δt used was equal to 0.05 ms, corresponding to an azimuthal increment between
 274 two subsequent steps of 0.045° and 0.12° at 150 and 400 rpm respectively. The largest was 50 times bigger,
 275 i.e. 2.5 ms, corresponding to a $\Delta\vartheta$ of 2.25° and 6° at 150 and 400 rpm respectively.

276 Globally, ten values were tested: 0.00005 s, 0.000075 s, 0.0001 s, 0.0002 s, 0.0003 s, 0.0004 s, 0.0005 s,
 277 0.0008 s, 0.00125 s and 0.0025 s.

278 3. Sensitivity analysis results

279 The complete set of simulations for a full sensitivity analysis would have globally required 200 runs,
 280 resulting from the combination of four revolution speeds, five meshes and ten timesteps. Not all the
 281 combinations were however simulated. In detail, some intermediate values were not considered if the
 282 independency was already achieved. Similar considerations were also applied at high revolution speeds,
 283 where the flow conditions are more favorable and independency is soon reached, so that the shortest
 284 timesteps and the finest grid ($M5$) were not used.

285 The assessment of the mesh and timestep independency is generally carried out in literature studies by
 286 simply monitoring the average torque (or torque coefficient) output. As an aggregate parameter, however, it
 287 could be deemed to hide differences between the simulations, due to undesired compensation between
 288 different zones of the torque profile. On this basis, the settings assessment was based both on the final
 289 average torque coefficient value (c_T) and on an evaluation of the matching of torque profiles. This latter
 290 aspect was addressed making use of the coefficient of determination R^2 [43], here defined as:

$$291 \quad R^2 = 1 - \frac{\sum_{\vartheta=0^\circ}^{360^\circ} (c_{T@ \vartheta} - c_{T_ref@ \vartheta})^2}{\sum_{\vartheta=0^\circ}^{360^\circ} (c_{T@ \vartheta} - c_{T_ave})^2} \quad (2)$$

292 A reference case (identified as “*ref*”) was selected as the baseline model to which the torque variations
 293 can be compared. The instantaneous torque of the reference case ($c_{Tref@ \vartheta}$) thus corresponds to the maximum
 294 refinement level for each rotating speed. c_{T_ave} represents the average torque coefficient over a revolution at
 295 the investigated tip-speed ratio.

296 The criteria adopted in the identification of the independent solution are based on the following
 297 thresholds with respect to the reference case:

$$298 \quad |c_{T_ave} - c_{T_ref_ave}| < 0.01 \cdot c_{T_ave_max} \quad (3)$$

$$299 \quad R^2 > 99.9\% \quad (4)$$

300 where the first condition imposes a difference between the average torques lower than 1% of the
 301 maximum torque at $TSR=3.3$ (max).

302 3.1 TSR=1.7

303 Figure 10 illustrates the results of the sensitivity analysis at TSR=1.7 in terms of both the torque
304 coefficient over a revolution and the coefficient of determination as a function of the global cells number,
305 which is expressed as a ratio to the elements number of the coarsest mesh (i.e. $N_E/N_{E,M1}$). The reference
306 torque for the evaluation of the R^2 is obtained with the $M5$ mesh and the timestep of 0.00005 s,
307 corresponding to a $\Delta\theta$ of 0.045° .

308 Upon examination of Figure 10a, one could readily notice that a grid independent behavior is achieved
309 with the $M4$ mesh, since no remarkable variation in the average torque output is detectable with the $M5$
310 refinement. The analysis in terms of R^2 reveals that the matching between the torque profiles is not
311 completely satisfactory, as also confirmed by Figure 11, where the instantaneous torque coefficient is plotted
312 for the $M3$, $M4$ and $M5$ meshes with analogous temporal discretization ($\Delta t = 0.000075$ s). Especially in the
313 second quadrant, where the torque is fluctuating due to stall, the local peaks are not perfectly corresponding
314 in terms of both amplitude and phase. The influence of the timestep can be pointed out analyzing Figure 10a.
315 It is readily noticeable that the results converge for a timestep less or equal to 0.000075 s, since an oscillatory
316 convergence is detected for $\Delta t = 0.0001$ s. A further increase of the timestep leads to an underestimation of
317 the torque output. The R^2 trends and the torque coefficient profiles of Figure 12 further demonstrate that the
318 use of the two smallest timesteps is equivalent. A slight discrepancy can be noticed only with a 0.0001 s or
319 greater Δt . In general, the influence of both the mesh and the timestep show a discontinuous behavior: the
320 results are almost consistent when the spatial and temporal resolution is sufficiently refined. As the
321 resolution is coarsened below a specific limit, an abrupt change in the dynamic response is observed.

322 3.2 TSR=2.2

323 The mesh size and timestep effects at TSR=2.2 are shown in Figure 13. The maximum refinement level
324 was again the configuration with the $M5$ mesh and the timestep of 0.00005 s, corresponding to an azimuthal
325 increment $\Delta\theta$ of 0.06° .

326 Focusing on the configurations with a timestep equal or greater to 0.0002 s (i.e. $\Delta\theta = 0.24^\circ$), the curves
327 of both c_T and R^2 diverge as the mesh elements size is reduced. The largest timestep ensuring a grid
328 independent behavior is 0.000075 s, and the use of the $M4$ mesh is sufficient to guarantee a reliable
329 estimation of the torque extraction, since the torque coefficient is perfectly predicted and $R^2 = 99.95\%$.

330 The above results clearly show that the coupling between the grid independency and the timestep
331 independency studies is necessary, due to the mutual influence of these two parameters on the accuracy and
332 stability of the results. It is indeed impossible to perform a mesh sensitivity analysis assuming a fixed value
333 of the timestep: if the value is too large, it is unsuitable to establish accurate results.

334 Figure 14 furthermore proves that choosing the “right” mesh ($M4$) with a “wrong” timestep does not
335 allow to correctly capturing the flow structures, since the agreement of the torque extraction profiles with a
336 timestep greater than 0.0001 s is not satisfactory. Especially in the second quadrant, the oscillation due to the
337 stall vortices is not adequately reproduced.

338 3.3 TSR=3.3

339 The outcomes of the sensitivity analysis at TSR=3.3 are resumed in Figure 15. As discussed, thanks to
340 the stable working conditions, it was here possible to avoid the use of the finest grid ($M5$) and to limit the
341 shortest tested timestep to 0.0001 s, i.e. $\Delta\theta = 0.18^\circ$.

342 As a matter of fact, the mesh and timestep independency is achieved with a lower refinement level, both
343 in space and time. Figure 15b indicates that all cases with a temporal discretization smaller than 0.0005 s
344 show a satisfactory matching in terms of torque distribution. Indeed, the R^2 values are greater than 99.98%,
345 largely above the tolerance threshold. Notwithstanding this, the diverging behavior of the c_T curves (Figure
346 15a) as the timestep increases is even more evident than the two previous analyzed cases. Using a large
347 timestep with a fine grid ($M4$) produces less accurate results than a coarse grid ($M2$), since the average
348 torque underestimation is greater and the matching between the curves is of poorer quality (lower R^2).

349 The instantaneous torque coefficient curves of Figure 16, plotted for different values of Δt with the same
350 mesh $M4$, are useful to understand the tendencies highlighted in Figure 15. By comparing the simulations
351 with the largest and the smallest timesteps (0.0008 s and 0.0001 s respectively), one can notice that the trend
352 is correctly reproduced (high values of R^2), but a constant slight underestimation can be observed in the
353 second and fourth quadrants, which is also responsible for the c_T underestimation.

354 3.4 TSR=4.4

355 Finally, the attention was focused on the simulations performed at the highest considered functioning
356 condition, i.e. TSR=4.4. Analogous to TSR=3.3 case, only four mesh refinements were analyzed, from mesh
357 *M1* to mesh *M4*, as well as the timestep was limited to 0.000075 s, corresponding to a $\Delta\theta = 0.18^\circ$ angular
358 timestep.

359 From a perusal of Figure 17, it is readily noticeable that the results are more consistent in terms of both
360 c_T and R^2 . The main difference with respect to all of the three previous cases is that this is the only working
361 condition stable enough to achieve a mesh independent behavior independently for the timestep. The torque
362 curves obtained with the meshes *M2*, *M3* and *M4* are almost coincident for each Δt considered. The
363 differences in terms of R^2 are less pronounced, therefore the configuration with *M2* and $\Delta t = 0.0002$ s is
364 assumed to be the optimal setup.

365 These aforementioned results globally show that a grid independency study must be necessarily
366 performed accounting also for the influence of the timestep, since the azimuthal increment between two
367 subsequent steps of analysis must be small enough to correctly describe every flow structure. Indeed, the
368 authors showed in [11] that in a transient calculation, if a reduction of the elements size is not combined with
369 a reduction of the timestep, the solution tends to become instable due to increase of the Courant Number
370 (Co) (Eq.2):

$$371 \quad Co = V \frac{\Delta t}{\Delta x} \quad (5)$$

372 The Courant Number expresses the ratio between the temporal timestep (Δt) and the time required by a
373 fluid particle moving with V velocity to be convected throughout a cell of dimension Δx .

374 Table 4 shows the selected mesh for each tested speed, along with the required timestep, expressed in
375 terms of both temporal and angular increments.

376 To summarize the main outcomes of the analysis, it was found that the discretization requirements can be
377 split into two different families, being the maximum torque speed approximately the boundary line. The
378 requirements for a calculation at a revolution speed higher than the limit, i.e. in the stable part of the torque
379 characteristic, are not extremely severe: in the present application, the grid density of the mesh *M2* (~800
380 nodes on the airfoil) was sufficient to guarantee accurate results and to correctly describe the torque profile if
381 it is simulated with an angular timestep of approximately $\Delta\theta = 0.5^\circ$. The temporal timestep must be
382 accordingly scaled, becoming directly proportional to the revolution speed of the rotor.

383 Focusing on the unstable part of the torque curve, the motion structures suffer of a sudden change,
384 leading to the enlargement of the high-vorticity region. The intensification of velocity and pressure gradients
385 imposes more strict requirements in terms of spatial discretization, as the mesh elements have to be small
386 enough to capture the vortices onset. The more intense are the vortices to be captured, the finer must be the
387 mesh, as confirmed by the necessity of adopting the *M5* mesh at TSR=1.7. The temporal discretization was
388 found to be broadly constant and drastically reduced, resulting in values lower than 0.1° in terms of
389 azimuthal increment between two steps.

390 4. Dimensionless Numbers

391 4.1 Grid-Reduced Vorticity

392 The findings reported in previous sections highlighted some conclusions that are of general validity in
393 the analysis of the unsteady aerodynamics of VAWTs, i.e.:

- 394 • The intensity of the vorticity field in the region surrounding the blade increases dramatically as the
395 rotating speed is reduced, due to the unstable working conditions;
- 396 • To avoid the increase of discretization errors, finer grids are required to correctly describe the higher
397 gradients of the flow quantities.

398 Although the grid independency study was useful to assess the most suitable mesh requirements for each
399 TSR, it is not sufficient to understand the relationship between the physical phenomena and the discretization
400 requirements in terms of cell dimensions. In this view, the authors decided to further assess the acceptability
401 of the selected mesh in capturing the flow structures by performing a systematic analysis of the results.
402 More specifically, the purpose was to define a quantitative parameter to be used as an indicator of the
403 suitability of the mesh refinement level. This parameter was deemed to correlate the flow properties, in terms

404 of gradients, to the mesh properties. The definition of a quantitative method in fact allows one to evaluate a
 405 priori the grid related error, without the necessity of performing an extensive and expensive analysis as the
 406 one shown in Paragraph 4.

407 The vorticity was considered as the most appropriate flow metric in quantifying the gradients. For 2-D
 408 flow fields it describes the rotation rate of a small fluid element about its vertical axis (Eq. 6):

$$409 \quad \vec{\omega} = \left(\frac{\Delta V_y}{\Delta x} - \frac{\Delta V_x}{\Delta y} \right) \vec{k} \quad (6)$$

410 Vorticity at a point within a flow is zero in case of translation and linear or angular deformation, i.e. in
 411 case of potential flows. On the other hand, complex non-uniform flow patterns (shear layers, transverse
 412 flows, eddies, etc.) are characterized by nonzero vorticity. Consequently, vorticity is a physically meaningful
 413 metric for measuring these spatially varying flows.

