A case study on the teaching of mathematics in the Italian Renaissance: Niccolò Tartaglia and his General Trattato

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Abstract

The abacus schools provided mathematical education to students aged 10-12 years destined to become merchants, engineers, soldiers and craftsmen. Nowadays many information are available on the abacus school system but a comprehensive and organic study about the way of teaching mathematics and of conceiving mathematical education in this environment is still missing.

This is a contribution to this topic, aimed to partially reconstruct, Niccolò Tartaglia's viewpoint on the teaching of mathematics. His viewpoint appears amazingly modern: his attention is often focused on the interpretation of problems more than on the resolution "by analogy", and on the way of resolution more than the numerical solution.

Niccolò Tartaglia (1449-1557) is well known in the history of mathematics thanks to the discovery of the solving algorithm of the third degree equation and also for the controversies about the authorship of this result, at first against Girolamo Cardano and then against his pupil Ludovico Ferrari.

Maybe less known, but certainly no less important, it is his activity as Abacus Master (*maestro d'abaco*), attested in Verona since from 1529 (Gabrieli 1997).

As it is known, the main disciplines taught in the abacus schools (also known as *botteghe d'abaco*) were arithmetic and practical geometry. These schools were born in 13th century Tuscany and thanks to the migration of teachers, they soon expanded in all those Italian regions characterized by a flourishing economy based on commerce, like the territories of Veneto. These institutions, however, were not only schools but also a kind of business consulting centres where one could have information on coins, weights and measures – that is, the most valuable information, because these elements changed from town to town.

Children admitted to abacus schools were usually aged 10 or 12 years; they had to be able to read and write and they were usually destined to commercial professions, or technical-engineering ones, or even to artistic ones.

Students attended the abacus school for approximately two years, depending on their learning capacity; the program was in fact segmented into *mute*, that is "didactic units" (i.e. numbers reading and writing, multiplication tables etc.). The switch from one unit to the following one depended by the achieved results.

Programs can be reconstructed on the basis of the over three hundred abacus extant treatises, although it is still unclear what role they might have really played. Certainly they were not textbooks in the modern sense of the term, since

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the book was a too valuable object to be left at the mercy of children, but it is plausible to think them as repertoires from which masters took information and which could be left on the shop counter, available for customers' consultation.

The style of the abacus treatise suggests that the provided education was oriented to the formation of a mnemonic-analogue-operational mind, instead of a logical-deductive one. In other words, the teaching was based on oral and written repetition of exercises, calculations, rules and typical cases, without nevertheless any attempt to identify general algorithms. As we will try to see below, the Tartaglia approach is quite different from this one and contains surprising elements of modernity.

We left Tartaglia as abacus Master in Verona in 1529, but shortly after he moved to Venice, changing definitively his career. Venice was in fact a town rich in opportunities, a culturally and scientifically lively place where one could attend both humanist entourage – like the centre of Ermolao Barbaro – and the engineering milieu like the Arsenale. In addition to his teaching activity, in Venice Tartaglia held public readings on Euclid's *Elements*, that earned him some fame.

Venice was also one of the most important typographical European centre of the time and here Tartaglia started to print his books, in particular

1537: *Nova Scientia*, considered the birth of modern ballistics;

1543: the Italian translation of Euclid's *Elements* (the first printed edition in a current language) and a selection of Archimedes' works in Latin;

1546: *Quesiti et inventioni diverse,* documenting his controversy with Cardano about the discovery of the formula to solve cubic equations;

1547-48: the twelve *Cartelli di matematica disfida*, documenting the challenge between Tartaglia and Ludovico Ferrari;

1551: *Ragionamenti sopra la travagliata inventione*, in which Archimedes' theory about the floating bodies is applied to the problem of making a sunken ship resurface.

The last work, partly posthumous, is the *General Trattato di numeri et misure*. Divided in six *Parti*, printed between 1556 and 1560, the *General Trattato* represent a real encyclopedia in which matters and methods of mercantile mathematics coexist with mathematical humanism, represented for example by the translation in vernacular of the First Book of the Archimedes' *Sfera e cilindro*.

It is in the *General Trattato* that we find many observations which help to retrace, even if partially, Tartaglia's ideas on the teaching of mathematics. As we said before, we are in front of a mathematical encyclopedia, consisting of the following six *Parti*:

1. practical arithmetic (elementary operations, rule of three, mercantile problems);

2. speculative arithmetic (in particular, arithmetical interpretation of Book II and Book X of the *Elements*);

3. *pratica minore*: commented definitions of Book I of the *Elements*, description of the units of measure used in different towns, matters on surveying;

4. area calculation of plane figures and calculation of solid figures' volume (including a chord table of Ptolemaic inspiration);

5. geometric problems (many of them taken from *Cartelli*, as the reconstruction of Euclid's *Elements* using only a ruler and a fixed opening compass) (Garibaldi 2010);

6. algebra: first and second degree equations (Giusti 2010).

To whom was directed such a composite and heterogeneous work?

In different parts of his work Tartaglia explicitly speaks to practitioners: artisans, merchants, architects, soldiers.

However, even if directed to a public interested in the daily and practical application of the results, the *General Trattato* is rich in different types of "demonstrations": from the purely abstract ones, inspired by Euclidean style, to simple number verifications.

