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The Revision theory of truth [*The Revision Theory of Truth*, MIT Press, 1993; MR1220222] is a proposal for dealing with semantic paradoxes by means of a sequence of two-valued, first-order models obtained by iterating a rule of revision, and extended transfinitely by means of a limit rule (i.e., a rule for dealing with stages indexed by limit ordinals). H. Herzberger's limit rule [Notes on naive semantics, *Journal of Philosophical Logic*, 11(1), 1982; MR0666657], corresponds to the mandate to collect at limit stages all and only the formulas that have become stably true in a revision sequence (i.e., formulas that are true at a previous stage in the sequence, and remain the same for all subsequent stages). The paper under review is devoted to developing a cut-free calculus that encapsulates Herzberger's limit rule. The paper is deeply connected to a variety of recent sources, and employs a number of different techniques to achieve the sought-for results. This contributes to making this extensive work an interesting piece of research.

The theory of formal truth the paper is centered around is the theory PosFS^+ from [Revision revisited, *Review of Symbolic Logic*, 5(4), 2012; MR2998931], which is related to the theory FS by H. Friedman and M. Sheard [An axiomatic approach to self-referential truth, *Annals of Pure and Applied Logic*, 33(1), 1987; MR0870684], but is formulated in a way that allows avoidance of ω-inconsistency. This theory is proved to be sound with respect to formulas that are stably true in every revision sequence. The main result of the paper is to show that this theory is equivalent to a labelled sequence calculus S_{HT}, devised along the lines of S. Negri's [Proof analysis in modal logic, *Journal of Philosophical Logic*, 34, 2005; MR2189371]. The rules of this calculus are designed to capture the properties of accessibility relations that can be used to re-state Herzberger's limit condition as applying in a possible world semantics. The system S_{HT} is proved to enjoy the admissibility of the cut rule, shown to prove each and every axiom of PosFS^+ and also to enjoy admissibility of the necessitation rule of PosFS^+. To prove that also the rules CONEC and CONEC$^+$ of PosFS^+ are similarly admissible in S_{HT}, a class of Herzberger-Kripke models are devised along with a validity relation to show that:

(i) if a formula is HK-valid, then it is stably true in every revision sequence;

(ii) S_{HT} is sound and complete with respect to validity in HK models;

(iii) rules CONEC and CONEC$^+$ of PosFS^+ are admissible in S_{HT}, i.e., S_{HT} proves (sequents corresponding to) the theorems of PosFS^+.

To establish the converse relation of (iii), neighborhood semantics models for Herzberger's limit rule (NH-models for short), are built along the lines of E. Pacuit's [*Neighborhood Semantics for Modal Logic*, MIT Press, 2017; MR3729253]. The equivalence between the two semantics is proved (i.e., a formula is HK-valid if and only if it is NH-valid), as weel as the completeness of PosFS^+ with respect to NH-validity, which give the desired equivalence between provability in PosFS^+ and S_{HT}.