414 The *Grid-Reduced Vorticity (GRV)* was introduced. The vorticity magnitude was rewritten in a
 415 dimensionless form through a proper scaling using characteristic length (L_0) and velocity (V_0) scales as
 416 follows (Eq. 7):

$$417 \quad GRV = \frac{\omega}{V_0/L_0} = \frac{L_0}{V_0} \left(\frac{\Delta V_y}{\Delta x} - \frac{\Delta V_x}{\Delta y} \right) \quad (7)$$

418 Since *GRV* represents a local quantity, the local element length and the local velocity were used as length
 419 and velocity scales, respectively. The element length was chosen as a parameter representing the
 420 discretization level. The local velocity was chosen since a ΔV_i variation has a higher relevance for the regions
 421 characterized by lower velocity.

422 The meaning of such a dimensionless parameter can be qualitatively explained by evaluating *GRV* for the
 423 simplified two-dimensional case of Figure 18. In the example, two adjacent square elements having the same
 424 size Δ and a general orientation in the X-Y plane are considered. The distance between the centroids of the
 425 two elements (Eq. 8) can be expressed as:

$$426 \quad \Delta = \sqrt{\Delta x^2 + \Delta y^2} \quad (8)$$

427 The velocity V_1 in the centroid of the first element is supposed to be aligned with the centroid of the
 428 second element. The velocity V_2 in the centroid of the second element differs from V_1 in both directions by
 429 generic amounts ε_x and ε_y (Eq. 9).

$$430 \quad \begin{cases} V_{x2} = V_{x1}(1 + \varepsilon_x) \\ V_{y2} = V_{y1}(1 + \varepsilon_y) \end{cases} \quad (9)$$

431 For the first centroid (Eq. 10) *GRV* can be calculated as:

$$432 \quad \omega_{BB} = \frac{\Delta}{V_1} \left(\frac{\Delta V_y}{\Delta x} - \frac{\Delta V_x}{\Delta y} \right) = \frac{\Delta}{V_1} \left(\frac{V_{y2} - V_{y1}}{\Delta x} - \frac{V_{x2} - V_{x1}}{\Delta y} \right) \quad (10)$$

433 The elements are supposed to form a 45° angle with both coordinate system axes, leading to the
 434 following further simplifications (Eq. 11):

$$435 \quad \begin{cases} \Delta x = \Delta y = \frac{\Delta}{\sqrt{2}} \\ V_{x1} = V_{y1} = V_1 \cos(45^\circ) = \frac{\sqrt{2}}{2} V_1 \end{cases} \quad (11)$$

$$436 \quad GRV = \frac{\Delta}{V_1} \left(\frac{V_{y1}(1 + \varepsilon_y) - V_{y1}}{\frac{\Delta}{\sqrt{2}}} - \frac{V_{x1}(1 + \varepsilon_x) - V_{x1}}{\frac{\Delta}{\sqrt{2}}} \right) = \frac{\sqrt{2}}{V_1} \left(\frac{\sqrt{2}}{2} V_1 \varepsilon_y - \frac{\sqrt{2}}{2} V_1 \varepsilon_x \right) = \varepsilon_y - \varepsilon_x \quad (12)$$

438 The worst condition is when $\varepsilon_y = -\varepsilon_x = \varepsilon$, which leads to (Eq. 13):

$$439 \quad GRV = 2\varepsilon \quad (13)$$

440 Basically, *GRV* gives an estimate of the velocity variation within a single element; therefore it represents
 441 the capability of the mesh itself of correctly computing the flow features. Under these particular
 442
 443
 444

445 simplifications, GRV corresponds to one when a 50% velocity variation between two adjacent cells occurs. It
446 is clear that errors in a computational cell arise when the local value of GRV is too high, while accurate
447 results can be achieved only when GRV is substantially smaller than one. The mesh sensitivity analysis and
448 the evaluation of GRV are therefore strictly related: grid independent results are obtained when the
449 discretization error becomes irrelevant, i.e. when GRV is “sufficiently” small.

450 In order to assess the criteria for guaranteeing that the mesh refinement is “sufficient”, the results of the
451 mesh sensitivity analysis were purposefully post-processed. The Grid-Reduced Vorticity was introduced as
452 an additional flow quantity to be evaluated in the entire computational domain during the rotor revolution.
453 Large GRV values were found only in the flow region surrounding the blade, due to the more complex and
454 intense flow structures. The regions away from the moving wall are generally characterized by low gradients
455 in relation to the size of the mesh elements.

456 The attention was focused on the region surrounding the blade within a distance of $0.5c$ from the blade
457 wall, being the more critical for the definition of the elements size and then that with the biggest impact on
458 the discretization errors. This region normally includes the flow structures with highest vorticity such as
459 eddies, wakes and boundary layers.

460 The average value of the GRV (GRV_{ave}) was computed in the selected area for all of the tested cases.
461 Figure 19 reports the trend of GRV_{ave} as a function of the blade azimuthal position for all of the four
462 considered regimes with analogous discretization properties. In particular, the $M2$ mesh was considered since
463 it was able to provide grid independent results only for high TSR, i.e. 3.3 and 4.4. The temporal
464 discretization does not have an influence in the present investigation; therefore, the timestep will not be
465 specified and discussed. The figure clearly explains the reason why the $M2$ mesh is not suitable in the case of
466 low rotating speeds: GRV_{ave} values in the order of 0.02 imply the presence of large areas characterized by
467 high discretization error. In roughly half of the region within a distance of $0.5c$, the velocity variation
468 between two subsequent cells is greater than 1%. On the contrary, a completely different range of GRV_{ave}
469 was obtained for $TSR=3.3$ and $TSR=4.4$, since the maximum values do not exceed the limit of 0.005 and it is
470 mostly lower than 0.003. Globally, the curves at $TSR=1.7$ and $TSR=2.2$ have values greater than the curves
471 at $TSR=3.3$ and $TSR=4.4$ for each angular position during one revolution.

472 To understand the influence of different refinement levels on the results in terms of GRV_{ave} , Figure 20
473 shows the GRV_{ave} trend for the simulations with four different meshes (from $M2$ to $M5$) at $TSR=1.7$. The
474 GRV_{ave} is reduced almost proportionally to the reduction of the average sizing of the elements on the blade
475 profile. Using the $M5$ mesh the values are lower than 0.005 for almost half of the revolution, with a
476 maximum peak slightly greater than 0.01.

477 The GRV_{ave} trends, considering the selected mesh as indicated in Table 4 for each TSR, are displayed in
478 Figure 21. The results are now consistent throughout the regimes, showing similar order of magnitude during
479 the largest part of the revolution. The maximum values are limited within $GRV_{ave}=0.01$, which can be
480 considered as a criterion to follow in order to estimate and reduce the source of grid-related errors.

481 From a theoretical point of view, the analytical solution of the system of partial differential equations is
482 approached with a refinement of the space discretization. The choice of the degree of resolution to discretize
483 the space is usually achieved through a grid convergence study: at least, three solutions on just as many
484 systematically refined grids are necessary. Moreover, as shown in previous sections, space and time
485 discretization have a mutual influence, with a notable increase in the number of runs to be carried out.

486 Although it is not practical to recommend in advance the most appropriate grid sizing (highly problem-
487 dependent), the proposed approach can be useful to evaluate a priori the mesh quality.

488 The suggested best practice to reduce the computational effort would be to calculate GRV for an initial
489 guess mesh, in order to be able to directly adjust the mesh size accordingly to proposed criterion. The mesh
490 resolution should then satisfy the following requirement during the revolution (Eq. 14):

491
492
$$GRV_{ave} < 0.01 \tag{14}$$

493
494 This verification is independent of the choice of the temporal timestep and does not require an exhaustive
495 mesh sensitivity study.

496 It is interesting to evaluate also the local distribution of GRV , to identify the most relevant contribution to
497 the source of errors. For each tested speed, the attention was focused on the angular position of maximum
498 GRV_{ave} . The local GRV values were computed in the region within a $0.5c$ distance from the blade and the
499 frequency distribution in terms of cumulative area was calculated and reported in Figure 22. As expected, the
500 spatial extent of the region with GRV greater than the proposed limit of 0.01 is largest at $TSR=1.7$, covering

501 almost 40% of the area. The extension of the area at $GRV > 0.01$ drops to almost 17% at $TSR = 2.2$ and less
502 than 10% at $TSR = 3.3$ and 4.4 . This is the motivation of the highest peak shown in Figure 21.
503 Notwithstanding this, the trend is inverted when considering the regions at high GRV . In particular, roughly
504 5% of the area is characterized by $GRV > 0.04$ for $TSR = 1.7$, $TSR = 3.3$ and $TSR = 4.4$, while the area
505 corresponds to 10% for $TSR = 2.2$. Moving to the extreme, the area at $GRV = 0.2$ is lower than 0.05% at
506 $TSR = 1.7$ and $TSR = 2.2$, while is in the order of 0.15% at $TSR = 3.3$ and $TSR = 4.4$.

507 In addition, Figure 23 shows the GRV field for the same cases depicted in Figure 5. For each rotating
508 speed, the results are displayed considering the final configuration mesh. Analogous levels of GRV can be
509 observed throughout the different regimes, whereas the vorticity levels of Figure 5 were not consistent. In
510 more detail, although the highest vorticity magnitude was observed in the wake region for $TSR = 3.3$ and
511 $TSR = 4.4$, the reduced element size in proximity of the blade wall guarantees low GRV values. On the
512 contrary, the vorticity magnitude of the large eddy originated from the leading edge at $TSR = 1.7$ is
513 substantially lower but the greater local length of the mesh elements leads to almost equal GRV levels.

514 Therefore, it is clear that the main issue for a correct computation of the flow structures at low $TSRs$ is to
515 generate a computational grid characterized by small element size in a wide region surrounding the blade, in
516 order to include the detaching vortices. Conversely, at high $TSRs$ is sufficient to accurately discretize the
517 boundary layer and the wake region. In particular, Figure 24 shows that the peaks of GRV at $TSR = 3.3$ are
518 located on the leading edge, where the acceleration of the flow is maximum, and downstream the leading
519 edge.

520 Finally, Figure 25 reports the GRV field at $TSR = 1.7$ for eight different azimuthal positions along a
521 complete rotor revolution. The second and third quadrants are the most critical for an accurate resolution
522 since the dynamic stall phenomena involve large regions with significant gradients. Therefore, the grid
523 coarsening with respect to the wall-closest grid is responsible of high values of GRV at distances even greater
524 than one chord from the blade surface.

525 4.2 Courant Number

526 Once the spatial discretization has been assessed based on the analysis of the GRV , the proper timestep
527 must be identified for each TSR , in order to ensure accurate results of the unsteady simulations. As discussed
528 by the authors in two previous works ([11] and [34]), the Courant number (Co) analysis can provide the
529 correct guideline for this selection. Based on its formulation (Eq. 5), this number expresses the ratio between
530 the temporal timestep (Δt) and the time required by a fluid particle moving with V velocity to be convected
531 throughout a cell of dimension Δx .

532 While in case of explicit schemes for temporal discretization the Courant-Friedrichs-Lewy (CFL)
533 criterion imposes a limit on the maximum allowed value of Co (i.e. $Co < 1$ [44]-[46]) to ensure the stability of
534 the calculation, implicit methods are thought to be unconditionally stable with respect to the timestep size
535 [25]. Although theoretically valid if the problem is studied with a linear stability analysis, when the timestep
536 is increased non-linearity effects would become prominent and oscillatory solutions may occur. On these
537 bases, the literature indicates that an operational Co between 5 and 10 for viscous turbomachinery flows,
538 solved with an implicit scheme, provides the best error damping properties ([45] and [46]).

539 According to Balduzzi et al. [11], in case of Darrieus VAWTs a specific analysis is suggested on the
540 Courant Number conditions in proximity of the blades, as a correct description of the flow in these zones is
541 in fact deemed to be the most restrictive requisite to accurately predict the torque output of the rotor.

542 As a general remark, the results of Table 4 highlighted that remarkably finer meshes are needed as long
543 as the TSR is reduced, in order to correctly capture the intensity and the extension of strong-gradients zones
544 in the flow. According to Eq. 5, in order to contain the Courant Number it has to be expected that the also the
545 timestep has to be reduced with TSR .

546 In particular, in order to define some general guidelines for the time-stepping strategy, a reference
547 Courant Number (Co^*) has been here defined, assuming that in Eq. 5 the reference length is represented by
548 the average nodes distance along the airfoil and the reference velocity is the peripheral speed of the airfoil.

549 Based on the present definition, Table 5 reports the resulting Co^* for the four investigated $TSRs$ using the
550 final settings obtained by the sensitivity analysis.

551 Upon examination of the table, it is worth noticing that a clear trend was highlighted. In particular, at low
552 $TSRs$, in which the presence of strong gradients makes the resolution requirements more strict, the Co^* must
553 be contained in the order of 5, whereas an increase up to approximately 10 can be tolerated at higher $TSRs$,
554 where the flow is mainly attached.