This is not a random choice or virtuosity, but a specific purpose of the author, who aimed at helping readers to 'carry out operations', but also to understand 'why' that had to be made in a certain way.

To bring his readers to a mathematics more speculative than the one they were used to, Tartaglia relied on a language rich in metaphors and similes taken from daily life. For example, when he has to explain the general meaning of 'measuring a surface' he evokes the figure of a shoemaker who 'measures' a piece of leather (the surface) placing upon it the model of a sole (the unit of measure) several times until its very end, so to see how many shoes he is able to make.

Definitions are similarly conceived and they are, when possible, anchored to the real world, so that they will be of some practical usefulness ("di qualche utilità al pratico"). This approach is easily successful with the geometric definitions of genetic type; indeed, in this instance we could think that Tartaglia is bringing back to their empirical origins some abstract geometrical objects (Gavagna 2014).

This is the case of the sphere, that is the solid which – according to the Euclidean definition in Book XI - is obtained from the revolution of a semicircle around its diameter. As Tartaglia notes, the stone-cutters actually used an iron semicircular profile to model stone balls from shapeless rocks.

The particular attention that Tartaglia payed to the language is also the fruit of his activity as both a translator and a teacher. It is in fact important to remember that he was the first to publish Euclid's *Elements* in a current language, that is the Italian vernacular. His translation was based on those which Tartaglia called "the two translations", in other words the one edited by Campano da Novara during the middle of the 13th century, but published in 1482, which was ascribed to the Arabic-Latin tradition, and the translation from Greek by the humanist Bartolomeo Zamberti, published in 1505.

The meeting point between Tartaglia-the-translator and Tartaglia-the-abacus-Master, gives rise to surprisingly modern reflections about possible obstacles arosed by the use of the common language in the learning of mathematics.

He argues, for example, how in the common language the term 'multiplication' always suggests an increase and it is no wonder that a student does not acquire spontaneously the fact that multiplying two rational numbers lesser than unit (the so-called *rotti* or 'broken', that is the proper fractions) you do not obtain a result greater than the factors.

According to Tartaglia, the origin of the problem derives from Campano, who wrongly used the verbs *multiplicare* e *ducere* as synonymous. In the Greek-Latin tradition, instead, the two terms had different meanings. The term *multiplicare* was associated only to the multiplication involving natural numbers; in that case there was a complete alignment with the common sense, because the multiplication was conceived as a repeated addition and necessarily the product was greater than factors. The term *ducere* instead referred to the operation between continuous magnitudes, like for example geometric magnitudes and fractions (the *rotti*) which are "by their very nature, of continuous quantity, a thing indefinitely divisible". To avoid the emergence of misconceptions, Tartaglia concludes, you must underline the conceptual difference between the two operations restoring the terminological distinction borrowed from the Greek-Latin Euclidean tradition. Therefore the more appropriate verb to describe the multiplication between *rotti* numbers would be *ducere o menare*, while the correct one to indicate the multiplication between natural ones it would be *multiplicare* (Gavagna 2010).

The attention that Tartaglia paid to the use of mathematical language in the process of learning, it is just one of the features of modernity that clearly emerges from the reading of his works. Another aspect of high interest concerns his approach to problem solving.

Facing a typical problem of surveying, as that one to determine the area of a triangle whose sides' length is known, Tartaglia presented different strategies of resolution, that are represented by different formulas to be chosen in relation to the concrete context in which we need to apply them[1].

[1] Tartaglia (1556-1560), *Quarta Parte*, cc. 3v-8v.

A first approach, for example, is to determinate the height of this triangle using the propositions 12 or 13 of Books II of Euclid's *Elements* depending on whether the triangle is obtuse-angle or acute-angle[2]. As an alternative, Tartaglia suggested the use of so-called "Heron's formula", of which he also provided the proof - an unusual mathematical 'object' in a practical geometry treatise – aimed "to satisfy speculative people". There is also another interesting expedient that Tartaglia used to focus his reader's attention on the resolution procedures.

The *escamotage* is to consider every example with the same numerical data, in other words to consider the triangle of 13, 14, 15 sides length. These numbers allow to make simple calculations and to not to deflect the attention from the comprehension of the resolution procedure. Only after that the procedure has been completely internalized, more complex calculation can be introduced.

To conclude, even if the *General Trattato* could not be considered a teaching handbook, the examples we have mentioned highlight some interesting ideas of the Master Niccolò da Brescia on teaching and learning mathematics. A purely mnemonic learning, in other words a learning not subordinated to the comprehension of the processes, it was liable to fade out in a short time without leaving any trace in the learners' mind. It is for this reason that Tartaglia offered to his readers also the possibility to explore the causes that are behind the rules.

Maybe the most significant pedagogical effort is the attempt to educate to abstract mathematical reasoning a public mainly interested in "useful" results, immediately employable in everyday life. Tartaglia addressed his work to these readers using a strongly evocative language and an approach rich in metaphors in which the readers can easily recognize the surrounding world and so can clearly perceive the mathematical laws hidden in their daily life. A message that after so many centuries has not yet lost its efficacy.

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[2] It is clear that in a right triangle the problem does not arise. Furthermore the distinction between acute-angle and obtuse-angle triangle is inaccurate: we would need to distinguish if we want to determinate the height related to a side opposite to an obtuse or acute angle (both coexisting in a obtuse-angle triangle).

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