555 In detail, the Co^* reduction at unstable regimes is mainly due to the onset of flow phenomena at multiple
556 frequencies (e.g. vortices detaching from the blades in the 2nd and 3rd quadrants), which therefore require a
557 different temporal resolution. For example, in Figure 26 the vorticity contours of a blade at TSR=2.2
558 between $\vartheta=114.3^\circ$ and $\vartheta=157.5^\circ$ are displayed consequently.

559 From Figure 26, it is apparent that, for high AoAs, the blade starts experiencing a vortex shedding quite
560 similar to that of a bluff body. In the present case, a characteristic frequency of approximately 14 Hz
561 (corresponding to a period of 0.07 s) was noticed both at TSR=1.7 and at TSR=2.2.

562 It was here then supposed that the required timestep reduction was mainly needed to correctly describe
563 this additional phenomenon, which introduced a frequency notably higher than the revolution speed. The
564 selected timestep of 0.000075 s, in particular, allowed the description of each single vortex shedding cycle
565 with approximately 1000 timesteps, which are in perfect agreement with the best literature prescriptions (e.g.
566 [47]).

567 As a final remark, the selection of the proper Courant Number (or Co^*), however, requires a specific
568 attention. In general, the Co should indeed be reduced as far as possible. Low Courant numbers can be,
569 however, easily achieved by increasing the element size (Δx in Eq. 5), but the coarsening of the mesh is
570 thought, on the other hand, to worsen the accuracy of the simulation. A time-step selection based on the Co^*
571 must therefore be carried out only after the mesh requirements definition, e.g. based on the GRV .

572 5. Conclusions

573 In the paper, a systematic analysis has been carried out to define a robust strategy for the assessment of
574 the meshing and time-stepping requirements in the CFD simulation of a Darrieus wind turbine. The spatial
575 and temporal discretization are indeed two of the most crucial sources of numerical error, due to the
576 difference between the exact solution of the analytical system of partial differential equations and the
577 numerical solution obtained with finite discretization. In the case of vertical-axis wind turbines, the proper
578 selection of the discretization strategy is even more complex because different aerodynamic phenomena have
579 to be described depending on the tip-speed ratio.

580 A study-case having a single blade was first derived from a real rotor, which was successfully simulated
581 in the past and verified with experiments. The optimal settings in terms of mesh and timestep were then
582 defined by means of a cross-coupled sensitivity analysis, which was pushed up to the physical limits of the
583 problem with no limitations imposed by the computing resources. Four functioning regimes were
584 investigated, corresponding to tip-speed ratios of 1.7, 2.2, 3.3 and 4.4.

585 Once the optimized settings for each TSR were defined, the computed flow fields were analyzed to
586 understand the main challenging phenomena for the simulation assessment. In particular, the extension and
587 intensity of high-vorticity zones were supposed to be the most requiring elements for the mesh refinement.

588 To verify this assumption, a dimensionless number was proposed, representing a dimensionless
589 expression of vorticity. Based on its definition, GRV in fact quantifies the velocity variation between two
590 cells. This new dimensionless number, calculated for different zones of the flow field, showed that low TSR
591 regimes are characterized by higher levels of vorticity in a larger part of the flow around the blades. In order
592 to describe correctly the gradients in those zones, finer meshes are therefore required. In particular, upon
593 comparison of the four optimal settings, it was found that average GRV within a proper mesh should be not
594 higher than 0.01 (i.e. $GRV_{ave} < 0.01$), corresponding to a maximum velocity variation between two adjacent
595 cells of $5 \cdot 10^{-3}$.

596 Once the mesh requirements have been assessed using the proposed criterion, the selection of the proper
597 timestep was again connected to the definition of the dimensionless number Co^* , i.e. a generalized version of
598 the Courant number based on the average elements length on the airfoil and the peripheral speed.
599 Consistency was again found between the results, highlighting the need of smaller timesteps at low TSRs,
600 where the presence of largely separated regions becomes more frequent. In particular, it was found that the
601 optimal timestep was that ensuring approximately 1000 points within the period of the vortex shedding
602 established on the blades for high AoAs in the second and third quadrants of the turbine.

603 In conclusion, the integrated approach presented in the paper, based on dimensionless numbers, is
604 thought to allow the assessment of the mesh and timestep requirements in the CFD simulation of a Darrieus
605 wind turbine. In this view, it is supposed to provide in the near future an important contribution to the
606 numerical analyses on these machines by setting a standard for these simulations and contemporarily
607 reducing the computational costs due to the preliminary sensitivity analyses.

608

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609

610

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611

7. Nomenclature

612

Acronyms

613

AR

Aspect Ratio

614

CFD

Computational Fluid Dynamics

615

CFL

Courant, Friedrichs and Levy criterion

616

GCI

Grid Convergence Index

617

GRE

General Richardson Extrapolation

618

SST

Shear Stress Transport

619

U-RANS

Unsteady Reynolds-Averaged Navier-Stokes

620

VAWT

Vertical-Axis Wind Turbine

621

622

Greek symbols

623

 Δ

Cell Dimension

[m]

624

 Δ_B

Average sizing of the elements on the blade profile

[m]

625

 $\Delta\theta$

Azimuthal Angle Increment

[deg]

626

 Δt

Temporal Timestep

[s]

627

 ε

Velocity Variation

628

 ϑ

Azimuthal Angle

[deg]

629

 σ

Turbine's Solidity

630

 ω

Vorticity

[s⁻¹]

631

 $\tilde{\omega}$

Dimensionless Vorticity

632

 Ω

Revolution Speed

[rad s⁻¹]

633

634

Latin symbols

635

 $A_{\tilde{\omega}}$

Dimensionless High Vorticity Area

636

 c

Blade's Chord

[c]

637

 c_T

Torque Coefficient

638

 c_P

Power Coefficient

639

 Co

Courant's Number

640

 Co^*

Reference Courant Number Based on Peripheral Speed

641

 y^+

Dimensionless Wall Distance

642

 D

Turbine's Diameter

[m]

643

 GRV

Grid-Reduced Vorticity

644

 L_0

Length Scale

[m]

645

 N_{BL}

Number of layers in the boundary layer

646

 N_N

Number of nodes on blade profile

647

 N_E

Total number of mesh elements

648

 R

Turbine's Radius

649

 R^2

Coefficient of determination

650

TSR

Tip-Speed Ratio

651

 V

Velocity

[m/s]

652

 V_0

Velocity Scale

[m/s]

653

8. References

654

655

656

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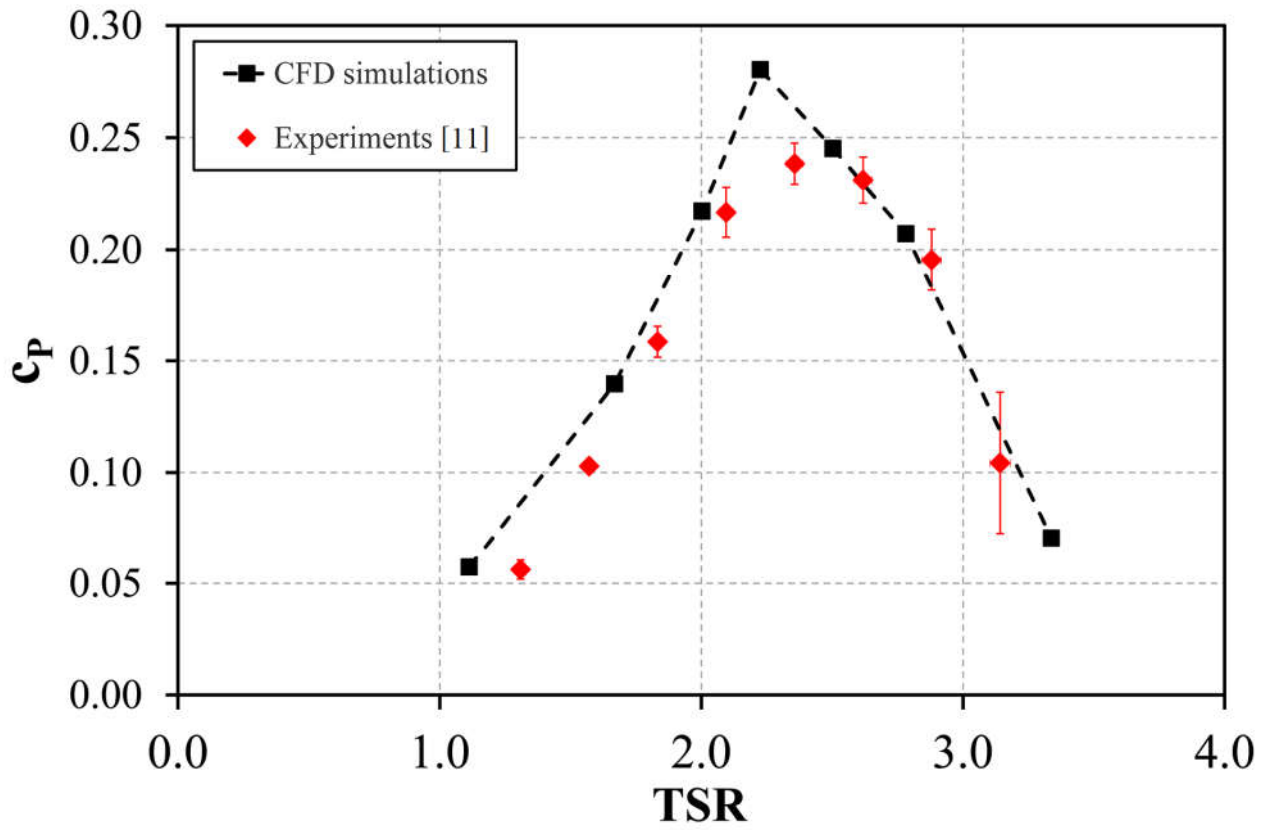
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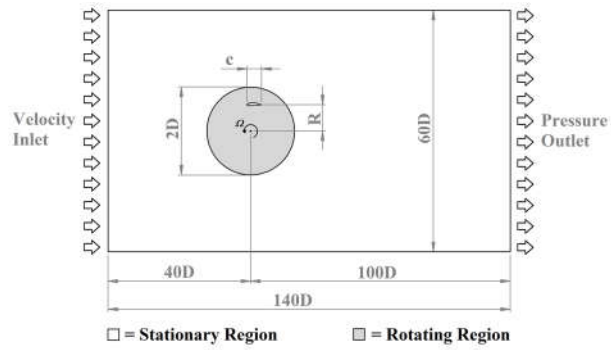
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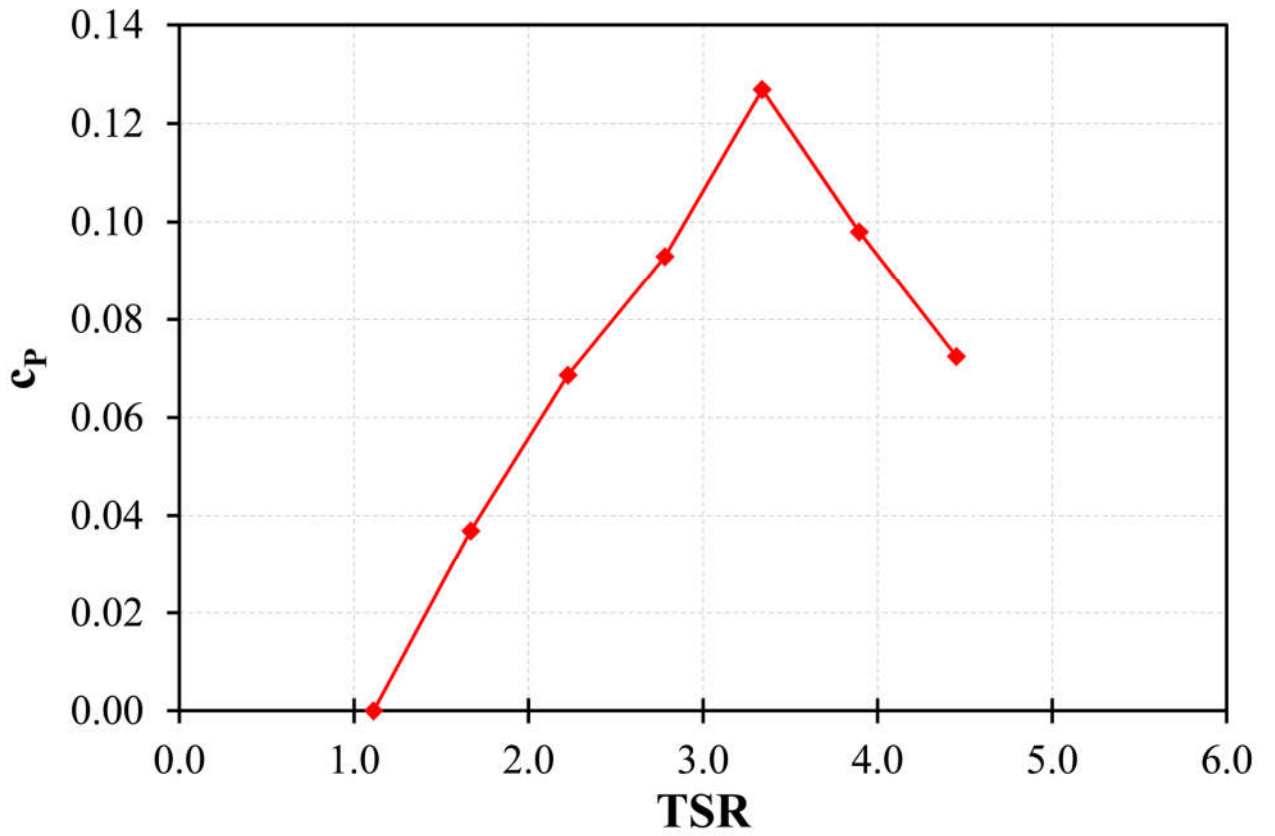
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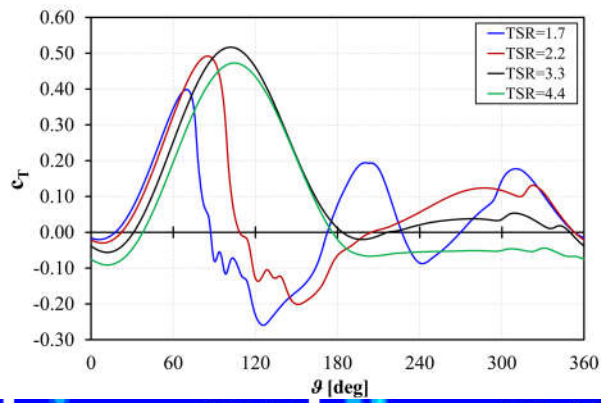
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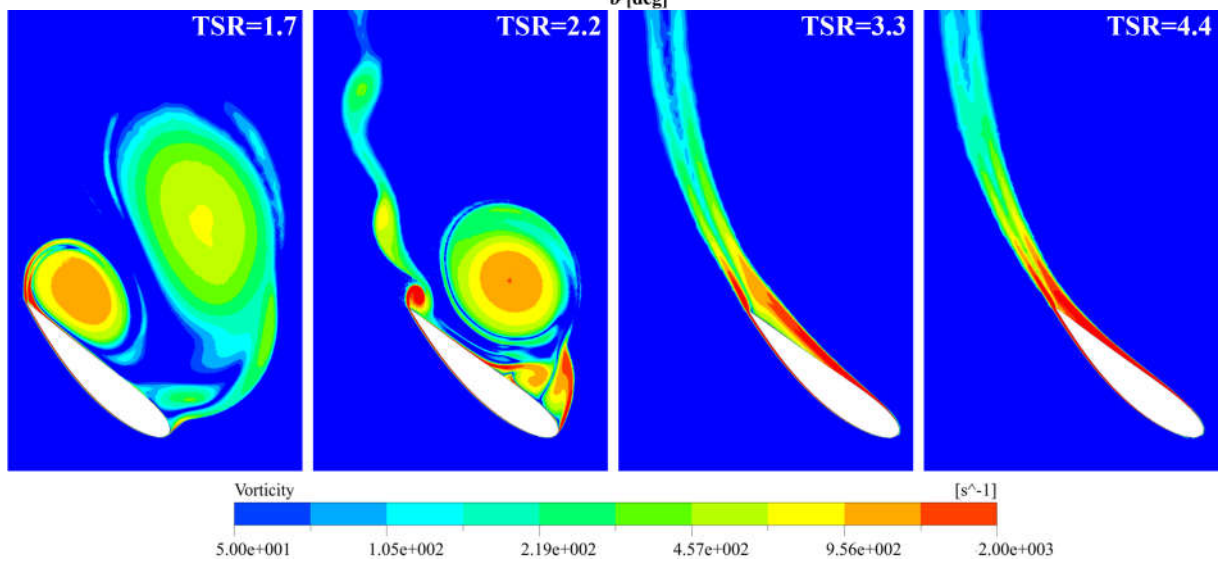
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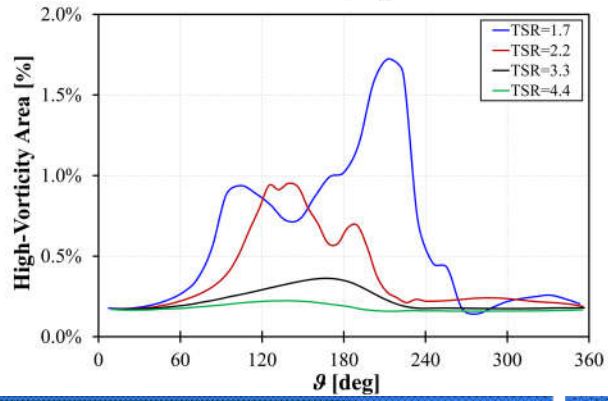
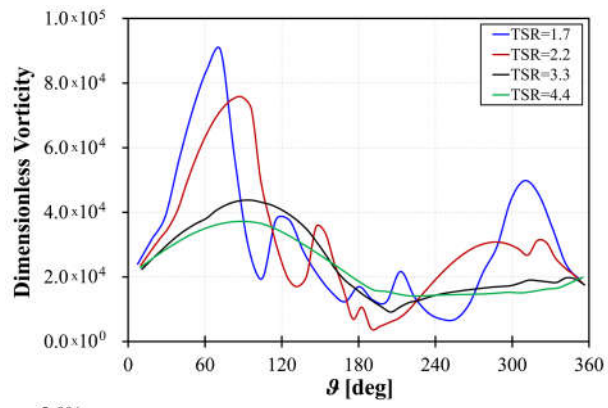
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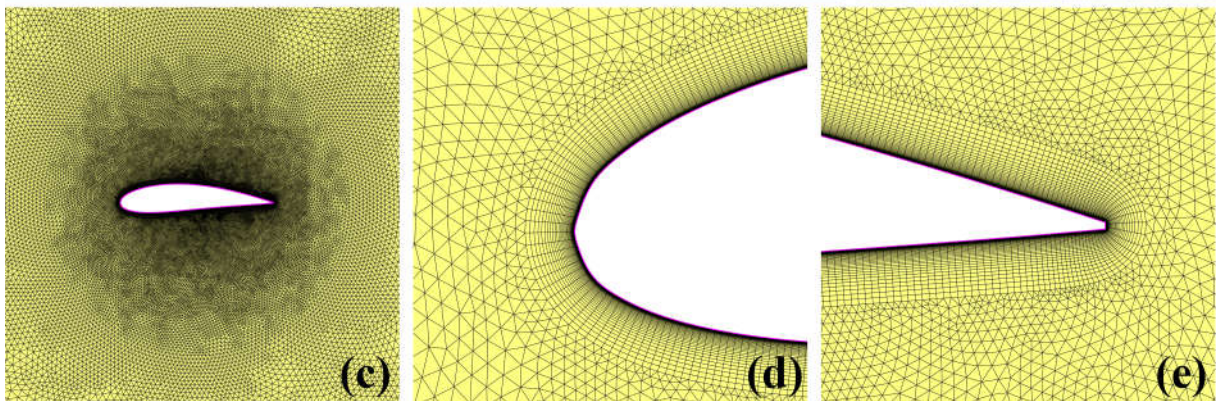
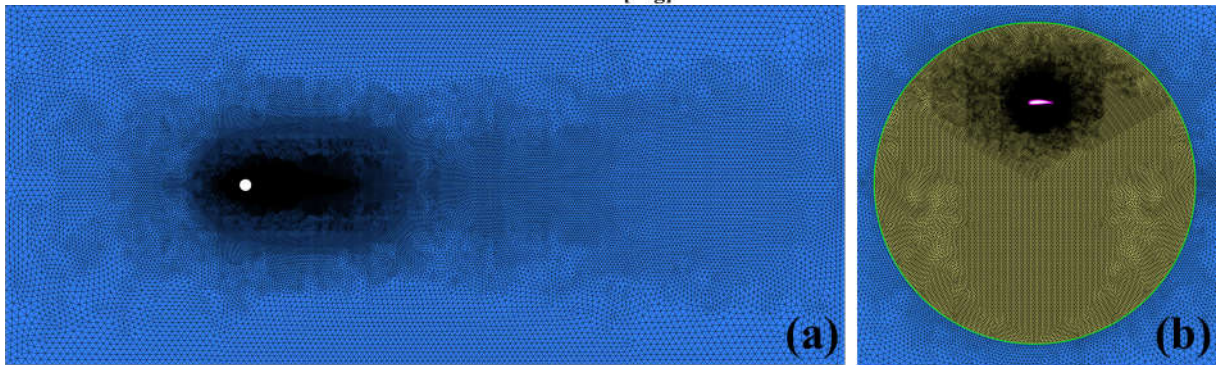


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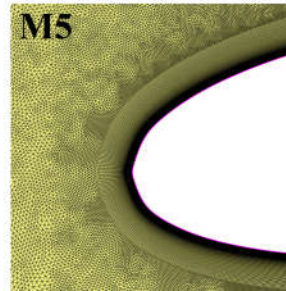
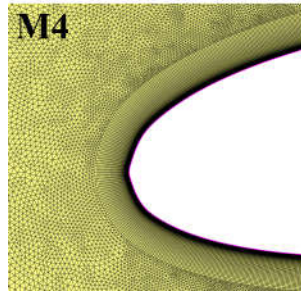
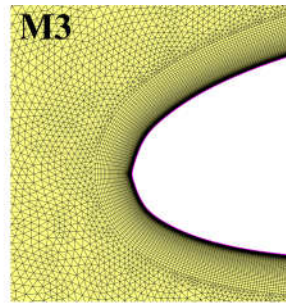
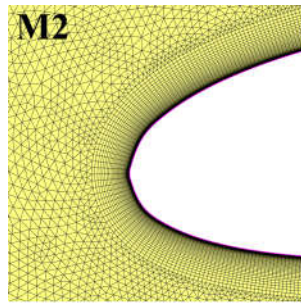


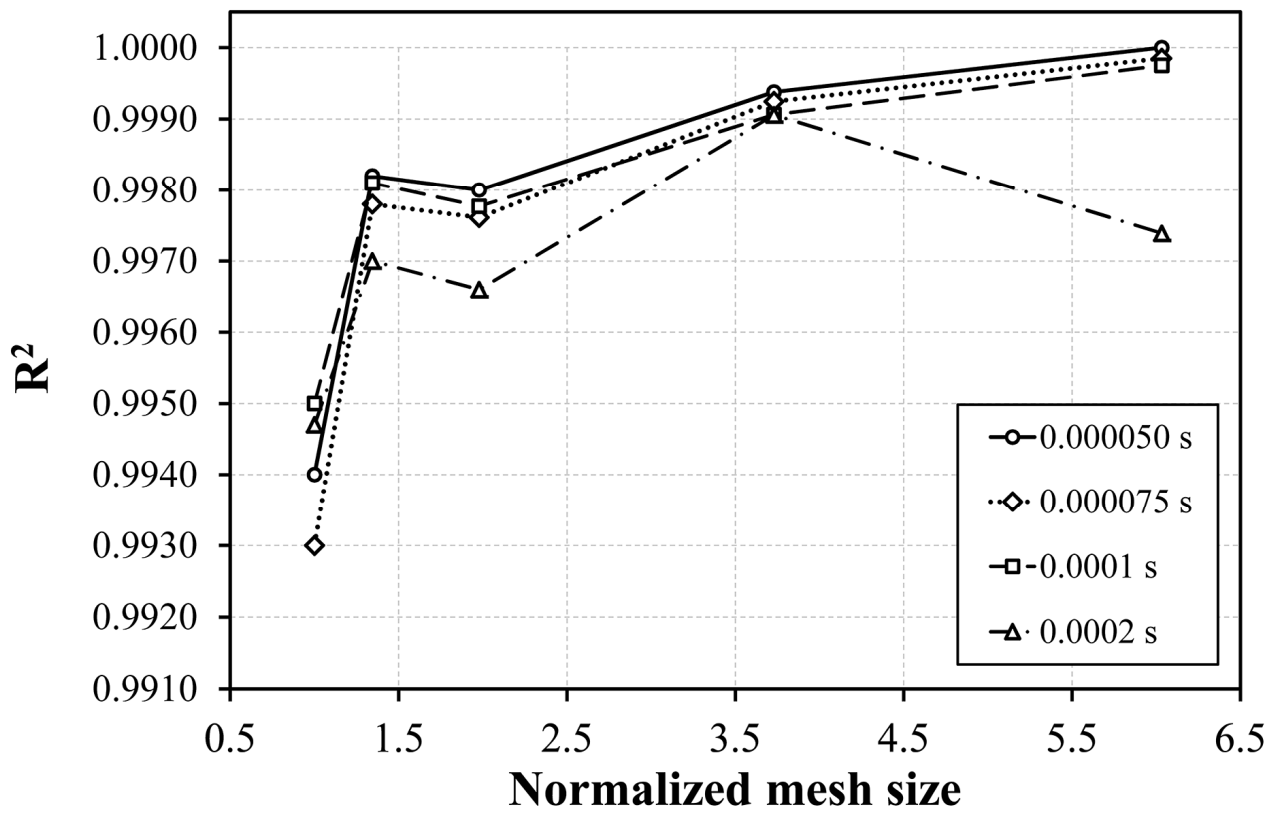
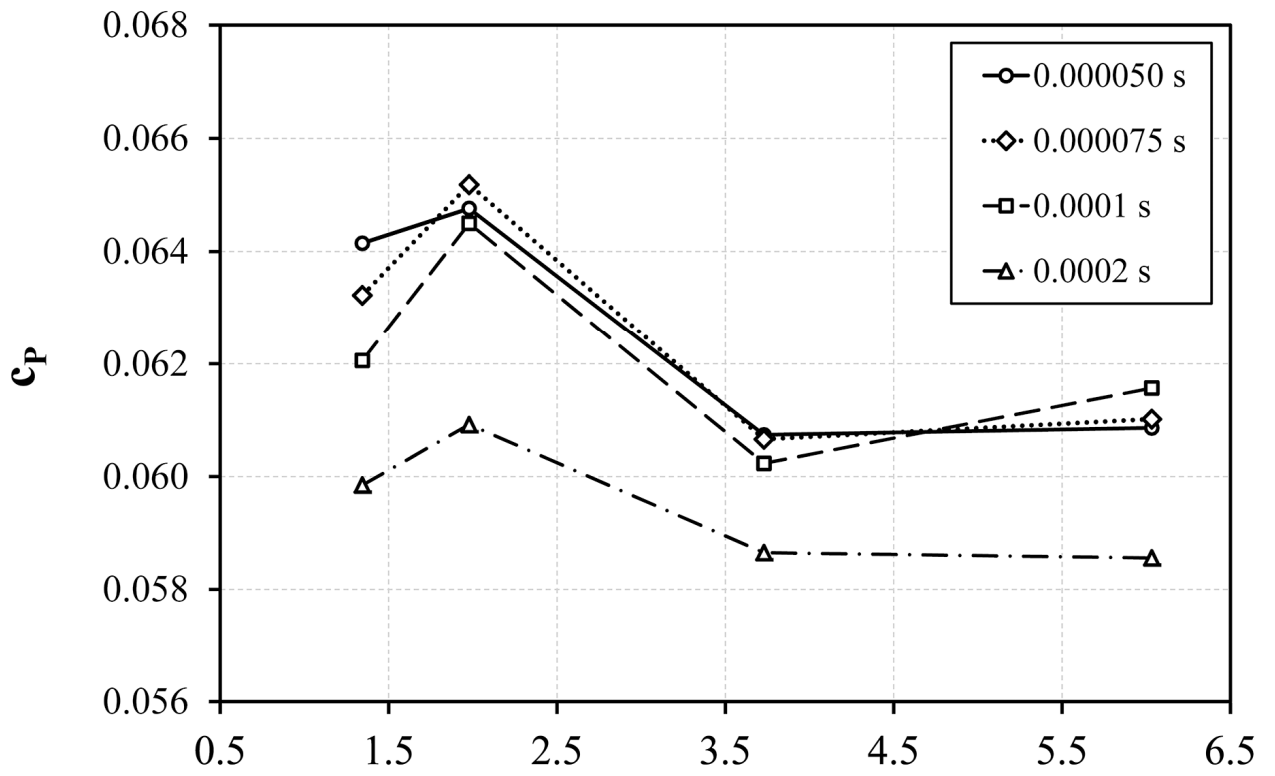
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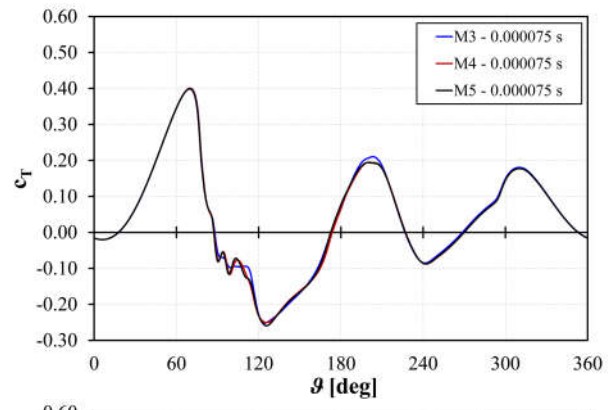


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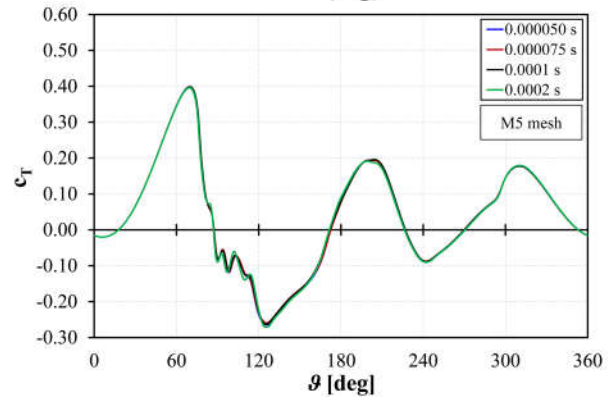


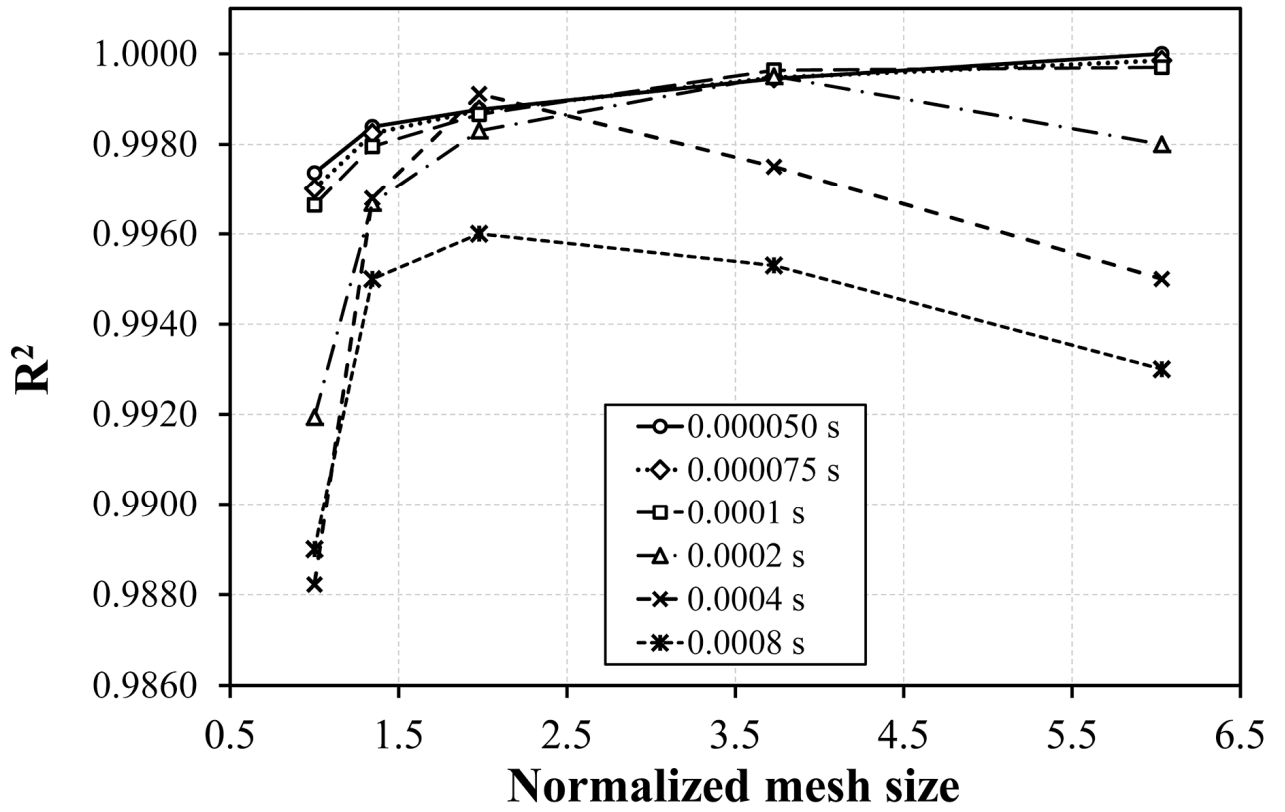
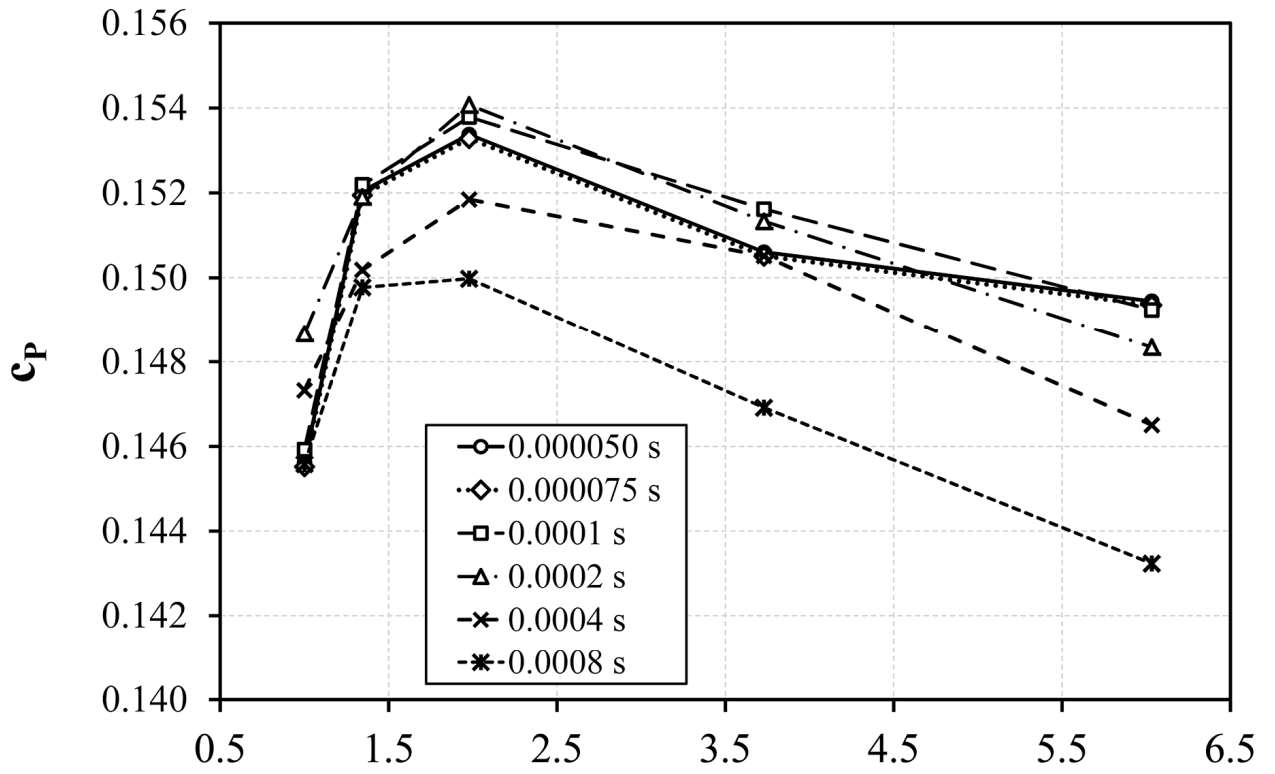


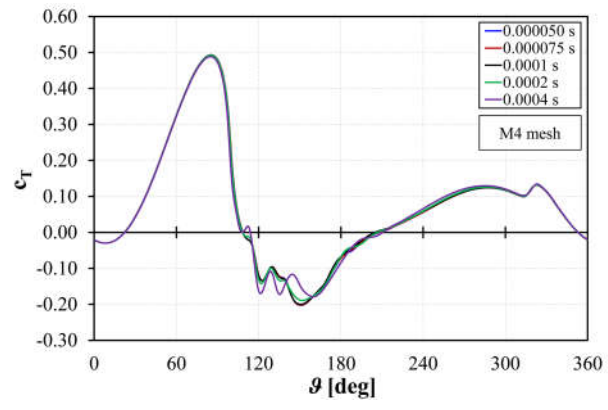
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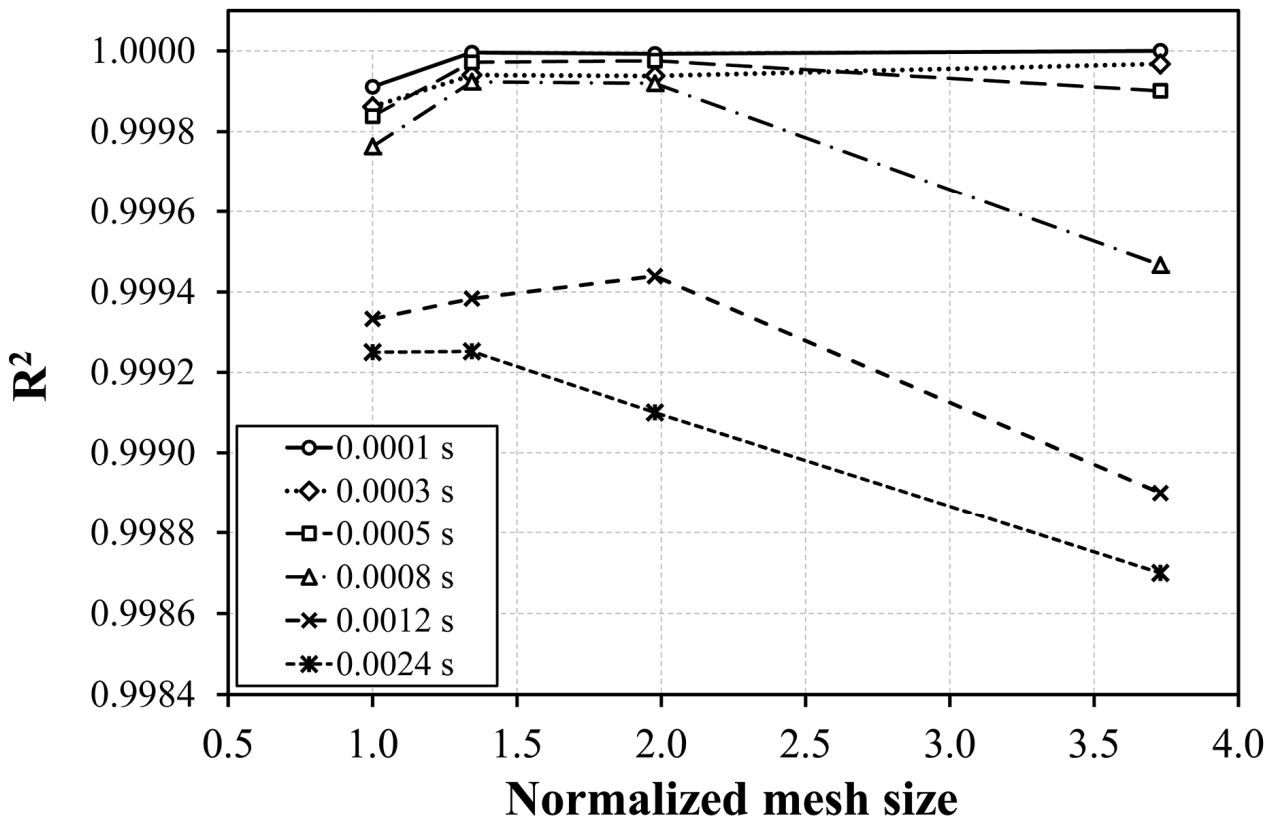
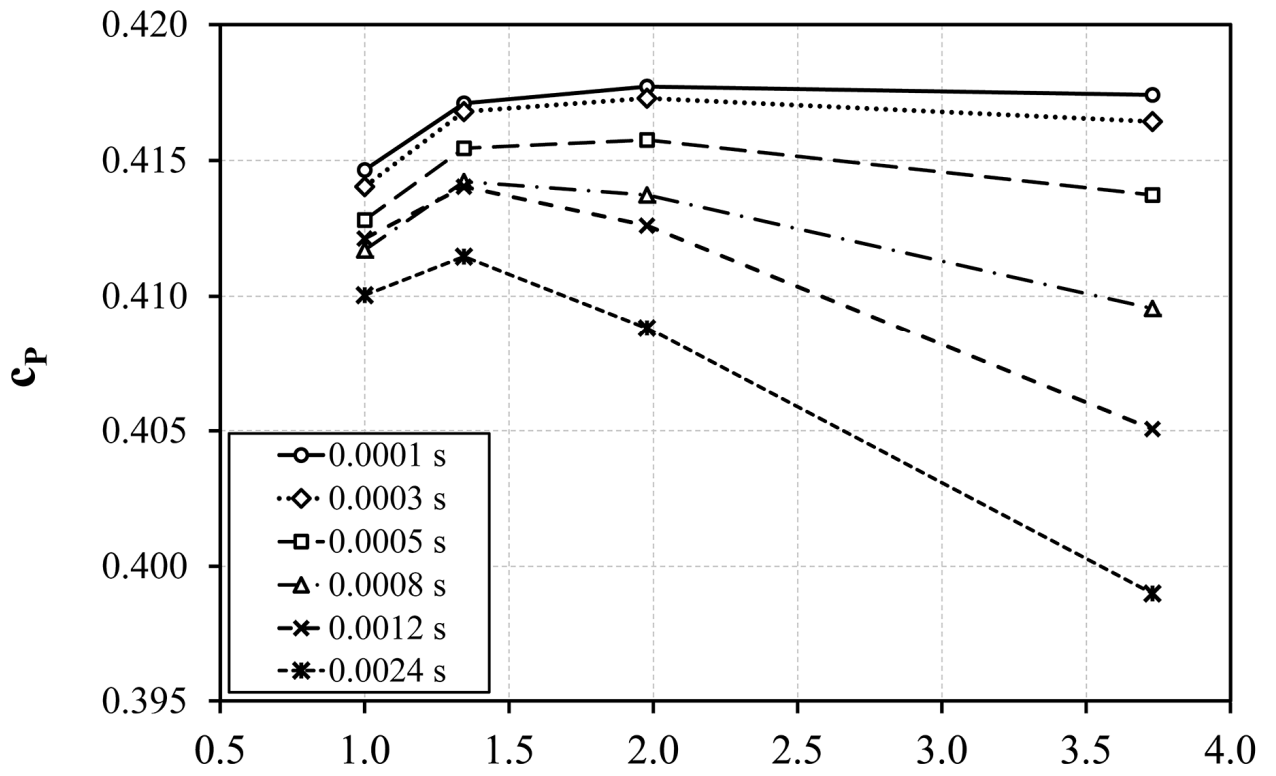


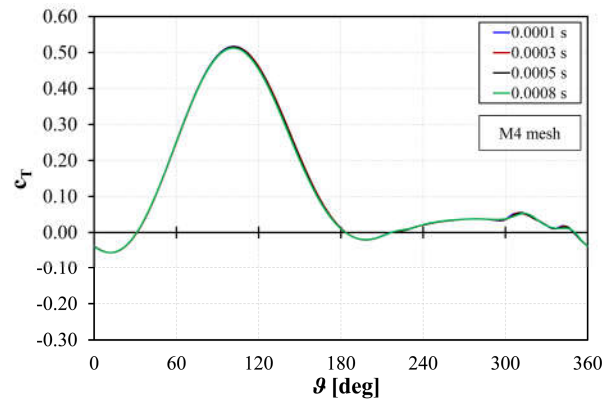
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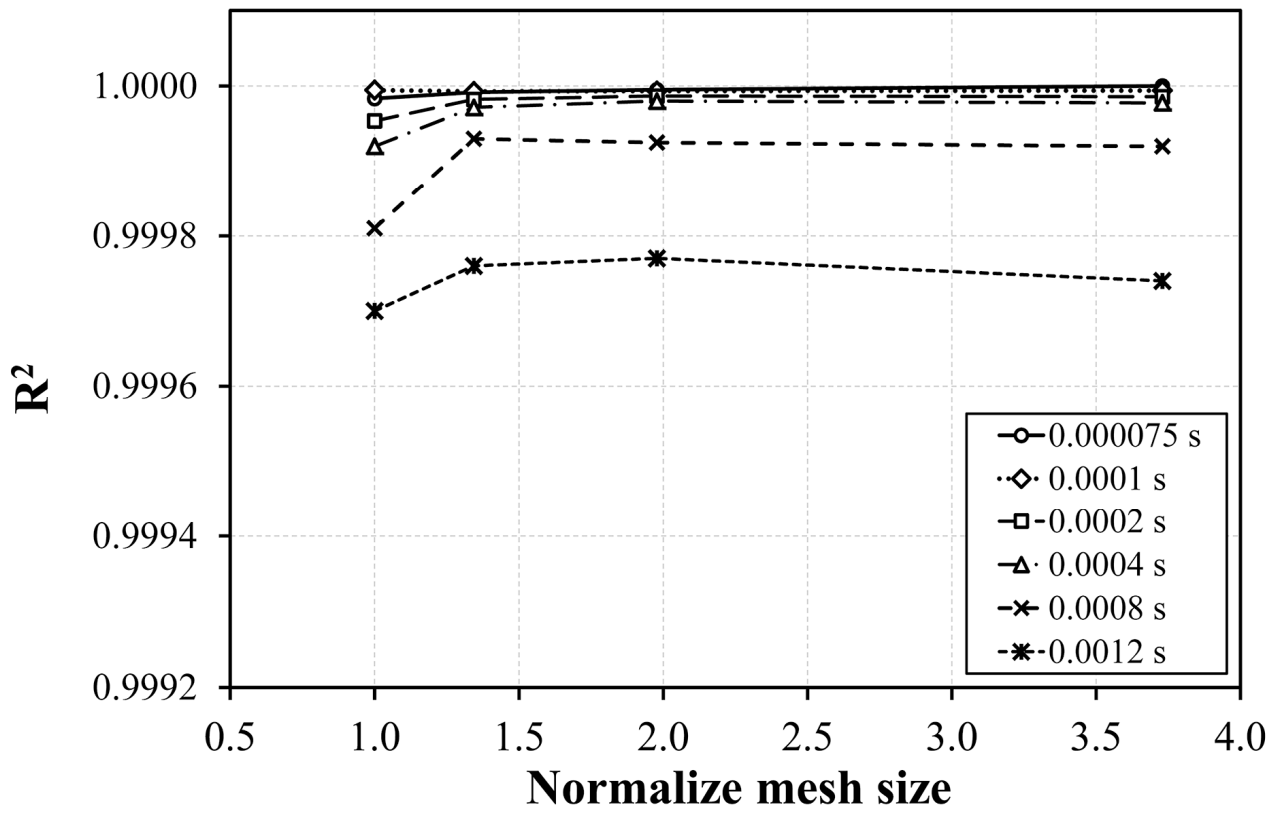
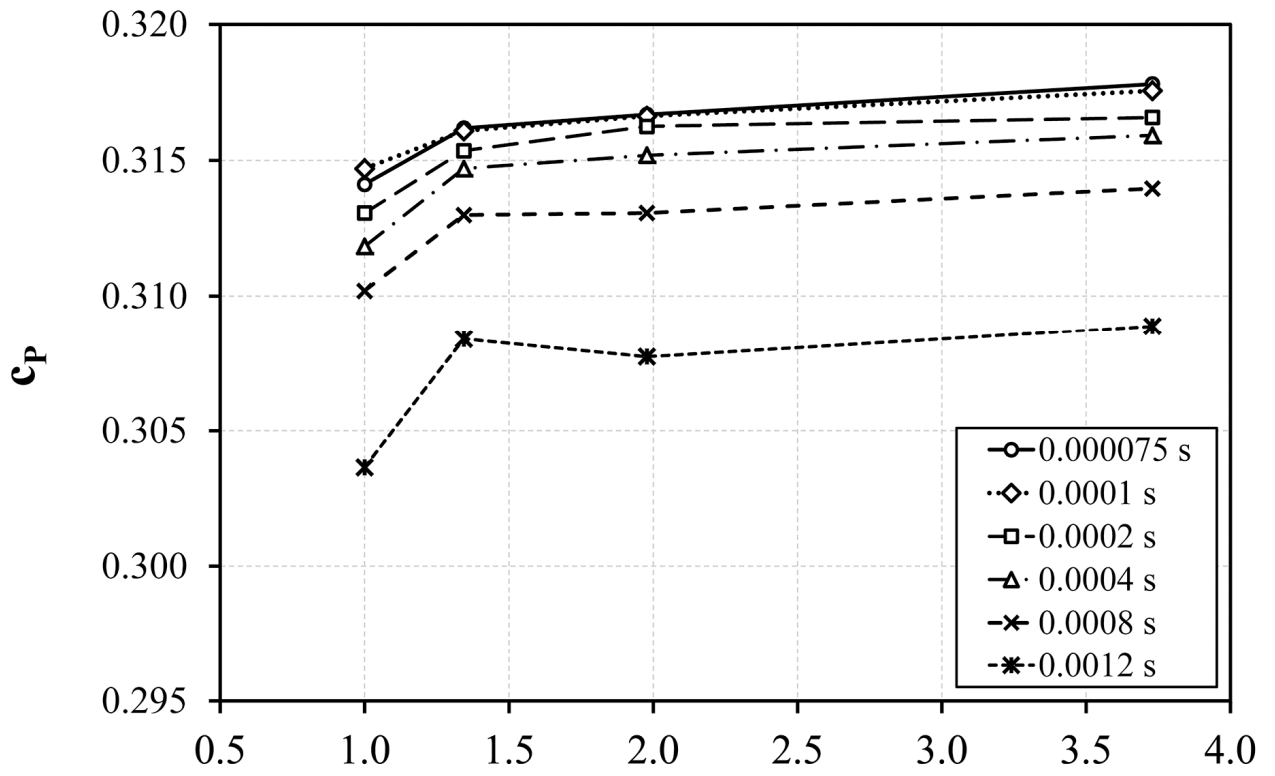




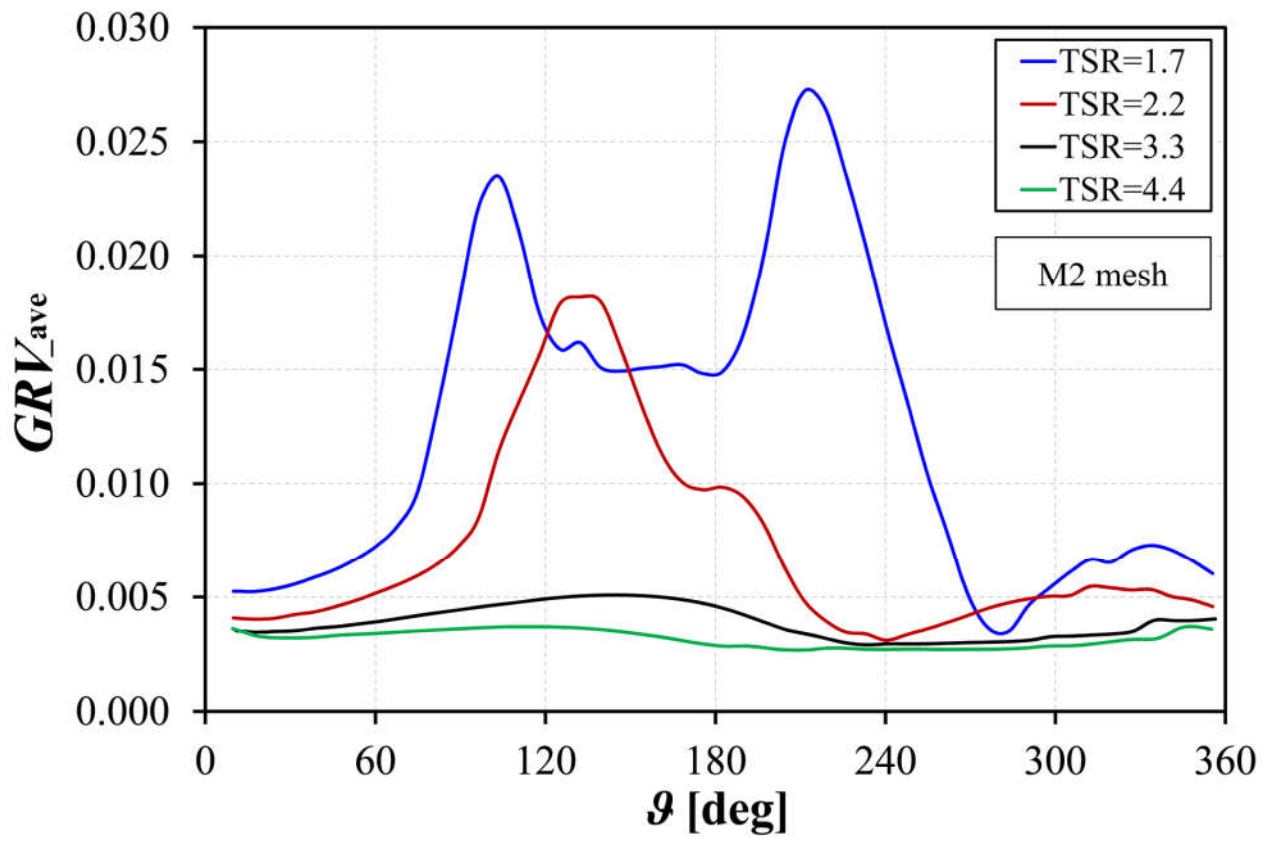
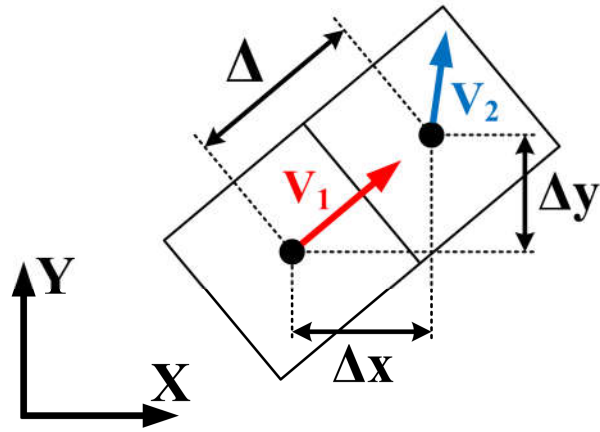




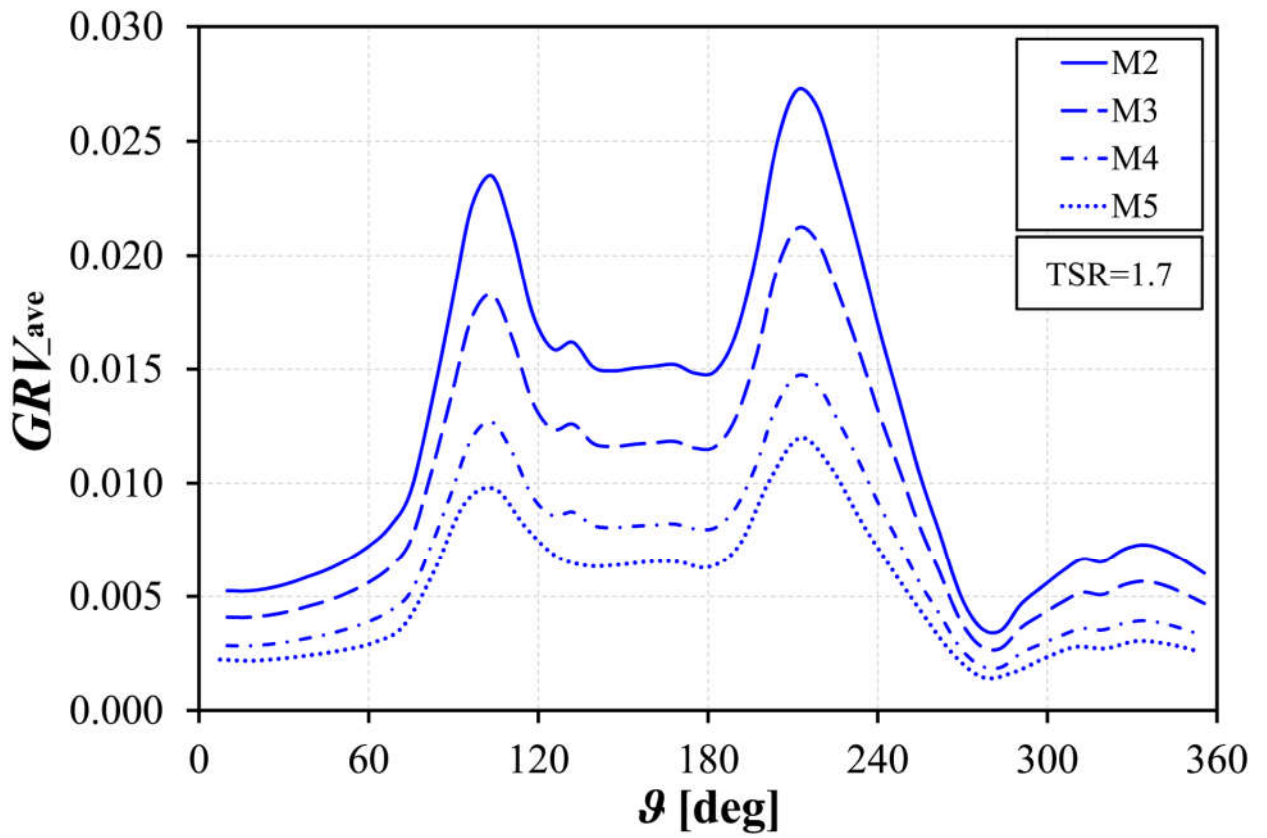




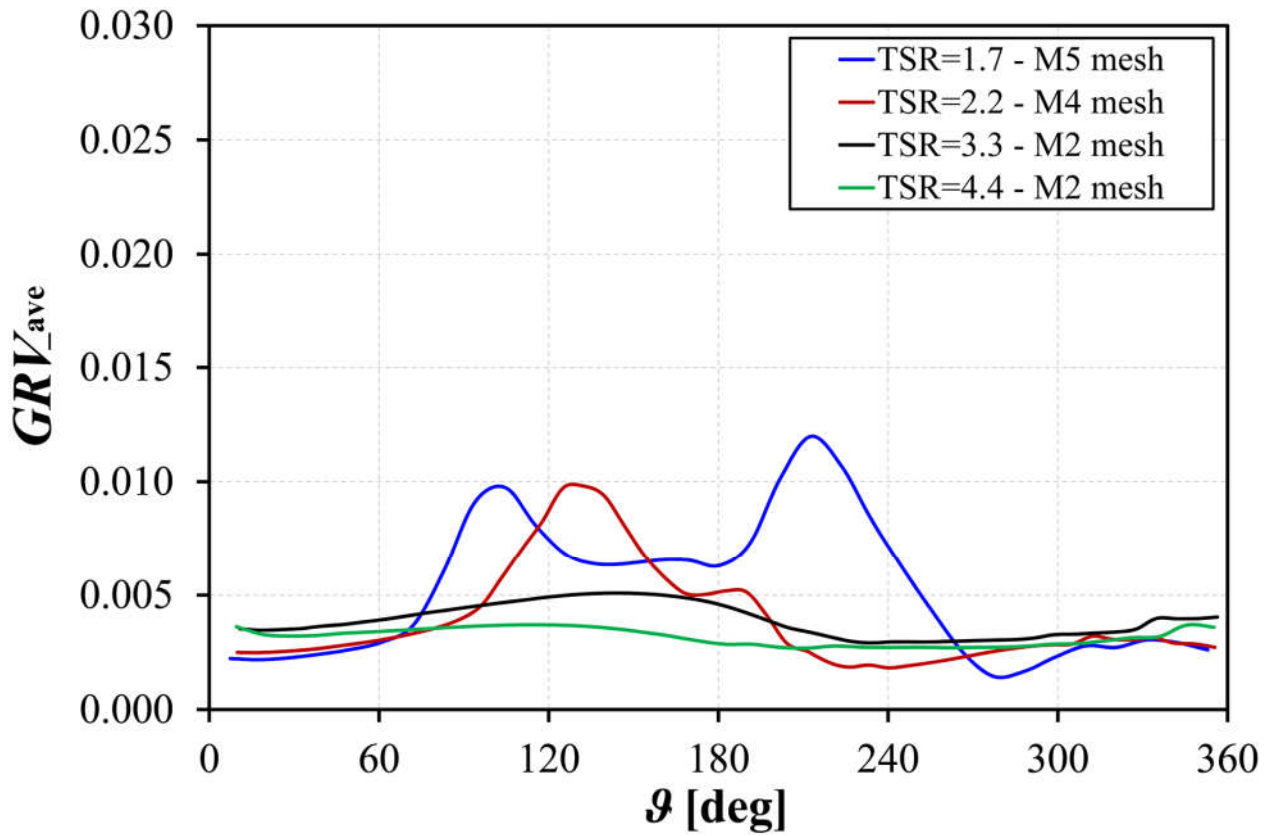
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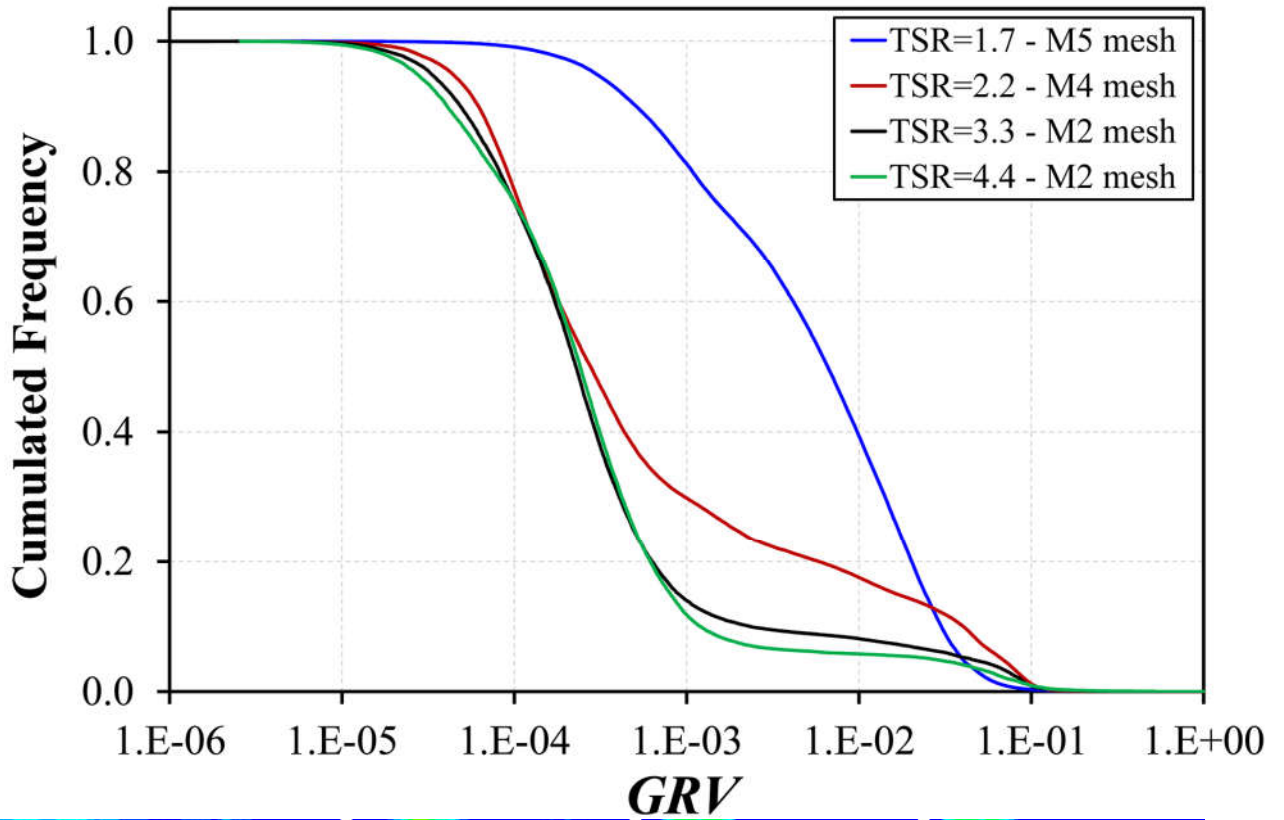
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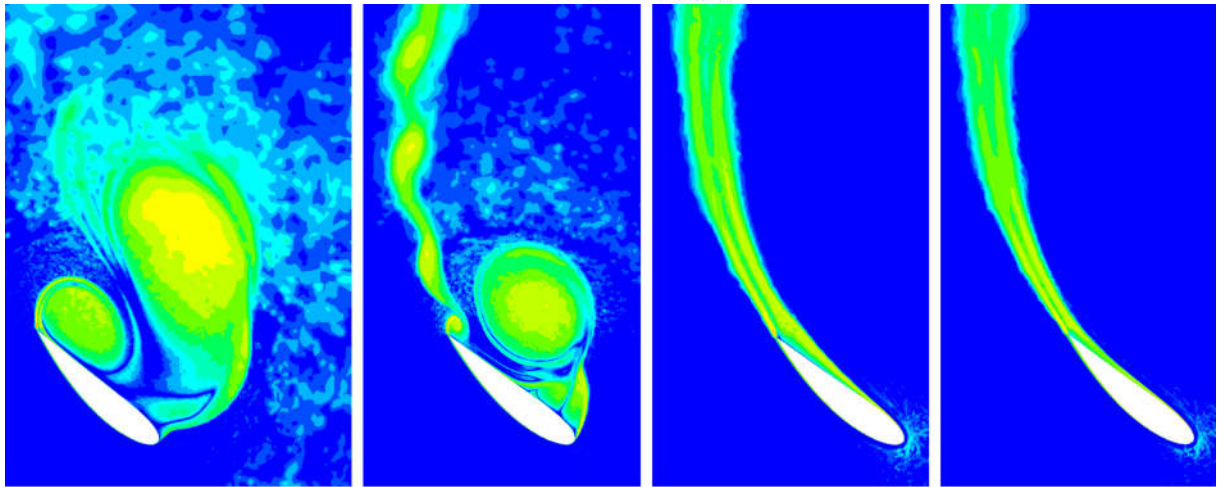
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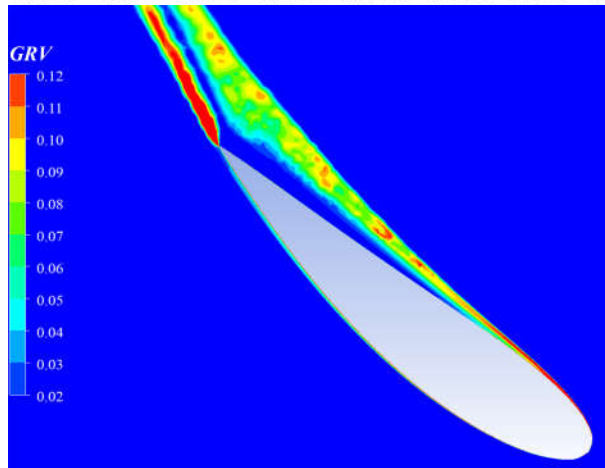
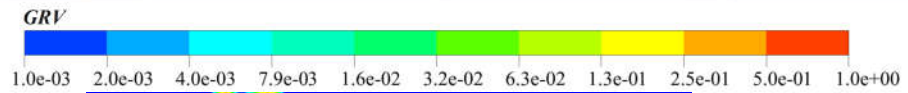
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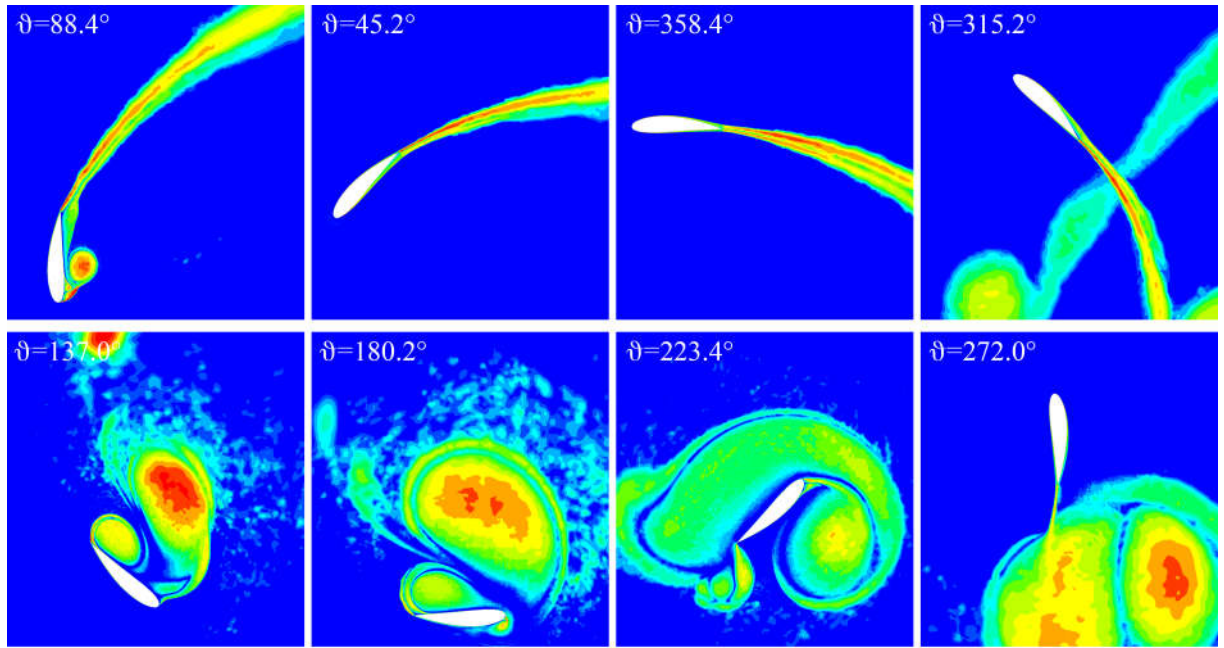
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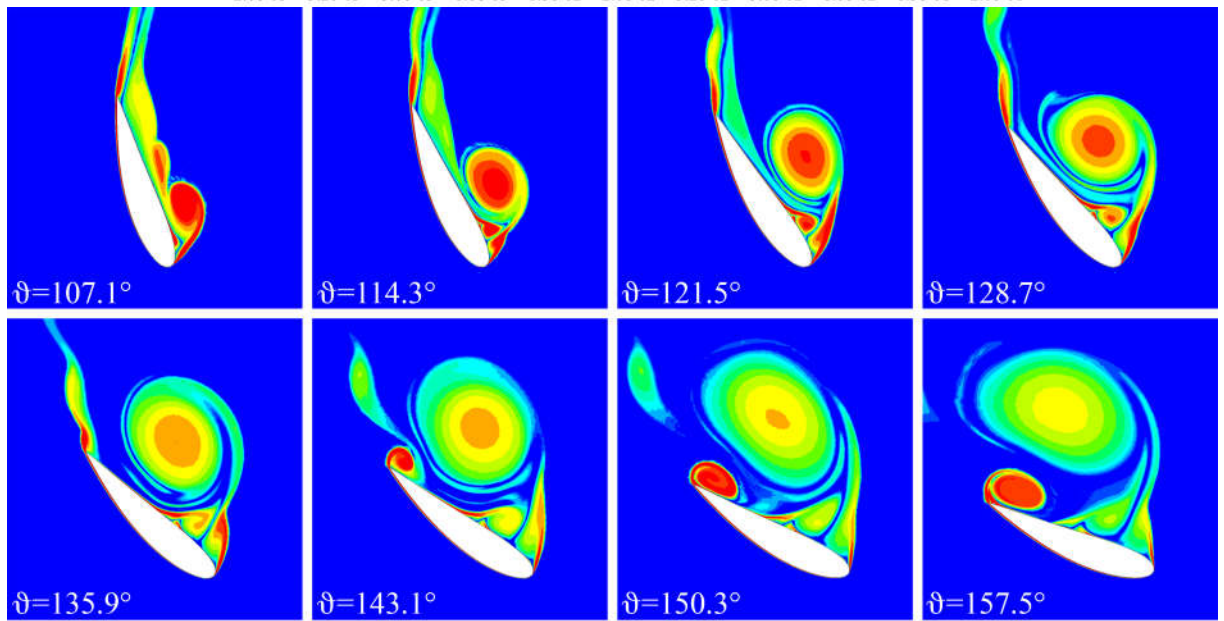
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